Dynamically generated hadron resonances

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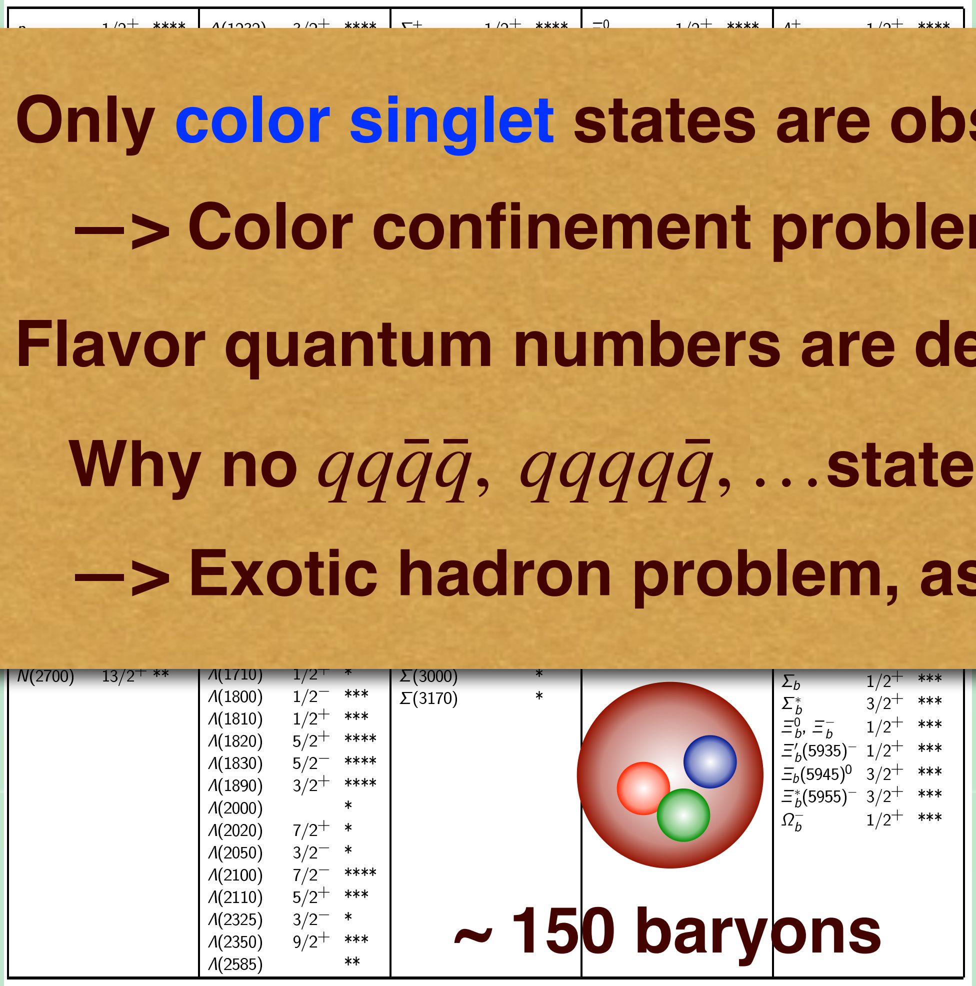
Contents

