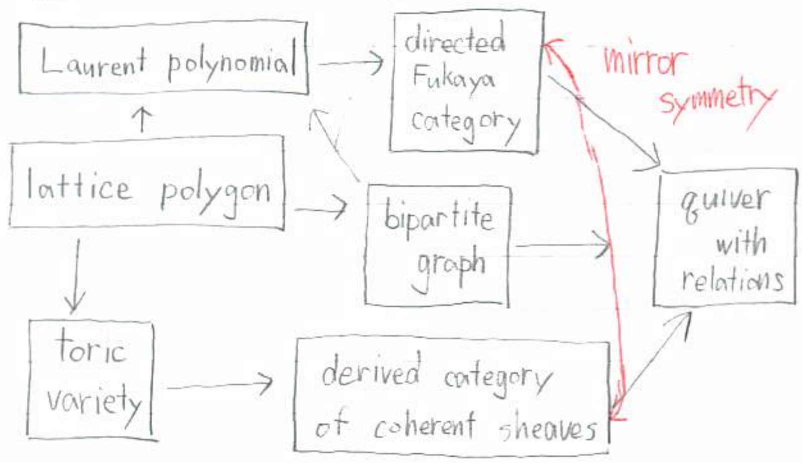
植田一石氏(大阪大) 代数多様体の藻

Feng - He - Kennaway - Vafa
hep-th / 0511287
"alga" 藻

characteristic polynomials

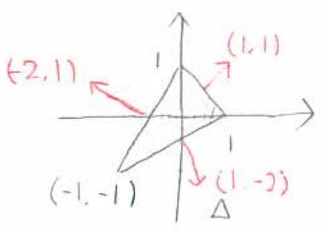


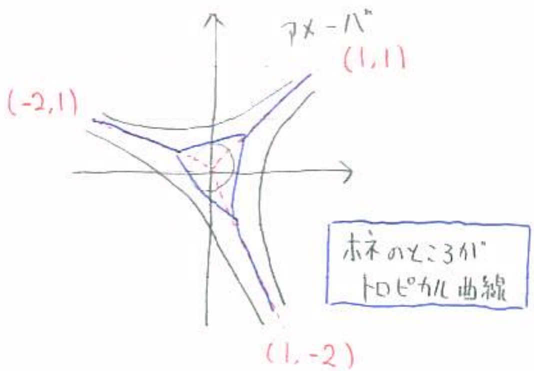
$W = \sum a_{ij} x^i y^j$: a Laurent polynomial

$\Delta = \mathrm{New}(W)$: the Newton polygon

$$\mathrm{Log} : (\mathbb{C}^\times)^2 \longrightarrow \mathbb{R}^2$$
$$(x, y) \longmapsto (\log|x|, \log|y|)$$

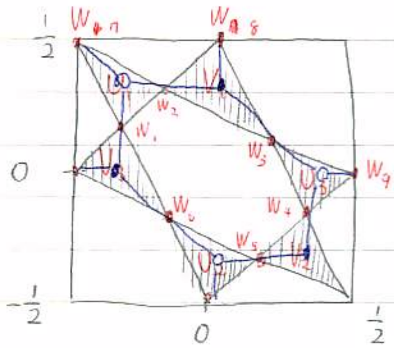
<u>Def.</u> (Gelfand - Kapranov - Zelevinski, 1994)
$\mathrm{Log}(W^{-1}(0))$: $W^{-1}(0) \subset (\mathbb{C}^\times)^2$ のアメーバ





<u>Def.</u> (…, Passare - Tsikh, Feng - He - Kennaway - Vafa)
$\mathrm{Arg}(W^{-1}(0))$: $W^{-1}(0)$ の alga (coamoeba)

$W = x + y + \frac{1}{xy}$



$\mathrm{Arg}(W^{-1}(0)) = \coprod_{i=1}^{3} U_i \cup \coprod_{i=1}^{3} V_i \cup \coprod_{i=1}^{9} W_i$

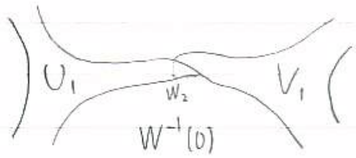
U_i, V_i : open triangles
W_i : vertices

$\mathrm{Arg}|_{\mathrm{Arg}^{-1}(U_i)} : \mathrm{Arg}^{-1}(U_i) \xrightarrow{\sim} U_i$; orientation-preserving diffeo.

