$\mathbb{P}^n$ n
M $\mathcal{O}_M$ M Ω^p_M M
p T_M Ω^1_M
M n Kähler Hodge

$$H^k(M,\mathbb{C}) \cong \oplus_{p+q=k} H^{p,q}(M) \ (\overline{H^{q,p}} = H^{p,q})$$

$h^{p,q} := \dim_{\mathbb{C}} H^{p,q}$ Hodge

?

$c_1 = 0$ Kähler mirror symmetry
Calabi-Yau
2σ Riemann Σ Riemann
(X, g) $\Phi : \Sigma \to X$ Σ, X
$(z, \bar{z})$, ϕ^I $K^{1/2}$ Ω^1_Σ

$$\psi^I_+ \in \Gamma(K^{1/2} \otimes \Phi^* T_X)$$
$$\psi^I_- \in \Gamma(\bar{K}^{1/2} \otimes \Phi^* T_X)$$

t Lagrangian

$$L = -2it \int dz \wedge d\bar{z} (\frac{1}{2} g_{IJ}(\Phi) \partial_z \phi^I \partial_{\bar{z}} \phi^J + \frac{i}{2} g_{IJ} \psi^I_- D_z \psi^J_- + \frac{i}{2} g_{IJ} \psi^I_+ D_{\bar{z}} \psi^J_+ + \frac{1}{4} R_{IJKL} \psi^I_+ \psi^J_+ \psi^K_- \psi^L_-)$$

$$D_{\bar{z}} \psi^J_+ = \frac{\partial}{\partial \bar{z}} \psi^J_+ + \frac{\partial \phi^K}{\partial \bar{z}} \Gamma^J_{KL} \psi^L_+$$

$D_z \psi^J_-$
X n Calabi-Yau
2 Witten A
B [W]

Typeset by $\mathcal{A}_{\mathcal{M}}\mathcal{S}$-TEX

A Calabi-Yau M $\oplus H^{p,p}(M)$ B ()
Calabi-Yau W $\oplus H^{n-p,p}(W)$ A
B Hodge
Jacobi
Calabi-Yau M, W
M, W A A_M B B_W
3
Hodge

Gepner
Calabi-Yau orbifold

$$M_\psi = \left\{ \sum X_i^5 = 5\psi \prod X_i \right\} \subset \mathbb{P}^4 \ (\psi \in \mathbb{P}^1)$$

3Kähler
) M_0 $\mathbb{P}^4$
Kähler t 3 () κ_{ttt} Hodge

$$\begin{aligned}\kappa_{ttt} &= 5 + 2875q + 4876875q^2 + 8564575000q^3 + 15517926796875q^4 + \cdots \\ &= 5 + 2875\frac{q}{1-q} + 609250\frac{2^3q^2}{1-q^2} + 317206375\frac{3^3q^3}{1-q^3} \\ &\quad + 242467530000\frac{4^3q^4}{1-q^4} + \cdots\end{aligned}$$

$\mathbb{P}^4$ 5 1, 2, 3,... ()
2875, 609250, 317206375, $\cdots$
[CdOGP]
Givental
[G]
Calabi-Yau[Mo]
rigid Calabi-Yau

Moonshine

n
Calabi-Yau M, W

(1) $e(W) = (-1)^n e(M)$
(2) () $= h^{n-1,1}$ Kähler cone $= h^{1,1}$

(3) $h^{p,q}(W) = h^{n-p,q}(M)$
(4) A $H^1(M, \Omega_M^1)$ B
$H^1(W, T_W) \cong H^1(W, \Omega_W^{n-1})$ M
W

Batyrev Gepner

M d Abel $\mathrm{Hom}_{\mathbb{Z}}(M,\mathbb{Z})$ N
$\langle x,y\rangle$
Δ $M_{\mathbb{Q}}$ ($=$)
Newton

. Δ M

$$\Delta^* = \{\, y \in N_{\mathbb{Q}} \mid \langle x,y\rangle \geq -1\}$$

N Δ (reflexive) [B]
Δ d $\mathbb{P}_\Delta$

$$k \qquad\qquad = k\Delta \cap M$$

Gorenstein
Δ [B] 2
Gorenstein $\mathbb{P}_\Delta$ crepant
$\tilde{\mathbb{P}}$ 3 3
Calabi-Yau
Δ
[B]
$\mathbb{P}_\Delta$ $\mathbb{P}_{\Delta^*}$
target
Kähler ω
Kähler

Remarks

I : chirality
IIA : chirality
IIB : chirality
Heterotic : 10 26 bosonic string
16 $\mathbb{R}^{16}$

$D = 10$ 4 6 N 1
6 $c_1 = 0$
Kähler
M $M' \to M$

.

$$M' \cong T \times \prod X_i \times \prod Y_j$$

T $\mathbb{C}^N$/lattice X_i, Y_j
$Sp(m_i/2)$, $SU(n_j)$ $(m_i = \dim X_i,\ n_j = \dim Y_j)$
3 3

(1) 3 T^3
(2) $\times$ K3
(3) $SU(3)$

(1) (2) Euler 0

$$\left|\frac{1}{2}e(M)\right| = \qquad \geq 3$$

6 (3)
n Kähler $SU(n)$
Calabi-Yau
$\mathbb{P}^N$ $\mathbb{P}^N$ (i.e.
) $N+1$ 3
(3)

