ハドロン分子状態の物理

Physics of hadronic molecules

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Clusters & Hierarchies

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Contents

Part I : Introduction

- What are “exotic hadrons”?
- Hadronic molecules and universality
- Unstable resonances

Part II : Structure of $\Lambda(1405)$ resonance

- Accurate $\bar{K}N$ scattering amplitude

  Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012)

- Compositeness of hadrons

  Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)

References

Review articles

- Compositeness

T. Hyodo, JPS journal Vol. 75 No. 8, 478 (2020)

- Exotic hadrons, etc.

T. Hyodo, M. Niiyama, arXiv: 2010.07592 [hep-ph], to appear in PPNP

Lecture notes (detailed calculation)

- English

Japanese

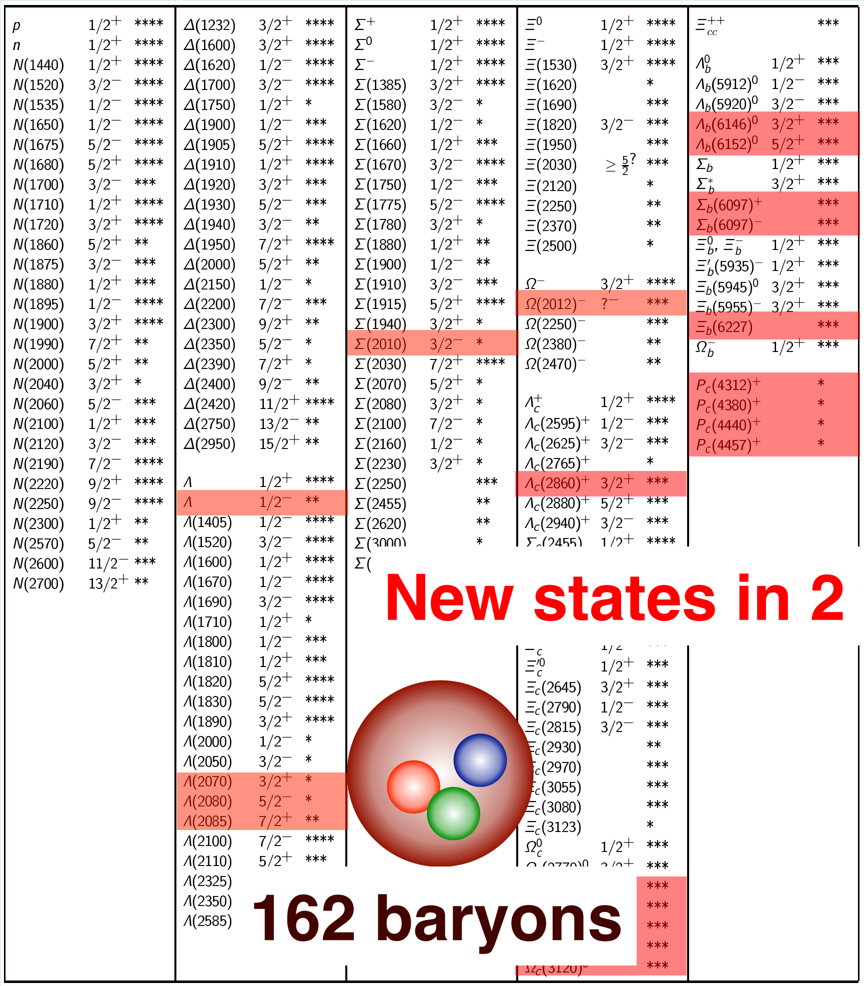
https://www.comp.tmu.ac.jp/hyodo/lecture_e.html#lec4

https://www.comp.tmu.ac.jp/hyodo/2020Tokuron.html

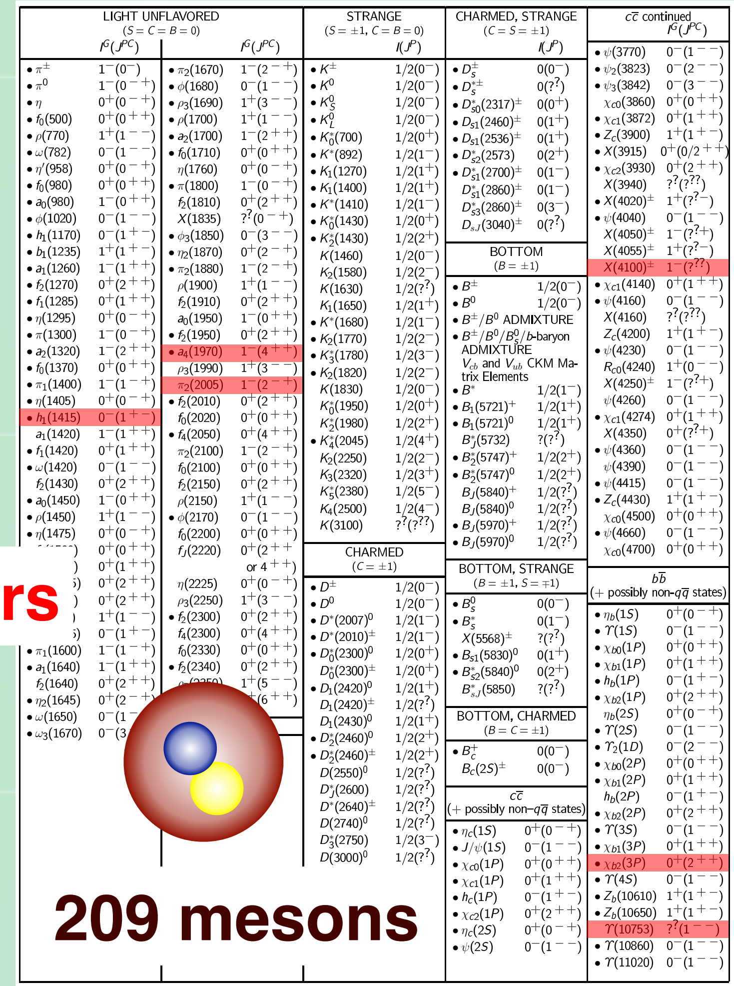
Observed hadrons

Particle Data Group (PDG) 2020 edition

http://pdg.lbl.gov/



New states in 2 years



All ~ 370 hadrons emerge from single QCD Lagrangian

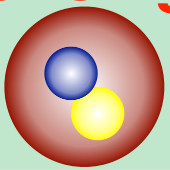
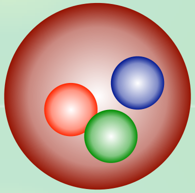
Color confinement

QCD Lagrangian : color $SU(3)$ symmetry

$$\mathscr{L}_{\mathrm{QCD}} = -\frac{1}{4} G^{a}_{\mu\nu} G^{\mu\nu}_{a} + \bar{q}_{\alpha}(i\gamma^{\mu} D^{\alpha\beta}_{\mu} - m_q \delta^{\alpha\beta}) q_{\beta}$$

- Quarks q_α : color $\mathbf{3}$ 
- Antiquarks $\bar{q}_\alpha$: color $\bar{\mathbf{3}}$ 
- Gluons A^a_μ : color $\mathbf{8}$

Color confinement: only color singlet states are observed

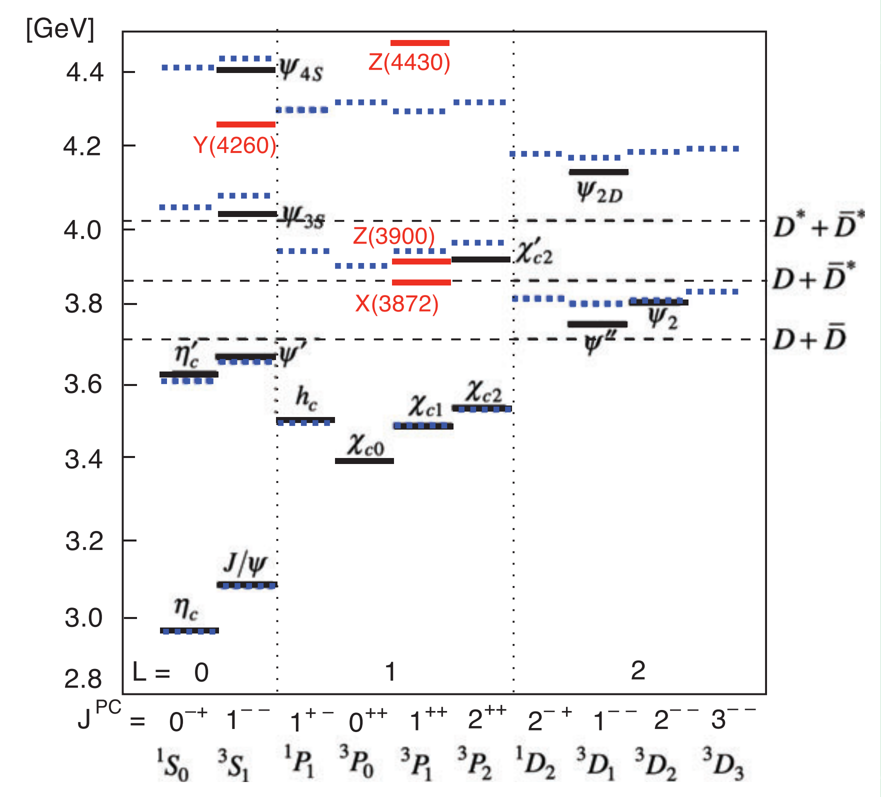
- Mesons $q\bar{q}$: $\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$ 
- Baryons qqq : $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10}$ 
- Why are the $\mathbf{3}, \mathbf{8}, \mathbf{10}, \dots$ states forbidden?

Experimental fact, but not understood from QCD

Exotic hadrons (naive)

Any other color singlet configurations?

- $qq\bar{q}\bar{q}$, $qqqq\bar{q}$, $q\bar{q}g$, gg, …
- Exotic hadrons : hadrons other than $q\bar{q}$, qqq
- How can we identify them?



Charmonium spectrum

A. Hosaka *et al.*, PTEP 2016, 062C01 (2016)

Standard criterion

- Constituent quark model
- Exotics : states which do not fit in the quark model

Problem : quark model is not QCD!

Exotic hadrons from symmetries 1

Spatial symmetries in QCD

- Parity Z_2 : $\mathbf{r} \to -\mathbf{r}$
- Rotation $SO(3)$: $\mathbf{r} \to R\mathbf{r}$

—> Hadrons are classified by spin-parity J^P

Internal symmetries in QCD

- Phase $U(1)_V$: $q \to e^{i\theta}q, \quad \bar{q} \to e^{-i\theta}\bar{q}$

—> Hadrons are classified by baryon number B

$B = 0$ state : meson, $B = 1$ state : baryon

Definition by conserved quantum number

- Note : $n_q - n_{\bar{q}}$ is conserved

—> One cannot distinguish $q\bar{q}$ from $qq\bar{q}\bar{q}$ by B

Exotic hadrons from symmetries 2

Flavor symmetry

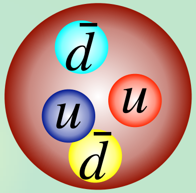
- Isospin $SU(2)$: $\begin{pmatrix} u \\ d \end{pmatrix} \to U \begin{pmatrix} u \\ d \end{pmatrix}$

—> Hadrons are classified by isospin I

I of mesons made from only u, d quarks ($q, \bar{q} : I = 1/2$)

- $q\bar{q}$: $I = 0,\ 1$
- $qq\bar{q}\bar{q}$: $I = 0,\ 1,\ 2$
- $qqq\bar{q}\bar{q}\bar{q}$: $I = 0,\ 1,\ 2,\ 3$

No annihilation of $q\bar{q}$



—> $I \geq 2$ meson (charge $|Q| \geq 2$) cannot be $q\bar{q}$: exotic!

Definition by conserved quantum number

Extension to SU(3) : exoticness

T. Hyodo, D. Jido, A. Hosaka, PRL97, 192002 (2006); PRD75, 034002 (2007)

Exotic hadrons from symmetries 3

Exotic hadrons defined by conserved quantum numbers

Mesons $B = 0$

- $J^P = 0^-, I = 1$
- $J^P = 1^-, I = 1$
- $J^P = 0^-, I = 1/2, S = \pm 1$
- ...

Baryons $B = 1$

- $J^P = 1/2^+, I = 1/2$
- $J^P = 3/2^+, I = 3/2$
- $J^P = 1/2^-, I = 0, S = -1$
- ...

Mesons:

- $J^P = 0^-, I = 2$
- ...