Classification of hadrons

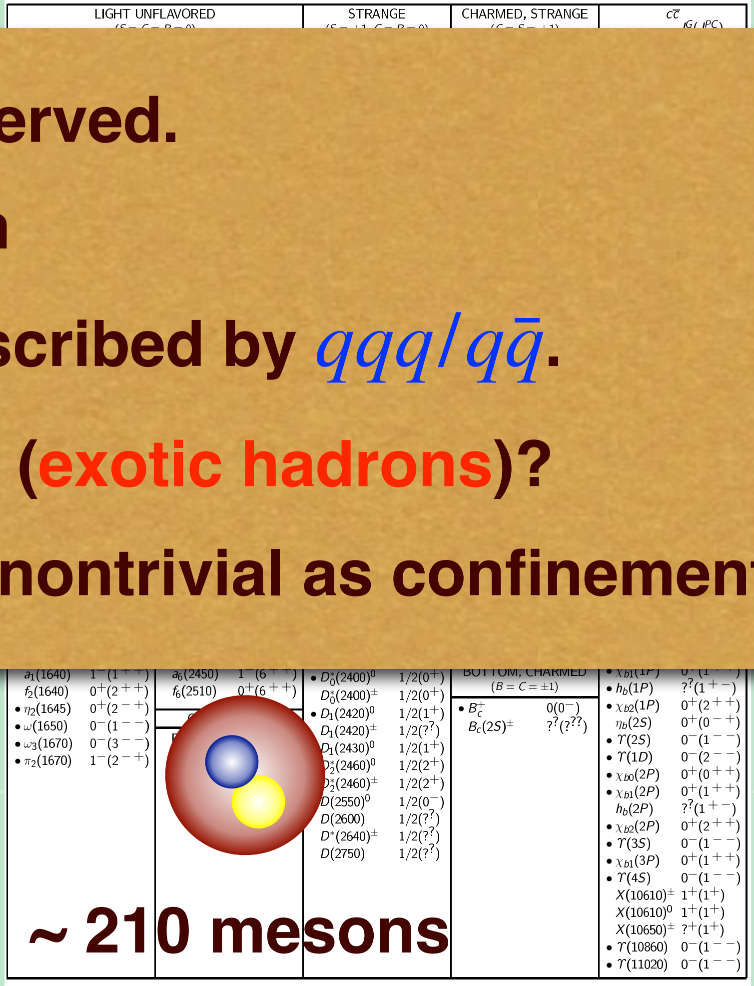
Observed hadrons

PDG2018 : http://pdg.lbl.gov/



$N(2700)$	$13/2^+$ **	$\Lambda(1710)$	$1/2^+$ *	$\Sigma(3000)$	*	Σ_b	$1/2^+$ ***
		$\Lambda(1800)$	$1/2^-$ ***	$\Sigma(3170)$	*	Σ_b^*	$3/2^+$ ***
		$\Lambda(1810)$	$1/2^+$ ***			Ξ_b^0, Ξ_b^-	$1/2^+$ ***
		$\Lambda(1820)$	$5/2^+$ ****			$\Xi'_b(5935)^-$	$1/2^+$ ***
		$\Lambda(1830)$	$5/2^-$ ****			$\Xi_b(5945)^0$	$3/2^+$ ***
		$\Lambda(1890)$	$3/2^+$ ****			$\Xi_b^*(5955)^-$	$3/2^+$ ***
		$\Lambda(2000)$	*			Ω_b^-	$1/2^+$ ***
		$\Lambda(2020)$	$7/2^+$ *				
		$\Lambda(2050)$	$3/2^-$ *				
		$\Lambda(2100)$	$7/2^-$ ****				
		$\Lambda(2110)$	$5/2^+$ ***				
		$\Lambda(2325)$	$3/2^-$ *				
		$\Lambda(2350)$	$9/2^+$ ***				
		$\Lambda(2585)$	**				

~ 150 baryons



LIGHT UNFLAVORED		STRANGE		CHARMED, STRANGE		$c\bar{c}$	
$a_1(1640)$	$1^-(1^{++})$	$a_6(2450)$	$1^-(6^{++})$	• $D_0^*(2400)^0$	$1/2(0^+)$	BOTTOM, CHARMED $(B = C = \pm1)$	
$f_2(1640)$	$0^+(2^{++})$	$f_6(2510)$	$0^+(6^{++})$	$D_0^*(2400)^\pm$	$1/2(0^+)$		
• $\eta_2(1645)$	$0^+(2^{-+})$			• $D_1(2420)^0$	$1/2(1^+)$	• B_c^+	$0(0^-)$
• $\omega(1650)$	$0^-(1^{--})$			$D_1(2420)^\pm$	$1/2(?^?)$	$B_c(2S)^\pm$	$?^?(?^{??})$
• $\omega_3(1670)$	$0^-(3^{--})$			$D_1(2430)^0$	$1/2(1^+)$		
• $\pi_2(1670)$	$1^-(2^{-+})$			$D_2^*(2460)^0$	$1/2(2^+)$		
				$D_2^*(2460)^\pm$	$1/2(2^+)$		
				$D(2550)^0$	$1/2(0^-)$		
				$D(2600)$	$1/2(?^?)$		
				$D^*(2640)^\pm$	$1/2(?^?)$		
				$D(2750)$	$1/2(?^?)$		

• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $h_b(1P)$	$?^?(1^{+-})$
• $\chi_{b2}(1P)$	$0^+(2^{++})$
$\eta_b(2S)$	$0^+(0^{-+})$
• $\Upsilon(2S)$	$0^-(1^{--})$
• $\Upsilon(1D)$	$0^-(2^{--})$
• $\chi_{b0}(2P)$	$0^+(0^{++})$
• $\chi_{b1}(2P)$	$0^+(1^{++})$
$h_b(2P)$	$?^?(1^{+-})$
• $\chi_{b2}(2P)$	$0^+(2^{++})$
• $\Upsilon(3S)$	$0^-(1^{--})$
• $\chi_{b1}(3P)$	$0^+(1^{++})$
• $\Upsilon(4S)$	$0^-(1^{--})$
$X(10610)^\pm$	$1^+(1^+)$
$X(10610)^0$	$1^+(1^+)$
$X(10650)^\pm$	$?^+(1^+)$
• $\Upsilon(10860)$	$0^-(1^{--})$
• $\Upsilon(11020)$	$0^-(1^{--})$

~ 210 mesons

Only color singlet states are observed.

—> Color confinement problem

Flavor quantum numbers are described by $qqq/q\bar{q}$.

Why no $qq\bar{q}\bar{q}$, $qqqq\bar{q}$, $\dots$states (exotic hadrons)?

—> Exotic hadron problem, as nontrivial as confinement!

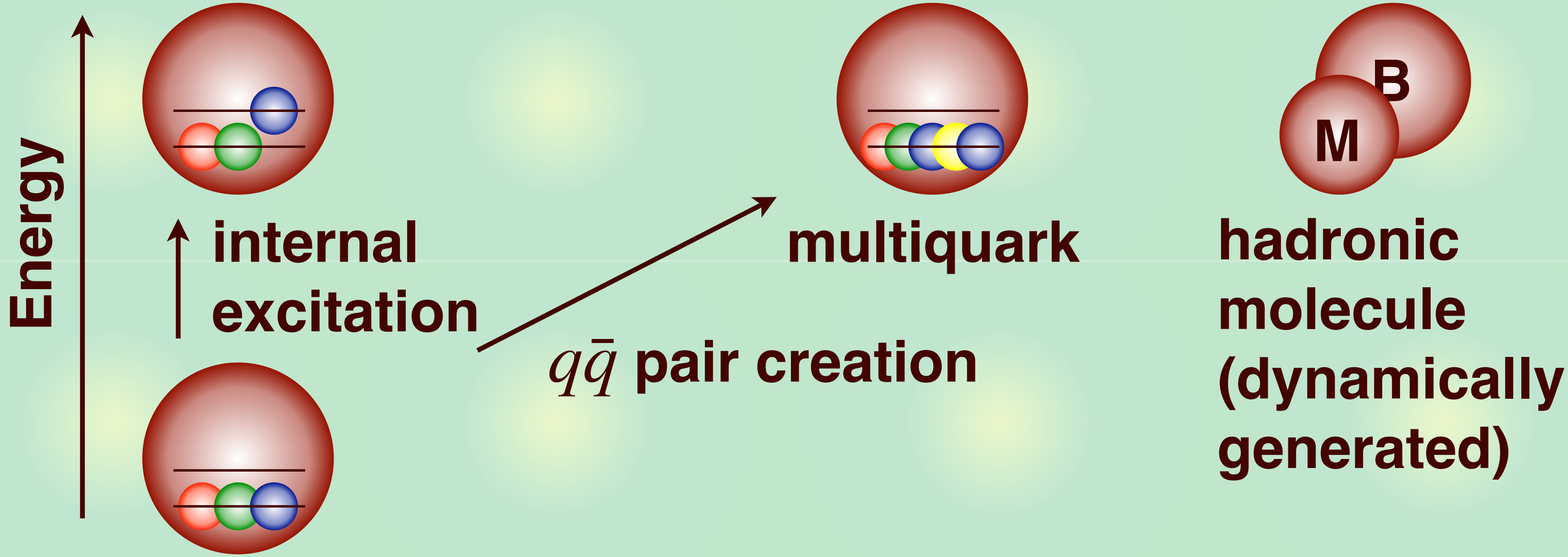
All ~ 360 hadrons emerge from single QCD Lagrangian.

