Λ(1405) as a hadronic molecule

Tetsuo Hyodo

Tokyo Metropolitan Univ.

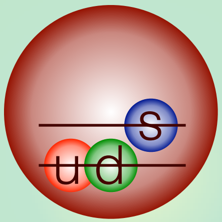
Clusters & Hierarchies

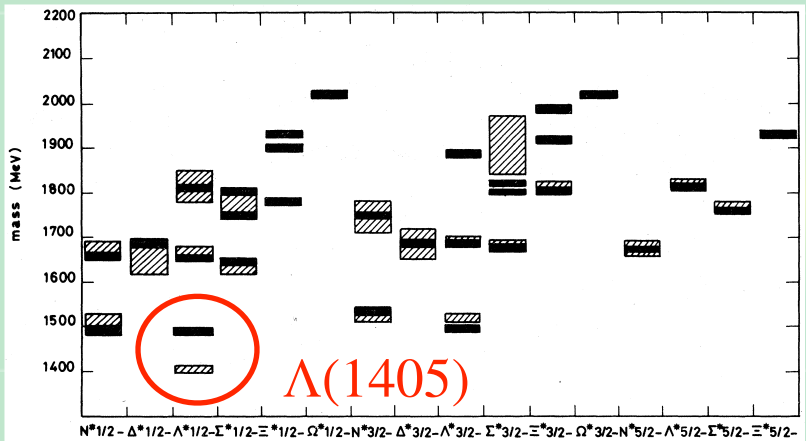
2022, Feb. 18th

Λ(1405) in meson-baryon scattering

Λ(1405) does not fit in standard picture —> exotic structure?

N. Isgur, G. Karl, PRD18, 4187 (1978)





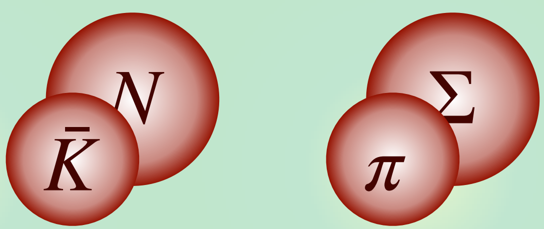
: theory

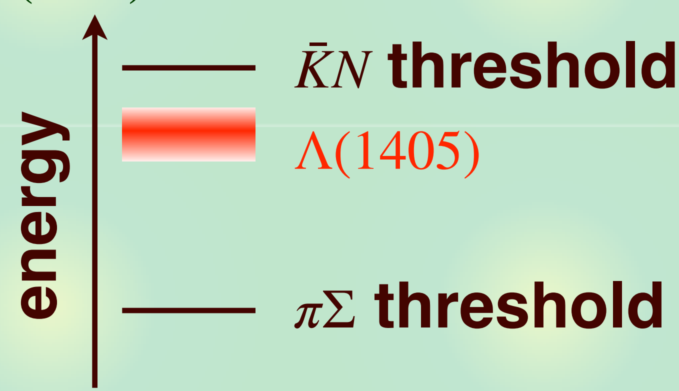
: experiment

Resonance in coupled-channel scattering

R.H. Dalitz, T.C. Wong, G. Rajasekaran, PR153, 1617 (1967)

