Compositeness of hadrons and its application to baryon resonances

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2023, Nov. 26th

Contents

- Introduction: structure of excited hadrons

- Compositeness with weak-binding relation

S. Weinberg, Phys. Rev. 137, B672 (1965);
T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013);
Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)
T. Kinugawa, T. Hyodo, PRC106, 015205 (2022)

- Compositeness of baryon resonances

T. Sekihara, T. Hyodo, D. Jido, PTEP2015, 063D04 (2015);
T. Sekihara, T. Arai, J. Yamagata-Sekihara, S. Yasui, PRC93, 035204 (2016)

- Summary

Observed hadrons (2020)

Particle Data Group (PDG) 2020 eddition

http://pdg.lbl.gov/

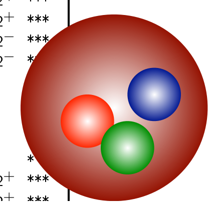
p	$1/2^+$	****
n	$1/2^+$	****
$N(1440)$	$1/2^+$	****
$N(1520)$	$3/2^-$	****
$N(1535)$	$1/2^-$	****
$N(1650)$	$1/2^-$	****
$N(1675)$	$5/2^-$	****
$N(1680)$	$5/2^+$	****
$N(1700)$	$3/2^-$	***
$N(1710)$	$1/2^+$	****
$N(1720)$	$3/2^+$	****
$N(1860)$	$5/2^+$	**
$N(1875)$	$3/2^-$	***
$N(1880)$	$1/2^+$	***
$N(1895)$	$1/2^-$	****
$N(1900)$	$3/2^+$	****
$N(1990)$	$7/2^+$	**
$N(2000)$	$5/2^+$	**
$N(2040)$	$3/2^+$	*
$N(2060)$	$5/2^-$	***
$N(2100)$	$1/2^+$	***
$N(2120)$	$3/2^-$	***
$N(2190)$	$7/2^-$	****
$N(2220)$	$9/2^+$	****
$N(2250)$	$9/2^-$	****
$N(2300)$	$1/2^+$	**
$N(2570)$	$5/2^-$	**
$N(2600)$	$11/2^-$	***
$N(2700)$	$13/2^+$	**

$\Delta(1232)$	$3/2^+$	****
$\Delta(1600)$	$3/2^+$	****
$\Delta(1620)$	$1/2^-$	****
$\Delta(1700)$	$3/2^-$	****
$\Delta(1750)$	$1/2^+$	*
$\Delta(1900)$	$1/2^-$	***
$\Delta(1905)$	$5/2^+$	****
$\Delta(1910)$	$1/2^+$	****
$\Delta(1920)$	$3/2^+$	***
$\Delta(1930)$	$5/2^-$	***
$\Delta(1940)$	$3/2^-$	**
$\Delta(1950)$	$7/2^+$	****
$\Delta(2000)$	$5/2^+$	**
$\Delta(2150)$	$1/2^-$	*
$\Delta(2200)$	$7/2^-$	***
$\Delta(2300)$	$9/2^+$	**
$\Delta(2350)$	$5/2^-$	*
$\Delta(2390)$	$7/2^+$	*
$\Delta(2400)$	$9/2^-$	**
$\Delta(2420)$	$11/2^+$	****
$\Delta(2750)$	$13/2^-$	**
$\Delta(2950)$	$15/2^+$	**
Λ	$1/2^+$	****
Λ	$1/2^-$	**
$\Lambda(1405)$	$1/2^-$	****
$\Lambda(1520)$	$3/2^-$	****
$\Lambda(1600)$	$1/2^+$	****
$\Lambda(1670)$	$1/2^-$	****
$\Lambda(1690)$	$3/2^-$	****
$\Lambda(1710)$	$1/2^+$	*
$\Lambda(1800)$	$1/2^-$	***
$\Lambda(1810)$	$1/2^+$	***
$\Lambda(1820)$	$5/2^+$	****
$\Lambda(1830)$	$5/2^-$	****
$\Lambda(1890)$	$3/2^+$	****
$\Lambda(2000)$	$1/2^-$	*
$\Lambda(2050)$	$3/2^-$	*
$\Lambda(2070)$	$3/2^+$	*
$\Lambda(2080)$	$5/2^-$	*
$\Lambda(2085)$	$7/2^+$	**
$\Lambda(2100)$	$7/2^-$	****
$\Lambda(2110)$	$5/2^+$	***
$\Lambda(2325)$	$3/2^-$	*
$\Lambda(2350)$	$9/2^+$	***
$\Lambda(2585)$		**

Σ^+	$1/2^+$	****
Σ^0	$1/2^+$	****
Σ^-	$1/2^+$	****
$\Sigma(1385)$	$3/2^+$	****
$\Sigma(1580)$	$3/2^-$	*
$\Sigma(1620)$	$1/2^-$	*
$\Sigma(1660)$	$1/2^+$	***
$\Sigma(1670)$	$3/2^-$	****
$\Sigma(1750)$	$1/2^-$	***
$\Sigma(1775)$	$5/2^-$	****
$\Sigma(1780)$	$3/2^+$	*
$\Sigma(1880)$	$1/2^+$	**
$\Sigma(1900)$	$1/2^-$	**
$\Sigma(1910)$	$3/2^-$	***
$\Sigma(1915)$	$5/2^+$	****
$\Sigma(1940)$	$3/2^+$	*
$\Sigma(2010)$	$3/2^-$	*
$\Sigma(2030)$	$7/2^+$	****
$\Sigma(2070)$	$5/2^+$	*
$\Sigma(2080)$	$3/2^+$	*
$\Sigma(2100)$	$7/2^-$	*
$\Sigma(2160)$	$1/2^-$	*
$\Sigma(2230)$	$3/2^+$	*
$\Sigma(2250)$		***
$\Sigma(2455)$		**
$\Sigma(2620)$		**
$\Sigma(3000)$		*
$\Sigma(3170)$		*

