Kaon-deuteron systems and femtoscopy

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Clusters & Hierarchies

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Contents

- $\bar{K}N$ potentials

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012);
K. Miyahara. T. Hyodo, PRC 93, 015201 (2016);

- Rigorous few-body calculation for K^-p and K^-d

T. Hoshino, S. Ohnishi, W. Horiuchi, T. Hyodo, W. Weise, PRC96, 045204 (2017)

  - Precision of Deser-Trueman formulae
  - Sensitivity to $I=1$ amplitude

- Femtoscopy of K^+d and K^-d
  - Rescattering and interaction strength

- Summary

Best-fit results

K at rest

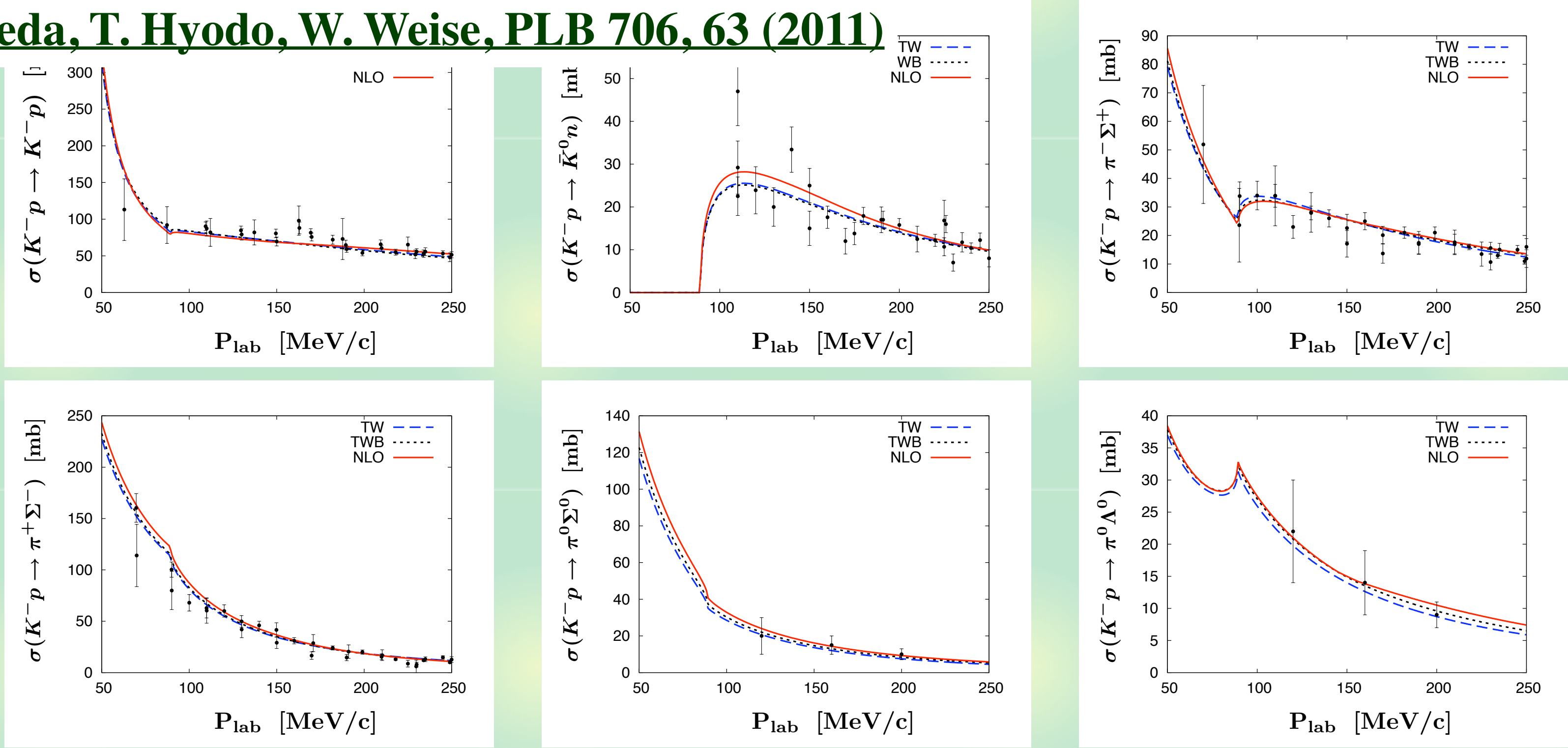
	TW	TWB	NLO	Experiment	
ΔE [eV]	373	377	306	$283 \pm 36 \pm 6$	[10]
Γ [eV]	495	514	591	$541 \pm 89 \pm 22$	[10]
γ	2.36	2.36	2.37	2.36 ± 0.04	[11]
R_n	0.20	0.19	0.19	0.189 ± 0.015	[11]
R_c	0.66	0.66	0.66	0.664 ± 0.011	[11]
χ^2/d.o.f	1.12	1.15	0.96		