- coupling to MB states

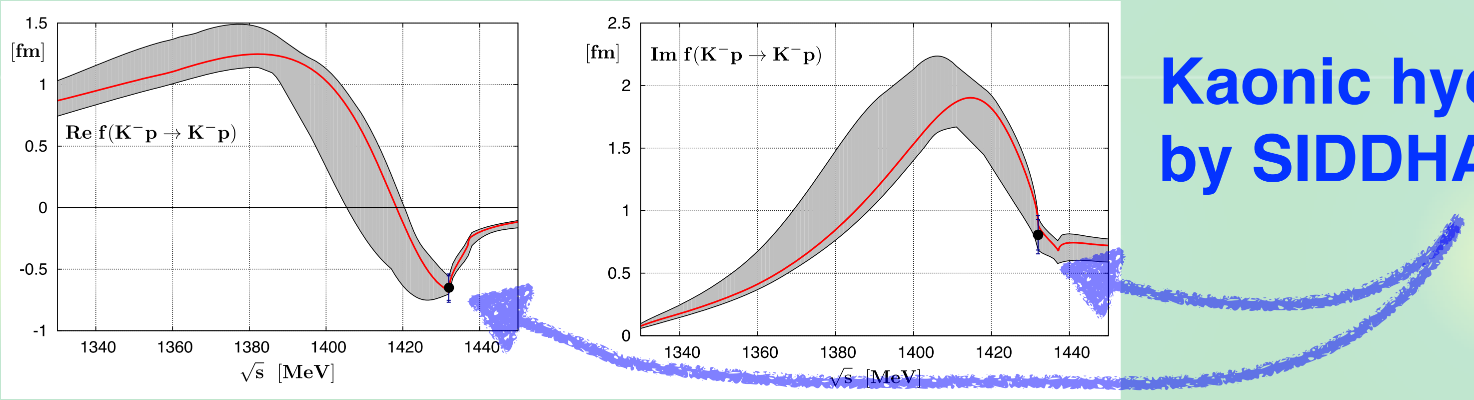




Detailed analysis of $\bar{K}N$-$\pi\Sigma$ scattering is necessary.

Pole positions determined

Recent analyses with chiral SU(3) dynamics at NLO



[14,15] Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012),
[17] Z.H. Guo, J.A. Oller, PRC 87, 035202 (2013),
[18] M. Mai, U.G. Meißner, EPJA 51, 30 (2015)

approach	pole 1 [MeV]	pole 2 [MeV]
Refs. [14, 15], NLO	$1424^{+7}_{-23} - i\ 26^{+3}_{-14}$	$1381^{+18}_{-6} - i\ 81^{+19}_{-8}$
Ref. [17], Fit II	$1421^{+3}_{-2} - i\ 19^{+8}_{-5}$	$1388^{+9}_{-9} - i\ 114^{+24}_{-25}$
Ref. [18], solution #2	$1434^{+2}_{-2} - i\ 10^{+2}_{-1}$	$1330^{+4}_{-5} - i\ 56^{+17}_{-11}$
Ref. [18], solution #4	$1429^{+8}_{-7} - i\ 12^{+2}_{-3}$	$1325^{+15}_{-15} - i\ 90^{+12}_{-18}$

- Two poles: superposition of two eigenstates

J.A. Oller, U.G. Meißner, PLB 500, 263 (2001);
D. Jido, J.A. Oller, E. Oset, A. Ramos, U.G. Meißner, NPA 723, 205 (2003)

Λ(1405) in PDG

2020 update of PDG

P.A. Zyla, *et al.*, PTEP 2020, 083C01 (2020); http://pdg.lbl.gov/

- Particle Listing section:

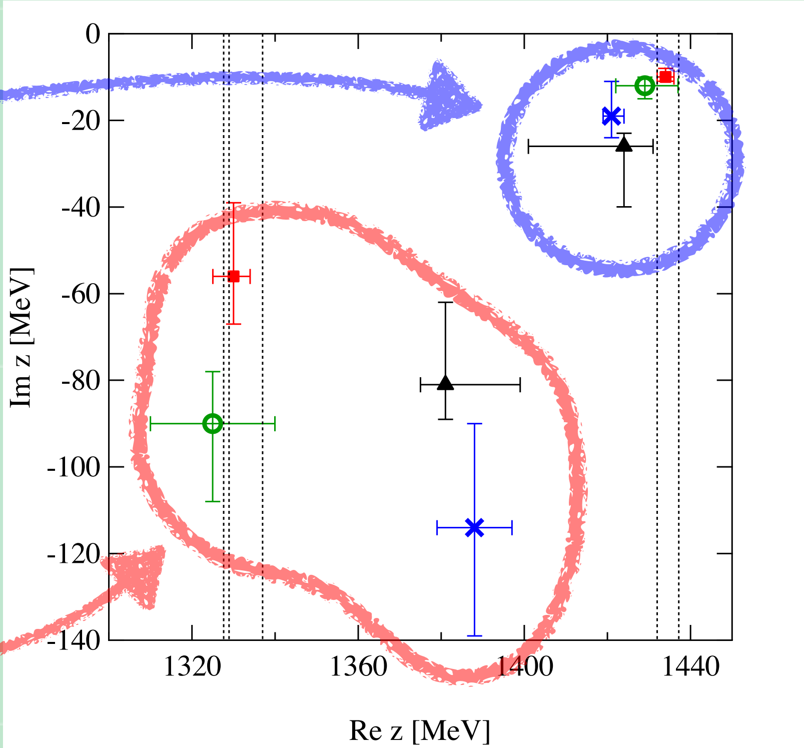
Citation: P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

