Periods of modular forms and special values of their L-functions

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For a modular group Γ let $S_k(\Gamma)$ denote the space of cusp forms of weight k with respect to Γ. In these lectures, the "period" of a cusp form F means the Petersson product $\langle F, F \rangle$ of F.

For a primitive form f in $S_k(SL_2(\mathbf{Z}))$, let $\hat{f}$ be a certain "lift" of f to $S_l(\Gamma)$ for some modular group Γ, namely, let $\hat{f}$ be a cuspidal Hecke eigenform in $S_l(\Gamma)$ whose spinor L-function or standard L-function can be expressed by a certain L-function of f. Then it is an interesting and important problem to express the ratio $\dfrac{\langle \hat{f}, \hat{f} \rangle}{\langle f, f \rangle^e}$ in terms arithmetic invariants of f, e.g. the special values of certain L-functions of f for some integer e.

In these lectures, I will give an survey of a result of W. Kohnen and N. Skoruppa, which gives an answer the above problem for the Saito-Kurokawa lift of f.

Furthermore, I will talk about my recent work joint with H. Kawamura, which gives a certain generalization of the above result to the Ikeda lift of f.