Ξ^0	$1/2^+$	****
Ξ^-	$1/2^+$	****
$\Xi(1530)$	$3/2^+$	****
$\Xi(1620)$		*
$\Xi(1690)$		***
$\Xi(1820)$	$3/2^-$	***
$\Xi(1950)$		***
$\Xi(2030)$	$\geq \frac{5}{2}^?$	***
$\Xi(2120)$		*
$\Xi(2250)$		**
$\Xi(2370)$		**
$\Xi(2500)$		*
Ω^-	$3/2^+$	****
$\Omega(2012)^-$	$?^-$	***
$\Omega(2250)^-$		***
$\Omega(2380)^-$		**
$\Omega(2470)^-$		**
Λ_c^+	$1/2^+$	****
$\Lambda_c(2595)^+$	$1/2^-$	***
$\Lambda_c(2625)^+$	$3/2^-$	***
$\Lambda_c(2765)^+$		*
$\Lambda_c(2860)^+$	$3/2^+$	***
$\Lambda_c(2880)^+$	$5/2^+$	***
$\Lambda_c(2940)^+$	$3/2^-$	***
$\Sigma_c(2455)$	$1/2^+$	****
$\Sigma_c(2520)$	$3/2^+$	***
$\Sigma_c(2800)$		***
Ξ_c^+	$1/2^+$	***
Ξ_c^0	$1/2^+$	****
$\Xi_c^{\prime +}$	$1/2^+$	***
$\Xi_c^{\prime 0}$	$1/2^+$	***
$\Xi_c(2645)$	$3/2^+$	***
$\Xi_c(2790)$	$1/2^-$	***
$\Xi_c(2815)$	$3/2^-$	*
$\Xi_c(2930)$		
$\Xi_c(2970)$		
$\Xi_c(3055)$		
$\Xi_c(3080)$		
$\Xi_c(3123)$		*
Ω_c^0	$1/2^+$	***
$\Omega_c(2770)^0$	$3/2^+$	***
$\Omega_c(3120)^0$		***

Ξ_{cc}^{++}		***
Λ_b^0	$1/2^+$	***
$\Lambda_b(5912)^0$	$1/2^-$	***
$\Lambda_b(5920)^0$	$3/2^-$	***
$\Lambda_b(6146)^0$	$3/2^+$	***
$\Lambda_b(6152)^0$	$5/2^+$	***
Σ_b	$1/2^+$	***
Σ_b^*	$3/2^+$	***
$\Sigma_b(6097)^+$		***
$\Sigma_b(6097)^-$		***
Ξ_b^0, Ξ_b^-	$1/2^+$	***
$\Xi'_b(5935)^-$	$1/2^+$	***
$\Xi_b(5945)^0$	$3/2^+$	***
$\Xi_b(5955)^-$	$3/2^+$	***
$\Xi_b(6227)$		***
Ω_b^-	$1/2^+$	***
$P_c(4312)^+$		*
$P_c(4380)^+$		*
$P_c(4440)^+$		*
$P_c(4457)^+$		*



~160 baryons

LIGHT UNFLAVORED ($S = C = B = 0$)

	$I^G(J^{PC})$		$I^G(J^{PC})$
• $\pi^\pm$	$1^-(0^-)$	• $\pi_2(1670)$	$1^-(2^{-+})$
• π^0	$1^-(0^{-+})$	• $\phi(1680)$	$0^-(1^{--})$
• η	$0^+(0^{-+})$	• $\rho_3(1690)$	$1^+(3^{--})$
• $f_0(500)$	$0^+(0^{++})$	• $\rho(1700)$	$1^+(1^{--})$
• $\rho(770)$	$1^+(1^{--})$	• $a_2(1700)$	$1^-(2^{++})$
• $\omega(782)$	$0^-(1^{--})$	• $f_0(1710)$	$0^+(0^{++})$
• $\eta'(958)$	$0^+(0^{-+})$	$\eta(1760)$	$0^+(0^{-+})$
• $f_0(980)$	$0^+(0^{++})$	• $\pi(1800)$	$1^-(0^{-+})$
• $a_0(980)$	$1^-(0^{++})$	$f_2(1810)$	$0^+(2^{++})$
• $\phi(1020)$	$0^-(1^{--})$	$X(1835)$	$?^?(0^{-+})$
• $h_1(1170)$	$0^-(1^{+-})$	• $\phi_3(1850)$	$0^-(3^{--})$
• $b_1(1235)$	$1^+(1^{+-})$	• $\eta_2(1870)$	$0^+(2^{-+})$
• $a_1(1260)$	$1^-(1^{++})$	• $\pi_2(1880)$	$1^-(2^{-+})$
• $f_2(1270)$	$0^+(2^{++})$	$\rho(1900)$	$1^+(1^{--})$
• $f_1(1285)$	$0^+(1^{++})$	$f_2(1910)$	$0^+(2^{++})$
• $\eta(1295)$	$0^+(0^{-+})$	$a_0(1950)$	$1^-(0^{++})$
• $\pi(1300)$	$1^-(0^{-+})$	• $f_2(1950)$	$0^+(2^{++})$
• $a_2(1320)$	$1^-(2^{++})$	• $a_4(1970)$	$1^-(4^{++})$
• $f_0(1370)$	$0^+(0^{++})$	$\rho_3(1990)$	$1^+(3^{--})$
• $\pi_1(1400)$	$1^-(1^{-+})$	$\pi_2(2005)$	$1^-(2^{-+})$
• $\eta(1405)$	$0^+(0^{-+})$	• $f_2(2010)$	$0^+(2^{++})$
• $h_1(1415)$	$0^-(1^{+-})$	$f_0(2020)$	$0^+(0^{++})$
$a_1(1420)$	$1^-(1^{++})$	• $f_4(2050)$	$0^+(4^{++})$
• $f_1(1420)$	$0^+(1^{++})$	$\pi_2(2100)$	$1^-(2^{-+})$
• $\omega(1420)$	$0^-(1^{--})$	$f_0(2100)$	$0^+(0^{++})$
$f_2(1430)$	$0^+(2^{++})$	$f_2(2150)$	$0^+(2^{++})$
• $a_0(1450)$	$1^-(0^{++})$	$\rho(2150)$	$1^+(1^{--})$
• $\rho(1450)$	$1^+(1^{--})$	• $\phi(2170)$	$0^-(1^{--})$
• $\eta(1475)$	$0^+(0^{-+})$	$f_0(2200)$	$0^+(0^{++})$
• $f_0(1500)$	$0^+(0^{++})$	$f_J(2220)$	$0^+(2^{++}$ or $4^{++})$
$f_1(1510)$	$0^+(1^{++})$		
• $f'_2(1525)$	$0^+(2^{++})$	$\eta(2225)$	$0^+(0^{-+})$
$f_2(1565)$	$0^+(2^{++})$	$\rho_3(2250)$	$1^+(3^{--})$
$\rho(1570)$	$1^+(1^{--})$	• $f_2(2300)$	$0^+(2^{++})$
$h_1(1595)$	$0^-(1^{+-})$	$f_4(2300)$	$0^+(4^{++})$
• $\pi_1(1600)$	$1^-(1^{-+})$	$f_0(2330)$	$0^+(0^{++})$
• $a_1(1640)$	$1^-(1^{++})$	• $f_2(2340)$	$0^+(2^{++})$
$f_2(1640)$	$0^+(2^{++})$	$\rho_5(2350)$	$1^+(5^{--})$
• $\eta_2(1645$		$f_6(2510)$	$0^+(6^{++})$
• $\omega(16$		OTHER LIGHT	
• ω_3		her States	