$\Lambda(1405)\ 1/2^-$ $\quad I(J^P) = 0(\frac{1}{2}^-)$ Status: ∗∗∗∗

Citation: P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

new!

$\Lambda(1380)\ 1/2^-$ $\quad J^P = \frac{1}{2}^-$ Status: ∗∗



T. Hyodo, M. Niiyama, PPNP 120, 103868 (2021);
T. Hyodo, W. Weise, arXiv:2202.06181 [nucl-th]

- Λ(1405) **is no longer at 1405 MeV but ~ 1420 MeV.**
- **Lower pole: two-star resonance** Λ(1380)

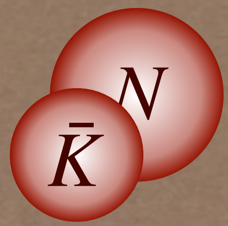
Next step: internal structure

- **Structure of $\Lambda(1405)$?**
- **Weak binding relation for stable bound states**

  S. Weinberg, Phys. Rev. 137, B672 (1965)
- **Generalization to unstable resonances**

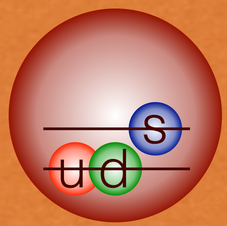
  Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)

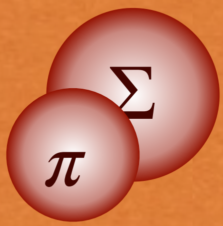
Compositeness X
threshold channel



or

"Elementarity" Z
other contributions





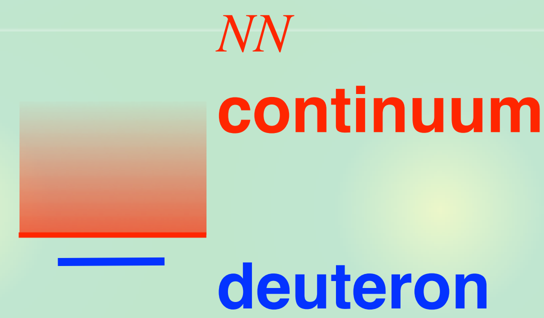
↑

observables (a_0, E_h)

Weak-binding relation for stable states

Compositeness X of s-wave weakly bound state $(R \gg R_{\rm typ})$

S. Weinberg, Phys. Rev. 137, B672 (1965);
T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)



$$|d\rangle = \sqrt{X}\,|NN\rangle + \sqrt{1-X}\,|\text{others}\rangle$$

range of interaction

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right)\right\}, \quad R = \frac{1}{\sqrt{2\mu B}}$$

scattering length (a_0) — radius of bound state (R)

- Deuteron is NN composite: $a_0 \sim R \Rightarrow X \sim 1$
- Internal structure from observables (a_0, B)

Quantitative discussion?

Detour: deuteron in more detail

Weak-binding relation

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right)\right\}, \quad R = \frac{1}{\sqrt{2\mu B}}$$

- empirical values for deuteron case

$$a_0 \sim 5.42 \text{ fm}, \quad R \sim 4.32 \text{ fm}$$

- neglecting $\mathcal{O}(R_{\rm typ}/R)$ term : contradiction with $0 \leq X \leq 1$?