$\}$ **SIDDHARTA**

$\}$ **Branching ratios**

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011)

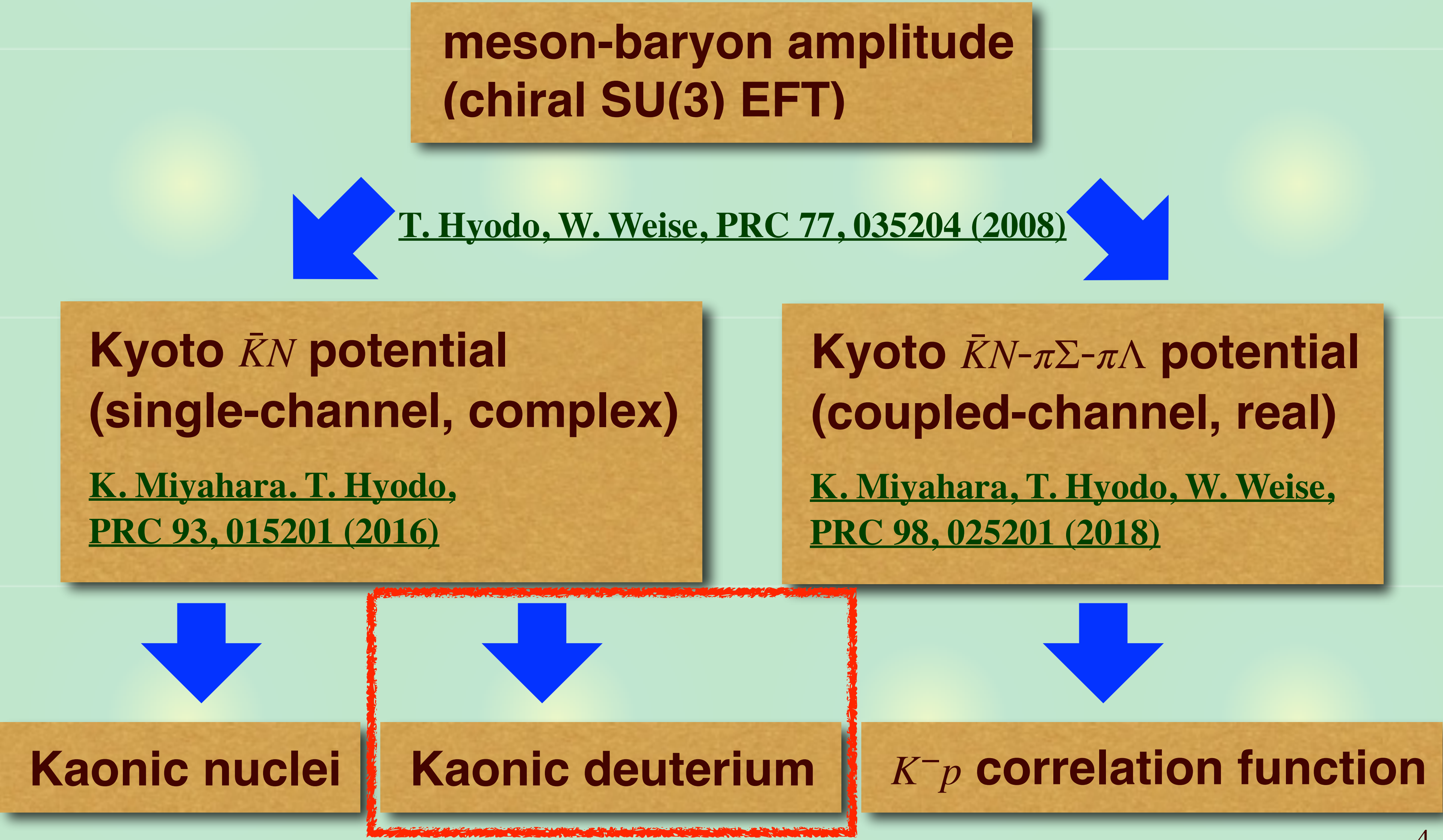
K^-p cross sections



Accurate description of all existing data (χ^2/d.o.f ~ 1)

Construction of $\bar{K}N$ potentials

Local $\bar{K}N$ potential is useful for various applications



meson-baryon amplitude
(chiral SU(3) EFT)

T. Hyodo, W. Weise, PRC 77, 035204 (2008)

Kyoto $\bar{K}N$ potential
(single-channel, complex)

K. Miyahara, T. Hyodo,
PRC 93, 015201 (2016)

Kyoto $\bar{K}N$-$\pi\Sigma$-$\pi\Lambda$ potential
(coupled-channel, real)