Baryons:

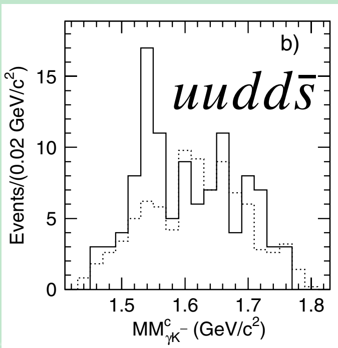
- $J^P = 1/2^+, I = 0, S = +1$
- ...

Quantum number exotics (require more than $\bar{q}q$, qqq)

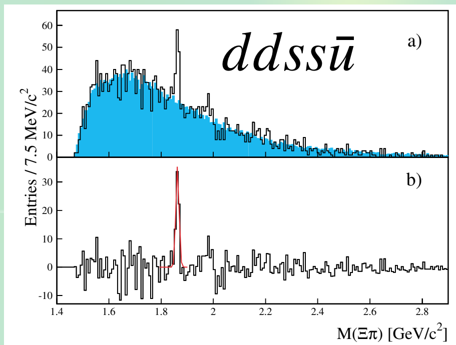
Exotic hadrons in experiments 1

Possible candidates of quantum number exotics

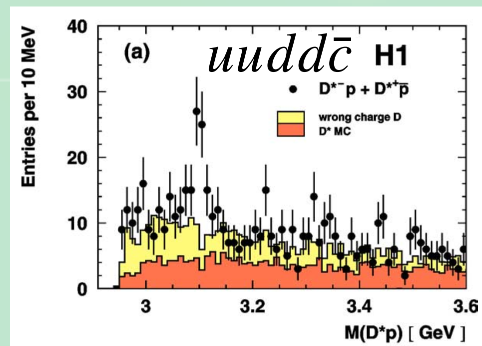
$\Theta^{+}(S=+1,\ B=1)$ **T. Nakano *et al.* (LEPS), PRL 91, 012002 (2003)**



$\Xi^{--}(Q=-2,\ B=1)$ **C. Alt *et al.* (NA49), PRL 92, 042003 (2004)**

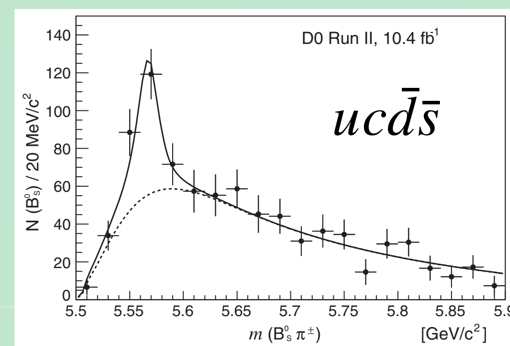


$\Theta_c(C=-1,\ B=1)$



A. Aktas *et al.* (H1), PLB 588, 17 (2004)

$X(C=+1,\ I=1)$



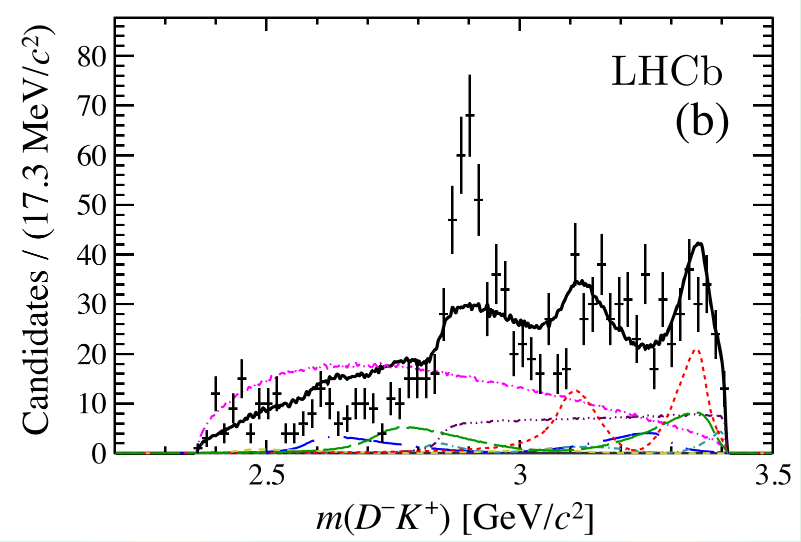
V.M. Abazov *et al.* (D0), PRL 117, 022003 (2016)

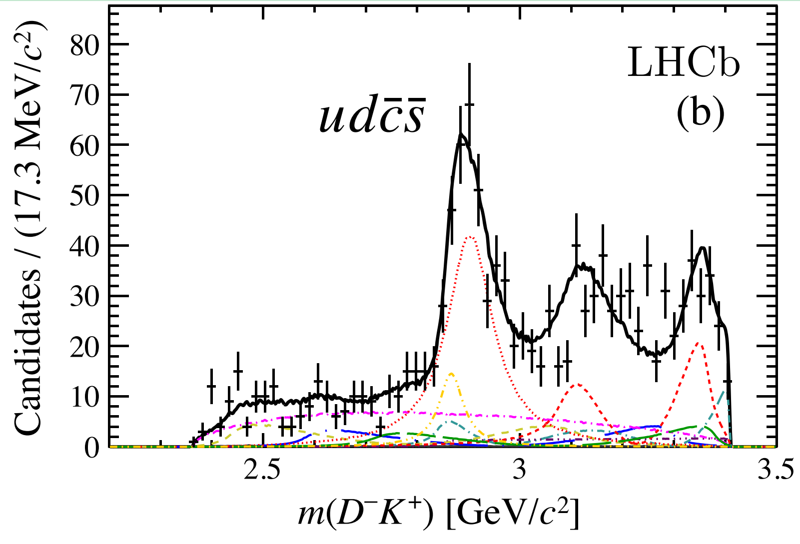
- $q\bar{q}$ annihilation is not possible : genuine exotic
- Excluded by higher statistics experiments

Exotic hadrons in experiments 2

New candidate $X(2900)$

$X(S=+1,\ C=-1)$ **R. Aaij *et al.* (LHCb), PRD 102, 112003 (2020)**





- Not included in PDG yet

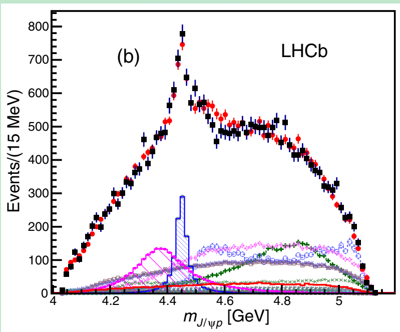
No quantum number exotics are established
(except for J^{PC} exotics : $\pi_1(1400)$ and $\pi_1(1600)$ with $J^{PC}=1^{-+}$)

Experimental fact, but not understood from QCD

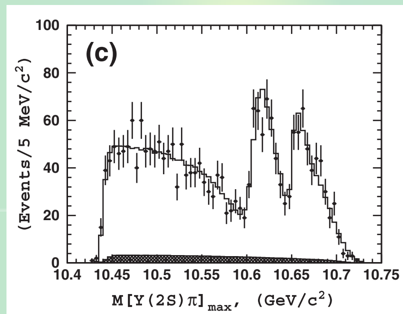
Quarkonium associated exotics

Pentaquarks P_c, P_{cs}, …, tetraquarks X, Y, Z_c, Z_b, Z_{cs}, … ?

$P_c \sim \bar{c}cuud$ R. Aaij, *et al.*, Phys. Rev. Lett. 115, 072001 (2015)



$Z_b \sim \bar{b}b\bar{u}d$ A. Bondar, *et al.*, Phys. Rev. Lett. 108, 122001 (2012)



…

- $\bar{c}c, \bar{b}b$ **can in principle be annihilated**

—> Quantum number is not exotic

$$P_c \sim \bar{c}cuud \sim uud \sim N, \quad Z_b \sim \bar{b}b\bar{u}d \sim \bar{u}d \sim b_1(J^{PC} = 1^{+-})$$

- **OZI rule : existence of heavy quark pair is almost certain**

—> Clues to understand the quantum number exotics

Summary of part I

- **Hadrons are classified by conserved quantum numbers. Non-observation of quantum number exotics is highly nontrivial.**

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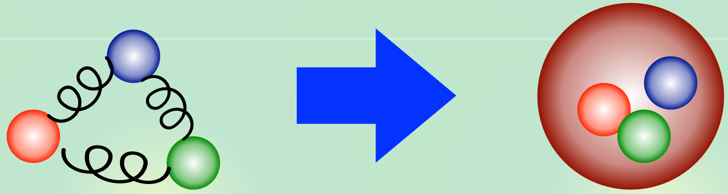
    Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012)
  - Compositeness of hadrons

    Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)

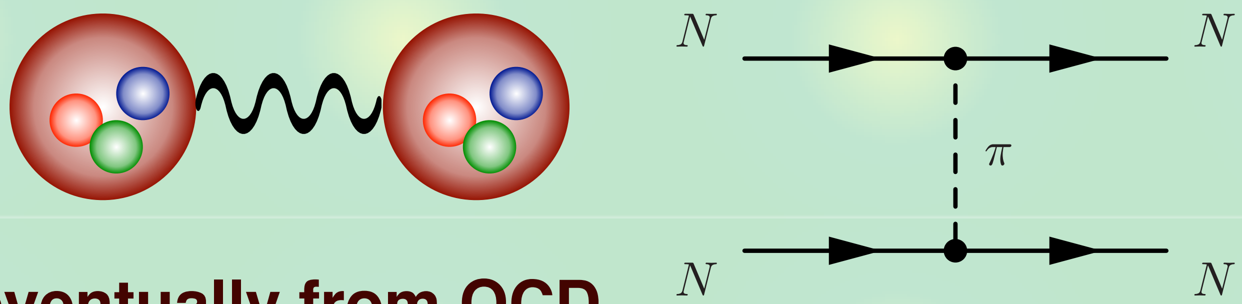
Hadronic interactions

Strong interaction : hadrons <— quarks



- c.f. EM interaction : atoms <— electrons and nucleus

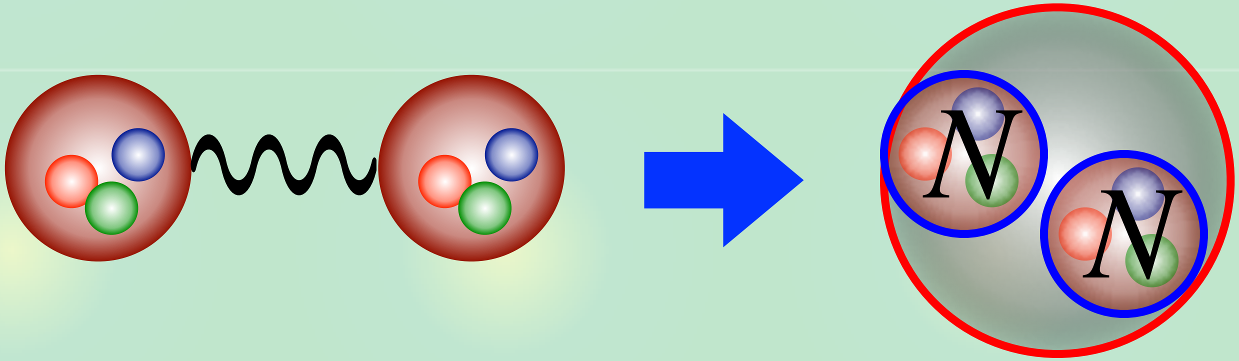
Hadron-hadron interaction (e.g. nuclear force)



- stems eventually from QCD
- Hadrons are (color) charge neutral : van der Waals force?
- Meson ($q\bar{q}$) exchange force, …

Hadronic molecules (naive)

If the hadronic interaction is sufficiently attractive…



- (quasi-)bound state can be formed : hadronic molecules
- Constituent hadrons keep their identity
- Clustering of quarks

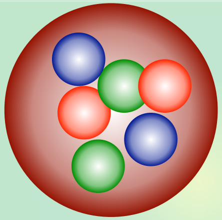
Known examples : deuteron d, nuclei

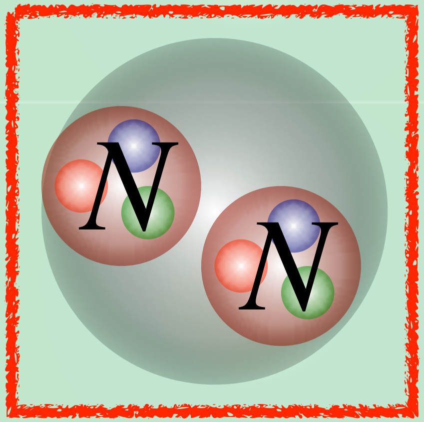
- Binding energy of nuclei : $\mathcal{O}(1)$ MeV
- Binding energy of quarks : ∞ or $\mathcal{O}(100)$ MeV

—> Hadronic molecules : shallow bound states

Compositeness

Can we distinguish hadronic molecules from others?