STRANGE ($S = \pm1$, $C = B = 0$)

	$I(J^P)$
• $K^\pm$	$1/2(0^-)$
• K^0	$1/2(0^-)$
• K_S^0	$1/2(0^-)$
• K_L^0	$1/2(0^-)$
• $K_0^*(700)$	$1/2(0^+)$
• $K^*(892)$	$1/2(1^-)$
• $K_1(1270)$	$1/2(1^+)$
• $K_1(1400)$	$1/2(1^+)$
• $K^*(1410)$	$1/2(1^-)$
• $K_0^*(1430)$	$1/2(0^+)$
• $K_2^*(1430)$	$1/2(2^+)$
$K(1460)$	$1/2(0^-)$
$K_2(1580)$	$1/2(2^-)$
$K(1630)$	$1/2(?^?)$
$K_1(1650)$	$1/2(1^+)$
• $K^*(1680)$	$1/2(1^-)$
• $K_2(1770)$	$1/2(2^-)$
• $K_3^*(1780)$	$1/2(3^-)$
• $K_2(1820)$	$1/2(2^-)$
$K(1830)$	$1/2(0^-)$
$K_0^*(1950)$	$1/2(0^+)$
$K_2^*(1980)$	$1/2(2^+)$
• $K_4^*(2045)$	$1/2(4^+)$
$K_2(2250)$	$1/2(2^-)$
$K_3(2320)$	$1/2(3^+)$
$K_5^*(2380)$	$1/2(5^-)$
$K_4(2500)$	$1/2(4^-)$
$K(3100)$	$?^?(?^{??})$

CHARMED ($C = \pm1$)

• $D^\pm$	$1/2(0^-)$
• D^0	$1/2(0^-)$
• $D^*(2007)^0$	$1/2(1^-)$
• $D^*(2010)^\pm$	$1/2(1^-)$
• $D_0^*(2300)^0$	$1/2(0^+)$
$D_0^*(2300)^\pm$	$1/2(0^+)$
• $D_1(2420)^0$	$1/2(1^+)$
$D_1(2420)^\pm$	$1/2(?^?)$
$D_1(2430)^0$	$1/2(1^+)$
• $D_2^*(2460)^0$	$1/2(2^+)$
• $D_2^*(2460)^\pm$	$1/2(2^+)$
$D(2550)^0$	$1/2(?^?)$
$D_J^*(2600)$	$1/2(?^?)$
$D^*(2640)^\pm$	$1/2(?^?)$
$D(2740)^0$	$1/2(?^?)$
$D_3^*(2750)$	$1/2(3^-)$
$D(3000)^0$	$1/2(?^?)$

CHARMED, STRANGE ($C = S = \pm1$)

	$I(J^P)$
• $D_s^\pm$	$0(0^-)$
• $D_s^{*\pm}$	$0(?^?)$
• $D_{s0}^*(2317)^\pm$	$0(0^+)$
• $D_{s1}(2460)^\pm$	$0(1^+)$
• $D_{s1}(2536)^\pm$	$0(1^+)$
• $D_{s2}^*(2573)$	$0(2^+)$
• $D_{s1}^*(2700)^\pm$	$0(1^-)$
$D_{s1}^*(2860)^\pm$	$0(1^-)$
$D_{s3}^*(2860)^\pm$	$0(3^-)$
$D_{sJ}(3040)^\pm$	$0(?^?)$

BOTTOM ($B = \pm1$)

• $B^\pm$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^\pm/B^0$ ADMIXTURE	
• $B^\pm/B^0/B_s^0/b$-baryon ADMIXTURE	
V_{cb} and V_{ub} CKM Matrix Elements	
• B^*	$1/2(1^-)$
• $B_1(5721)^+$	$1/2(1^+)$
• $B_1(5721)^0$	$1/2(1^+)$
$B_J^*(5732)$	$?(?^?)$
• $B_2^*(5747)^+$	$1/2(2^+)$
• $B_2^*(5747)^0$	$1/2(2^+)$
$B_J(5840)^+$	$1/2(?^?)$
$B_J(5840)^0$	$1/2(?^?)$
• $B_J(5970)^+$	$1/2(?^?)$
• $B_J(5970)^0$	$1/2(?^?)$

BOTTOM, STRANGE ($B = \pm1$, $S = \mp1$)

• B_s^0	$0(0^-)$
• B_s^*	$0(1^-)$
$X(5568)^\pm$	$?(?^?)$
• $B_{s1}(5830)^0$	$0(1^+)$
• $B_{s2}^*(5840)^0$	$0(2^+)$
$B_{sJ}^*(5850)$	$?(?^?)$

BOTTOM, CHARMED ($B = C = \pm1$)

• B_c^+	$0(0^-)$
$B_c(2S)^\pm$	$0(0^-)$

$c\bar{c}$ (+ possibly non-$q\bar{q}$ states)

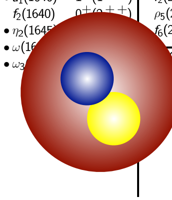
• $\eta_c(1S)$	$0^+(0^{-+})$
• $J/\psi(1S)$	$0^-(1^{--})$
• $\chi_{c0}(1P)$	$0^+(0^{++})$
$c1(1P)$	$0^+(1^{++})$
$(1P)$	$0^-(1^{+-})$
$c2(1P)$	$0^+(2^{++})$
$(2S)$	$0^+(0^{-+})$
$(2S)$	$0^-(1^{--})$

$c\bar{c}$ continued

	$I^G(J^{PC})$
• $\psi(3770)$	$0^-(1^{--})$
• $\psi_2(3823)$	$0^-(2^{--})$
• $\psi_3(3842)$	$0^-(3^{--})$
$\chi_{c0}(3860)$	$0^+(0^{++})$
• $\chi_{c1}(3872)$	$0^+(1^{++})$
• $Z_c(3900)$	$1^+(1^{+-})$
• $X(3915)$	$0^+(0/2^{++})$
• $\chi_{c2}(3930)$	$0^+(2^{++})$
$X(3940)$	$?^?(?^{??})$
• $X(4020)^\pm$	$1^+(?^{?-})$
• $\psi(4040)$	$0^-(1^{--})$
$X(4050)^\pm$	$1^-(?^{?+})$
$X(4055)^\pm$	$1^+(?^{?-})$
$X(4100)^\pm$	$1^-(?^{??})$
• $\chi_{c1}(4140)$	$0^+(1^{++})$
• $\psi(4160)$	$0^-(1^{--})$
$X(4160)$	$?^?(?^{??})$
$Z_c(4200)$	$1^+(1^{+-})$
• $\psi(4230)$	$0^-(1^{--})$
$R_{c0}(4240)$	$1^+(0^{--})$
$X(4250)^\pm$	$1^-(?^{?+})$
$\psi(4260)$	$0^-(1^{--})$
• $\chi_{c1}(4274)$	$0^+(1^{++})$
$X(4350)$	$0^+(?^{?+})$
• $\psi(4360)$	$0^-(1^{--})$
$\psi(4390)$	$0^-(1^{--})$
• $\psi(4415)$	$0^-(1^{--})$
• $Z_c(4430)$	$1^+(1^{+-})$
$\chi_{c0}(4500)$	$0^+(0^{++})$
• $\psi(4660)$	$0^-(1^{--})$
$\chi_{c0}(4700)$	$0^+(0^{++})$