K. Miyahara, T. Hyodo, W. Weise,
PRC 98, 025201 (2018)

Kaonic nuclei

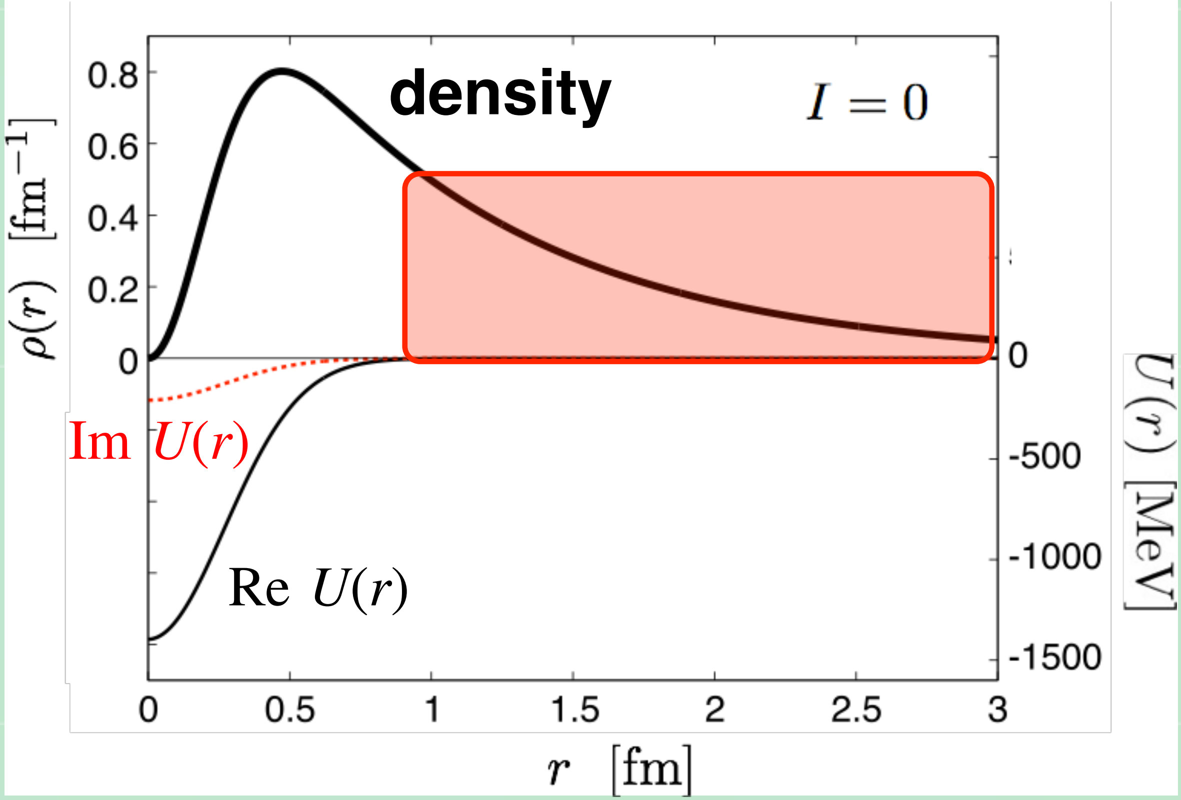
Kaonic deuterium

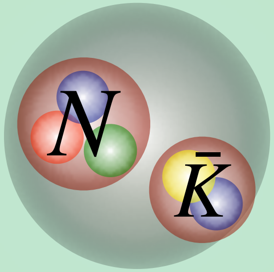
$K^{-}p$ correlation function

Spatial structure of $\Lambda(1405)$

$\bar{K}N$ wave function at $\Lambda(1405)$ pole

K. Miyahara. T. Hyodo, PRC93, 015201 (2016)



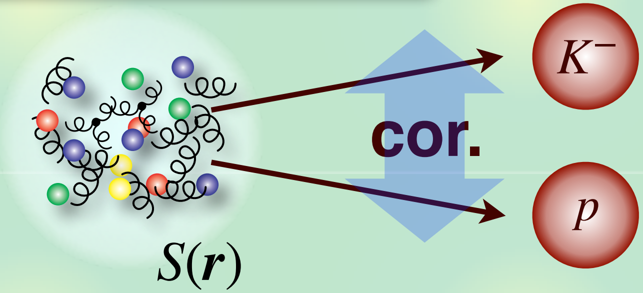


- substantial distribution at $r > 1$ fm
- root mean squared radius $\sqrt{\langle r^2 \rangle} = 1.44$ fm