- Same conserved quantum numbers $J^P = 1^+, I = 0, B = 2$
- Physical state = superposition of possible configurations

$$|d\rangle \overset{?}{=} C_{6q}|qqqqqq\rangle + C_{NN}|NN\rangle + \cdots$$

- Orthogonal basis?

Expansion in terms of hadrons $|d\rangle = \sqrt{Z}|d_0\rangle + \sqrt{X}|NN\rangle + \cdots$

- Hadrons are asymptotic states of QCD

—> compositeness X (part II)

Two-body universal physics

Shallow s-wave bound states : low-energy universality

E. Braaten, H.-W. Hammer, Phys. Rept. 428, 259 (2006);
P. Naidon, S. Endo, Rept. Prog. Phys. 80, 056001 (2017)

- Scattering length $|a| \gg$ interaction range $R_{\rm typ}$
- Size of (quasi-)bound state $R \sim a$: loosely bound
- Relation with eigenenergy E

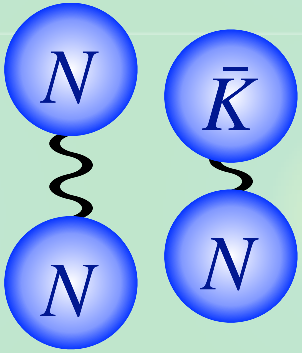
$$a(E) \sim \frac{i}{k} = \frac{i}{\sqrt{2\mu E}} \quad (|E| \ll 1)$$

vdW

Examples: d, $\Lambda(1405)$, ^{4}He **dimer**

	NN [fm]	$\bar{K}N$ [fm]	^{4}He [a_0]
$a(E)$	4.3	$1.2 - 0.8i$	178
$a_{\rm emp}$	5.1	$1.4 - 0.9i$	189
$R_{\rm typ}$	1.4	0.4	10

strong



^{4}He

Wave function

Why is a weakly bound s-wave state spatially large?

- Radial Schrödinger equation

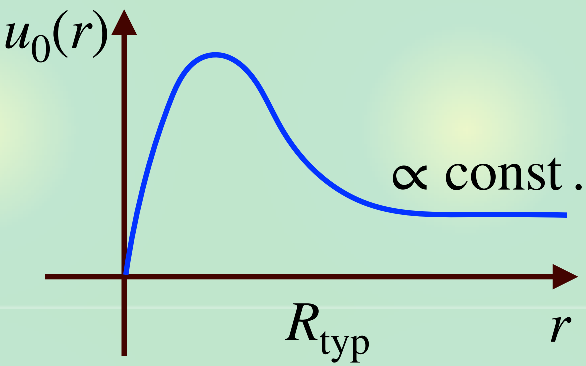
$$-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}u_0(r) + V(r)u_0(r) = Eu_0(r), \quad \psi_0(r) = Y_0^0\frac{u_0(r)}{r}$$

- At large distance ($V(r) = 0$) with zero energy ($E = 0$)

$$u_0(r) = C(r - a), \quad (r > R_{\rm typ})$$

- Scattering length a : intercept of $u_0(r)$

- Bound state with $B = 0 \Rightarrow |a| = \infty$



Wave function is not normalizable

$$\int d^3r\,|\psi_0(r)|^2 = \int_0^\infty dr\,|u_0(r)|^2 = \infty$$

$B = 0$ **state is not a bound state (zero-energy resonance)**

Consequences

Mean squared radius

$$\langle r^2 \rangle = \int d^3r \ r^2 |\psi_0(r)|^2 = \int_0^\infty dr \ r^2 |u_0(r)|^2 = \infty$$

—> Size of $B = 0$ state is infinitely large ($a = R$)

Compositeness X (weight of the composite component)

T. Hyodo, Phys. Rev. C 90, 055208 (2014)

$$\Psi = \begin{pmatrix} \tilde{\psi}_0(p) \\ c \\ \vdots \end{pmatrix}$$

$$X = \int \frac{d^3p}{(2\pi)^3} |\tilde{\psi}_0(p)|^2 = \int d^3r \ |\psi_0(r)|^2$$ infinite

$$Z = |c|^2 + \cdots$$ finite

$$X + Z = 1 \quad \leftarrow |\Psi|^2 = 1$$

—> $B = 0$ state is completely composite ($X = 1, \ \ Z = 0$)

Weakly bound state ($B \neq 0$, except for fine tuning)

- Deviation from $a = R$: weak-binding relation

Summary of part I

- Hadrons are classified by conserved quantum numbers. Non-observation of quantum number exotics is highly nontrivial.

- Hadronic molecules are shallow (quasi-)bound states of hadrons. Different hierarchies can be related by low-energy universality.

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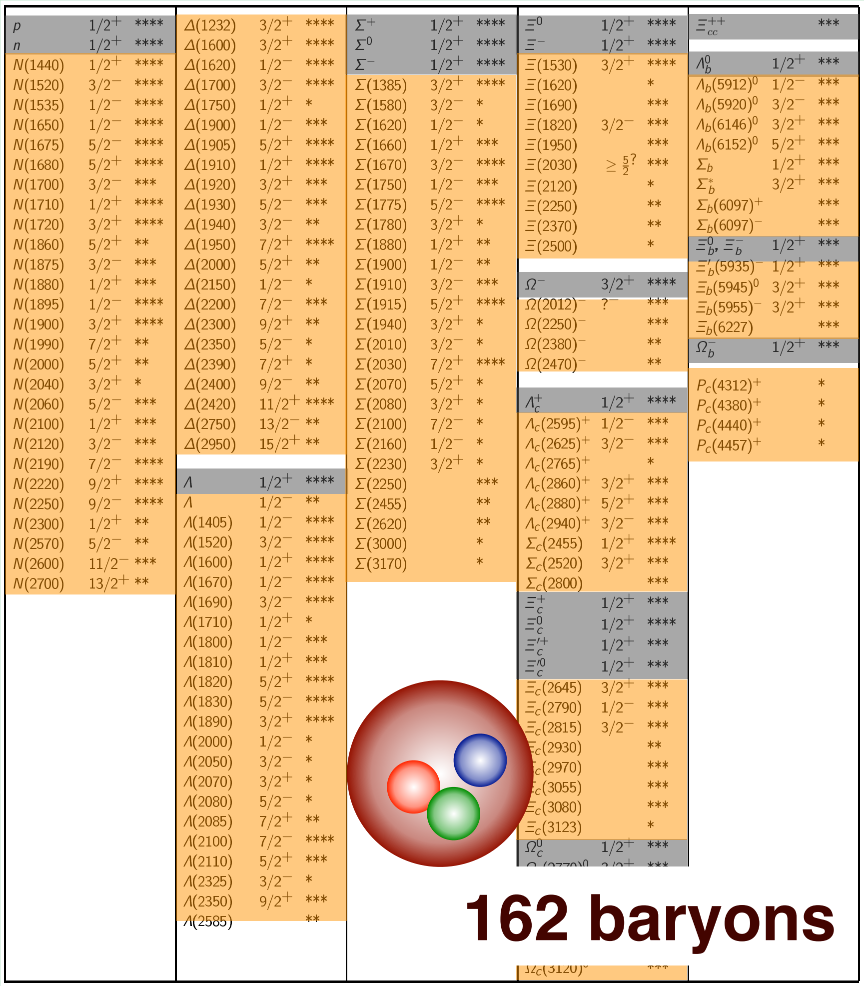
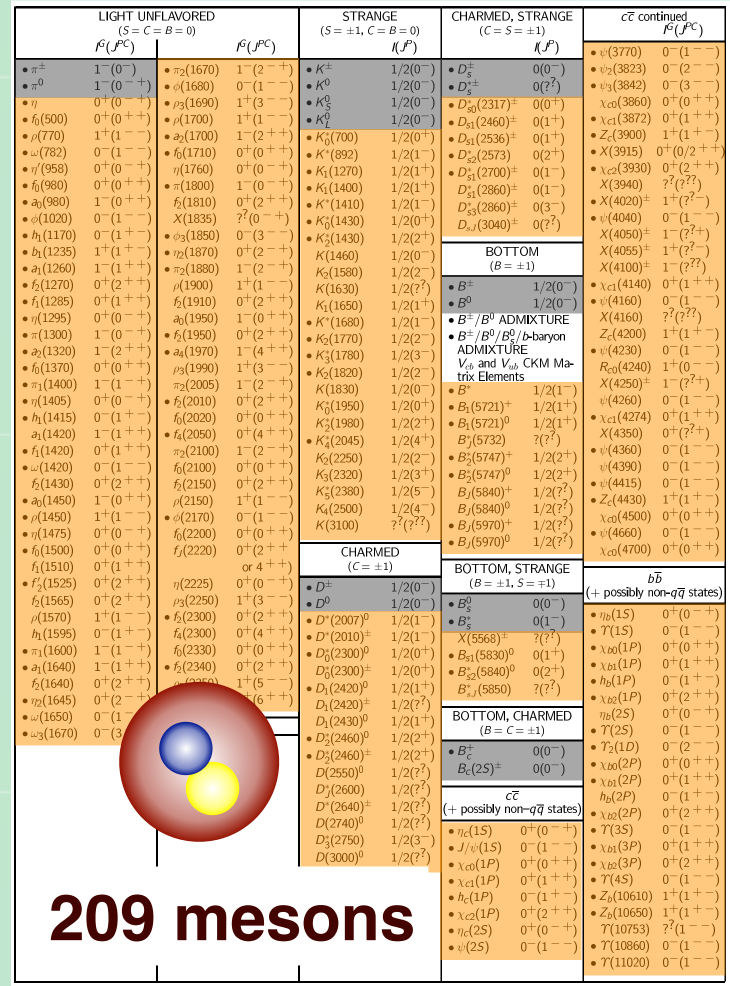
Unstable states via strong interaction

Stable/unstable hadrons

http://pdg.lbl.gov/

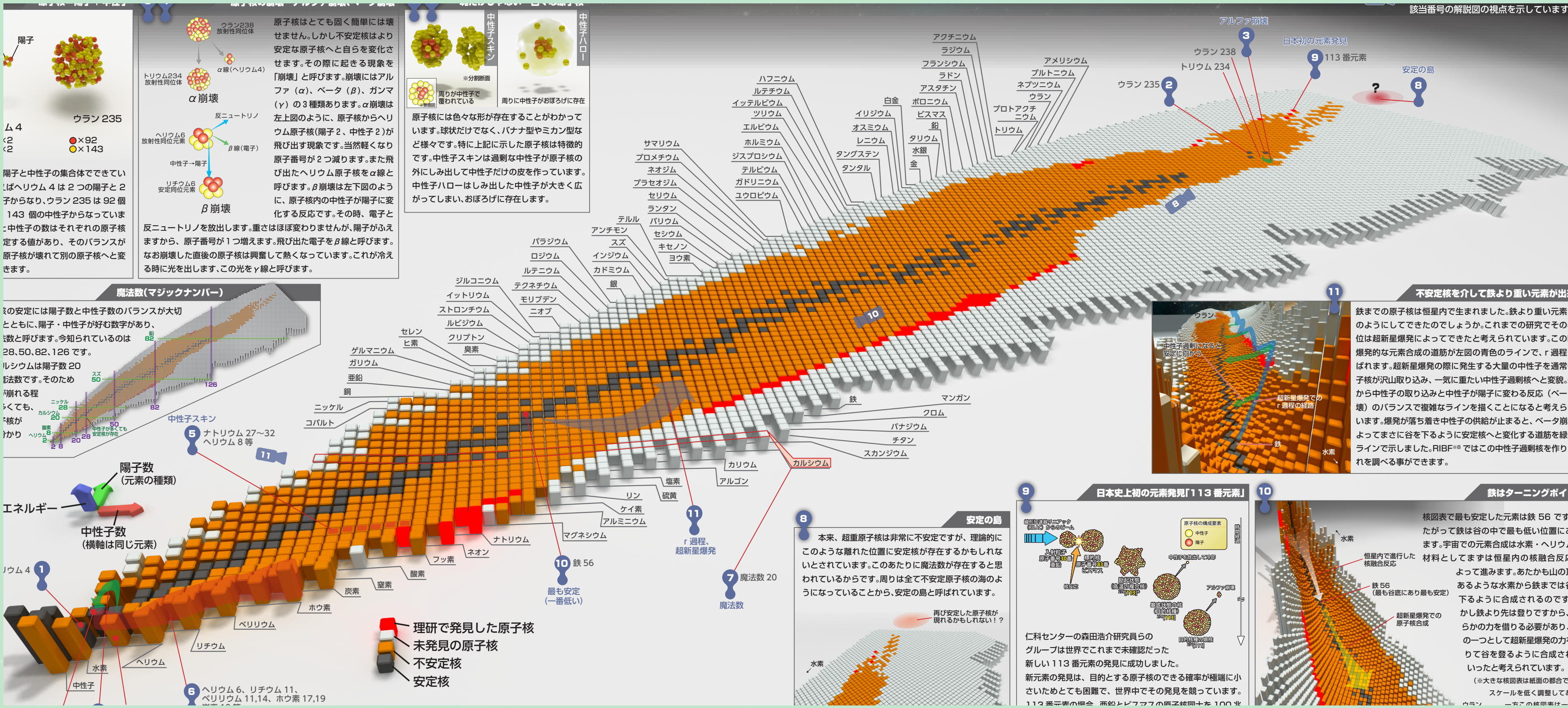




Most of hadrons are unstable (above two-hadron threshold)