Various hadronic excitations

Description of excited baryons

Conventional structure

Exotic structures



internal excitation

$q\bar{q}$ pair creation

multiquark

hadronic molecule (dynamically generated)

In QCD, non-qqq structures naturally arise.

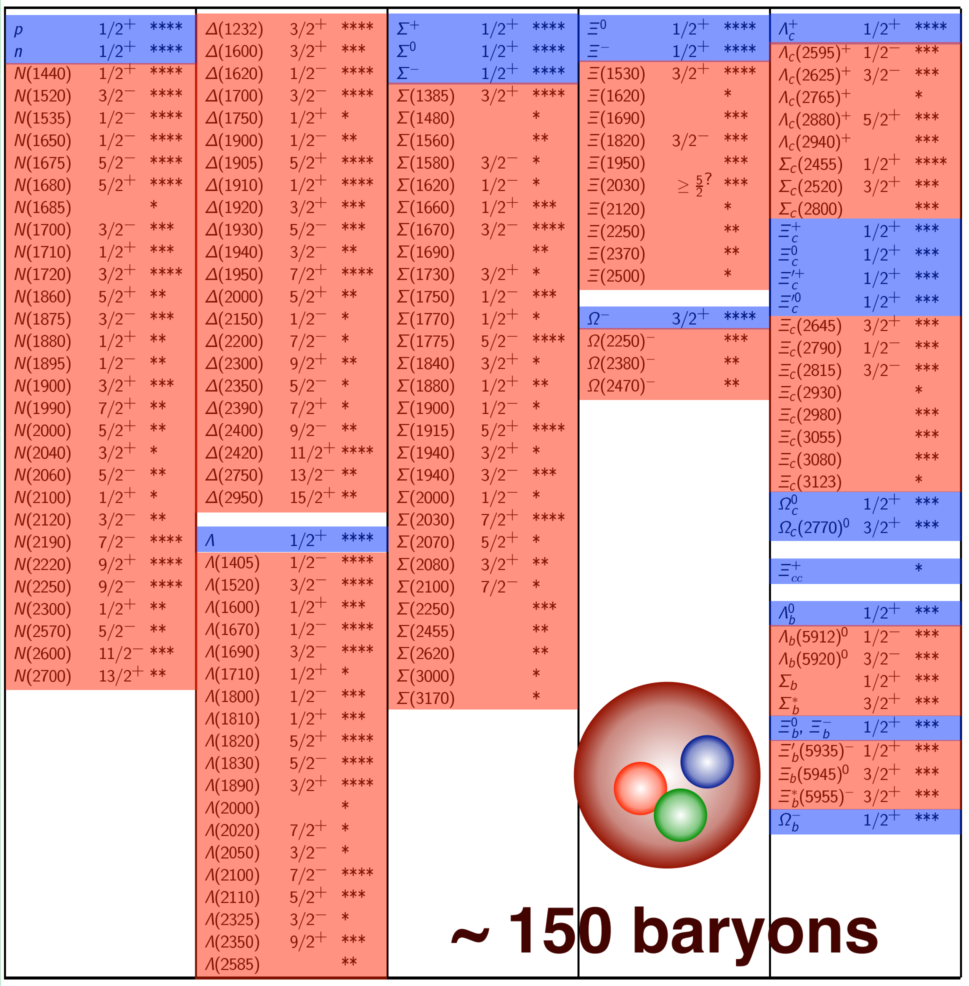
- Baryons: superposition of qqq + exotic structures