The size of $\Lambda(1405)$ is much larger than ordinary hadrons

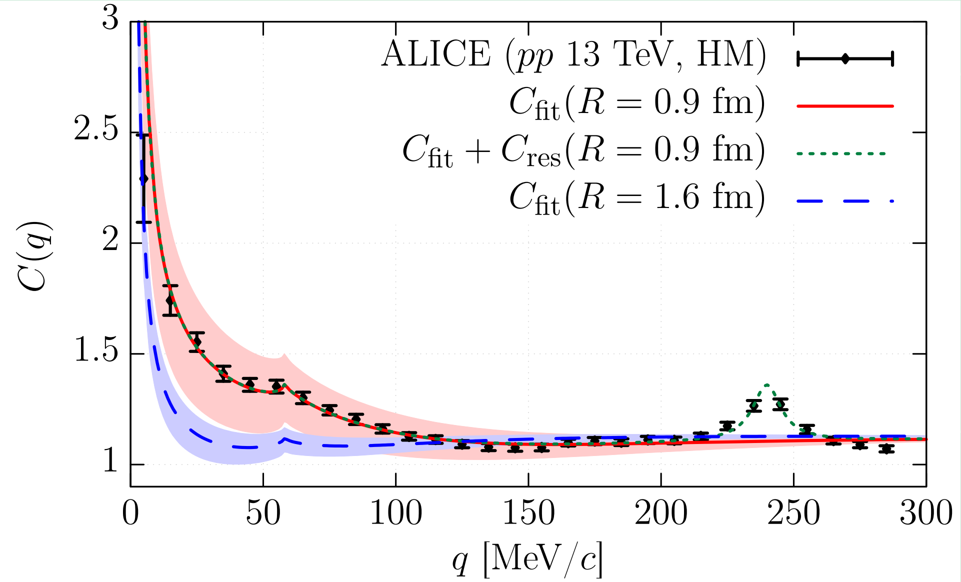
Correlation function and femtoscopy

$K^{-}p$ correlation function $C(q)$



$$C(\boldsymbol{q}) = \frac{N_{K^{-}p}(\boldsymbol{p}_{K^{-}}, \boldsymbol{p}_{p})}{N_{K^{-}}(\boldsymbol{p}_{K^{-}})N_{p}(\boldsymbol{p}_{p})} \simeq \int d^{3}\boldsymbol{r}\, S(\boldsymbol{r})\,|\,\Psi_{\boldsymbol{q}}^{(-)}(\boldsymbol{r})\,|^{2}$$

- Wave function $\Psi_{\boldsymbol{q}}^{(-)}(\boldsymbol{r})$: coupled-channel $\bar{K}N$-$\pi\Sigma$-$\pi\Lambda$ potential



—> Talks by Y. Kamiya and R. Lea

Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi, W. Weise. PRL124, 132501 (2020)

Correlation function is well reproduced

Contents

- $\bar{K}N$ potentials

  Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012);
  K. Miyahara. T. Hyodo, PRC 93, 015201 (2016);

- Few-body calculations for K^-p and K^-d

  T. Hoshino, S. Ohnishi, W. Horiuchi, T. Hyodo, W. Weise, PRC96, 045204 (2017)

  - Precision of DT formulae
  - Sensitivity to $I = 1$ potential

- Femtoscopy of K^+d and K^-d

  - Rescattering and correlation functions

- Summary

Check of kaonic hydrogen

Kaonic hydrogen (K^-p) should be checked


SIDDHARTA

MB amplitude

Kyoto $\bar{K}N$ potential

?

DT formula

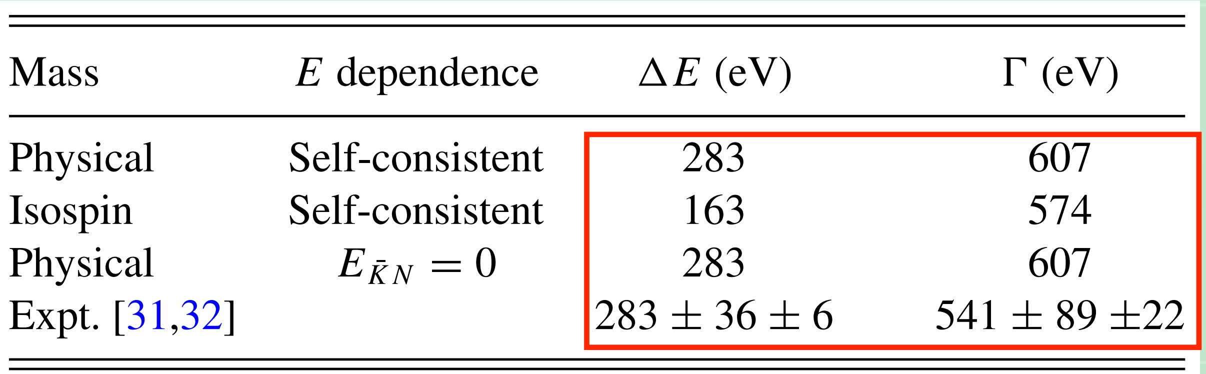
isospin symmetric

Two-body calculation with physical masses

$$\begin{pmatrix} \hat{T}+\hat{V}^{\bar{K}N}+\hat{V}^{\mathrm{EM}} & \hat{V}^{\bar{K}N} \\ \hat{V}^{\bar{K}N} & \hat{T}+\hat{V}^{\bar{K}N}+\Delta m \end{pmatrix} \begin{pmatrix} |K^-p\rangle \\ |\bar{K}^0 n\rangle \end{pmatrix} = E \begin{pmatrix} |K^-p\rangle \\ |\bar{K}^0 n\rangle \end{pmatrix}$$