Relation to unstable nuclei

Stable nuclei (~300), unstable nuclei (~2000)



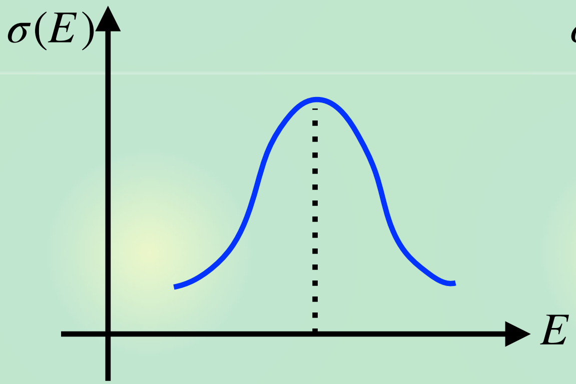
https://www.nishina.riken.jp/enjoy/kakuzu/index.html

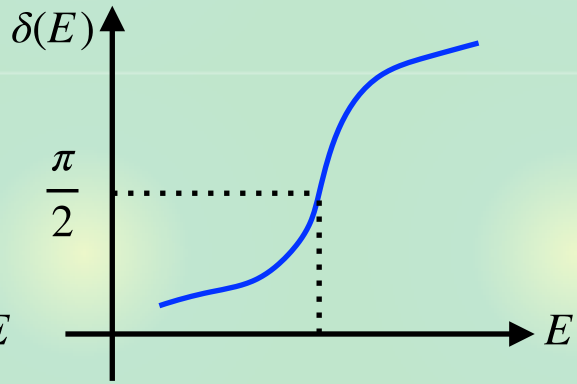
Structure of unstable nuclei

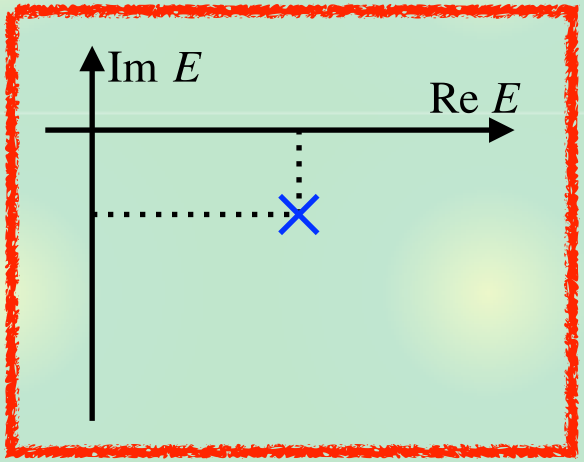
- Clustering, halo nuclei, Efimov effect, …

Pole of resonances

Signals of a resonance







Well-defined characterization : pole of scattering amplitude

T. Hyodo, M. Niiyama, arXiv: 2010.07592 [hep-ph], to appear in PPNP

Schrödinger eq. + outgoing b.c. at energy E $(p = \sqrt{2\mu E})$

- bound states $(E < 0)$

$$p = i\kappa \quad (\kappa > 0)$$

- resonances $(E \in \mathbb{C})$

$$p \in \mathbb{C} \quad (\text{Im } p < 0)$$

$\Leftrightarrow$

zero of Jost function

$$\mathscr{f}_\ell(p) = 0$$

$\Leftrightarrow$

pole of s-matrix/ scattering amplitude

$$|f_\ell(p)| \to \infty$$
$$|s_\ell(p)| \to \infty$$

Theoretical analysis to pin down the pole position

Nature of resonances

Resonance as an “eigenstate” of Hamiltonian

- **Complex energy**

G. Gamow, Z. Phys. 51, 204 (1928)

Zur Quantentheorie des Atomkernes.

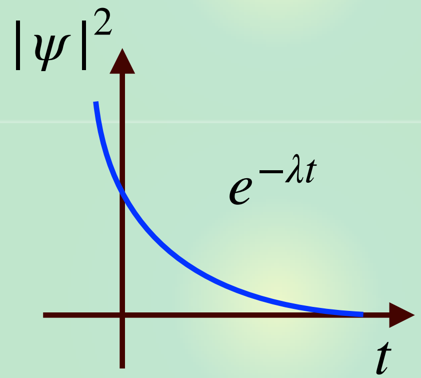
Von G. Gamow, z. Zt. in Göttingen.

Mit 5 Abbildungen. (Eingegangen am 2. August 1928.)

Um diese Schwierigkeit zu überwinden, müssen wir annehmen, daß die Schwingungen gedämpft sind, und E komplex setzen:

$$E = E_0 + i\,\frac{h\lambda}{4\pi},$$

wo E_0 die gewöhnliche Energie ist und λ das Dämpfungsdekrement (Zerfallskonstante). Dann sehen wir aber aus den Relationen (2 a) und (2 b),

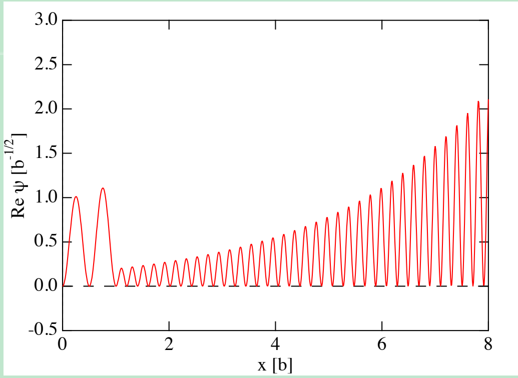


- **Time dependence : probability decreasing**

$$\psi = \Psi(q)\,.\,e^{+\frac{2\pi i E}{h}t}, \quad \propto e^{+2\pi i E_0 t/h} e^{-(\lambda/2)t}, \quad |\psi|^2 \propto e^{-\lambda t}$$

- **Spatial distribution : divergence at large r**

$$\Psi(r) \sim e^{ikr}, \quad |\Psi(r)|^2 \sim e^{2k_I r}, \quad k = k_R - ik_I$$



- **complex expectation value (norm, $\langle r^2 \rangle$)**
- **interpretation problem…**

Summary of part I

- Hadrons are classified by conserved quantum numbers. Non-observation of quantum number exotics is highly nontrivial.

- Hadronic molecules are shallow (quasi-)bound states of hadrons. Different hierarchies can be related by low-energy universality.

- Most of hadrons are unstable against the strong decay. Internal structure of hadrons should be discussed with unstable nature.

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Part II : Structure of $\Lambda(1405)$ resonance

- $\bar{K}N$ scattering and $\Lambda(1405)$ poles

  Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012)

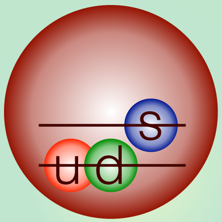
- Compositeness of $\Lambda(1405)$

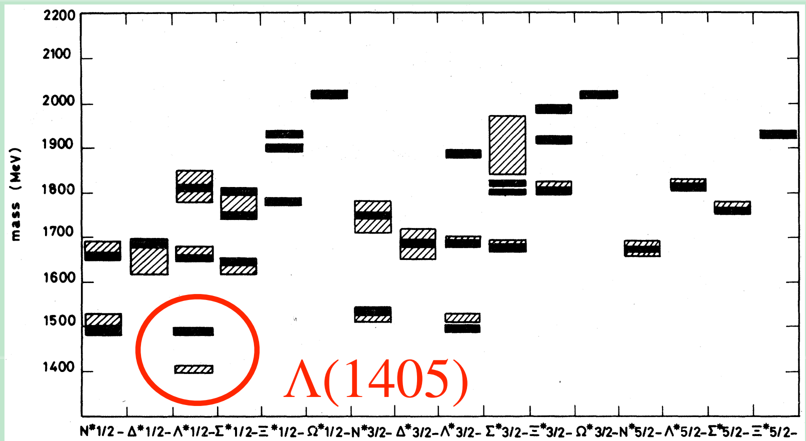
  Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)

$\Lambda(1405)$ and $\bar{K}N$ scattering

$\Lambda(1405)$ does not fit in standard picture —> exotic candidate

N. Isgur and G. Karl, Phys. Rev. D18, 4187 (1978)



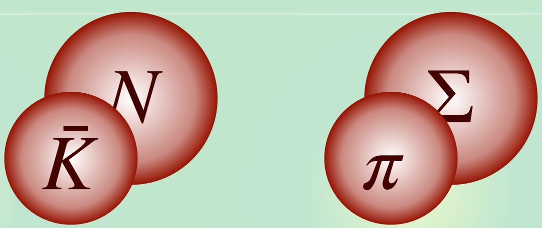


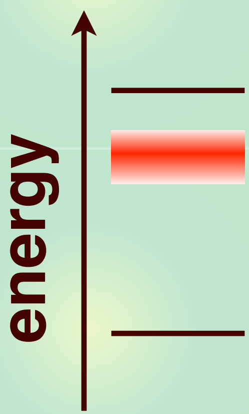
: theory

: experiment

Resonance in coupled-channel scattering

- Coupling to MB states





Detailed analysis of $\bar{K}N$-$\pi\Sigma$ scattering is necessary

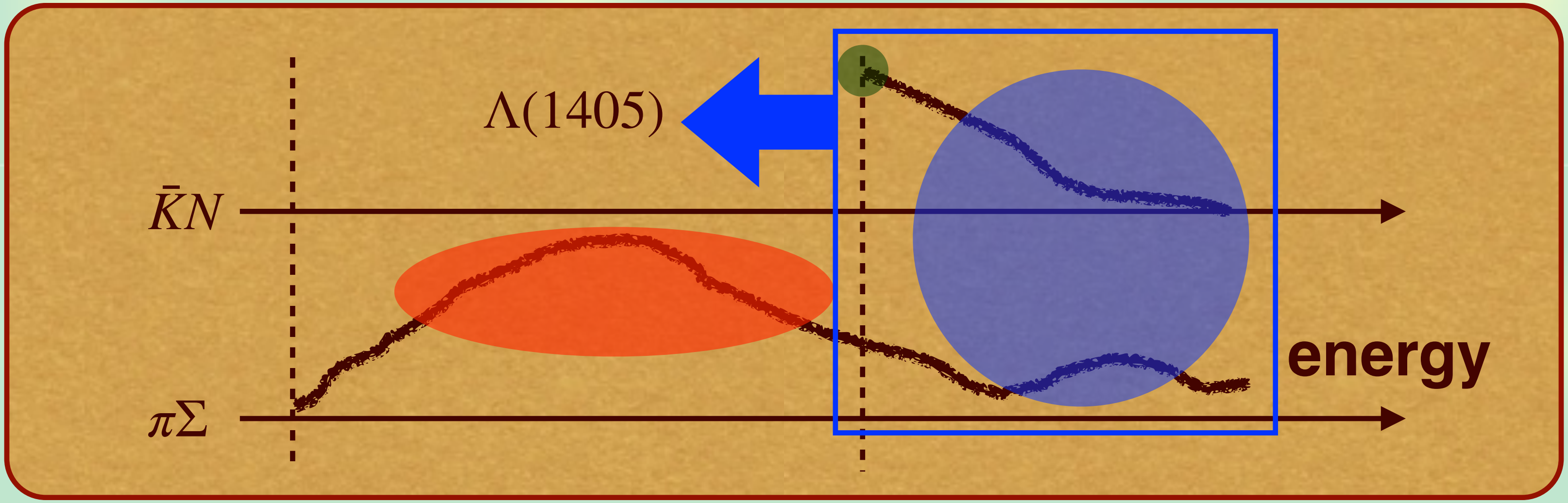
Strategy for $\bar{K}N$ interaction

Above the $\bar{K}N$ threshold : direct constraints

- K^-p **total cross sections** (old data)
- $\bar{K}N$ **threshold branching ratios** (old data)
- K^-p **scattering length** (new data : SIDDHARTA)

Below the $\bar{K}N$ threshold: indirect constraints

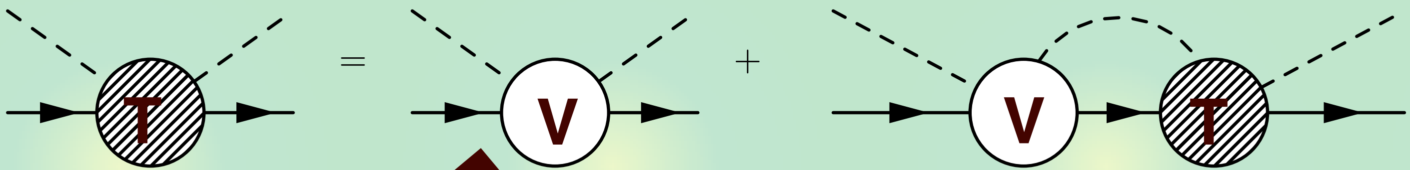
- $\pi\Sigma$ **mass spectra** (new data : LEPS, CLAS, HADES, ...)