—> How can we **disentangle different components?**

Unstable states via strong interaction

Stable/unstable hadrons

PDG2018 : http://pdg.lbl.gov/



p	$1/2^+$	****
n	$1/2^+$	****
$N(1440)$	$1/2^+$	****
$N(1520)$	$3/2^-$	****
$N(1535)$	$1/2^-$	****
$N(1650)$	$1/2^-$	****
$N(1675)$	$5/2^-$	****
$N(1680)$	$5/2^+$	****
$N(1685)$		*
$N(1700)$	$3/2^-$	***
$N(1710)$	$1/2^+$	***
$N(1720)$	$3/2^+$	****
$N(1860)$	$5/2^+$	**
$N(1875)$	$3/2^-$	***
$N(1880)$	$1/2^+$	**
$N(1895)$	$1/2^-$	**
$N(1900)$	$3/2^+$	***
$N(1990)$	$7/2^+$	**
$N(2000)$	$5/2^+$	**
$N(2040)$	$3/2^+$	*
$N(2060)$	$5/2^-$	**
$N(2100)$	$1/2^+$	*
$N(2120)$	$3/2^-$	**
$N(2190)$	$7/2^-$	****
$N(2220)$	$9/2^+$	****
$N(2250)$	$9/2^-$	****
$N(2300)$	$1/2^+$	**
$N(2570)$	$5/2^-$	**
$N(2600)$	$11/2^-$	***
$N(2700)$	$13/2^+$	**

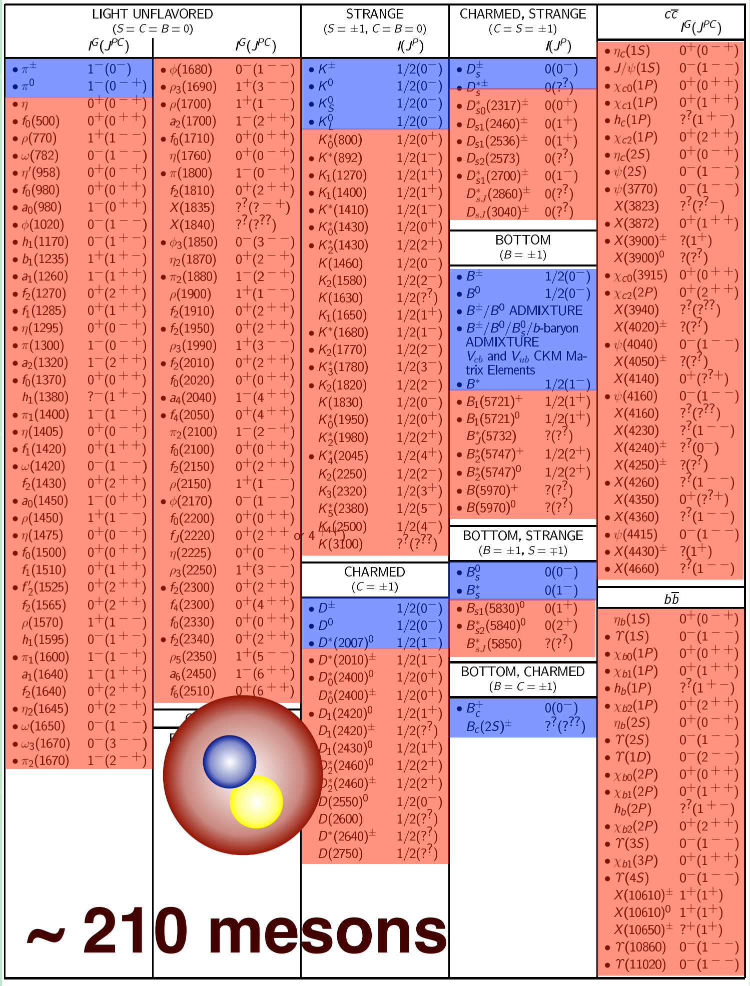
$\Delta(1232)$	$3/2^+$	****
$\Delta(1600)$	$3/2^+$	***
$\Delta(1620)$	$1/2^-$	****
$\Delta(1700)$	$3/2^-$	****
$\Delta(1750)$	$1/2^+$	*
$\Delta(1900)$	$1/2^-$	**
$\Delta(1905)$	$5/2^+$	****
$\Delta(1910)$	$1/2^+$	****
$\Delta(1920)$	$3/2^+$	***
$\Delta(1930)$	$5/2^-$	***
$\Delta(1940)$	$3/2^-$	**
$\Delta(1950)$	$7/2^+$	****
$\Delta(2000)$	$5/2^+$	**
$\Delta(2150)$	$1/2^-$	*
$\Delta(2200)$	$7/2^-$	*
$\Delta(2300)$	$9/2^+$	**
$\Delta(2350)$	$5/2^-$	*
$\Delta(2390)$	$7/2^+$	*
$\Delta(2400)$	$9/2^-$	**
$\Delta(2420)$	$11/2^+$	****
$\Delta(2750)$	$13/2^-$	**
$\Delta(2950)$	$15/2^+$	**
Λ	$1/2^+$	****
$\Lambda(1405)$	$1/2^-$	****
$\Lambda(1520)$	$3/2^-$	****
$\Lambda(1600)$	$1/2^+$	***
$\Lambda(1670)$	$1/2^-$	****
$\Lambda(1690)$	$3/2^-$	****
$\Lambda(1710)$	$1/2^+$	*
$\Lambda(1800)$	$1/2^-$	***
$\Lambda(1810)$	$1/2^+$	***
$\Lambda(1820)$	$5/2^+$	****
$\Lambda(1830)$	$5/2^-$	****
$\Lambda(1890)$	$3/2^+$	****
$\Lambda(2000)$		*
$\Lambda(2020)$	$7/2^+$	*
$\Lambda(2050)$	$3/2^-$	*
$\Lambda(2100)$	$7/2^-$	****
$\Lambda(2110)$	$5/2^+$	***
$\Lambda(2325)$	$3/2^-$	*
$\Lambda(2350)$	$9/2^+$	***
$\Lambda(2585)$		**