Result reproduces SIDDHARTA (with physical mass)

S. Ohnishi, W. Horiuchi, T. Hoshino, K. Miyahara. T. Hyodo, PRC95, 065202 (2017)



Mass	E dependence	ΔE (eV)	Γ (eV)
Physical	Self-consistent	283	607
Isospin	Self-consistent	163	574
Physical	$E_{\bar{K}N} = 0$	283	607
Expt. [31,32]		283 ± 36 ± 6	541 ± 89 ±22

Deser-Trueman formulae for kaonic hydrogen

(Improved) Deser-Trueman formulae for K^-p

S. Deser, *et al.*, PR96, 774 (1954); T.L. Trueman, NP26, 57 (1961)

$$\Delta E - \frac{i\Gamma}{2} = -2\mu_K^2\alpha^3 a_{K^-p} \times \begin{cases} \left[1 - 2\mu_K\alpha(\ln\alpha - 1)a_{K^-p}\right] \\ \left[1 + 2\mu_K\alpha(\ln\alpha - 1)a_{K^-p}\right]^{-1} \end{cases}$$

Improved

U.G. Meißner, U. Raha, A. Rusetsky, EPJC35, 349 (2004)

V. Baru, E. Epelbaum, A. Rusetsky, EPJA42, 111 (2009)

Resummed

	ΔE (eV)	Γ (eV)	δ (eV)
DT	272	734	64
Improved DT	293	596	11
Resummed DT	284	605	1
Exact	283	607	-

deviation from Exact

T. Hoshino, S. Ohnishi, W. Horiuchi, T. Hyodo, W. Weise, PRC96, 045204 (2017)

Ressumed DT formula works well for K^-p

c.f. N.V. Shevchenko, FBS, 63, 22 (2022)

Formulation

Three-body calculation of K^-d with physical masses

T. Hoshino, S. Ohnishi, W. Horiuchi, T. Hyodo, W. Weise, PRC96, 045204 (2017)

$$\begin{pmatrix} \hat{H}_{K^-pn} & \hat{V}^{\bar{K}N}_{12} + \hat{V}^{\bar{K}N}_{13} \\ \hat{V}^{\bar{K}N}_{12} + \hat{V}^{\bar{K}N}_{13} & \hat{H}_{\bar{K}^0nn} \end{pmatrix} \begin{pmatrix} |K^-pn\rangle \\ |\bar{K}^0nn\rangle \end{pmatrix} = E \begin{pmatrix} |K^-pn\rangle \\ |\bar{K}^0nn\rangle \end{pmatrix}$$

$$\hat{H}_{K^-pn} = \sum_{i=1}^{3} \hat{T}_i - \hat{T}_{\text{cm}} + \hat{V}^{NN}_{23} + \sum_{i=2}^{3} (\hat{V}^{\bar{K}N}_{1i} + \underline{\hat{V}^{\text{EM}}_{1i}})$$ **Coulomb**

$$\hat{H}_{\bar{K}^0nn} = \sum_{i=1}^{3} \hat{T}_i - \hat{T}_{\text{cm}} + \hat{V}^{NN}_{23} + \sum_{i=2}^{3} \hat{V}^{\bar{K}N}_{1i} + \underline{\Delta m}$$ **threshold difference**