$b\bar{b}$ (+ possibly non-$q\bar{q}$ states)

• $\eta_b(1S)$	$0^+(0^{-+})$
• $\Upsilon(1S)$	$0^-(1^{--})$
• $\chi_{b0}(1P)$	$0^+(0^{++})$
• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $h_b(1P)$	$0^-(1^{+-})$
• $\chi_{b2}(1P)$	$0^+(2^{++})$
$\eta_b(2S)$	$0^+(0^{-+})$
• $\Upsilon(2S)$	$0^-(1^{--})$
• $\Upsilon_2(1D)$	$0^-(2^{--})$
• $\chi_{b0}(2P)$	$0^+(0^{++})$
• $\chi_{b1}(2P)$	$0^+(1^{++})$
$h_b(2P)$	$0^-(1^{+-})$
• $\chi_{b2}(2P)$	$0^+(2^{++})$
• $\Upsilon(3S)$	$0^-(1^{--})$
• $\chi_{b1}(3P)$	$0^+(1^{++})$
• $\chi_{b2}(3P)$	$0^+(2^{++})$
• $\Upsilon(4S)$	$0^-(1^{--})$
• $Z_b(10610)$	$1^+(1^{+-})$
• $Z_b(10650)$	$1^+(1^{+-})$
$\Upsilon(10753)$	$?^?(1^{--})$
• $\Upsilon(10860)$	$0^-(1^{--})$
• $\Upsilon(11020)$	$0^-(1^{--})$

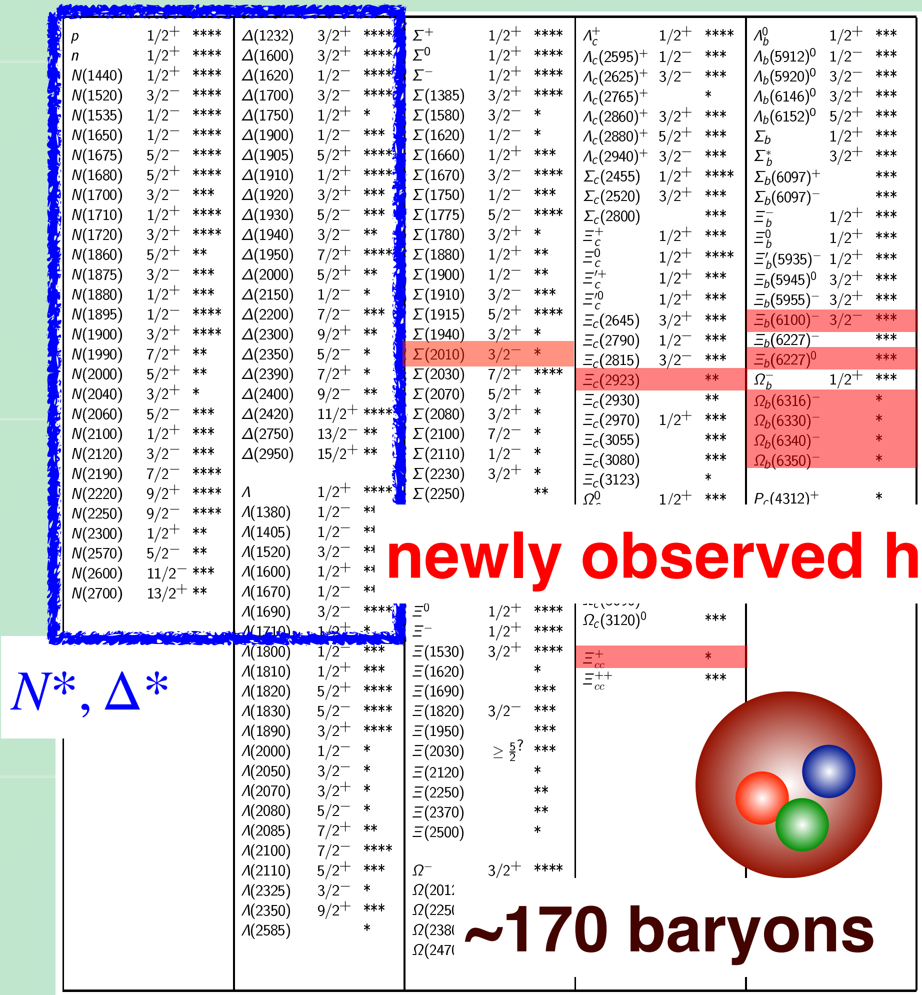


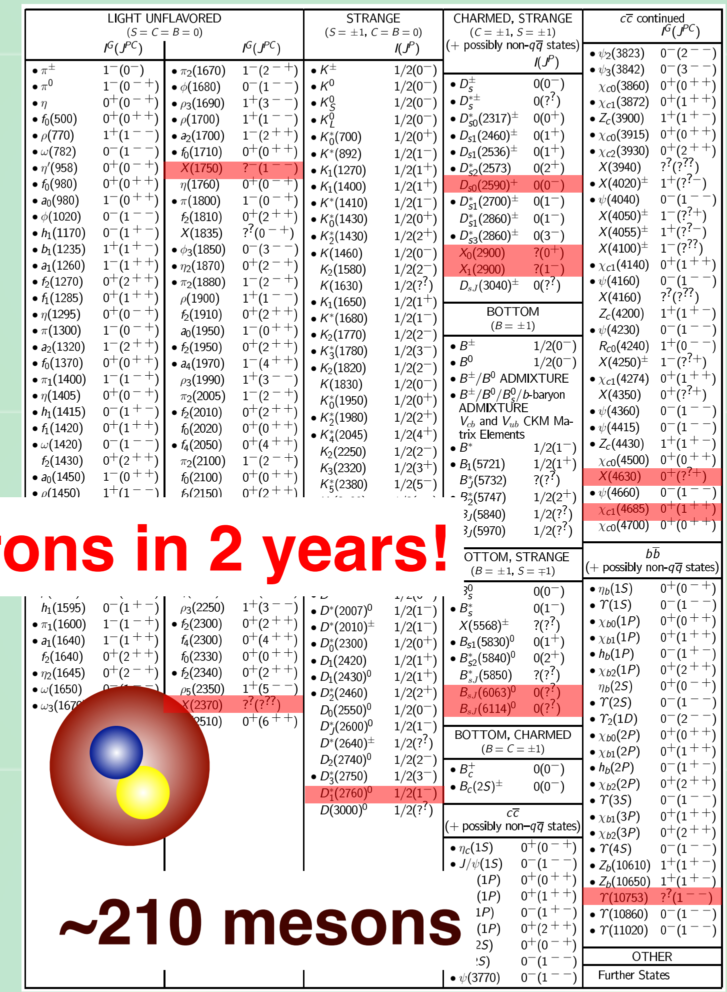
~210 mesons

Observed hadrons (2022)