$\mathrm{Arg}|_{\mathrm{Arg}^{-1}(V_i)} : \mathrm{Arg}^{-1}(V_i) \xrightarrow{\sim} V_i$; ori.- reversing diffeo.

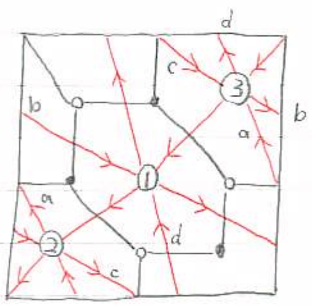
$\mathrm{Arg}^{-1}(W_i) \xrightarrow{\sim} (0,1)$

$\because$ $W^{-1}(0)$: U_iたちとV_iたちと開区間たちを頂点で貼ってつなぐ

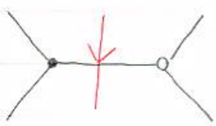


$\boxed{\text{夫田干し}}$

U_i の重心に白丸
V_i 〃 に黒丸
W_i に辺 を対応



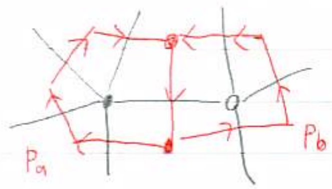
<u>箙</u>
面 → 頂点
辺 → 矢印 s.t.



関係

辺 $\longrightarrow$ イデアルの生成元

$p_a - p_b$



Rem.

P : the path alg. with rel.

$\mathrm{mod}\, P \simeq \mathrm{Coh}^{\mathbb{Z}/3\mathbb{Z}}\, \mathbb{C}^3$

$D^b \mathrm{mod}\, P \simeq D^b \mathrm{coh}\, \mathcal{O}_{\mathbb{P}^2}(-3)$

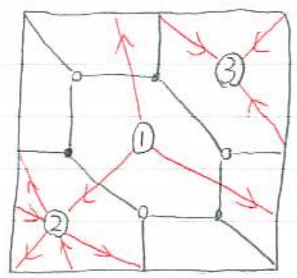
$ba = dc$ (path の合成)

directed subquiver

$i > j$ のとき

ⓘ $\longrightarrow$ ⓙ という矢印を消す

$\Rightarrow$



Observation

$$P^{\rightarrow} = \mathrm{End}\left(\bigoplus_{i=0}^{2} \mathcal{O}_{\mathbb{P}^2}(i)\right)$$

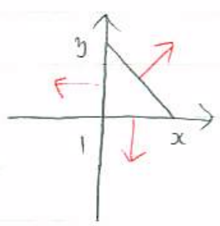
$$\mathcal{O}(1) \overset{6}{\Rrightarrow} \mathcal{O}(2)$$

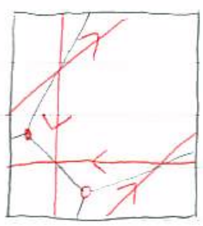
2色グラフのことを ダイマー模型 とも言う

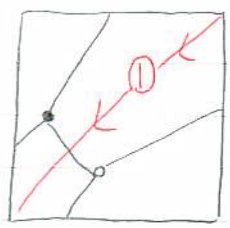
Def.