Σ^+	$1/2^+$	****
Σ^0	$1/2^+$	****
Σ^-	$1/2^+$	****
$\Sigma(1385)$	$3/2^+$	****
$\Sigma(1480)$		*
$\Sigma(1560)$		**
$\Sigma(1580)$	$3/2^-$	*
$\Sigma(1620)$	$1/2^-$	*
$\Sigma(1660)$	$1/2^+$	***
$\Sigma(1670)$	$3/2^-$	****
$\Sigma(1690)$		**
$\Sigma(1730)$	$3/2^+$	*
$\Sigma(1750)$	$1/2^-$	***
$\Sigma(1770)$	$1/2^+$	*
$\Sigma(1775)$	$5/2^-$	****
$\Sigma(1840)$	$3/2^+$	*
$\Sigma(1880)$	$1/2^+$	**
$\Sigma(1900)$	$1/2^-$	*
$\Sigma(1915)$	$5/2^+$	****
$\Sigma(1940)$	$3/2^+$	*
$\Sigma(1940)$	$3/2^-$	***
$\Sigma(2000)$	$1/2^-$	*
$\Sigma(2030)$	$7/2^+$	****
$\Sigma(2070)$	$5/2^+$	*
$\Sigma(2080)$	$3/2^+$	**
$\Sigma(2100)$	$7/2^-$	*
$\Sigma(2250)$		***
$\Sigma(2455)$		**
$\Sigma(2620)$		**
$\Sigma(3000)$		*
$\Sigma(3170)$		*

Ξ^0	$1/2^+$	****
Ξ^-	$1/2^+$	****
$\Xi(1530)$	$3/2^+$	****
$\Xi(1620)$		*
$\Xi(1690)$		***
$\Xi(1820)$	$3/2^-$	***
$\Xi(1950)$		***
$\Xi(2030)$	$\geq \frac{5}{2}^?$	***
$\Xi(2120)$		*
$\Xi(2250)$		**
$\Xi(2370)$		**
$\Xi(2500)$		*
Ω^-	$3/2^+$	****
$\Omega(2250)^-$		***
$\Omega(2380)^-$		**
$\Omega(2470)^-$		**