Particle Data Group (PDG) 2022 eddition

http://pdg.lbl.gov/





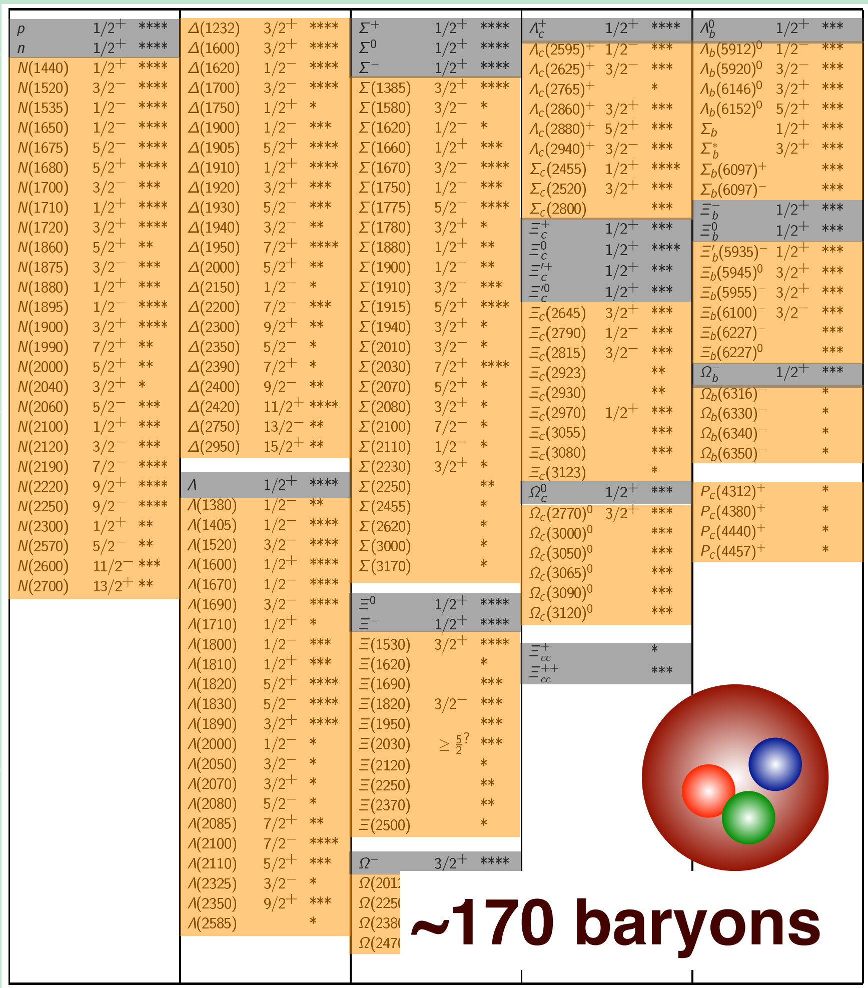
newly observed hadrons in 2 years!

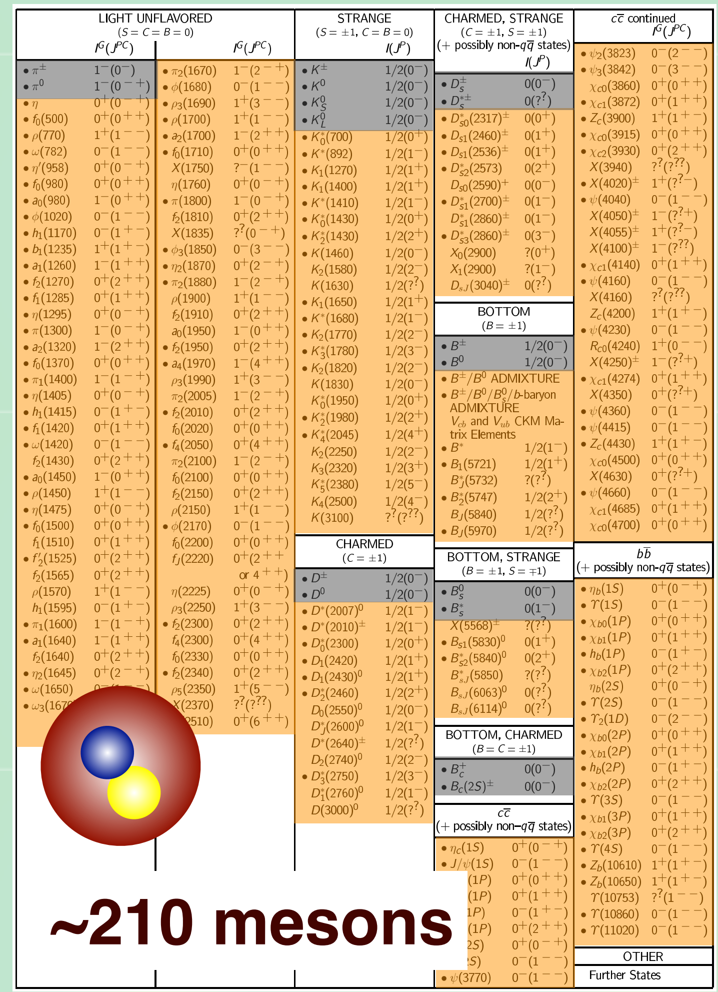
All ~ 380 hadrons emerge from single QCD Lagrangian

