Compositeness of hadrons from effective field theory

Clusters & Hierarchies

Tomona Kinugawa, Tetsuo Hyodo

Tokyo Metropolitan Univ.

2023, Feb. 28th

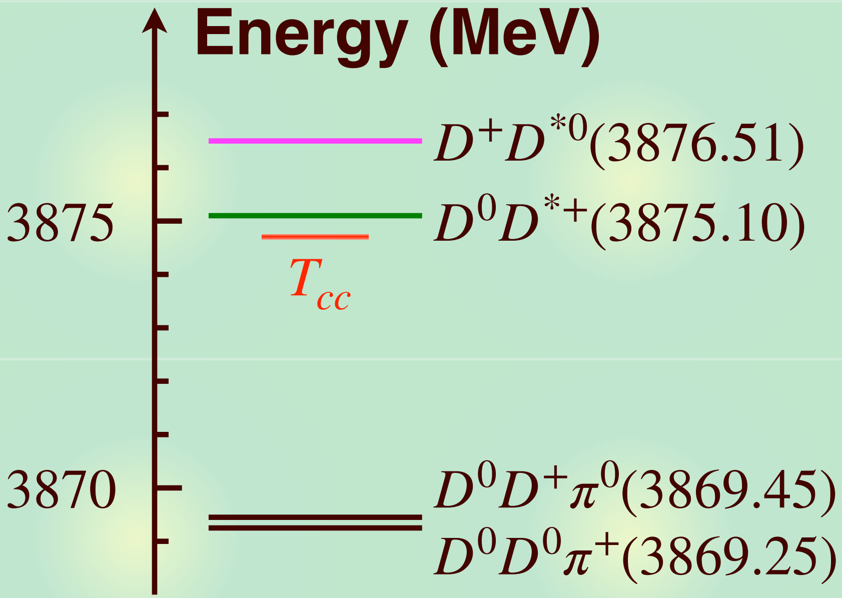
Contents

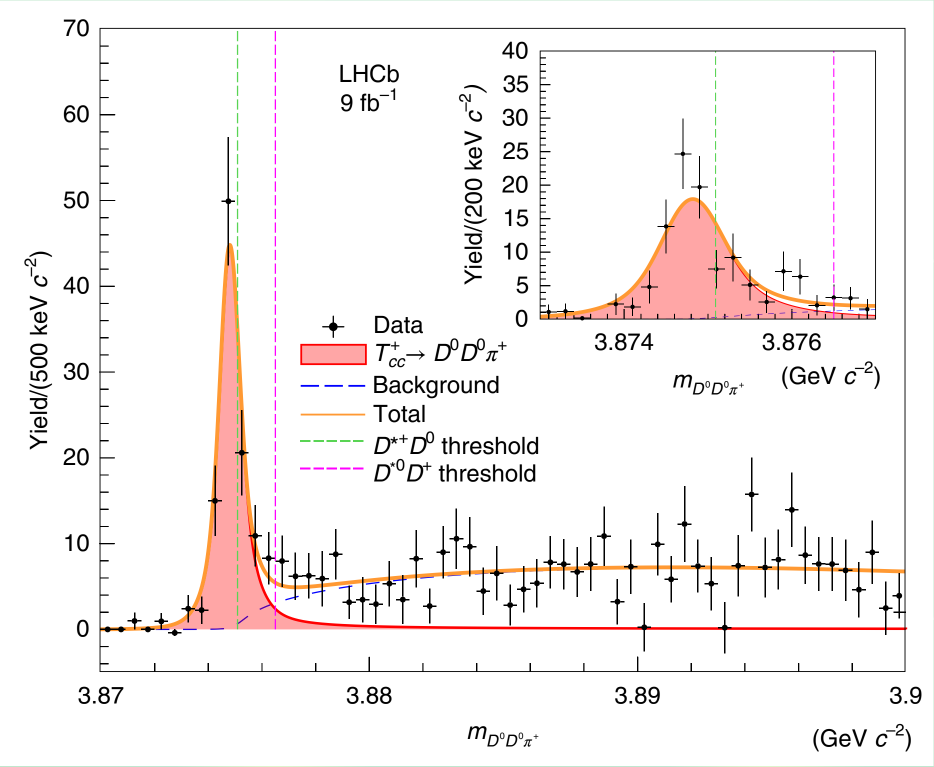
Observation of T_{cc}

T_{cc} observed in $D^0D^0\pi^+$ spectrum

LHCb collaboration, Nature Phys., 18, 751 (2022); Nature Comm., 13, 3351 (2022)

- **Signal near DD^* threshold**
- **Charm $C = +2 : \sim cc\bar{u}\bar{d}$**
- **Level structure**