Λ_c^+	$1/2^+$	****
$\Lambda_c(2595)^+$	$1/2^-$	***
$\Lambda_c(2625)^+$	$3/2^-$	***
$\Lambda_c(2765)^+$		*
$\Lambda_c(2880)^+$	$5/2^+$	***
$\Lambda_c(2940)^+$		***
$\Sigma_c(2455)$	$1/2^+$	****
$\Sigma_c(2520)$	$3/2^+$	***
$\Sigma_c(2800)$		***
Ξ_c^+	$1/2^+$	***
Ξ_c^0	$1/2^+$	***
$\Xi_c'^+$	$1/2^+$	***
$\Xi_c'^0$	$1/2^+$	***
$\Xi_c(2645)$	$3/2^+$	***
$\Xi_c(2790)$	$1/2^-$	***
$\Xi_c(2815)$	$3/2^-$	***
$\Xi_c(2930)$		*
$\Xi_c(2980)$		***
$\Xi_c(3055)$		***
$\Xi_c(3080)$		***
$\Xi_c(3123)$		*
Ω_c^0	$1/2^+$	***
$\Omega_c(2770)^0$	$3/2^+$	***
Ξ_{cc}^+		*
Λ_b^0	$1/2^+$	***
$\Lambda_b(5912)^0$	$1/2^-$	***
$\Lambda_b(5920)^0$	$3/2^-$	***
Σ_b	$1/2^+$	***
Σ_b^*	$3/2^+$	***
Ξ_b^0, Ξ_b^-	$1/2^+$	***
$\Xi_b'(5935)^-$	$1/2^+$	***
$\Xi_b(5945)^0$	$3/2^+$	***
$\Xi_b^*(5955)^-$	$3/2^+$	***
Ω_b^-	$1/2^+$	***

~ 150 baryons



LIGHT UNFLAVORED ($S = C = B = 0$)

	$I^G(J^{PC})$		$I^G(J^{PC})$
• $\pi^\pm$	$1^-(0^-)$	• $\phi(1680)$	$0^-(1^{--})$
• π^0	$1^-(0^{-+})$	• $\rho_3(1690)$	$1^+(3^{--})$
• η	$0^+(0^{-+})$	• $\rho(1700)$	$1^+(1^{--})$
• $f_0(500)$	$0^+(0^{++})$	$a_2(1700)$	$1^-(2^{++})$
• $\rho(770)$	$1^+(1^{--})$	• $f_0(1710)$	$0^+(0^{++})$
• $\omega(782)$	$0^-(1^{--})$	$\eta(1760)$	$0^+(0^{-+})$
• $\eta'(958)$	$0^+(0^{-+})$	• $\pi(1800)$	$1^-(0^{-+})$
• $f_0(980)$	$0^+(0^{++})$	$f_2(1810)$	$0^+(2^{++})$
• $a_0(980)$	$1^-(0^{++})$	$X(1835)$	$?^?(?^{-+})$
• $\phi(1020)$	$0^-(1^{--})$	$X(1840)$	$?^?(?^{??})$
• $h_1(1170)$	$0^-(1^{+-})$	• $\phi_3(1850)$	$0^-(3^{--})$
• $b_1(1235)$	$1^+(1^{+-})$	$\eta_2(1870)$	$0^+(2^{-+})$
• $a_1(1260)$	$1^-(1^{++})$	• $\pi_2(1880)$	$1^-(2^{-+})$
• $f_2(1270)$	$0^+(2^{++})$	$\rho(1900)$	$1^+(1^{--})$
• $f_1(1285)$	$0^+(1^{++})$	$f_2(1910)$	$0^+(2^{++})$
• $\eta(1295)$	$0^+(0^{-+})$	• $f_2(1950)$	$0^+(2^{++})$
• $\pi(1300)$	$1^-(0^{-+})$	$\rho_3(1990)$	$1^+(3^{--})$
• $a_2(1320)$	$1^-(2^{++})$	• $f_2(2010)$	$0^+(2^{++})$
• $f_0(1370)$	$0^+(0^{++})$	$f_0(2020)$	$0^+(0^{++})$
$h_1(1380)$	$?^-(1^{+-})$	• $a_4(2040)$	$1^-(4^{++})$
• $\pi_1(1400)$	$1^-(1^{-+})$	• $f_4(2050)$	$0^+(4^{++})$
• $\eta(1405)$	$0^+(0^{-+})$	$\pi_2(2100)$	$1^-(2^{-+})$
• $f_1(1420)$	$0^+(1^{++})$	$f_0(2100)$	$0^+(0^{++})$
• $\omega(1420)$	$0^-(1^{--})$	$f_2(2150)$	$0^+(2^{++})$
$f_2(1430)$	$0^+(2^{++})$	$\rho(2150)$	$1^+(1^{--})$
• $a_0(1450)$	$1^-(0^{++})$	• $\phi(2170)$	$0^-(1^{--})$
• $\rho(1450)$	$1^+(1^{--})$	$f_0(2200)$	$0^+(0^{++})$
• $\eta(1475)$	$0^+(0^{-+})$	$f_J(2220)$	$0^+(2^{++}$ or $4^{++})$
• $f_0(1500)$	$0^+(0^{++})$	$\eta(2225)$	$0^+(0^{-+})$
$f_1(1510)$	$0^+(1^{++})$	$\rho_3(2250)$	$1^+(3^{--})$
• $f_2'(1525)$	$0^+(2^{++})$	• $f_2(2300)$	$0^+(2^{++})$
$f_2(1565)$	$0^+(2^{++})$	$f_4(2300)$	$0^+(4^{++})$
$\rho(1570)$	$1^+(1^{--})$	$f_0(2330)$	$0^+(0^{++})$
$h_1(1595)$	$0^-(1^{+-})$	• $f_2(2340)$	$0^+(2^{++})$
• $\pi_1(1600)$	$1^-(1^{-+})$	$\rho_5(2350)$	$1^+(5^{--})$
$a_1(1640)$	$1^-(1^{++})$	$a_6(2450)$	$1^-(6^{++})$
$f_2(1640)$	$0^+(2^{++})$	$f_6(2510)$	$0^+(6^{++})$
• $\eta_2(1645)$	$0^+(2^{-+})$		
• $\omega(1650)$	$0^-(1^{--})$		
• $\omega_3(1670)$	$0^-(3^{--})$		
• $\pi_2(1670)$	$1^-(2^{-+})$		