Construction of the realistic amplitude

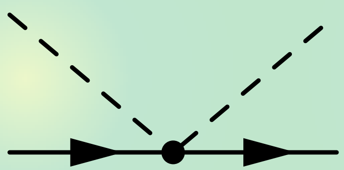
Chiral SU(3) coupled-channels $(\bar{K}N, \pi\Sigma, \pi\Lambda, \eta\Lambda, \eta\Sigma, K\Xi)$ approach

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881 98 (2012)



Chiral perturbation theory

1) TW term

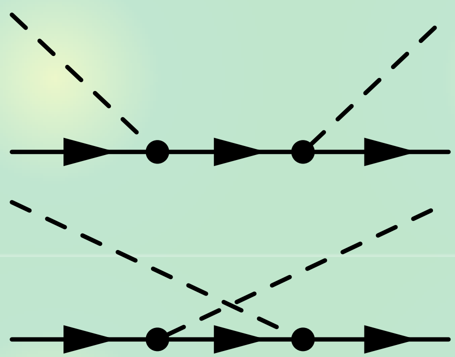


$\mathcal{O}(p)$

6 cutoffs

TW model

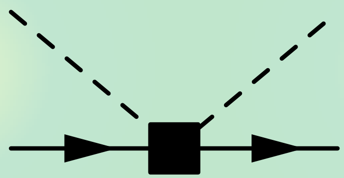
2) Born terms



$\mathcal{O}(p)$

TWB model

3) NLO terms



$\mathcal{O}(p^2)$

7 LECs

NLO model

Best-fit results

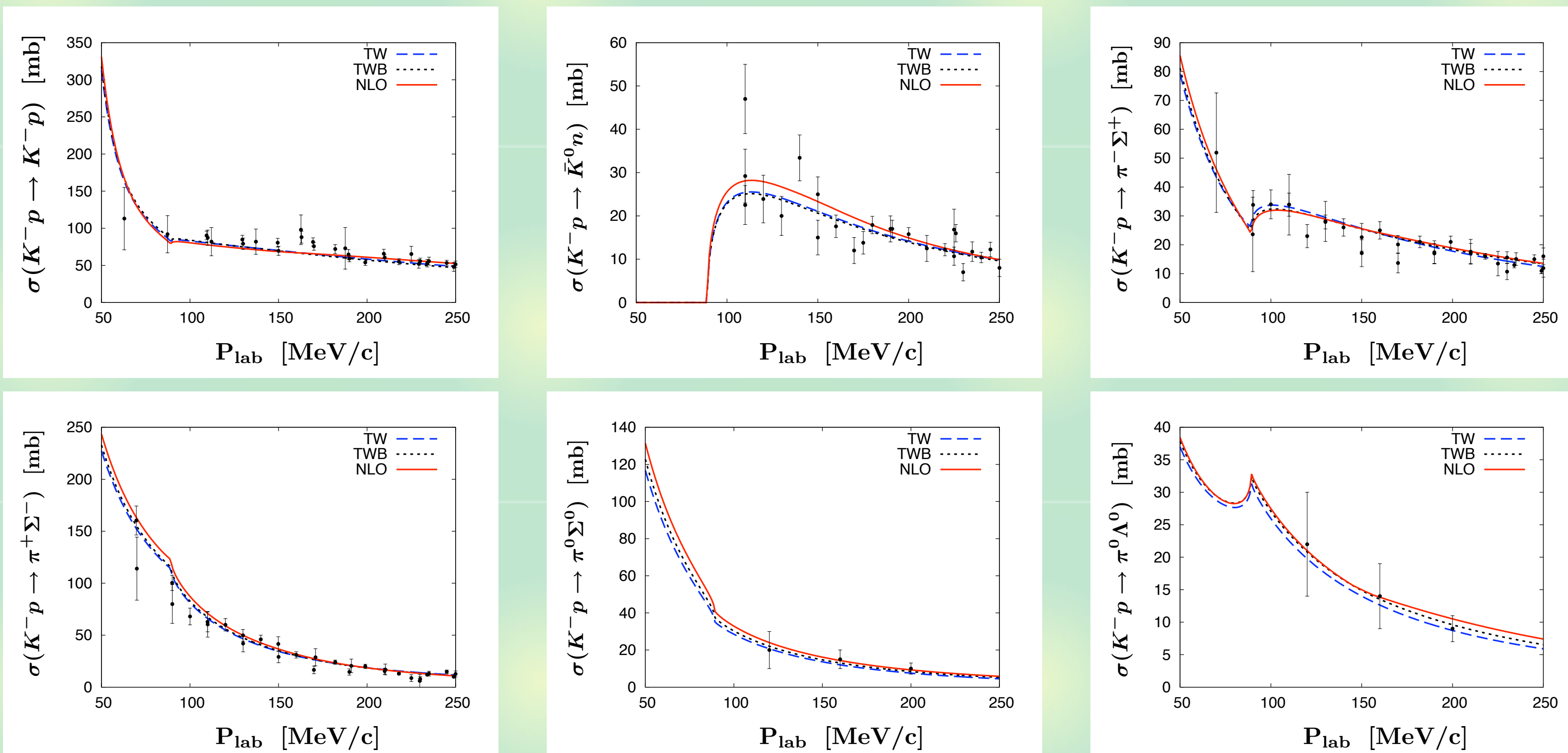
K at rest

	TW	TWB	NLO	Experiment	
ΔE [eV]	373	377	306	$283 \pm 36 \pm 6$	[10]
Γ [eV]	495	514	591	$541 \pm 89 \pm 22$	[10]
γ	2.36	2.36	2.37	2.36 ± 0.04	[11]
R_n	0.20	0.19	0.19	0.189 ± 0.015	[11]
R_c	0.66	0.66	0.66	0.664 ± 0.011	[11]
χ^2/d.o.f	1.12	1.15	0.96		

} **SIDDHARTA** (ΔE, Γ)

} **Branching ratios** (γ, R_n, R_c)

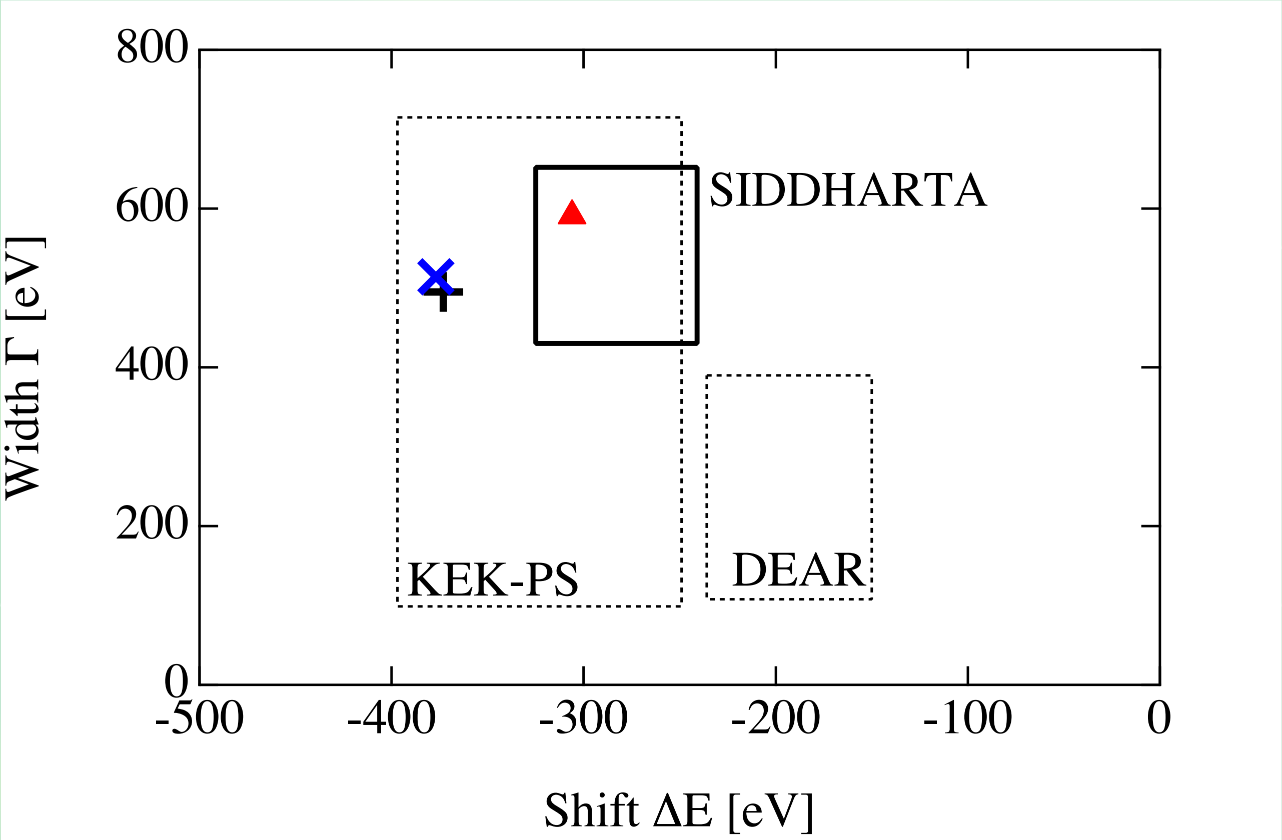
$K^- p$ cross sections



Accurate description of all existing data (χ^2/d.o.f ~ 1)