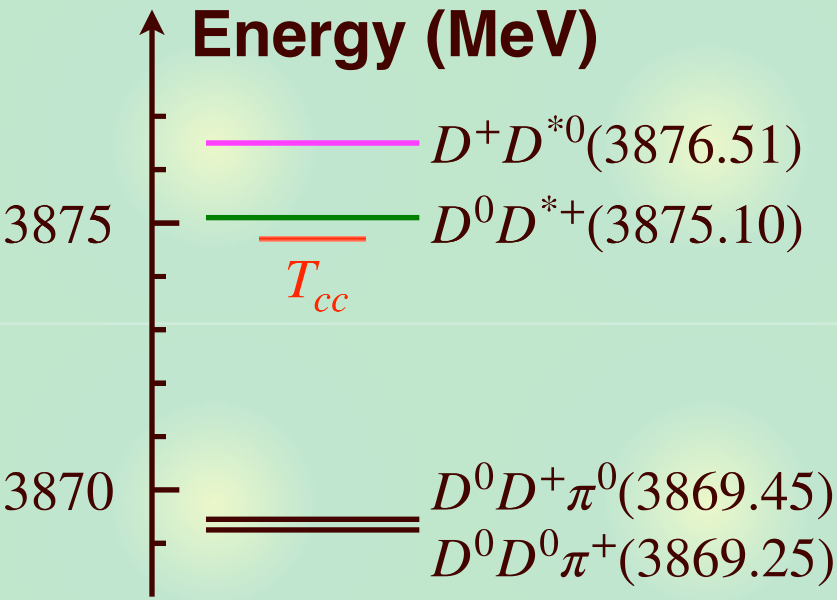


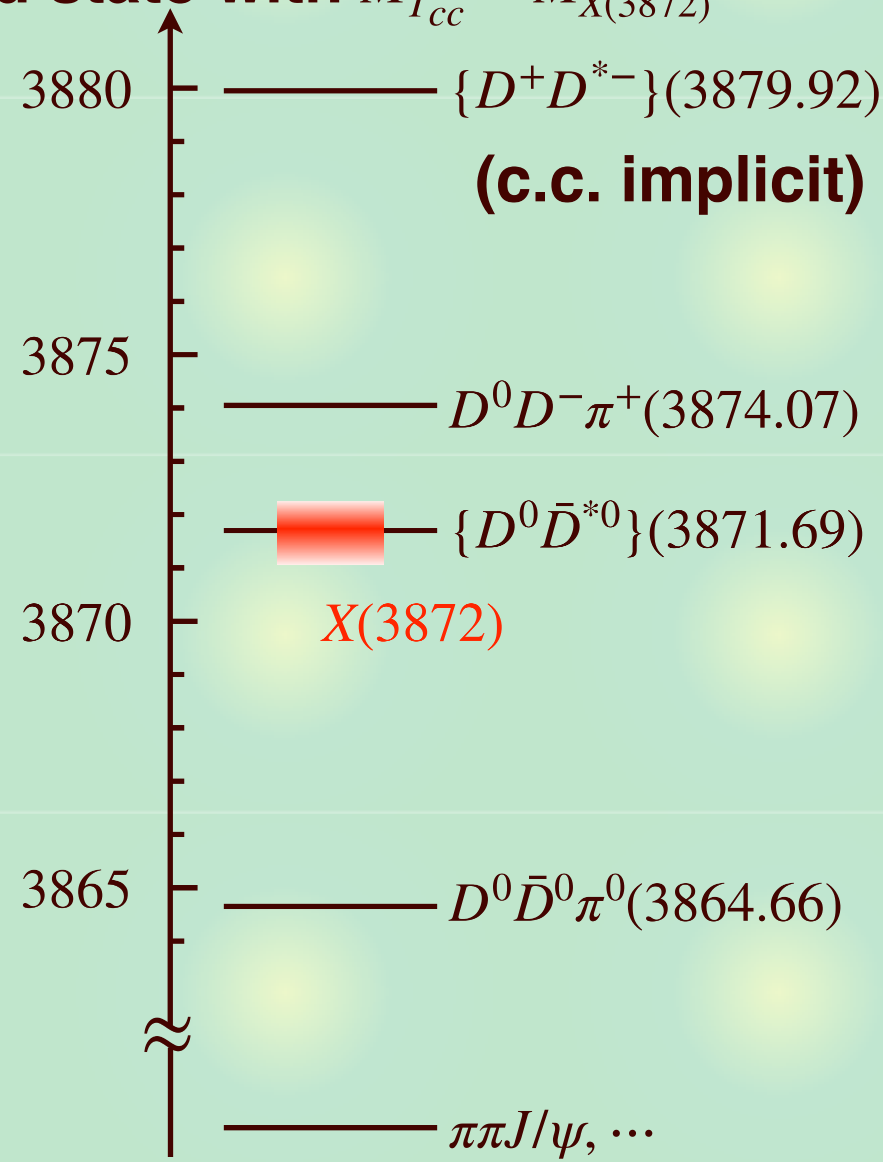


- **Very small (few MeV ~ keV) energy scale involved**

T_{cc} and $X(3872)$

$X(3872)$: **another near-threshold state with** $M_{T_{cc}} \sim M_{X(3872)}$

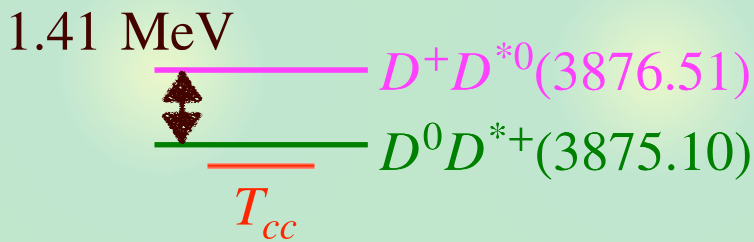




- T_{cc}, $X(3872)$ **near** $DD^*, D\bar{D}^*$
- **Various thresholds**
- $X(3872)$ **has decay channels**

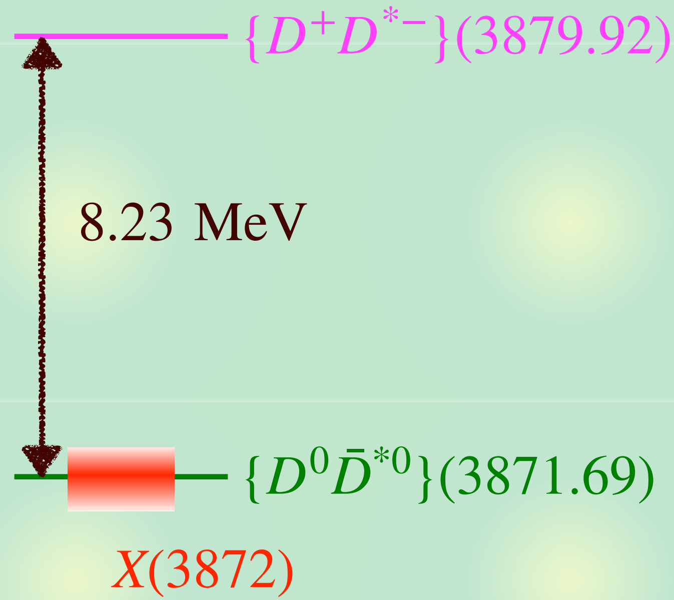
Simplified picture

In this talk, we consider two-body channels



$$E_h = -0.36 - i\frac{0.048}{2}\ \text{MeV}$$

(pole mass by LHCb)



$$E_h = -0.04 - i\frac{1.19}{2}\ \text{MeV}$$

(mass and width by PDG)

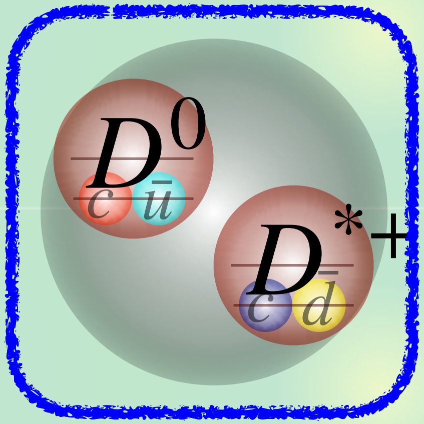
- **Binding energy** : $T_{cc} > X(3872)$
- **Decay width** : $T_{cc} < X(3872)$
- **Threshold energy difference** : $T_{cc} < X(3872)$

Plan of this talk

Goal : structure of T_{cc} and $X(3872)$

multiquark

hadronic molecule



- Near threshold —> two-body composite states?

Questions

1) Why composite state is expected?

<— In $B \to 0$ limit, state must be fully composite

2) Is it possible to have non-composite state with $B \neq 0$?

<— Yes, it is always possible

3) If so, how can we expect composite state for $B \neq 0$?