Unstable states via strong interaction

Stable/unstable hadrons

http://pdg.lbl.gov/

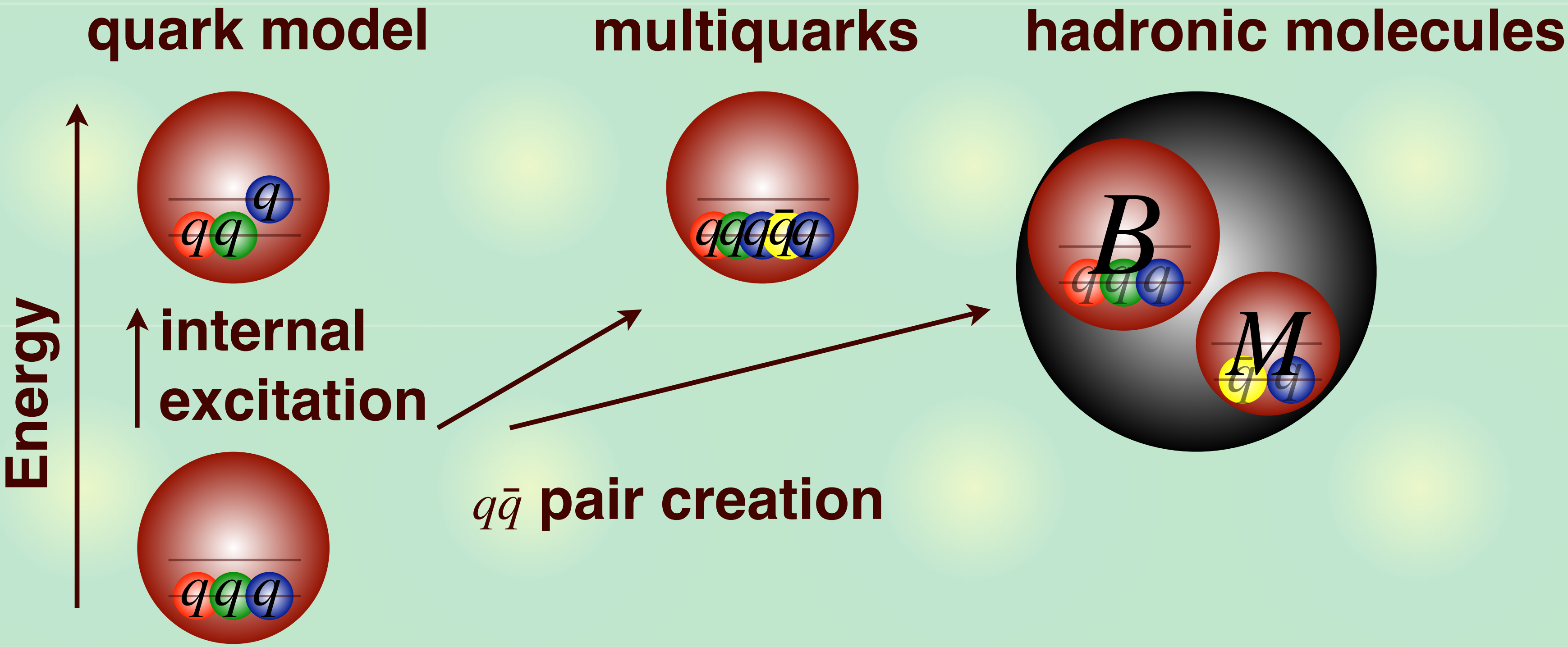




Most of hadrons are unstable (above two-hadron threshold)

Aim of this talk

Various excitations of hadrons



Issues:

- **Quantitative** discussion of internal structure
- **Unstable** nature of excited hadrons

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- Introduction: structure of excited hadrons
- Compositeness with weak-binding relation

  S. Weinberg, Phys. Rev. 137, B672 (1965);
  T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013);
  Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)
  T. Kinugawa, T. Hyodo, PRC106, 015205 (2022)

- Compositeness of baryon resonances

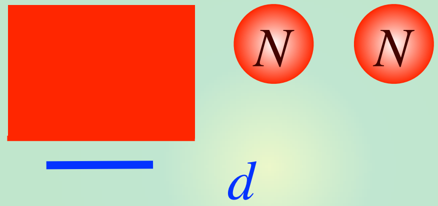
  T. Sekihara, T. Hyodo, D. Jido, PTEP2015, 063D04 (2015);
  T. Sekihara, T. Arai, J. Yamagata-Sekihara, S. Yasui, PRC93, 035204 (2016)

- Summary

Weak-binding relation for stable states

Compositeness X of **stable** bound state

S. Weinberg, Phys. Rev. 137, B672 (1965);
T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)



$$|d\rangle = \sqrt{X}\,|NN\rangle + \sqrt{Z}\,|\text{others}\rangle, \quad X + Z = 1, \quad 0 \le X \le 1$$

range of interaction

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\text{typ}}}{R}\right)\right\}, \quad R = \frac{1}{\sqrt{2\mu B}}$$

scattering length — radius of bound state

- for shallow bound state $R \gg R_{\text{typ}}$, $X \leftarrow (a_0, B)$

(i) The particle must be stable; else Z is undefined. (However, it may be an adequate approximation to ignore the decay modes of a very narrow resonance.)
(ii) The particle must couple to a two-particle channel with threshold not too much above the particle mass.
(iii) It is crucial that this two-body channel have zero orbital angular momentum l, since for $l \neq 0$ the factor $(E)^{1/2}$ in the integrands of (24) and (32) would be $E^{l+(1/2)}$, and the integrals could not be approximated by their low-energy parts.

Problem1: applicable only to stable states

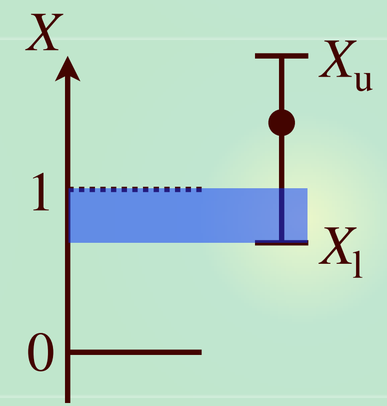
Problem2: empirical $(a_0, B) \rightarrow X = 1.68$ **?**

Uncertainty and interpretation

Uncertainty estimation with $\mathcal{O}(R_{\rm typ}/R)$ term

Y. Kamiya, T. Hyodo, PTEP2017, 023D02 (2017)

$$X_{\rm u} = \frac{a_0/R + \xi}{2 - a_0/R - \xi}, \quad X_{\rm l} = \frac{a_0/R - \xi}{2 - a_0/R + \xi}, \quad \xi = \frac{R_{\rm typ}}{R}$$



Interpretation (with finite range correction)

T. Kinugawa, T. Hyodo, PRC 106, 015205 (2022)

- exclude region outside $0 \leq X \leq 1$

$$R_{\rm typ} = \max\{R_{\rm int}, R_{\rm eff}\}$$

Bound state	Compositeness X
d	$0.74 \leqslant X \leqslant 1$
$X(3872)$	$0.53 \leqslant X \leqslant 1$
$D^*_{s0}(2317)$	$0.81 \leqslant X \leqslant 1$
$D_{s1}(2460)$	$0.55 \leqslant X \leqslant 1$
$N\Omega$ dibaryon	$0.80 \leqslant X \leqslant 1$
$\Omega\Omega$ dibaryon	$0.79 \leqslant X \leqslant 1$
${}^3_\Lambda$H	$0.74 \leqslant X \leqslant 1$
^{4}He dimer	$0.93 \leqslant X \leqslant 1$