Y. Kamiya, T. Hyodo, PoS INPC2016, 270 (2017),
Y. Li, F.K. Guo, J.Y. Pang, J.J. Wu, arXiv:2110.02766 [hep-ph],
J. Song, L.R. Dai, E, Oset arXiv:2201.04414 [hep-ph].

$$X \sim 1.68$$

If $0 \leq X \leq 1$, then $a_0 < R$

—> For systems with $a_0 > R$, $\mathcal{O}(R_{\rm typ}/R)$ term is important.

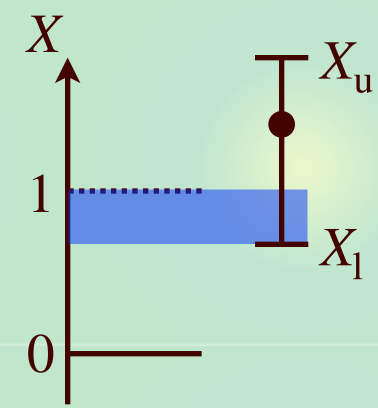
T. Kinugawa, T. Hyodo, in preparation

Detour: range correction

Uncertainty estimation with $\mathcal{O}(R_{\rm typ}/R)$ term

Y. Kamiya, T. Hyodo, PTEP2017, 023D02 (2017)

$$X_{\rm u} = \frac{a_0/R + \xi}{2 - a_0/R - \xi}, \quad X_{\rm l} = \frac{a_0/R - \xi}{2 - a_0/R + \xi}, \quad \xi = \frac{R_{\rm typ}}{R}$$



- exclude region outside $0 \leq X \leq 1$

Application and finite range correction

$$R_{\rm typ} = \max\{R_{\rm int}, R_{\rm eff}\}$$

T. Kinugawa, T. Hyodo, arXiv:2111.06619; 2112.00249; 2201.04283 [hep-ph]

Bound state	$R_{\rm typ} = R_{\rm eff}$	$R_{\rm typ} = R_{\rm int}$	This work
d	$1.68^{+3.19}_{-0.943}$	$1.68^{+2.14}_{-0.823}$	$0.738 \leq X \leq 1$
$X(3872)$	$0.743^{+0.282}_{-0.213}$	$0.743^{+0.0675}_{-0.0627}$	$0.530 \leq X \leq 1$
$N\Omega$ dibaryon	$1.40^{+1.20}_{-0.600}$	$1.40^{+0.523}_{-0.364}$	$0.801 \leq X \leq 1$
$\Omega\Omega$ dibaryon	$1.56^{+1.95}_{-0.773}$	$1.56^{+1.22}_{-0.626}$	$0.791 \leq X \leq 1$
${}^3_\Lambda$H	$1.35^{+0.531}_{-0.366}$	$1.35^{+1.241}_{-0.603}$	$0.745 \leq X \leq 1$
^{4}He dimer	$1.08^{+0.179}_{-0.152}$	$1.08^{+0.129}_{-0.115}$	$0.926 \leq X \leq 1$

Effective field theory

Low-energy scattering with near-threshold bound state

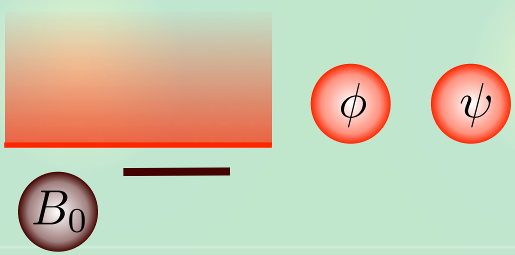
- **Nonrelativistic EFT with contact interaction**