<— Probability to realize non-composite state is tiny

Contents

Formulation

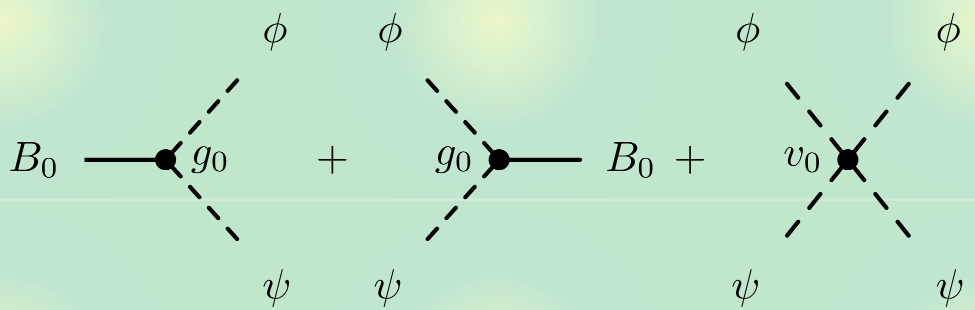
Effective field theory (bare state + scattering states)

Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)

$$H_{\text{free}} = \int d\mathbf{r} \left[\frac{1}{2M} \nabla \psi^{\dagger} \cdot \nabla \psi + \frac{1}{2m} \nabla \phi^{\dagger} \cdot \nabla \phi + \frac{1}{2M_0} \nabla B_0^{\dagger} \cdot \nabla B_0 + \omega_0 B_0^{\dagger} B_0 \right]$$

- Eigenstates of fee Hamiltonian

$$H_{\text{free}} | B_0 \rangle = \omega_0 | B_0 \rangle, \quad H_{\text{free}} | \boldsymbol{p} \rangle = \frac{\boldsymbol{p}^2}{2\mu} | \boldsymbol{p} \rangle$$



- Contact interactions

$$H_{\text{int}} = \int d\mathbf{r} \left[g_0 \left(B_0^{\dagger} \phi \psi + \psi^{\dagger} \phi^{\dagger} B_0 \right) + v_0 \psi^{\dagger} \phi^{\dagger} \phi \psi \right]$$

Compositeness and elementairty

Eigenstate of full Hamiltonian : bound state

$$(H_{\rm free} + H_{\rm int})\,|B\rangle = -\,B\,|B\rangle$$

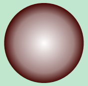
- Normalization of $|B\rangle$ + completeness relation

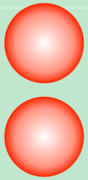
$$\langle B|B\rangle = 1, \quad 1 = |B_0\rangle\langle B_0| + \int \frac{d\boldsymbol{p}}{(2\pi)^3}|\boldsymbol{p}\rangle\langle\boldsymbol{p}|$$

- Overlap with free eigenstates

$$1 = Z + X, \quad Z \equiv |\langle B_0|B\rangle|^2, \quad X \equiv \int \frac{d\boldsymbol{p}}{(2\pi)^3}|\langle\boldsymbol{p}|B\rangle|^2$$

"elementarity" **compositeness**





Z, X : **real and nonnegative —> interpreted as probability**

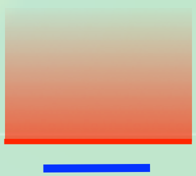
Weak-binding relation for stable states

Compositeness X of s-wave weakly bound state $(R \gg R_{\rm typ})$

S. Weinberg, Phys. Rev. 137, B672 (1965);

Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)

$$|d\rangle = \sqrt{X}\,|NN\rangle + \sqrt{Z}\,|\text{others}\rangle$$



range of interaction

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right)\right\}, \quad R = \frac{1}{\sqrt{2\mu B}}$$

scattering length (a_0) — **radius of bound state** (R)

- **Deuteron is NN composite :** $a_0 \sim R \Rightarrow X \sim 1$
- **Internal structure from observables** (a_0, B)

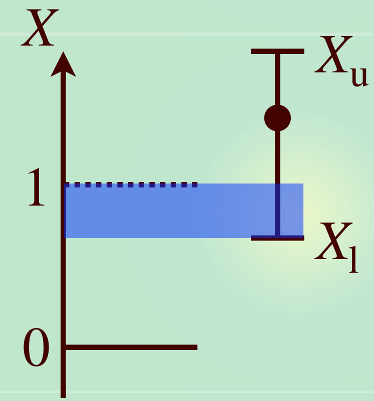
Problem: $a_0 = 5.42$ fm, $R = 4.32$ fm $\Rightarrow X = 1.68 > 1$?

Application to bound states

Uncertainty estimation with $\mathcal{O}(R_{\rm typ}/R)$ term

Y. Kamiya, T. Hyodo, PTEP2017, 023D02 (2017)

$$X_{\rm u} = \frac{a_0/R + \xi}{2 - a_0/R - \xi}, \quad X_{\rm l} = \frac{a_0/R - \xi}{2 - a_0/R + \xi}, \quad \xi = \frac{R_{\rm typ}}{R}$$



Application with finite range correction

T. Kinugawa, T. Hyodo, PRC 106, 015205 (2022)

- exclude region outside $0 \leq X \leq 1$

$$R_{\rm typ} = \max\{R_{\rm int}, R_{\rm eff}\}$$

- X of hadrons, nuclei, and atoms
- X of deuteron is reasonable
- $X \geq 0.5$ in all cases studied

Bound state	Compositeness X
d	$0.74 \leqslant X \leqslant 1$
$X(3872)$	$0.53 \leqslant X \leqslant 1$
$D^*_{s0}(2317)$	$0.81 \leqslant X \leqslant 1$
$D_{s1}(2460)$	$0.55 \leqslant X \leqslant 1$
$N\Omega$ dibaryon	$0.80 \leqslant X \leqslant 1$
$\Omega\Omega$ dibaryon	$0.79 \leqslant X \leqslant 1$
${}^3_\Lambda$H	$0.74 \leqslant X \leqslant 1$
^{4}He dimer	$0.93 \leqslant X \leqslant 1$

Near-threshold states are mostly composite

Contents

Original motivation

Systematic expansion of hadron masses

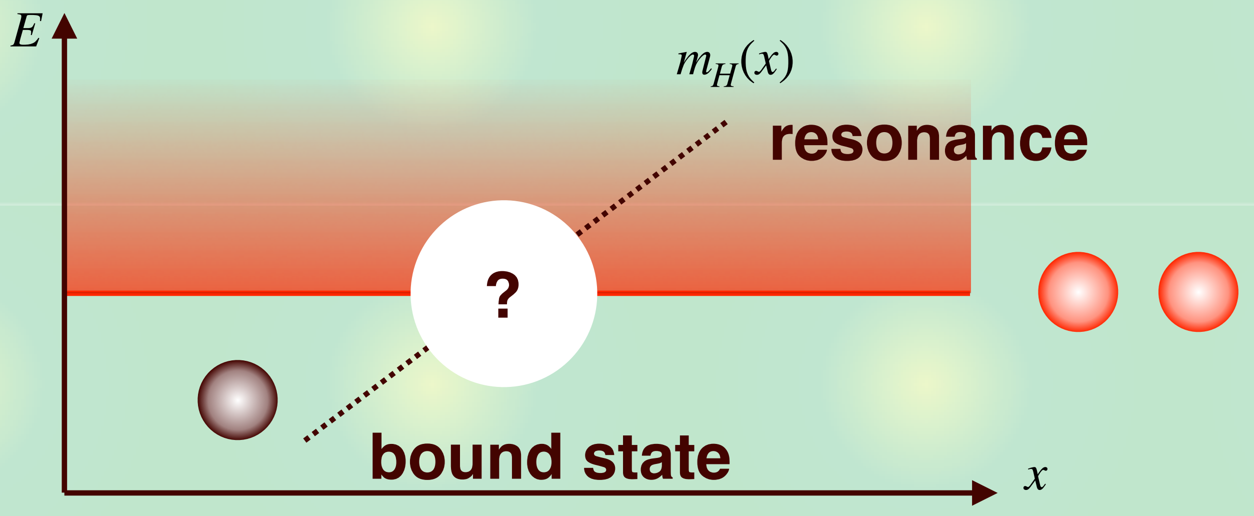
T. Hyodo, PRC90, 055208 (2014)