Kähler
Kähler

. X Kähler $H^2(X,\mathbb{C})$ $\omega = B + iJ$
J Kähler

1 Calabi-Yau =

τ

$$\tau = \int_\beta \Omega \Big/ \int_\alpha \Omega \ (\alpha, \beta \quad H_1(E,\mathbb{Z}) \qquad\qquad , \Omega \neq 0 \in H^0(\Omega^1_E))$$

τ

$$E = E_\tau = \mathbb{C}/(\mathbb{Z} \oplus \mathbb{Z}\tau) \ (\tau \in \mathbb{H})$$

τ

$H^2(X,\mathbb{Z}) \cong \mathbb{Z}$

$$\{ \text{ Kähler } \qquad\qquad \} = \{B + iJ | B \in \mathbb{R}, J > 0\} \cong \mathbb{H}$$

Kähler (τ, ω)

$$\mu : (\tau, \omega) \to (\omega, \tau)$$

μ [DVV] μ

$PSL(2,\mathbb{Z})$ $\mathbb{H}$
Kähler $PSL(2,\mathbb{Z})$

$$S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

B 0 J $1/J$

$$T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

B 1

$$\int \sum_{\alpha \in H_2(X,\mathbb{Z})} e^{2\pi i \langle \omega, \alpha \rangle} \cdots \ (k \in \mathbb{Z})$$

B $H^2(\mathbb{Z})$

K3

4 (c 6) Calabi-Yau M 2 K3

. M $h^1(\mathcal{O}_M) = 0, \Omega^2_M \cong \mathcal{O}_M$
K3

K3 [BPV]

. M *K3* $h^{0,0} = h^{2,0} = h^{0,2} = h^{2,2} = 1,\ h^{1,1} = 20,$
Hodge 0.

2 Calabi-Yau $h^{0,1} = 0$ K3 K3
()K3 Hyperkähler Verbitsky
[V]

. *K3 Kähler*

K3 M

$$\phi : H^2(M,\mathbb{Z}) \to L := E_8(-1)^{\oplus 2} \oplus U^{\oplus 3}$$

(M,ϕ) ϕ marking $h^{2,0}(M) = 1$ $\phi_{\mathbb{C}}(H^{2,0})$
$L_{\mathbb{C}}$ $\mathbb{P}(L_{\mathbb{C}})$

$$\{(M,\phi)\}/\sim \to \mathbb{P}(L_{\mathbb{C}})$$

K3 M 0 2 Ω

$$\Omega \cdot \Omega = 0,\ \Omega \cdot \bar{\Omega} > 0$$

L' L $x^2 = 0,\ x\bar{x} > 0$
$\mathbb{P}(L'_{\mathbb{C}})$

. *K3*

Torelli K3
K3 lattice L

. M_1, M_2 *K3* $\varphi : H^2(M_2, \mathbb{Z}) \to H^2(M_1, \mathbb{Z})$ *effective Hodge*
$f : M_1 \to M_2$ $\varphi = f^*$

effective effective effective
Hodge Hodge
A-model :

. *K3*

B Hodge
2 2

Arnold
[K][D]
M K3 $L = H^2(M, \mathbb{Z})$ Kähler $\omega \in L_{\mathbb{C}}$
$\Omega \neq 0 \in H^{2,0}\} \subset L_{\mathbb{C}}$

. L $\omega^\perp \cap \Omega^\perp$

. G L ω, Ω G
Aut$M$

. G ω effective
Ω Hodge Torelli □

K3 lattice ω, Ω
Kähler K3 marking
Kähler 2 K3

$\omega \cdot \omega$ 0 ω Ω
H^0 H^4 ω [AM] [AM]
Kähler $H^*(M, \mathbb{Z})$
Ω $\bar{\Omega}$ 2-plane Kähler (mod $\mathbb{Z}$) $\mho$ $\bar{\mho}$
2-plane K3

K3

Calabi-Yau (
) IIA IIB [Mo]

heterotic IIA
[KV] K3 T^2 $N = 2$ heterotic
K3-fibration 3 Calabi-Yau IIA string

K3-fibration K3
3 Calabi-Yau
$\mathbb{P}^1$ 4 Calabi-Yau K3-fibration
[KLM][HLY2]
K3heterotic
K3 pencil 3
Calabi-Yau IIA [CF]
Calabi-Yau
Calabi-Yau moduli K3 moduli
K3 3 Calabi-Yau
Euler $e(M)$

$|e|$ Calabi-Yau $\mathbb{P}^2$
ellipitic fibration Calabi-Yau Euler -540
[N] 4-
Calabi-Yau -960 960
[CLS] $\mathbb{P}(1,1,12,28,42)$ 84
Calabi-Yau $\mathbb{P}(41,42,498,1162,1743)$ 3486

Euler $-1712, -2592$
[HS] $e = -960$ Calabi-Yau K3
pencil K3
42 heterotic IIA
IIA Euler
44, 66 K3
3 Calabi-Yau
moduli

ordinary double point (ODP) ODP $\mathbb{P}^1$
3 Calabi-Yau moduli
[R]
ODP

$$\{xy + zw = 0\} \subset \mathbb{C}^4$$

[S][GMS] ODP
ODP

heterotic/II [LS]

Batyrev (
)

GKZ

[HLY1]
Fano $\mathbb{P}^2$ A B
pencil+[Ma]

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