- X of hadrons, nuclei, and atoms
- X of deuteron is reasonable
- $X \geq 0.5$ in all cases studied

Near-threshold bound states are mostly composite

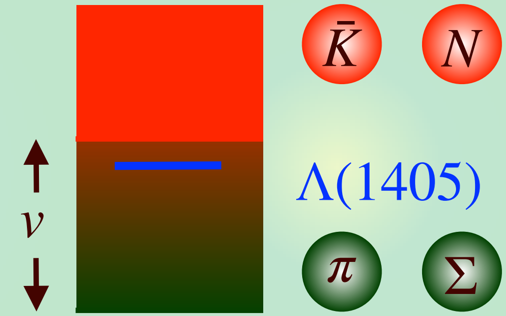
Weak-binding relation for unstable states

Compositeness X of unstable quasibound state

Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)

- complex eigenenergy: $-B \to E_h \in \mathbb{C}$

$$|\Lambda(1405)\rangle = \sqrt{X}\,|\bar{K}N\rangle + \sqrt{Z}\,|\text{others}\rangle, \quad X + Z = 1$$



- complex a_0, X

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\text{typ}}}{R}\right|\right) + \mathcal{O}\left(\left|\frac{\ell}{R}\right|^3\right)\right\}, \quad R = \frac{1}{\sqrt{-2\mu E_h}}, \quad \ell \equiv \frac{1}{\sqrt{2\mu\nu}}$$

- correction from threshold energy difference

- for near-threshold quasibound state $|R| \gg (R_{\text{typ}}, \ell)$, $X \leftarrow (a_0, E_h)$

Interpretation of complex X

$$\tilde{X} = \frac{1-|Z|+|X|}{2}, \quad \tilde{Z} = \frac{1-|X|+|Z|}{2}, \quad \tilde{X} + \tilde{Z} = 1, \quad 0 \le \tilde{X} \le 1$$

Compositeness of $\Lambda(1405)$: central values

Generalized weak-binding relation

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\rm typ}}{R}\right|\right) + \mathcal{O}\left(\left|\frac{\ell}{R}\right|^3\right)\right\}, \quad R = \frac{1}{\sqrt{-2\mu E_h}}, \quad \ell \equiv \frac{1}{\sqrt{2\mu\nu}}$$

(a_0, E_h) determinations by several groups

- Neglecting correction terms:

	E_h [MeV]	a_0 [fm]	$X_{\bar{K}N}$	$\tilde{X}_{\bar{K}N}$	$U/2$
Set 1 [35]	$-10 - i26$	$1.39 - i0.85$	$1.2 + i0.1$	1.0	0.3
Set 2 [36]	$-4 - i8$	$1.81 - i0.92$	$0.6 + i0.1$	0.6	0.0
Set 3 [37]	$-13 - i20$	$1.30 - i0.85$	$0.9 - i0.2$	0.9	0.1
Set 4 [38]	$2 - i10$	$1.21 - i1.47$	$0.6 + i0.0$	0.6	0.0
Set 5 [38]	$-3 - i12$	$1.52 - i1.85$	$1.0 + i0.5$	0.8	0.3

- In all cases, $X \sim 1$ and $\tilde{X} \sim 1$

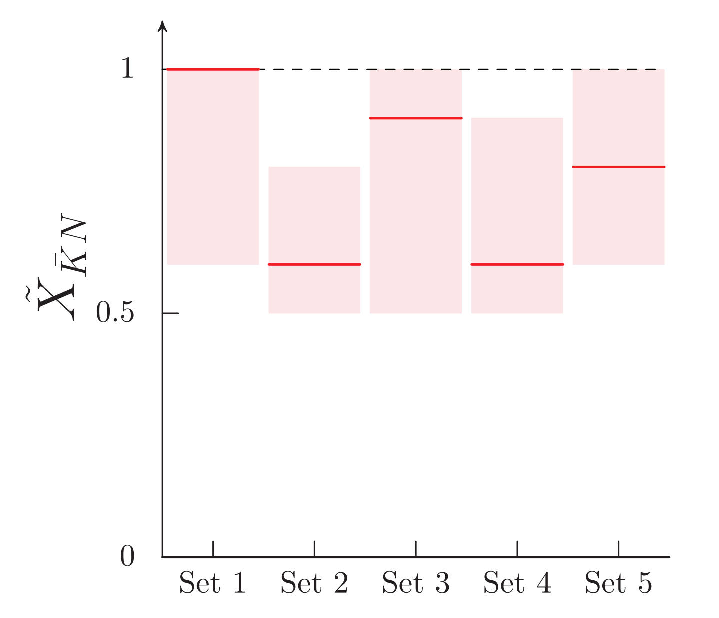
$\Lambda(1405)$: $\bar{K}N$ composite dominance <— observables

Compositeness of $\Lambda(1405)$: uncertainties

Estimation of correction terms: $|R| \sim 2$ fm

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\rm typ}}{R}\right|\right) + \mathcal{O}\left(\left|\frac{\ell}{R}\right|^3\right)\right\}, \quad R = \frac{1}{\sqrt{-2\mu E_h}}, \quad \ell \equiv \frac{1}{\sqrt{2\mu\nu}}$$

- **ρ meson exchange picture:** $R_{\rm typ} \sim 0.25$ fm
- **Energy difference from $\pi\Sigma$:** $\ell \sim 1.08$ fm



$\bar{K}N$ composite dominance holds even with correction terms

Contents

- Introduction: structure of excited hadrons

- Compositeness with weak-binding relation

  S. Weinberg, Phys. Rev. 137, B672 (1965);
  T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013);
  Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)
  T. Kinugawa, T. Hyodo, PRC106, 015205 (2022)

- Compositeness of baryon resonances

  T. Sekihara, T. Hyodo, D. Jido, PTEP2015, 063D04 (2015);
  T. Sekihara, T. Arai, J. Yamagata-Sekihara, S. Yasui, PRC93, 035204 (2016)

- Summary

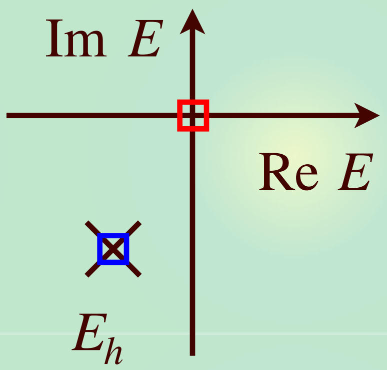
Two methods to evaluate compositeness

Weak-binding relation

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\rm typ}}{R}\right|\right) + \mathcal{O}\left(\left|\frac{\ell}{R}\right|^3\right)\right\}, \quad R = \frac{1}{\sqrt{-2\mu E_h}}, \quad \ell \equiv \frac{1}{\sqrt{2\mu\nu}}$$