- ChPT : light quark mass m_q
- HQET : heavy quark mass m_Q
- Large Nc : number of colors N_c

Hadron mass scaling

$$m_H(x), \quad x = \frac{m_q}{\Lambda}, \frac{\Lambda}{m_Q}, \frac{1}{N_c}$$

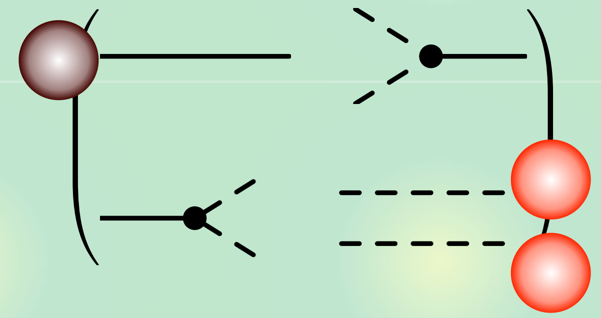
What happens at two-body threshold?



Formulation

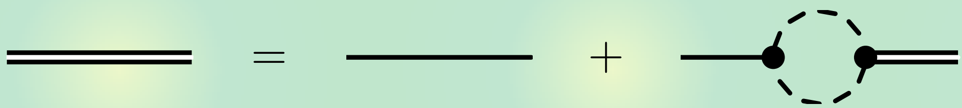
Coupled-channel Hamiltonian (discrete state + continuum)

$$\begin{pmatrix} M_0 & \hat{V} \\ \hat{V} & \frac{p^2}{2\mu} \end{pmatrix} |\Psi\rangle = E|\Psi\rangle, \quad |\Psi\rangle = \begin{pmatrix} c(E)|\psi_0\rangle \\ \int d\boldsymbol{p}\ \chi_E(\boldsymbol{p})|\boldsymbol{p}\rangle \end{pmatrix}$$



- Exactly solvable, equivalent to EFT

Eigenenergy $E_h = -B$ <— Dyson equation (pole condition)



$$\Rightarrow \quad 0 = E_h - M_0 - \Sigma(E_h), \quad \Sigma(E) = \int \frac{\langle\psi_0|\hat{V}|\boldsymbol{q}\rangle\langle\boldsymbol{q}|\hat{V}|\psi_0\rangle}{E - q^2/(2\mu) + i0^+} d\boldsymbol{q}$$

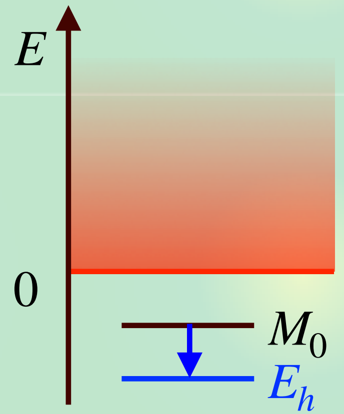
- Elementarity (wavefunction renormalization)

$$Z = |\langle\Psi|\psi_0\rangle|^2 = |c(E_h)|^2 = \frac{1}{1 - \Sigma'(E_h)}, \quad \Sigma'(E) = \frac{d\Sigma(E)}{dE}$$

Eigenstate at threshold

For weak coupling : perturbative estimation

$$E_h = M_0 + \Sigma(M_0) = M_0 + \int \frac{|\langle \psi_0 | \hat{V} | \boldsymbol{q} \rangle|^2}{M_0 - q^2/(2\mu) + i0^+} d\boldsymbol{q}$$

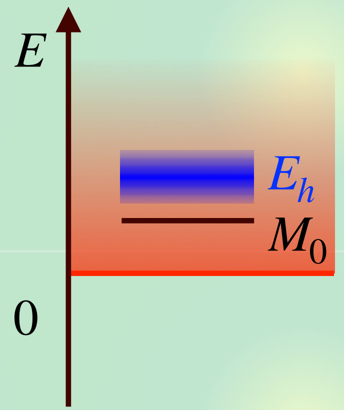


- $M_0 \leq 0$: second order perturbation

$$\Sigma(M_0) < 0 \quad \Rightarrow \quad E_h < M_0$$

- $M_0 > 0$: complex eigenenergy <— decay

$$\Sigma(M_0) \in \mathbb{C} \quad \Rightarrow \quad E_h \in \mathbb{C}$$



—> No solution for $E_h = 0$

Solution for $E_h = 0$

- Nonperturbtaive calculation (self-consistent solution)

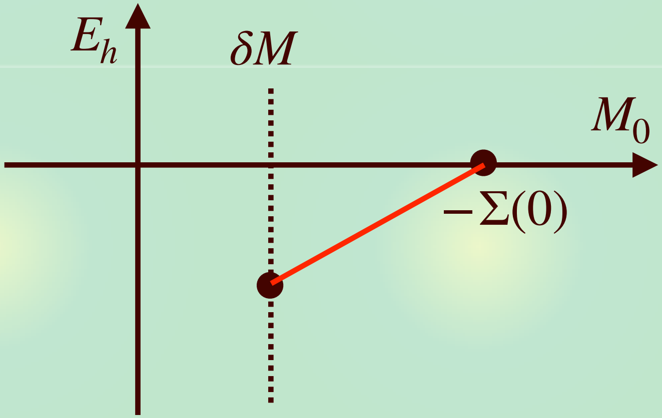
$$0 = M_0 + \Sigma(0) \quad \Rightarrow \quad M_0 = -\Sigma(0)$$

Slope and elementarity

M_0 dependence across the threshold

- Introduce $\delta M < 0$ to $M_0 = -\Sigma(0)$ for $E_h = 0$

$$E_h = -\Sigma(0) + \delta M + \Sigma(E_h)$$



- For sufficiently small δM,

$$E_h = \frac{1}{1 - \Sigma'(0)} \delta M$$

$= Z(0)$: elementarity of $E_h = 0$ state

$$Z(E_h) = \frac{1}{1 - \Sigma'(E_h)}$$

- Slope at $E_h = 0$ is given by $Z(0)$

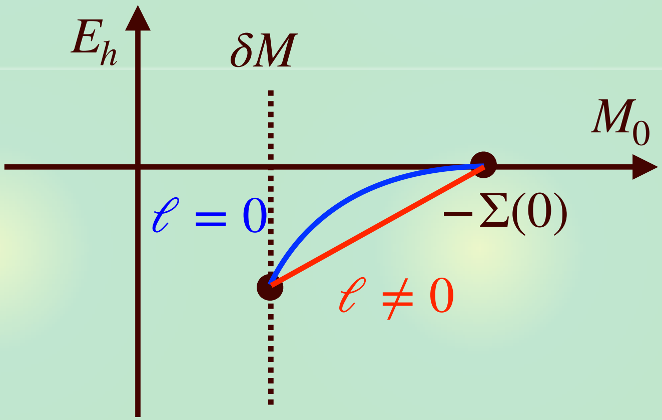
Elementarity $Z(E_h)$ at $E_h \to 0$?