- Kyoto $\bar{K}N$ potential

Few-body technique

- a large number of correlated gaussian basis

Y. Suzuki, K. Varga, Lect. Notes Phys. M54, (1998)

Kaonic deuterium: shift and width

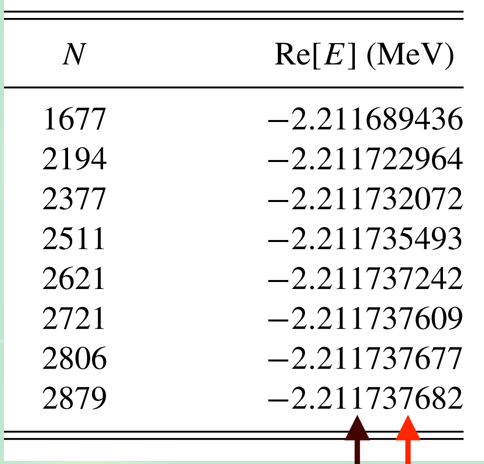
Rigorous three-body calculation

T. Hoshino, S. Ohnishi, W. Horiuchi, T. Hyodo, W. Weise, PRC96, 045204 (2017)

- energy convergence

<— large number of basis

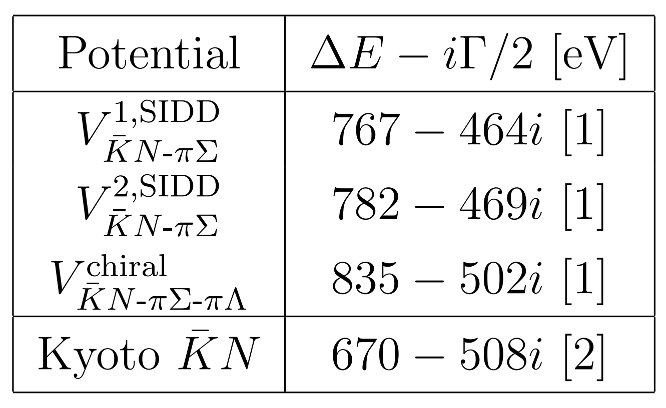
- No shift in $2P$ state is shown
  by explicit calculation.



N	Re[E] (MeV)
1677	−2.211689436
2194	−2.211722964
2377	−2.211732072
2511	−2.211735493
2621	−2.211737242
2721	−2.211737609
2806	−2.211737677
2879	−2.211737682

keV **eV!**

Results



Potential	$\Delta E - i\Gamma/2$ [eV]
$V^{1,\mathrm{SIDD}}_{\bar{K}N\text{-}\pi\Sigma}$	$767 - 464i$ [1]
$V^{2,\mathrm{SIDD}}_{\bar{K}N\text{-}\pi\Sigma}$	$782 - 469i$ [1]
$V^{\mathrm{chiral}}_{\bar{K}N\text{-}\pi\Sigma\text{-}\pi\Lambda}$	$835 - 502i$ [1]
Kyoto $\bar{K}N$	$670 - 508i$ [2]

[1] J. Revai, PRC 94, 054001 (2016)

[2] T. Hoshino, S. Ohnishi, W. Horiuchi, T. Hyodo, W. Weise, PRC96, 045204 (2017)

Deser-Trueman formulae for kaonic deuterium

(Improved) Deser-Trueman formulae for K^-d

S. Deser, *et al.*, PR96, 774 (1954); T.L. Trueman, NP26, 57 (1961)

$$\Delta E - \frac{i\Gamma}{2} = -2\mu_{Kd}^2\alpha^3 a_{K\text{-}d} \times \begin{cases} \left[1 - 2\mu_{Kd}\alpha(\ln\alpha - 1)a_{K\text{-}d}\right] \\ \left[1 + 2\mu_{Kd}\alpha(\ln\alpha - 1)a_{K\text{-}d}\right]^{-1} \end{cases}$$

Improved

U.G. Meißner, U. Raha, A. Rusetsky, EPJC35, 349 (2004)

V. Baru, E. Epelbaum, A. Rusetsky, EPJA42, 111 (2009)

Resummed

	ΔE (eV)	Γ (eV)	δ (eV)
DT	854	1925	490
Improved DT	910	989	241
Resummed DT	818	1188	171
Exact	670	1016	-

deviation from Exact

T. Hoshino, S. Ohnishi, W. Horiuchi, T. Hyodo, W. Weise, PRC96, 045204 (2017)

DT formulae do not work accurately for K^-d

c.f. J. Revai, PRC 94, 054001 (2016), N.V. Shevchenko, FBS, 63, 22 (2022)

$I = 1$ dependence

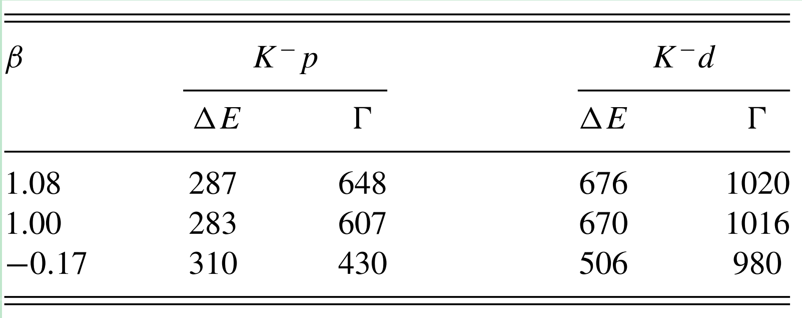
Study sensitivity to $I = 1$ interaction

- **introduce parameter β to control the potential strength**

$$\mathrm{Re}\ \hat{V}^{\bar{K}N(I=1)} \to \beta \times \mathrm{Re}\ \hat{V}^{\bar{K}N(I=1)}$$

Vary β within SIDDHARTA uncertainty of K^-p

- **allowed region: $-0.17 < \beta < 1.08$**
  (negative β may contradict with scattering data)