Comparison with SIDDHARTA

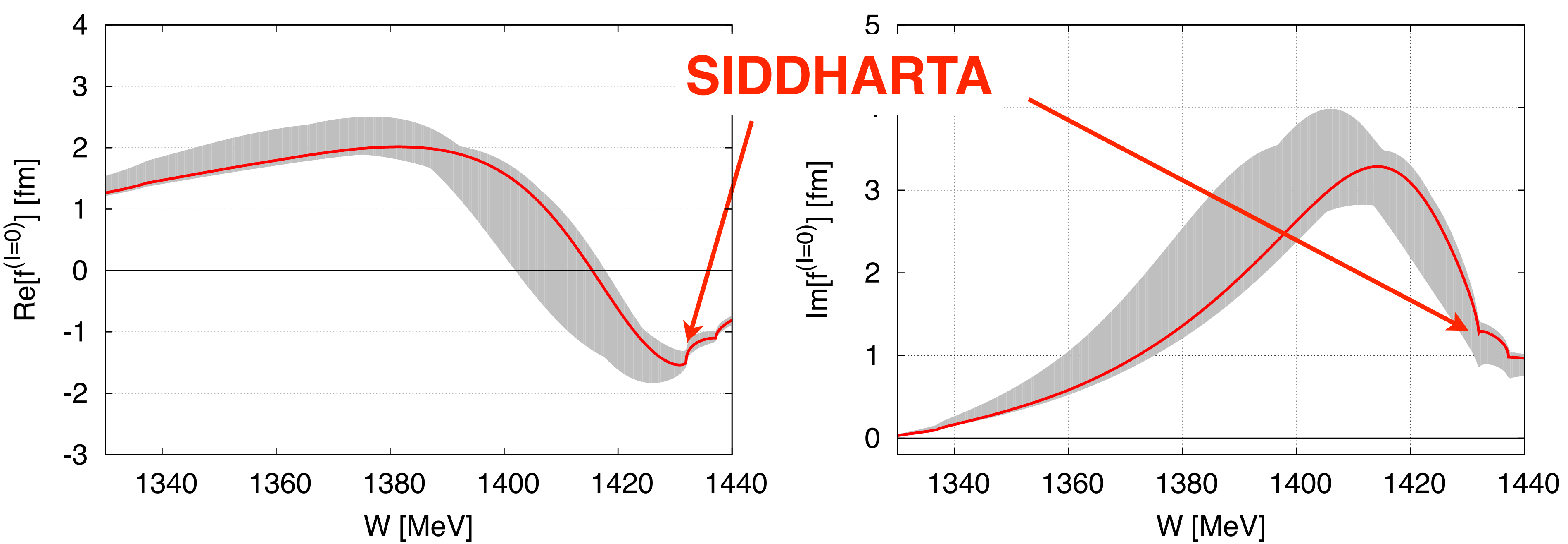
	TW	TWB	NLO
χ^2/d.o.f.	1.12	1.15	0.957



TW and TWB are reasonable, while best-fit requires NLO

Subthreshold extrapolation

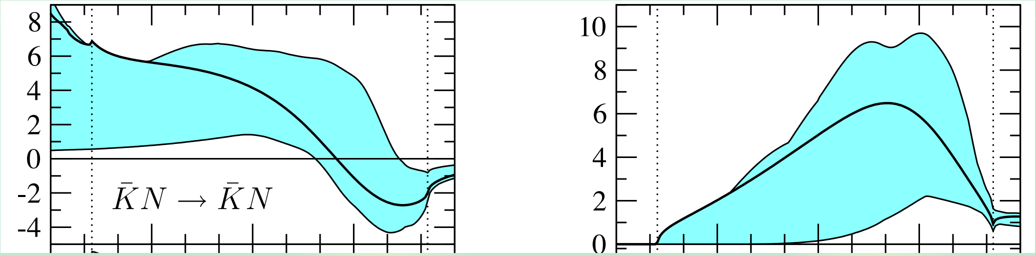
Uncertainty of $\bar{K}N \to \bar{K}N(I=0)$ amplitude below threshold



Y. Kamiya, K. Miyahara, S. Ohnishi, Y. Ikeda, T. Hyodo, E. Oset, W. Weise, NPA 954, 41 (2016)

- c.f. without SIDDHARTA

R. Nissler, Doctoral Thesis (2007)

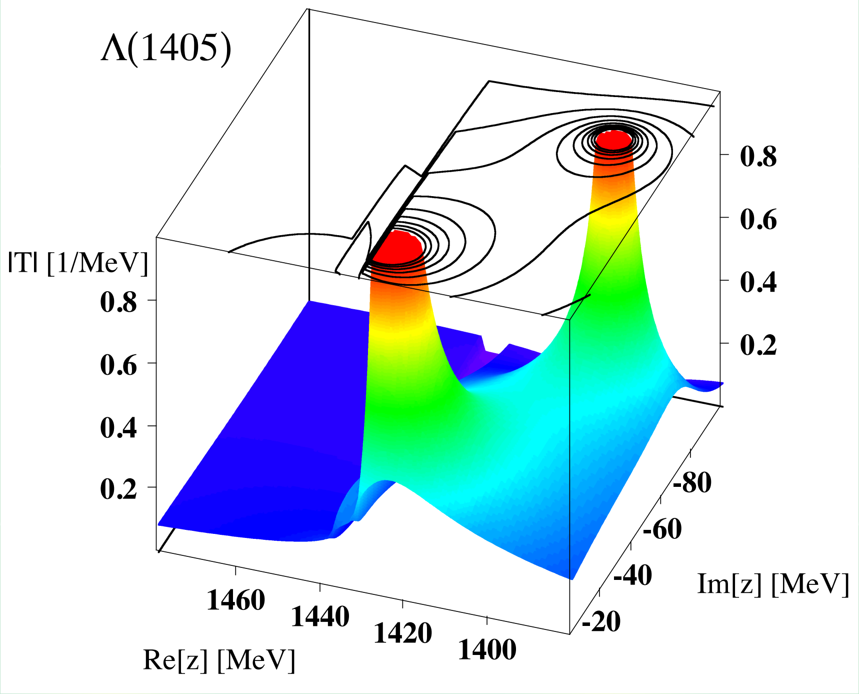


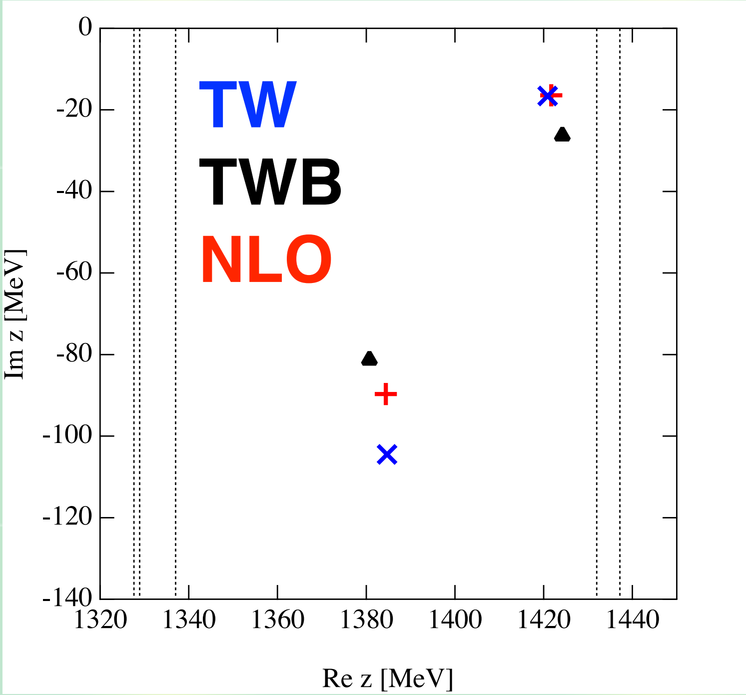
SIDDHARTA is essential for subthreshold extrapolation

Extrapolation to complex energy: two poles

Two poles : superposition of two eigenstates

J.A. Oller, U.G. Meißner, PLB 500, 263 (2001);
D. Jido, J.A. Oller, E. Oset, A. Ramos, U.G. Meißner, NPA 723, 205 (2003);
U.G. Meißner, Symmetry 12, 981 (2020); M. Mai, arXiv: 2010.00056 [nucl-th];
T. Hyodo, M. Niiyama, arXiv: 2010.07592 [hep-ph]





T. Hyodo, D. Jido, Prog. Part. Nucl. Phys. 67, 55 (2012)

NLO analysis confirms the two-pole structure

PDG has changed

2020 update of PDG

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012); ▲

Z.H. Guo, J.A. Oller, PRC87, 035202 (2013); ×

M. Mai, U.G. Meißner, EPJA51, 30 (2015) ■ ○

- Particle Listing section:

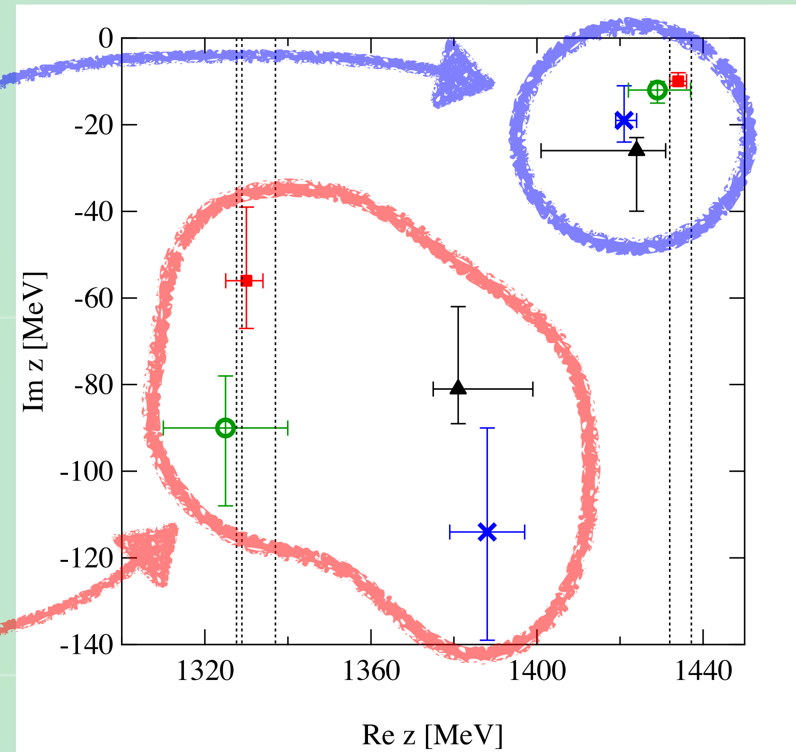
Citation: P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

$\Lambda(1405)\ 1/2^-$ $I(J^P) = 0(\frac{1}{2}^-)$ Status: ∗∗∗∗

Citation: P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

$\Lambda(1380)\ 1/2^-$ $J^P = \frac{1}{2}^-$ Status: ∗∗

new!



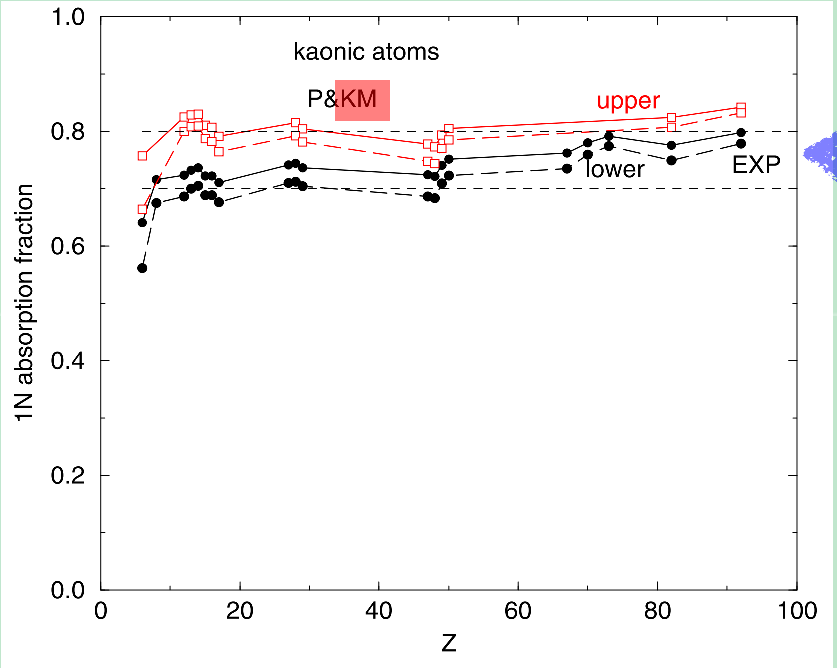
T. Hyodo, M. Niiyama, arXiv: 2010.07592 [hep-ph], to appear in PPNP

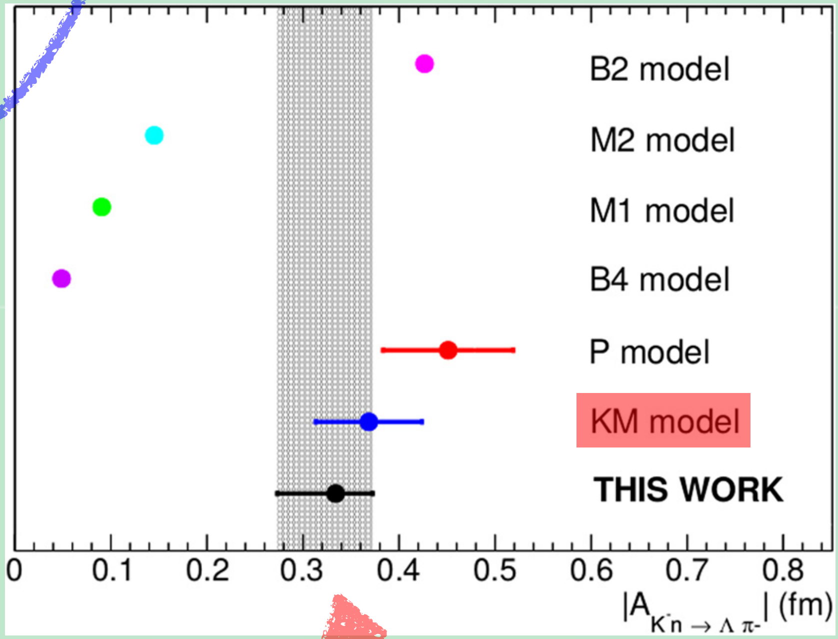
- "$\Lambda(1405)$" is no longer at 1405 MeV but ~ 1420 MeV.
- Lower pole : two-star resonance $\Lambda(1380)$