Compositeness theorem

Self-energy for small $E_h \to 0$ (g_0 : coupling constant)

$$\Sigma(E_h) \sim C g_0^2 (-E_h)^{1/2+\ell} + \cdots$$

$$\Sigma'(E_h) \sim D g_0^2 (-E_h)^{-1/2+\ell} + \cdots \to \begin{cases} \infty & \ell = 0 \\ \text{finite} & \ell \neq 0 \end{cases}$$



—> $Z(0) = 0$ **follows in** $E_h \to 0$

$$Z(E_h) = \frac{1}{1 - \Sigma'(E_h)} \to 0 \quad (E_h \to 0 \text{ with } g_0 \neq 0,\ \ell = 0)$$

Compositeness theorem :

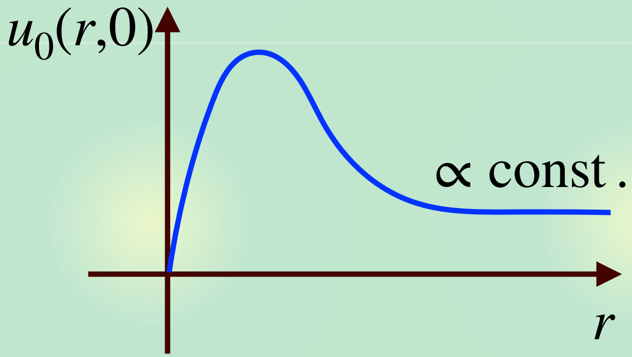
T. Hyodo, PRC90, 055208 (2014)

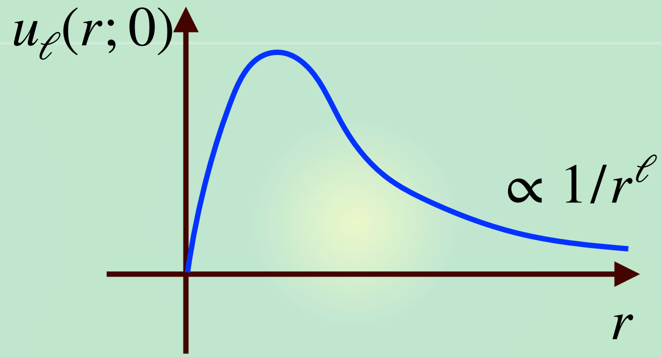
If the s-wave scattering amplitude has a pole exactly at the threshold with a finite range interaction, then the field renormalization constant vanishes.

$Z(0) = 0 \Leftrightarrow$ **state at** $E_h = 0$ **is fully composite**

Intuitive picture of compositeness theorem

Wavefunction of $E_h = 0$ state is not normalizable ($\ell = 0$)





—> Compositeness $X \gg Z$

$$1 = |\langle \Psi | \psi_0 \rangle|^2 + \int d\boldsymbol{q} \, |\langle \Psi | \boldsymbol{q} \rangle|^2 = |\langle \Psi | \psi_0 \rangle|^2 + \int d\boldsymbol{r} \, |\Psi(\boldsymbol{r})|^2$$

- Divergence of scattering length, low-energy universality

E. Braaten, H.-W. Hammer, Phys. Rept. 428, 259 (2006);
P. Naidon, S. Endo, Rept. Prog. Phys. 80, 056001 (2017)

- Threshold rule of cluster nuclei

H. Horiuchi, K. Ikeda, Y. Suzuki, PTPS 55, 89 (1972)

Contents

- Introduction — T_{cc} and $X(3872)$
- Compositeness from effective field theory

  Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017);
  T. Kinugawa, T. Hyodo, PRC 106, 015205 (2022)
- Compositeness theorem ($B \to 0$)

  T. Hyodo, PRC90, 055208 (2014)
- Structure of near-threshold states ($B \neq 0$)

  - Probability to realize elementary states
  - Decay and coupled-channel effects

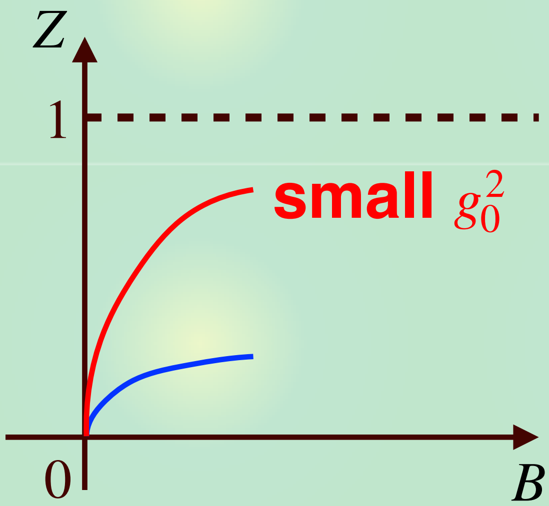
  T. Kinugawa, T. Hyodo, in preparation
- Summary

Finite binding case

Elementarity of bound state with small but finite $B = -E_h$

$$Z(-B) = \frac{1}{1 - \Sigma'(-B)} \sim \frac{1}{1 - Dg_0^2/\sqrt{B}} \sim -\frac{\sqrt{B}}{Dg_0^2} + \cdots \neq 0$$

- **B dependence**
- **$Z(0) = 0$ is fixed**
- **$Z \ll 1$ for small B (composite)**



For sufficiently small g_0^2, $\sqrt{B}/g_0^2 \sim \mathcal{O}(1)$ for small B

—> sizable Z for small B by fine tuning of parameter g_0^2

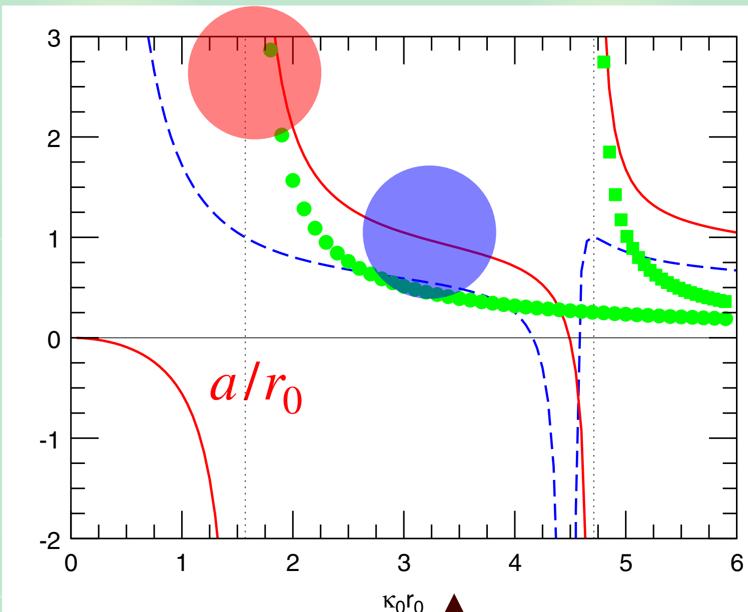
How probable is such fine tuning?