β	K^-p		K^-d	
	ΔE	Γ	ΔE	Γ
1.08	287	648	676	1020
1.00	283	607	670	1016
−0.17	310	430	506	980

- **deviation of ΔE of K^-d ~ 170 eV**
- **Planned precision: 60 eV (30 eV) at J-PARC (SIDDHARTA-2)**

Measurement of K^-d will provide strong constraint on $I = 1$

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K. Miyahara. T. Hyodo, PRC 93, 015201 (2016);

- **Few-body calculations for $K^{-}p$ and $K^{-}d$**

T. Hoshino, S. Ohnishi, W. Horiuchi, T. Hyodo, W. Weise, PRC96, 045204 (2017)

  - Precision of DT formulae
  - Sensitivity to $I = 1$ potential

- **Femtoscopy of $K^{+}d$ and $K^{-}d$**
  - Rescattering and correlation functions

- **Summary**

$K^{\pm}d$ scattering length

$K^{\pm}d$ scattering length by fixed-center approximation

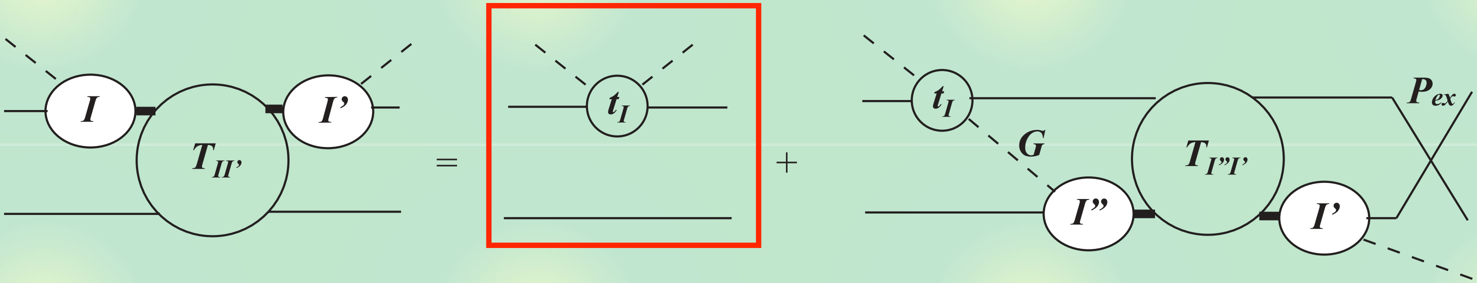
S.S. Kamalov, E. Oset, A. Ramos, NPA 690, 494 (2001)

T. Hoshino, S. Ohnishi, W. Horiuchi, T. Hyodo, W. Weise, PRC96, 045204 (2017)

$$a_{K^{\pm}d} = \frac{\mu_{K^{\pm}d}}{M_{K^{\pm}}} \int d^3\boldsymbol{r}\rho_d(r) \frac{\boxed{\tilde{a}_p + \tilde{a}_n} + \frac{2\tilde{a}_p\tilde{a}_n - \tilde{b}_x^2(r)}{r} - \frac{2\tilde{a}_{p/n}\tilde{b}_x^2(r)}{r^2}}{1 - \frac{\tilde{a}_p\tilde{a}_n}{r^2} + \frac{\tilde{a}_{p/n}\tilde{b}_x^2(r)}{r^3}}, \quad \tilde{b}_x^2(r) = \frac{\tilde{a}_x^2}{1 + \frac{\tilde{a}_0}{r}}$$

- good approximation around $K^{\pm}d$ threshold

Diagrammatically: **Impulse** **Rescattering**



- Weak 2-body t (scattering length) : impulse should work
- Strong 2-body t : rescattering becomes important

Comparison of K^+d and K^-d

Two-body scattering lengths

K. Aoki, D. Jido, PTEP 2019, 013D01 (2019)

T. Hoshino, S. Ohnishi, W. Horiuchi, T. Hyodo, W. Weise, PRC96, 045204 (2017)

	a_p [fm]	a_n [fm]	a_x [fm]	a_0 [fm]
K^+d	-0.310	-0.195	-0.115	-0.195
K^-d	$-0.66+i0.89$	$-0.58+i0.78$	$-0.85+i0.26$	$-0.40+i1.03$