Further check of amplitude

Single-nucleon absorption on kaonic atoms

E. Friedman, A. Gal, NPA959, 66 (2017)





$|f_{K^- n\to\pi^-\Lambda}|$ from K^- absorption on ^{4}He at DAΦNE

K. Piscicchia, *et al.*, PLB782, 339 (2018)

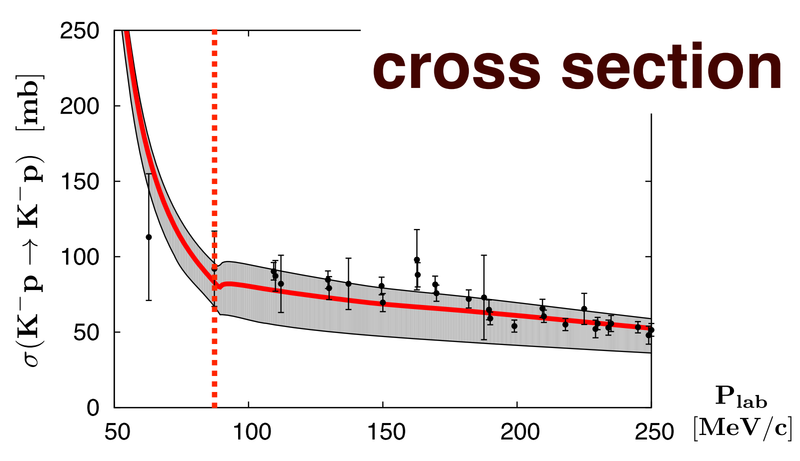
Our amplitude (KM model) is compatible with these analyses

New data : K^-p correlation function

K^-p total cross sections

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011)

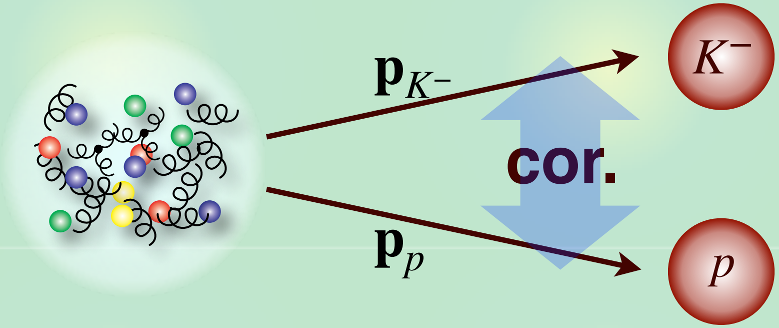
- Old bubble chamber data

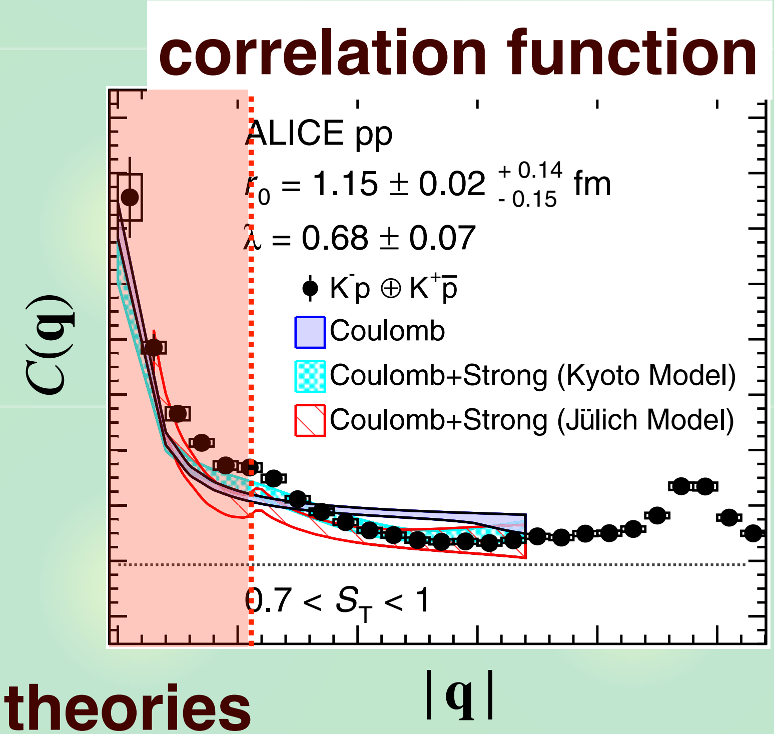


K^-p correlation function

S. Acharya *et al.* (ALICE), PRL 124, 092301 (2020)

$$C(\mathbf{q}) = \frac{N_{K^-p}(\mathbf{p}_{K^-}, \mathbf{p}_p)}{N_{K^-}(\mathbf{p}_{K^-})N_p(\mathbf{p}_p)}$$





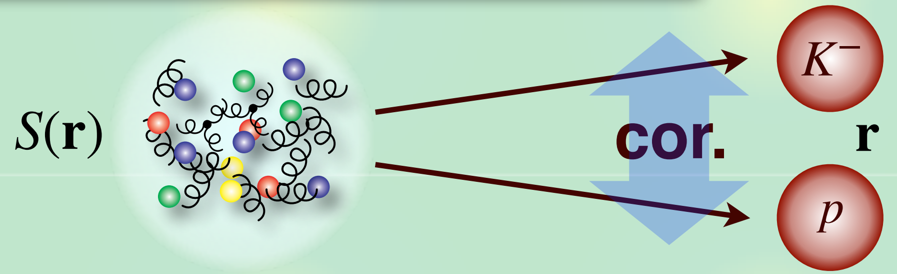
- Excellent precision ($\bar{K}^0n$ cusp)
- Low-energy data below $\bar{K}^0n$

—> Important constraint on $\Lambda(1405)$ theories

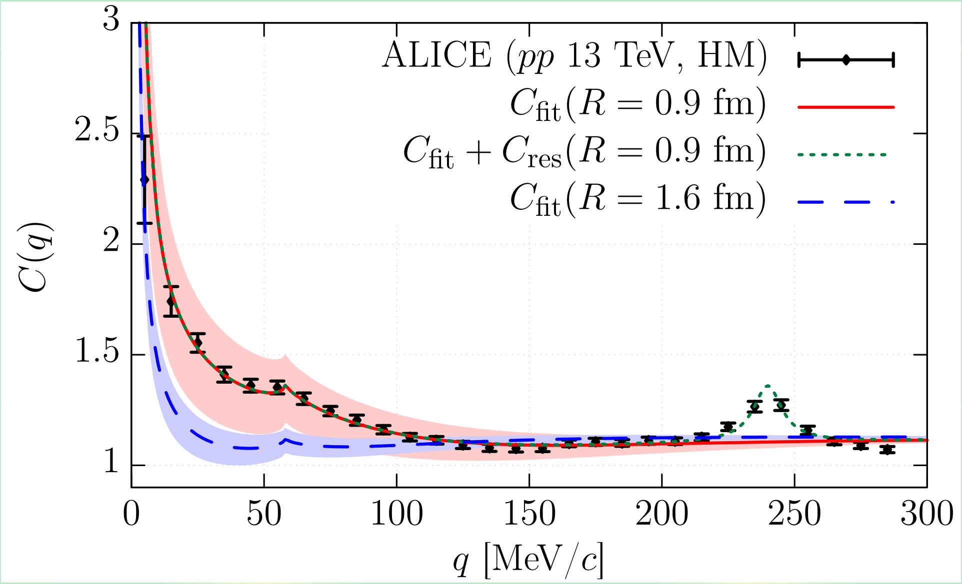
Prediction from chiral SU(3) dynamics

Theoretical calculation of $C(q)$

$$C(\mathbf{q}) \simeq \int d^3\mathbf{r}\, S(\mathbf{r})\, |\Psi_{\mathbf{q}}^{(-)}(\mathbf{r})|^2$$



- Wave function $\Psi_{\mathbf{q}}^{(-)}(\mathbf{r})$: coupled-channel $\bar{K}N$-$\pi\Sigma$-$\pi\Lambda$ potential
- Source function $S(\mathbf{r})$: estimated by K^+p data



Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi, W. Weise. PRL124, 132501 (2020)

Correlation function is well reproduced

Contents

- **Part I : Introduction**
  - What are “exotic hadrons”?
  - Hadronic molecules and universality
  - Unstable resonances
- **Part II : Structure of $\Lambda(1405)$ resonance**
  - $\bar{K}N$ scattering and $\Lambda(1405)$ poles

    Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012)
  - Compositeness of $\Lambda(1405)$

    Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)

Compositeness of hadrons

- **Structure of a given resonance (pole)?**
- **Weak binding relation for stable bound states**

S. Weinberg, Phys. Rev. 137, B672 (1965)

Compositeness X
threshold channel

or

"Elementariness" Z
other contributions

↑
observables (a_0, B)

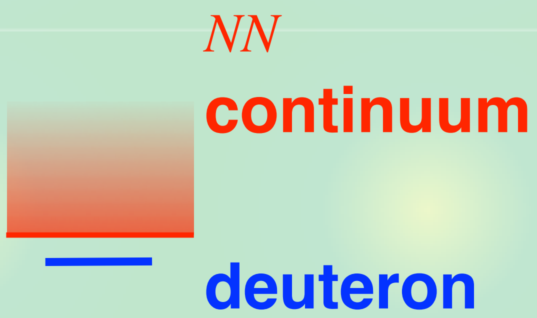
- **Effective field theory —> description of low-energy scattering amplitude, generalization to unstable resonances**

Weak-binding relation for stable states

Compositeness X of s-wave **weakly bound** state $(R \gg R_{\rm typ})$

S. Weinberg, Phys. Rev. 137, B672 (1965);
T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)

$$|d\rangle = \sqrt{X}\,|NN\rangle + \sqrt{1-X}\,|\text{others}\rangle$$



range of interaction

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right)\right\}, \quad R = \frac{1}{\sqrt{2\mu B}}$$

scattering length (a_0) — radius of state (R)

- Deuteron is NN composite : $a_0 \sim R \Rightarrow X \sim 1$
- Internal structure from **observable** (a_0, B)

Problem: applicable only for stable states

Effective field theory

Low-energy scattering with near-threshold bound state

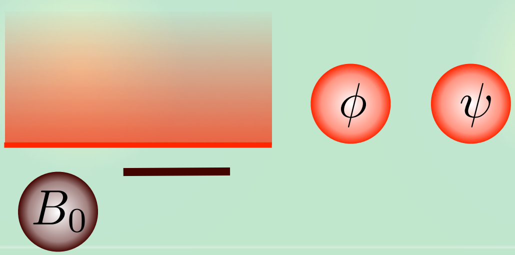
- **Nonrelativistic EFT with contact interaction**

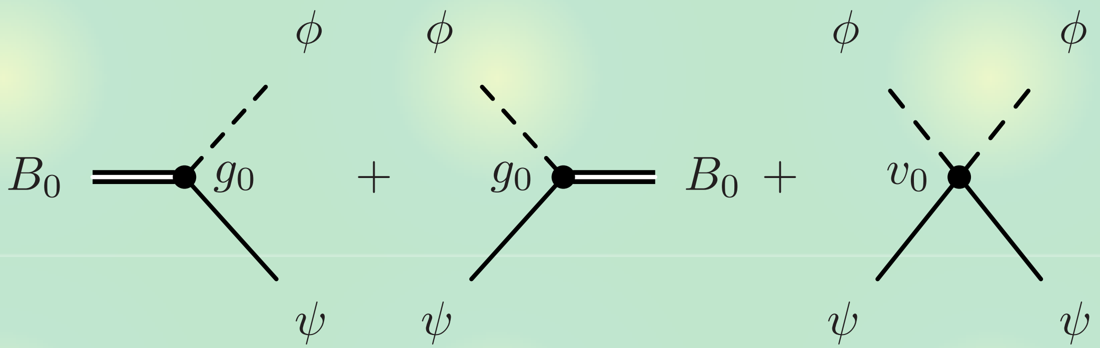
D.B. Kaplan, Nucl. Phys. B494, 471 (1997)