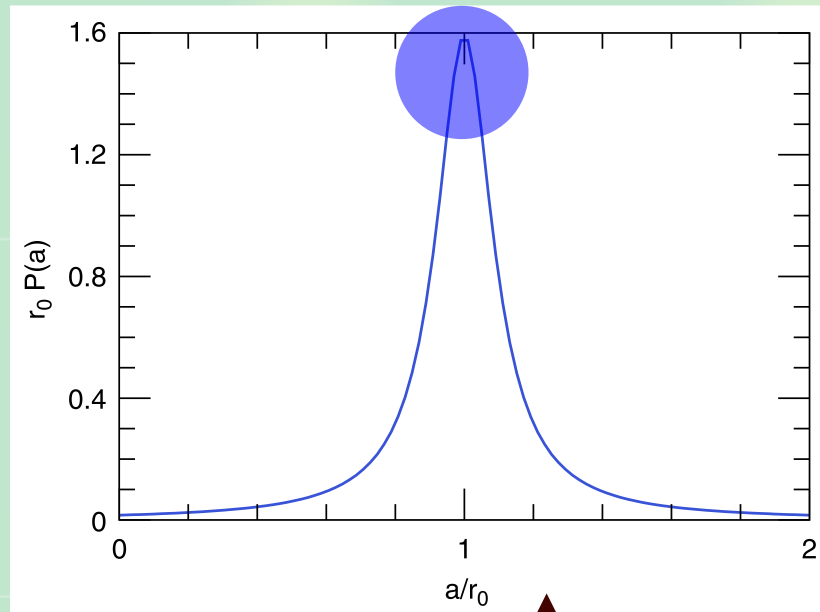
Quantifying fine tuning

Shallow bound state already requires fine tuning

E. Braaten, H.-W. Hammer, Phys. Rept. 428, 259 (2006)

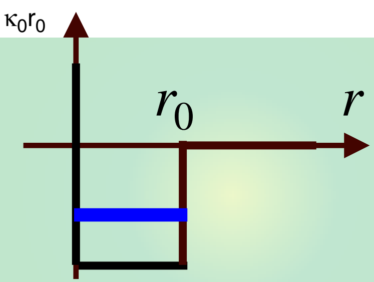
- probability distribution of a of square-well potential





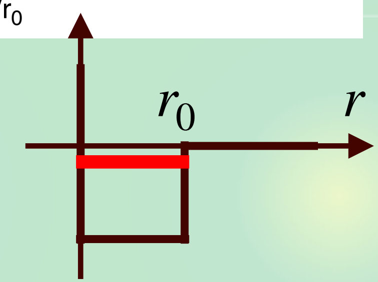
typical

: $a/r_0 \sim 1$



shallow

: $a/r_0 \gg 1$

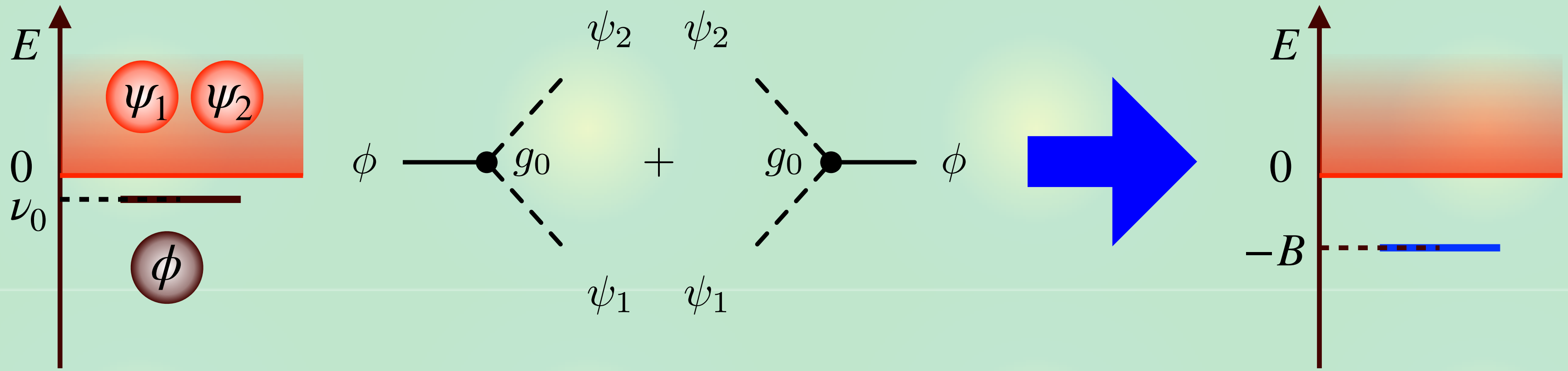


Fine-tuning can be quantified by parameter dependence

Model setup

EFT with bare state + scattering states (no direct int.)

T. Kinugawa, T. Hyodo, in preparation



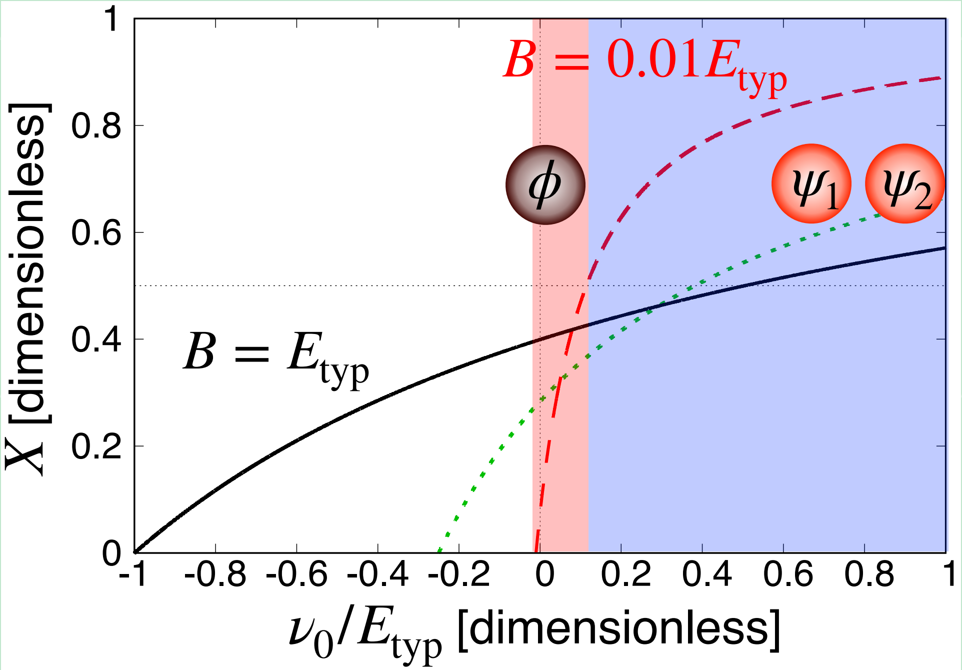
Parameters : coupling g_0, bare energy ν_0 (cutoff Λ —> scale)