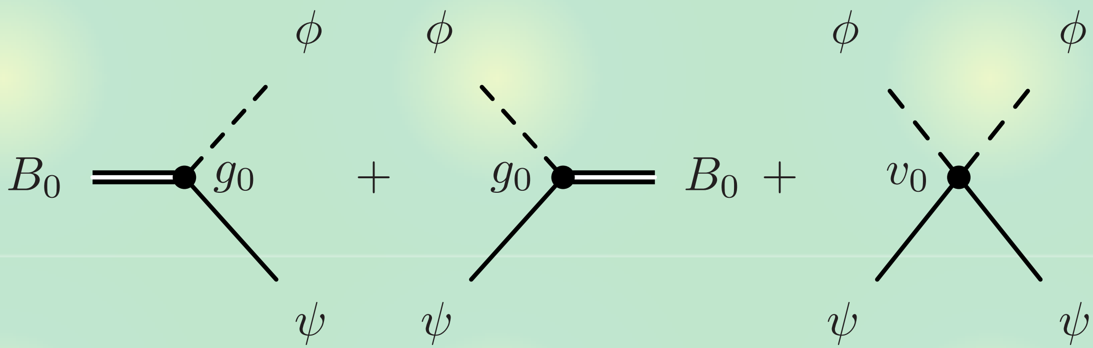
D.B. Kaplan, Nucl. Phys. B494, 471 (1997)

E. Braaten, M. Kusunoki, D. Zhang, Annals Phys. 323, 1770 (2008)

$$H_{\text{free}} = \int d\mathbf{r} \left[\frac{1}{2M} \nabla \psi^{\dagger} \cdot \nabla \psi + \frac{1}{2m} \nabla \phi^{\dagger} \cdot \nabla \phi + \frac{1}{2M_0} \nabla B_0^{\dagger} \cdot \nabla B_0 + \omega_0 B_0^{\dagger} B_0 \right]$$

$$H_{\text{int}} = \int d\mathbf{r} \left[g_0 \left(B_0^{\dagger} \phi \psi + \psi^{\dagger} \phi^{\dagger} B_0 \right) + v_0 \psi^{\dagger} \phi^{\dagger} \phi \psi \right]$$





- **cutoff**: $\Lambda \sim 1/R_{\text{typ}}$ **(interaction range of microscopic theory)**
- **At low momentum** $p \ll \Lambda$, **interaction ~ contact**

Compositeness and "elementarity"

Eigenstates

$$H_{\text{free}}|B_0\rangle = \omega_0|B_0\rangle, \quad H_{\text{free}}|\mathbf{p}\rangle = \frac{\mathbf{p}^2}{2\mu}|\mathbf{p}\rangle$$

free (discrete + continuum)

$$(H_{\text{free}} + H_{\text{int}})|B\rangle = -B|B\rangle$$

full (bound state)

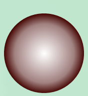
- normalization of $|B\rangle$ + completeness relation

$$\langle B|B\rangle = 1, \quad 1 = |B_0\rangle\langle B_0| + \int \frac{d\mathbf{p}}{(2\pi)^3}|\mathbf{p}\rangle\langle\mathbf{p}|$$

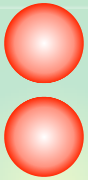
- projections onto free eigenstates

$$1 = Z + X, \quad Z \equiv |\langle B_0|B\rangle|^2, \quad X \equiv \int \frac{d\mathbf{p}}{(2\pi)^3}|\langle\mathbf{p}|B\rangle|^2$$

"elementarity"



compositeness

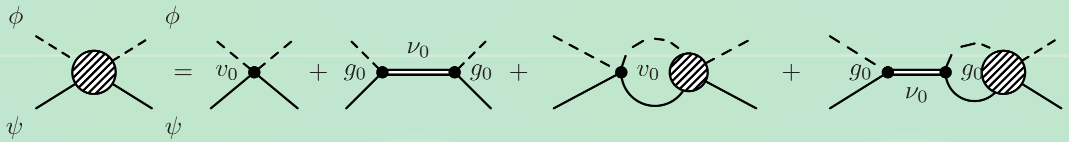


Z, X**: real and nonnegative —> interpreted as probability**

Weak binding relation

$\psi\phi$ scattering amplitude (exact result)

$$f(E) = -\frac{\mu}{2\pi}\frac{1}{[v(E)]^{-1} - G(E)}$$



$$v(E) = v_0 + \frac{g_0^2}{E-\omega_0}, \quad G(E) = \frac{1}{2\pi^2}\int_0^\Lambda dp \frac{p^2}{E - p^2/(2\mu) + i0^+}$$