- K^-d system has stronger 2-body interactions than K^+d

$K^{\pm}d$ scattering lengths

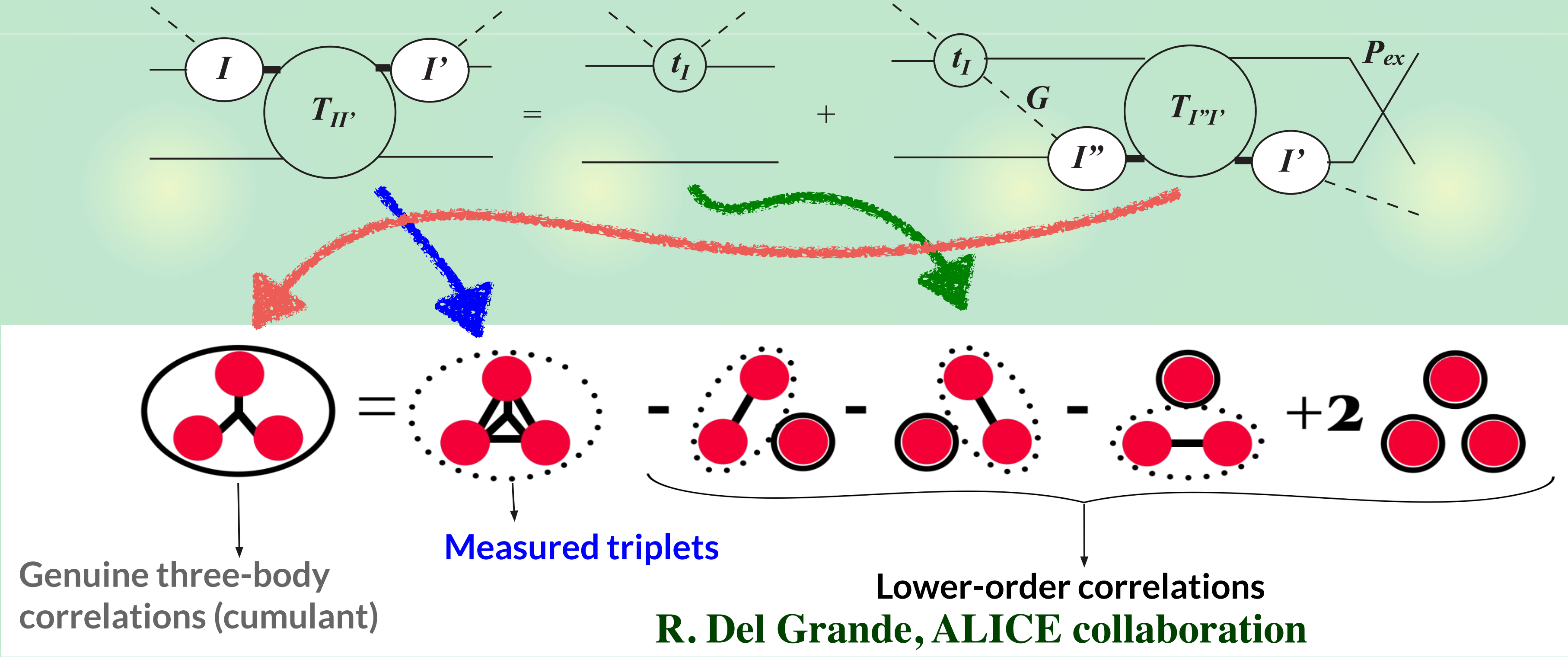
	Impluse [fm]	Full [fm]
K^+d	-0.61	-0.54
K^-d	$-0.10+2.02i$	$-1.42+1.61i$

- Impulse works for K^+d (weak), but not for K^-d (strong)

LL formula suitable for K^+d correlation function?

Relation to correlation functions?

3-body equation and correlation functions



Genuine three-body correlations (cumulant)

Measured triplets

Lower-order correlations

R. Del Grande, ALICE collaboration

"Genuine three-body correlation"

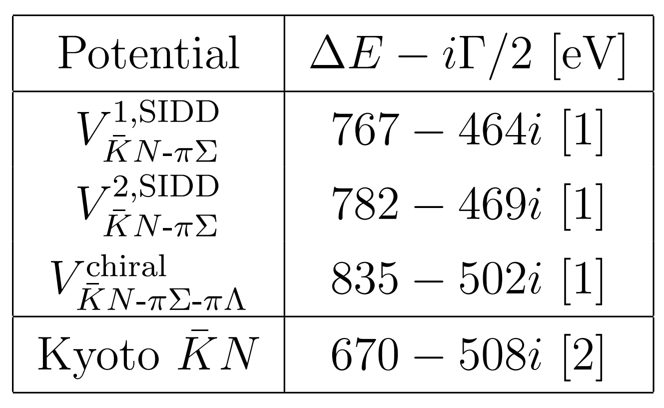
- **multiple rescattering of 2-body interaction?**
- **3-body force (act only in 3-body system)?**

Summary

- **Realistic $\bar{K}N$ potentials are constructed**

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012);
K. Miyahara. T. Hyodo, PRC 93, 015201 (2016);

- **Rigorous Few-body calculations of $K^{-}d$**



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[2] T. Hoshino, S. Ohnishi, W. Horiuchi, T. Hyodo, W. Weise, PRC96, 045204 (2017)

- DT formulae work well for $K^{-}p$, but not for $K^{-}d$

- $\Delta E - i\Gamma/2$ is sensitive to $\bar{K}N(I=1)$ potential