- **Fix binding energy** B **—>** $g_0(\nu_0; \Lambda, B)$
- $E_{\rm typ} = \Lambda^2/(2\mu)$ **: typical energy scale**
- **Allowed parameter region :** $-B \leq \nu_0 \leq E_{\rm typ}$

Vary ν_0 and calculate compositeness X of bound state

Structure of bound state

Compositeness X in the allowed ν_0 region



- **Typical bound state $B = E_{typ}$: mostly elementary**

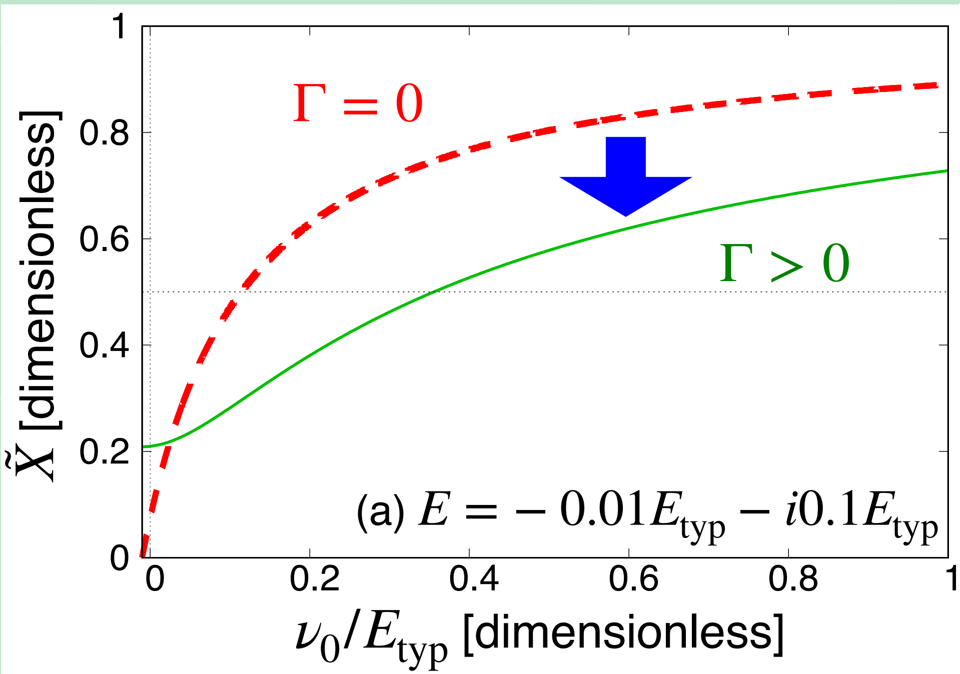
- **Shallow bound state $B = 0.01E_{typ}$: mostly composite**

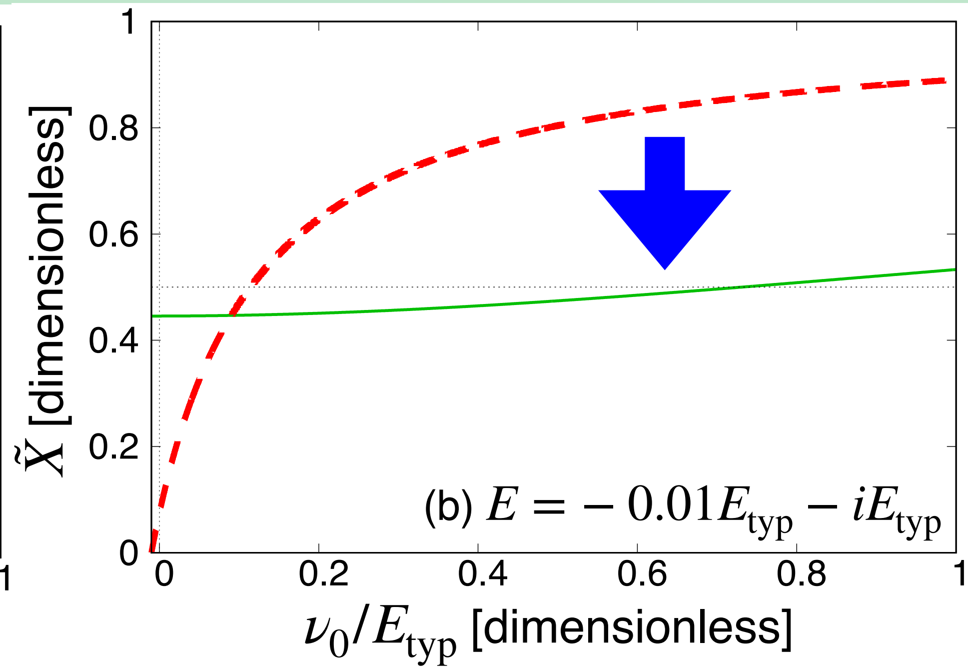
Shallow elementary state :