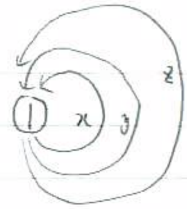
A perfect matching
(or a dimer configuration)
is a choice of a subset E of the set of edges so that for any vertex v, there is a unique edge $e \in E$ connected to v

algaを知らなくても ダイマー模型 が書ける (Ref. Arxiv. joint with Masahito Yamazaki)







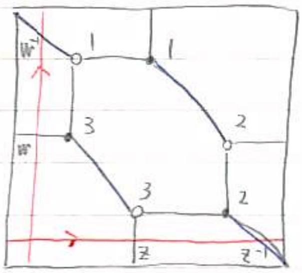


$xy - yx$
$xz - zx$
$yz - zy$

$P \simeq \mathbb{C}[x, y, z]$

Kasteleyn 行列

トーラスのαサイクルとβサイクルを
1つ選ぶ
辺との交点に、色に応じて
z, z^{-1}, w, w^{-1} を置く



α サイクル：

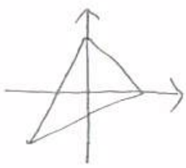
β 〃 ：

weighted adjacency matrix を書く．

$$K = \begin{array}{c} \\ \circ 1 \\ \circ 2 \\ \circ 3 \end{array} \begin{array}{c} \begin{array}{ccc} \bullet 1 & \bullet 2 & \bullet 3 \end{array} \\ \begin{pmatrix} 1 & z^{-1}w^{-1} & 1 \\ 1 & 1 & w \\ z & 1 & 1 \end{pmatrix} \end{array}$$

$$\Rightarrow |K| = 3 - z - w - z^{-1}w^{-1}$$

New $|K|$ =



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定理 (U - Yamazaki)

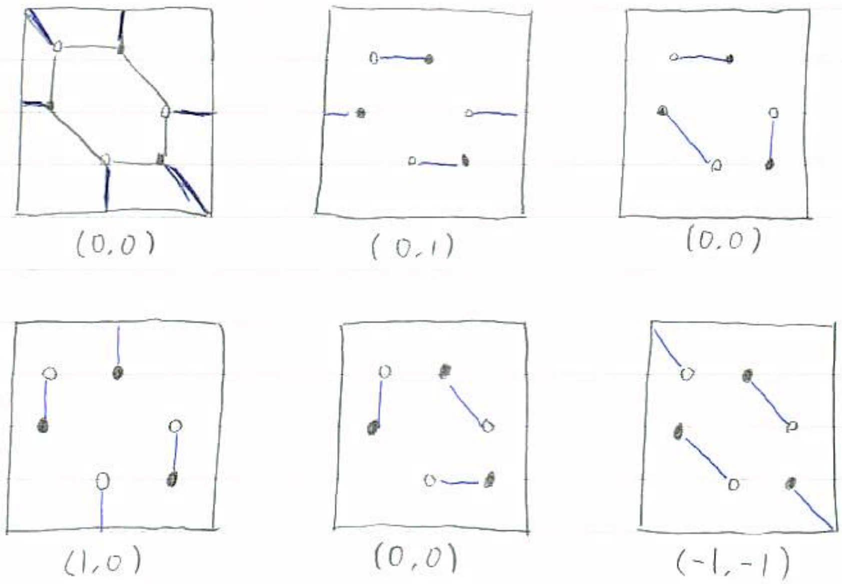
Δ：格子三角形の時
alga を使って定義されるダイマー模型と
normal vector の情報を使って定まる (Hanany - Vegh のアルゴリズム)
ダイマー模型は一致し、そのKasteleyn 行列の determinant の Newton 多角形
はもとの三角形になる．しかもここから得られる relation付きquiverは
Mckay quiver になる．

Kasteleyn 行列(式)って何？

dimer config. ⇝ height change (i, j)

$i = \#(\text{—}) - \#(\text{—})$

j も同様



$$\sum_E \underbrace{(-1)^i}_{?} z^i w^j = |K|$$

No. Date . .