- Pro: model independent, determined by observables
- Con: uncertainty, near-threshold s-wave state only

Evaluation from residue of resonance pole



T. Hyodo, D. Jido, A. Hosaka, PRC85, 015201 (2012);
F. Aceti. E. Oset, PRD86, 014012 (2012)

$$X = -g^2 \frac{dG(E)}{dE}\bigg|_{E=E_h}$$

- Pro: no uncertainty, applicable to any states (e.g. p wave)
- Con: model dependent (off-shell nature)

Two methods are complementary with each other

Comparison of two methods

Compositeness of $\Lambda(1405)$ with NLO chiral SU(3) dynamics

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA881, 98 (2012)

$$E_h = -10 - 26i \ [\text{MeV}]$$

- **Weak-binding relation**

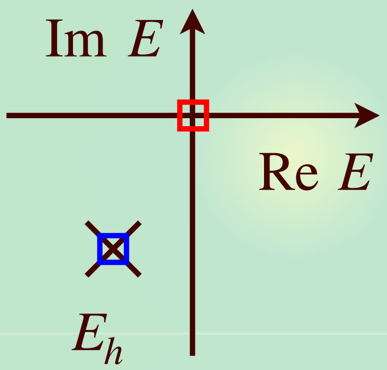
Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)

$$X = 1.2 + 0.1i$$

- **Evaluation from residue of resonance pole**

T. Sekihara, T. Hyodo, D. Jido, PTEP2015, 063D04 (2015)

$$X = 1.14 + 0.01i$$



Good agreement <— $\Lambda(1405)$ is sufficiently close to threshold

- **model dependence/uncertainty reduces as $|E_h| \to 0$**

see also T. Kinugawa, T. Hyodo, arXiv:2303.07038 [hep-ph]

Compositeness of baryon resonances

Unitarized NLO cihral (coupled-channel) amplitude

T. Sekihara, T. Arai, J. Yamagata-Sekihara, S. Yasui, PRC93, 035204 (2016)

TABLE II. Properties of Δ(1232) and $N(940)$. We do not calculate U, $\tilde{X}_{\pi N}$, and $\tilde{Z}$ for $N(940)$ because it is a stable state.

	Naive		Constrained	
	Δ(1232)	$N(940)$	Δ(1232)	$N(940)$
w_{pole} (MeV)	$1209.8 - 47.6i$	938.9	$1206.9 - 49.6i$	938.9
g (MeV$^{-1/2}$)	$0.383 - 0.053i$	0.560	$0.395 - 0.061i$	0.516
$X_{\pi N}$	$0.69 + 0.39i$	−0.18	$0.87 + 0.35i$	0.00
Z	$0.31 - 0.39i$	1.18	$0.13 - 0.35i$	1.00
U	0.30	–	0.31	–
$\tilde{X}_{\pi N}$	0.61	–	0.71	–
$\tilde{Z}$	0.39	–	0.29	–

TABLE IV. Properties of $N(1535)$ and $N(1650)$.

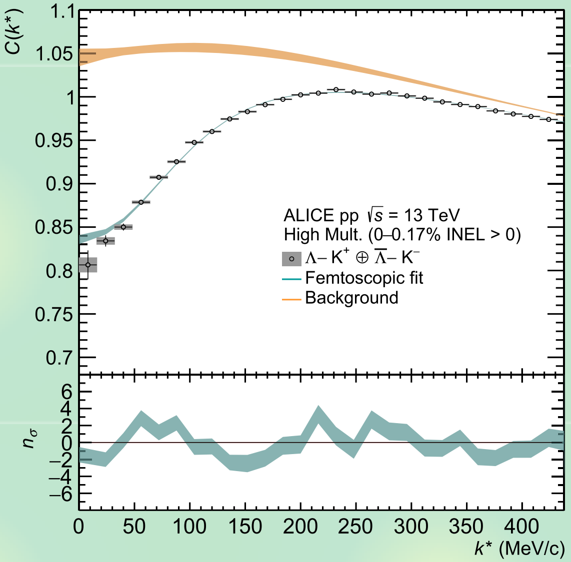
	$N(1535)$	$N(1650)$
w_{pole} (MeV)	$1496.4 - 58.7i$	$1660.7 - 70.0i$
$g_{\pi N}$ (MeV$^{1/2}$)	$47.1 - 7.3i$	$49.8 - 23.1i$
$g_{\eta N}$ (MeV$^{1/2}$)	$68.9 - 42.4i$	$-19.0 + 11.1i$
$g_{K\Lambda}$ (MeV$^{1/2}$)	$85.0 + 14.4i$	$-29.9 + 37.1i$
$g_{K\Sigma}$ (MeV$^{1/2}$)	$-31.4 + 17.5i$	$-73.8 + 6.0i$
$X_{\pi N}$	$-0.02 + 0.03i$	$0.00 + 0.04i$
$X_{\eta N}$	$0.04 + 0.37i$	$0.00 + 0.01i$
$X_{K\Lambda}$	$0.14 + 0.00i$	$0.08 + 0.05i$
$X_{K\Sigma}$	$0.01 - 0.02i$	$0.09 - 0.12i$
Z	$0.84 - 0.38i$	$0.84 + 0.01i$
U	0.48	0.13
$\tilde{X}_{\pi N}$	0.03	0.04
$\tilde{X}_{\eta N}$	0.25	0.01
$\tilde{X}_{K\Lambda}$	0.09	0.08
$\tilde{X}_{K\Sigma}$	0.01	0.13
$\tilde{Z}$	0.62	0.74

- $N(940)$: Z **dominance (qqq like)**
- Δ(1232): $X_{\pi N}$ **dominance (molecule like)**
- $N(1535)$, $N(1650)$: Z **dominance (qqq like)**

What to measure

Determination of (partial wave) scattering amplitude

- cross sections, angular dependence, …
- pole position (eigenenergy) —> weak-binding relation
- dynamical coupled-channel model —> residue method



Determination of scattering length

- ΛK^+ scattering length by femtoscopy

$$a_0^{\Lambda K^+} = 0.61 - 0.23i \text{ [fm]}$$

ALICE collaboration, PRC 103, 055201 (2021); PLB 845, 138145 (2023)

Accumulation of data will sharpen the evaluation of X

Summary

- Structure of hadrons should be studied
  - in quantitative manner, and
  - with unstable nature taken into account.
- Compositeness is a quantitative measure of hadron structure, applicable to unstable states.
  - weak-binding relation (model-independent)
  - residue method (no uncertainty)
- Compositeness of baryon resonances have been evaluated. More experimental data are welcome to improve the estimations.