- **can be realized, but only with fine tuning = unlikely**

Decay effect

Effect of finite decay width





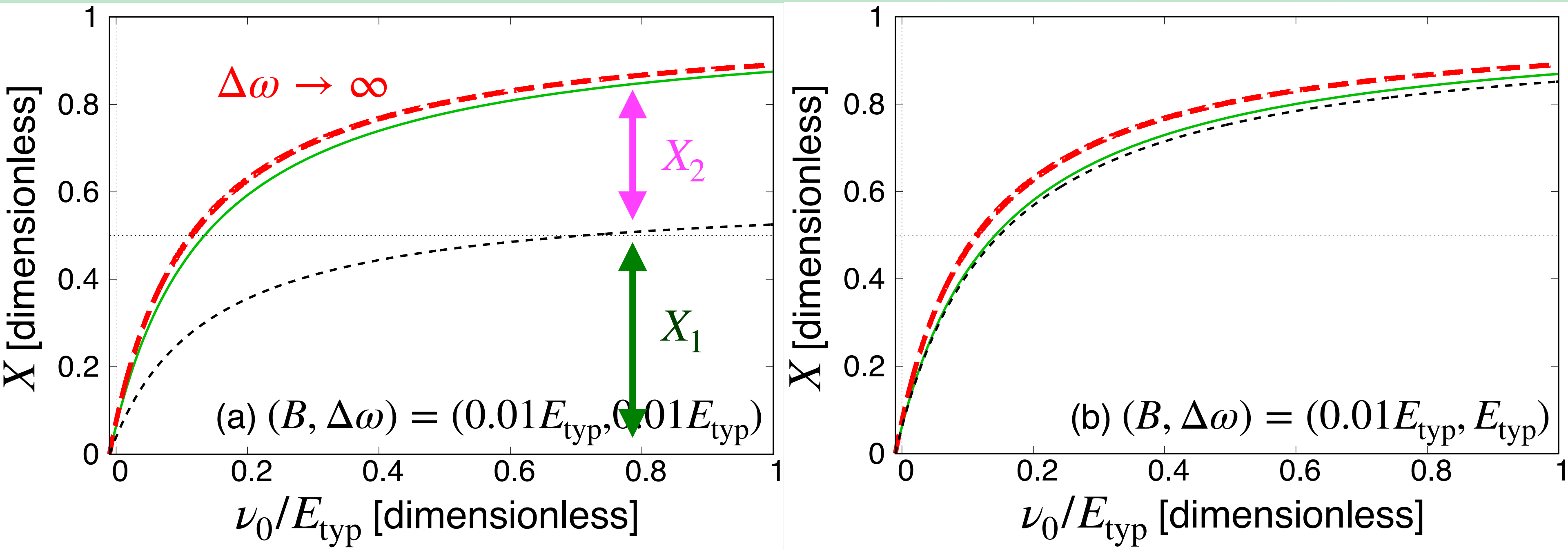
Narrow width **Broad width**

X is reduced —> decay channel component

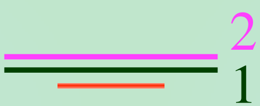
Broad (narrow) width : large (small) reduction of X

Coupled channel effect

Introduce coupled channel with $\Delta\omega$ above the threshold



small $\Delta\omega$



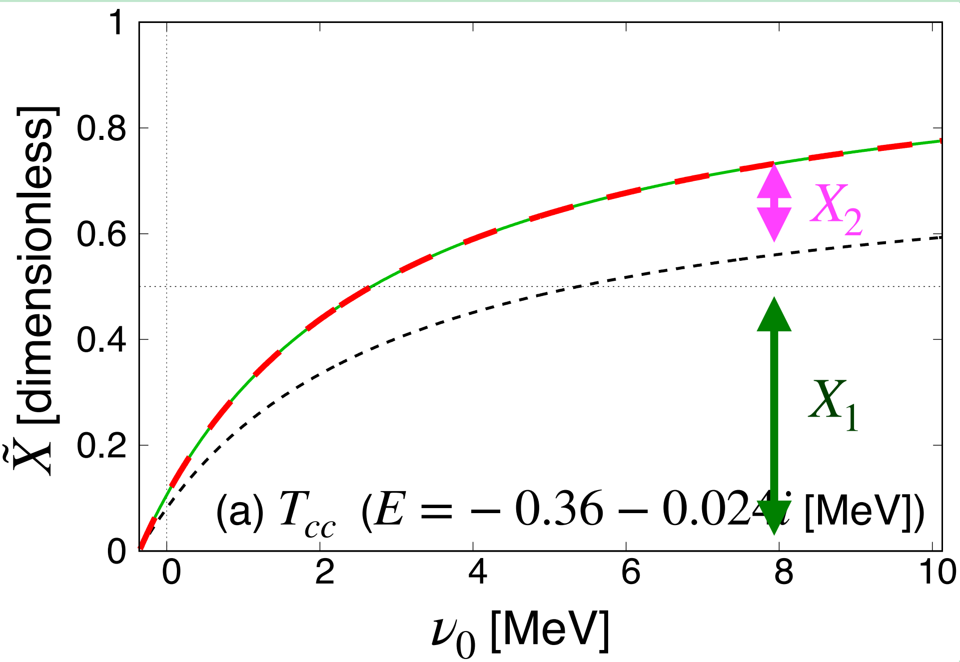
large $\Delta\omega$

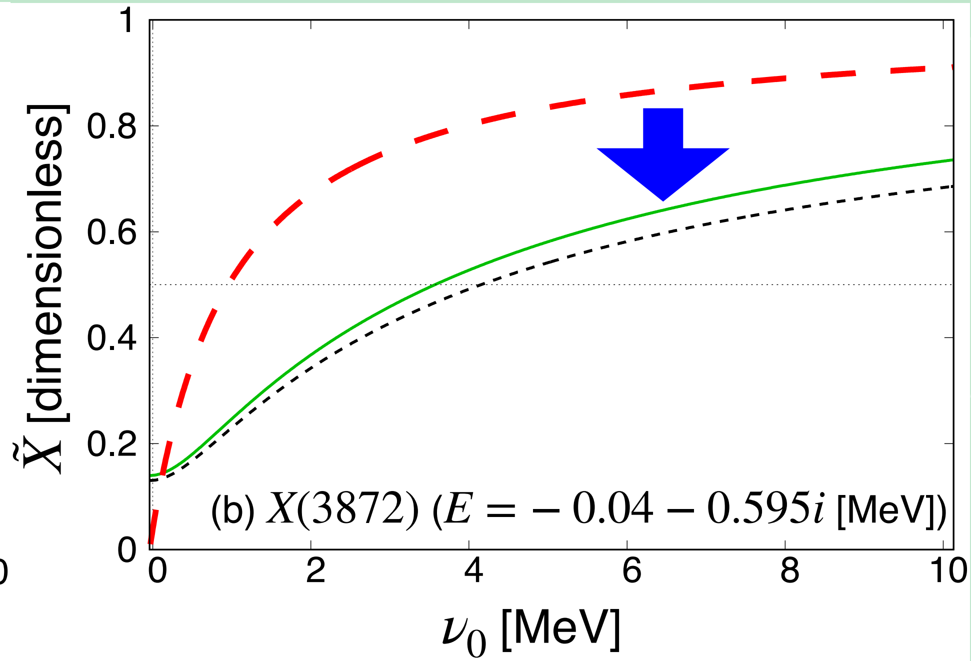
X_1 is **reduced** —> coupled channel component (X_2)

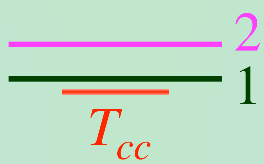
Small (large) $\Delta\omega$: large (small) reduction of X_1

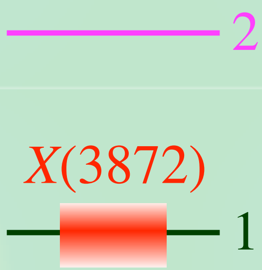
T_{cc} and $X(3872)$

T_{cc} and $X(3872)$: decay + coupled-channel effects









Coupled-channel (decay) effect is important for T_{cc} ($X(3872)$)

Summary

- **Bound state is fully composite in $B \to 0$ limit**

  T. Hyodo, PRC90, 055208 (2014)

- **Non-composite state with $B \neq 0$ <— fine tuning**

  - naive expectation of near-threshold molecule

- **Compositeness is reduced by decay and coupled-channel effects**

- **Important effect for exotic hadron candidates**

  - T_{cc} : coupled-channel effect
  - $X(3872)$: decay effect

  T. Kinugawa, T. Hyodo, in preparation