E. Braaten, M. Kusunoki, D. Zhang, Annals Phys. 323, 1770 (2008)

$$H_{\text{free}} = \int d\mathbf{r}\left[\frac{1}{2M}\nabla\psi^{\dagger}\cdot\nabla\psi + \frac{1}{2m}\nabla\phi^{\dagger}\cdot\nabla\phi + \frac{1}{2M_0}\nabla B_0^{\dagger}\cdot\nabla B_0 + \omega_0 B_0^{\dagger}B_0\right]$$

$$H_{\text{int}} = \int d\mathbf{r}\left[g_0\left(B_0^{\dagger}\phi\psi + \psi^{\dagger}\phi^{\dagger}B_0\right) + v_0\psi^{\dagger}\phi^{\dagger}\phi\psi\right]$$





- **Cutoff** : $\Lambda \sim 1/R_{\text{typ}}$ **(interaction range of microscopic theory)**
- **At low momentum** $p \ll \Lambda$**, interaction ~ contact**

Compositeness and “elementariness”

Eigenstates

$$H_{\rm free}|B_0\rangle = \omega_0|B_0\rangle, \quad H_{\rm free}|\mathbf{p}\rangle = \frac{\mathbf{p}^2}{2\mu}|\mathbf{p}\rangle$$

free (discrete + continuum)

$$(H_{\rm free} + H_{\rm int})|B\rangle = -B|B\rangle$$

full (bound state)

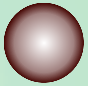
- Normalization of $|B\rangle$ + completeness relation

$$\langle B|B\rangle = 1, \quad 1 = |B_0\rangle\langle B_0| + \int \frac{d\mathbf{p}}{(2\pi)^3}|\mathbf{p}\rangle\langle\mathbf{p}|$$

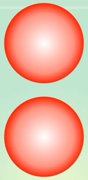
- Projections onto free eigenstates

$$1 = Z + X, \quad Z \equiv |\langle B_0|B\rangle|^2, \quad X \equiv \int \frac{d\mathbf{p}}{(2\pi)^3}|\langle\mathbf{p}|B\rangle|^2$$

“elementarity”



compositeness

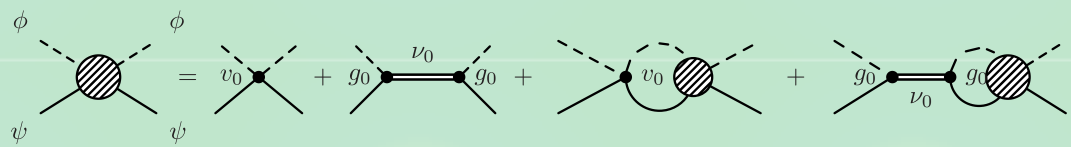


Z, X **: real and nonnegative —> interpreted as probability**

Weak binding relation

$\psi\phi$ scattering amplitude (exact result)

$$f(E) = -\frac{\mu}{2\pi}\frac{1}{[v(E)]^{-1} - G(E)}$$



$$v(E) = v_0 + \frac{g_0^2}{E - \omega_0}, \quad G(E) = \frac{1}{2\pi^2}\int_0^{\Lambda} dp \frac{p^2}{E - p^2/(2\mu) + i0^+}$$

Compositeness $X \leftarrow v(E), G(E)$

$$X = \frac{G'(-B)}{G'(-B) - [1/v(-B)]'}$$

$1/R = \sqrt{2\mu B}$ expansion of scattering length a_0

$$a_0 = -f(E=0) = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right)\right\}$$

renormalization dependent

renormalization independent

If $R \gg R_{\rm typ}$, correction terms neglected: $X \leftarrow (a_0, B)$

Inclusion of decay channel

Introduce decay channel

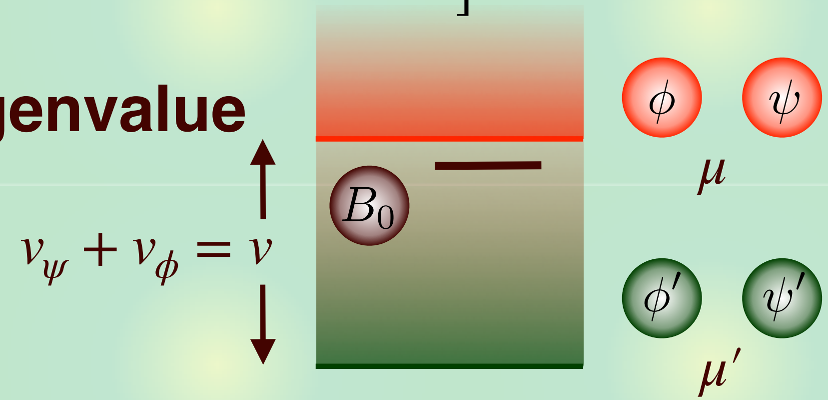
$$H'_{\text{free}} = \int d\mathbf{r} \left[\frac{1}{2M'} \nabla \psi'^{\dagger} \cdot \nabla \psi' - \nu_{\psi} \psi'^{\dagger} \psi' + \frac{1}{2m'} \nabla \phi'^{\dagger} \cdot \nabla \phi' - \nu_{\phi} \phi'^{\dagger} \phi' \right]$$

$$H'_{\text{int}} = \int d\mathbf{r} \left[g'_0 \left(B_0^{\dagger} \phi' \psi' + \psi'^{\dagger} \phi'^{\dagger} B_0 \right) + v'_0 \psi'^{\dagger} \phi'^{\dagger} \phi' \psi' + v_0^t (\psi^{\dagger} \phi^{\dagger} \phi' \psi' + \psi'^{\dagger} \phi'^{\dagger} \phi \psi) \right]$$

Quasi-bound state : complex eigenvalue

$$H = H_{\text{free}} + H'_{\text{free}} + H_{\text{int}} + H'_{\text{int}}$$

$$H|h\rangle = E_h |h\rangle, \quad E_h \in \mathbb{C}$$



Generalized relation : correction from threshold difference

$$a_0 = R \left\{ \frac{2X}{1+X} + \mathcal{O}\left(\left| \frac{R_{\text{typ}}}{R} \right| \right) + \underline{\mathcal{O}\left(\left| \frac{\ell}{R} \right|^3 \right)} \right\}, \quad R = \frac{1}{\sqrt{-2\mu E_h}}, \quad \ell \equiv \frac{1}{\sqrt{2\mu\nu}}$$

Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)

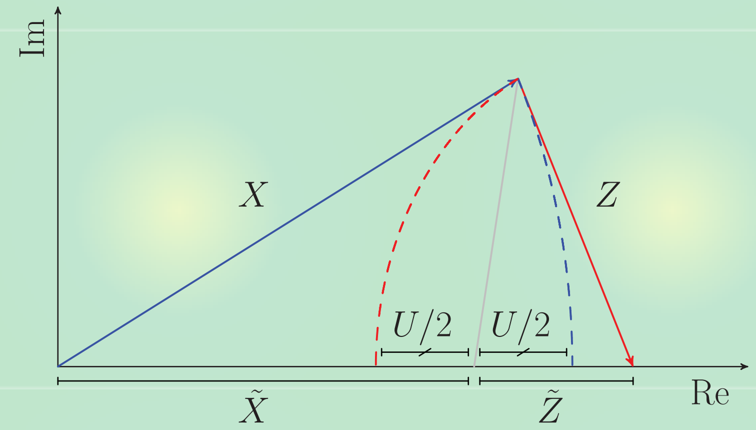
If $|R| \gg (R_{\text{typ}}, \ell)$, correction terms neglected: $X \leftarrow (a_0, E_h)$

Complex compositeness

Unstable states —> complex Z and X

$$Z + X = 1, \quad Z, X \in \mathbb{C}$$

- Probabilistic interpretation?



New definition

$$\tilde{Z} = \frac{1 - |X| + |Z|}{2}, \quad \tilde{X} = \frac{1 - |Z| + |X|}{2}$$

- Interpreted as probabilities $\tilde{Z} + \tilde{X} = 1, \quad \tilde{Z}, \tilde{X} \in [0, 1]$
- reduces to Z and X in the bound state limit

$U/2$: uncertainty of interpretation

$$U = |Z| + |X| - 1$$

c.f. T. Berggren, Phys. Lett. 33B, 547 (1970)

- Sensible interpretation only for small $U/2$ case

Evaluation of compositeness

Generalized weak-binding relation

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\rm typ}}{R}\right|\right) + \mathcal{O}\left(\left|\frac{\ell}{R}\right|^3\right)\right\}, \quad R = \frac{1}{\sqrt{-2\mu E_h}}, \quad \ell \equiv \frac{1}{\sqrt{2\mu\nu}}$$

(a_0, E_h) determinations by several groups

- Neglecting correction terms:

	E_h [MeV]	a_0 [fm]	$X_{\bar{K}N}$	$\tilde{X}_{\bar{K}N}$	$U/2$
Set 1 [35]	$-10 - i26$	$1.39 - i0.85$	$1.2 + i0.1$	1.0	0.3
Set 2 [36]	$-4 - i8$	$1.81 - i0.92$	$0.6 + i0.1$	0.6	0.0
Set 3 [37]	$-13 - i20$	$1.30 - i0.85$	$0.9 - i0.2$	0.9	0.1
Set 4 [38]	$2 - i10$	$1.21 - i1.47$	$0.6 + i0.0$	0.6	0.0
Set 5 [38]	$-3 - i12$	$1.52 - i1.85$	$1.0 + i0.5$	0.8	0.3

- In all cases, $X \sim 1$ with small $U/2$ (complex nature)

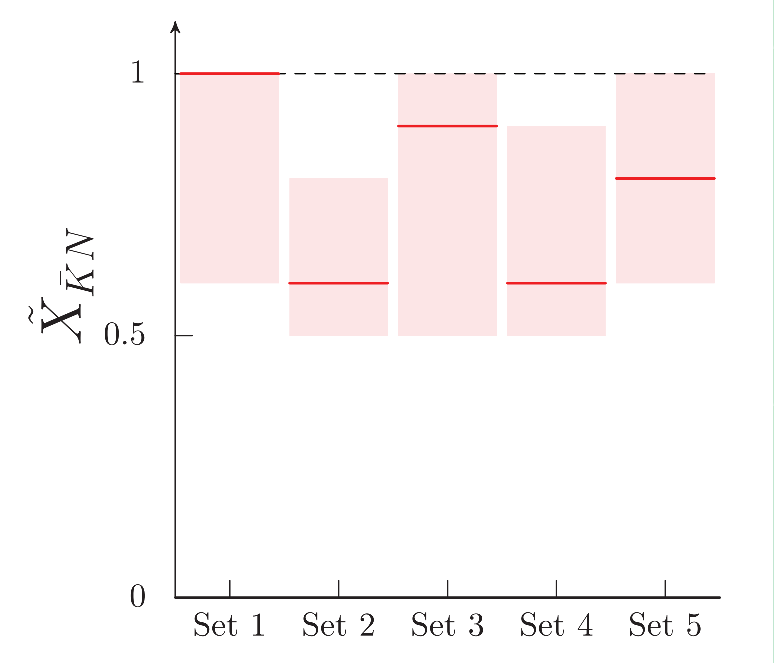
$\Lambda(1405)$: $\bar{K}N$ composite dominance <— observables

Uncertainty estimation

Estimation of correction terms: $|R| \sim 2$ fm

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\text{typ}}}{R}\right|\right) + \mathcal{O}\left(\left|\frac{\ell}{R}\right|^3\right)\right\}, \quad R = \frac{1}{\sqrt{-2\mu E_h}}, \quad \ell \equiv \frac{1}{\sqrt{2\mu\nu}}$$

- **ρ meson exchange picture:** $R_{\text{typ}} \sim 0.25$ fm
- **Energy difference from $\pi\Sigma$:** $\ell \sim 1.08$ fm



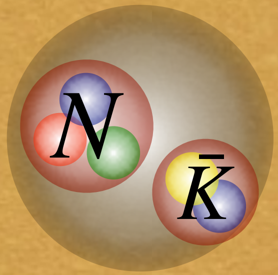
$\bar{K}N$ composite dominance holds even with correction terms.

Summary of part II

- **Pole structure of the $\Lambda(1405)$ region is now well constrained by the experimental data.**
  "$\Lambda(1405)$" **—>** $\Lambda(1405)$ **and** $\Lambda(1380)$

  Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012)

- **Compositeness of hadrons can be studied by observables through the weak-binding relation. Generalized weak-binding relation shows that (higher-energy) $\Lambda(1405)$ is dominated by $\bar{K}N$ molecular component.**



Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)