Compositeness $X \leftarrow v(E), G(E)$

$$X = \frac{G'(-B)}{G'(-B) - [1/v(-B)]'}$$

$1/R = \sqrt{2\mu B}$ expansion of scattering length a_0

$$a_0 = -f(E=0) = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right)\right\}$$

renormalization dependent

renormalization independent

If $R \gg R_{\rm typ}$, correction terms neglected: $X \leftarrow (a_0, B)$

Inclusion of decay channel

Introduce decay channel

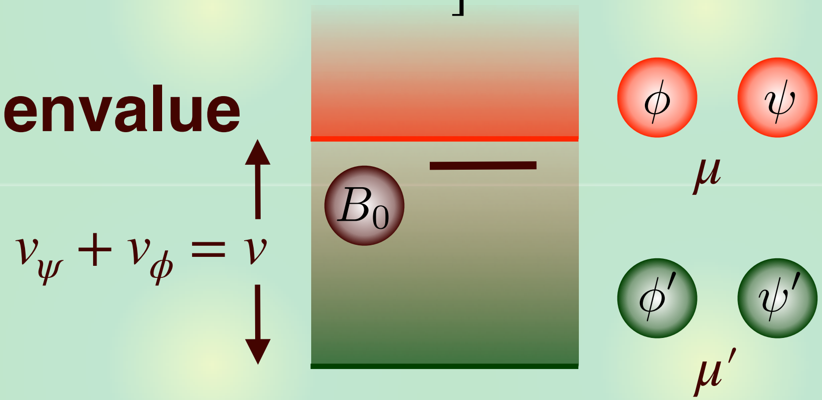
$$H'_{\rm free} = \int d\mathbf{r} \left[\frac{1}{2M'} \nabla \psi'^{\dagger} \cdot \nabla \psi' - \nu_{\psi} \psi'^{\dagger} \psi' + \frac{1}{2m'} \nabla \phi'^{\dagger} \cdot \nabla \phi' - \nu_{\phi} \phi'^{\dagger} \phi' \right]$$

$$H'_{\rm int} = \int d\mathbf{r} \left[g'_0 \left(B_0^{\dagger} \phi' \psi' + \psi'^{\dagger} \phi'^{\dagger} B_0 \right) + v'_0 \psi'^{\dagger} \phi'^{\dagger} \phi' \psi' + v_0^t (\psi^{\dagger} \phi^{\dagger} \phi' \psi' + \psi'^{\dagger} \phi'^{\dagger} \phi \psi) \right]$$

Quasi-bound state : complex eigenvalue

$$H = H_{\rm free} + H'_{\rm free} + H_{\rm int} + H'_{\rm int}$$

$$H | h \rangle = E_h | h \rangle, \quad E_h \in \mathbb{C}$$



Generalized relation : correction from threshold difference

$$a_0 = R \left\{ \frac{2X}{1+X} + \mathcal{O}\left(\left| \frac{R_{\rm typ}}{R} \right| \right) + \mathcal{O}\left(\left| \frac{\ell}{R} \right|^3 \right) \right\}, \quad R = \frac{1}{\sqrt{-2\mu E_h}}, \quad \ell \equiv \frac{1}{\sqrt{2\mu\nu}}$$

Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)

If $|R| \gg (R_{\rm typ}, \ell)$, correction terms neglected: $X \leftarrow (a_0, E_h)$

Evaluation of compositeness

Generalized weak-binding relation

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\rm typ}}{R}\right|\right) + \mathcal{O}\left(\left|\frac{\ell}{R}\right|^3\right)\right\}, \quad R = \frac{1}{\sqrt{-2\mu E_h}}, \quad \ell \equiv \frac{1}{\sqrt{2\mu\nu}}$$

(a_0, E_h) determinations by several groups

- neglecting correction terms:

	E_h [MeV]	a_0 [fm]	$X_{\bar{K}N}$	$\tilde{X}_{\bar{K}N}$	$U/2$
Set 1 [35]	$-10-i26$	$1.39-i0.85$	$1.2+i0.1$	1.0	0.3
Set 2 [36]	$-4-i8$	$1.81-i0.92$	$0.6+i0.1$	0.6	0.0
Set 3 [37]	$-13-i20$	$1.30-i0.85$	$0.9-i0.2$	0.9	0.1
Set 4 [38]	$2-i10$	$1.21-i1.47$	$0.6+i0.0$	0.6	0.0
Set 5 [38]	$-3-i12$	$1.52-i1.85$	$1.0+i0.5$	0.8	0.3

- In all cases, $X \sim 1$ with small $U/2$ (complex nature)

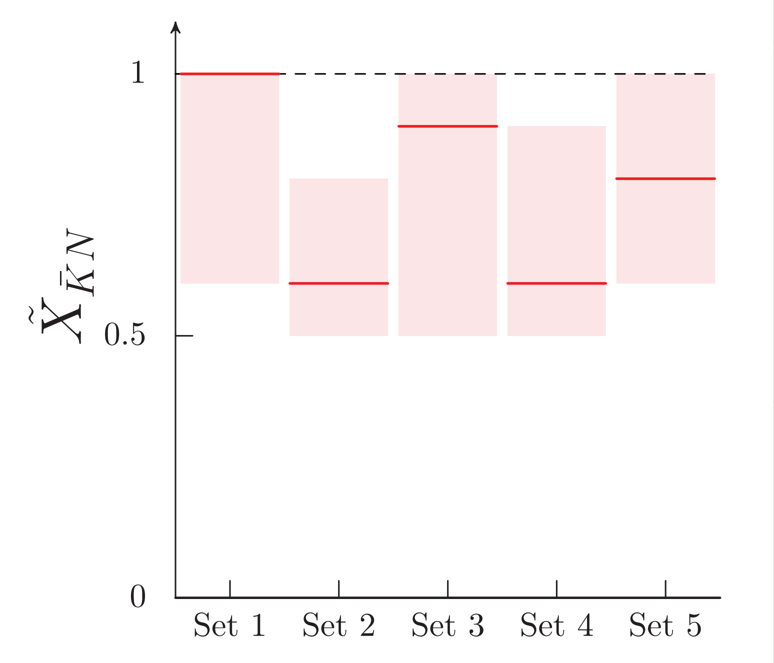
$\Lambda(1405)$: $\bar{K}N$ composite dominance <— observables

Uncertainty estimation

Estimation of correction terms: $|R| \sim 2$ fm

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\rm typ}}{R}\right|\right) + \mathcal{O}\left(\left|\frac{\ell}{R}\right|^3\right)\right\}, \quad R = \frac{1}{\sqrt{-2\mu E_h}}, \quad \ell \equiv \frac{1}{\sqrt{2\mu\nu}}$$

- ρ **meson exchange picture:** $R_{\rm typ} \sim 0.25$ fm
- **energy difference from** $\pi\Sigma$**:** $\ell \sim 1.08$ fm



$\bar{K}N$ **composite dominance holds even with correction terms.**

Summary

- **Pole structure of the $\Lambda(1405)$ region is now well constrained: "$\Lambda(1405)$" —> $\Lambda(1405)$ and $\Lambda(1380)$.**

  Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012);
  P.A. Zyla, *et al.* (Particle Data Group), PTEP 2020, 083C01 (2020)
  T. Hyodo, M. Niiyama, PPNP 120, 103868 (2021);
  T. Hyodo, W. Weise, arXiv:2202.06181 [nucl-th]

- **With the uncertainty estimation and range correction, deuteron is quantitatively shown to be composite.**

  T. Kinugawa, T. Hyodo, arXiv:2111.06619; 2112.00249; 2201.04283 [hep-ph]

- **Generalized weak-binding relation shows that $\Lambda(1405)$ is dominated by molecular $\bar{K}N$ state.**

  Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)