STRANGE ($S = \pm1$, $C = B = 0$)

	$I(J^P)$
• $K^\pm$	$1/2(0^-)$
• K^0	$1/2(0^-)$
• K_S^0	$1/2(0^-)$
• K_L^0	$1/2(0^-)$
$K_0^*(800)$	$1/2(0^+)$
• $K^*(892)$	$1/2(1^-)$
• $K_1(1270)$	$1/2(1^+)$
• $K_1(1400)$	$1/2(1^+)$
• $K^*(1410)$	$1/2(1^-)$
• $K_0^*(1430)$	$1/2(0^+)$
• $K_2^*(1430)$	$1/2(2^+)$
$K(1460)$	$1/2(0^-)$
$K_2(1580)$	$1/2(2^-)$
$K(1630)$	$1/2(?^?)$
$K_1(1650)$	$1/2(1^+)$
• $K^*(1680)$	$1/2(1^-)$
• $K_2(1770)$	$1/2(2^-)$
• $K_3^*(1780)$	$1/2(3^-)$
• $K_2(1820)$	$1/2(2^-)$
$K(1830)$	$1/2(0^-)$
$K_0^*(1950)$	$1/2(0^+)$
$K_2^*(1980)$	$1/2(2^+)$
• $K_4^*(2045)$	$1/2(4^+)$
$K_2(2250)$	$1/2(2^-)$
$K_3(2320)$	$1/2(3^+)$
$K_5^*(2380)$	$1/2(5^-)$
$K_4(2500)$	$1/2(4^-)$
$K(3100)$	$?^?(?^{??})$

CHARMED ($C = \pm1$)

• $D^\pm$	$1/2(0^-)$
• D^0	$1/2(0^-)$
• $D^*(2007)^0$	$1/2(1^-)$
• $D^*(2010)^\pm$	$1/2(1^-)$
• $D_0^*(2400)^0$	$1/2(0^+)$
$D_0^*(2400)^\pm$	$1/2(0^+)$
• $D_1(2420)^0$	$1/2(1^+)$
$D_1(2420)^\pm$	$1/2(?^?)$
$D_1(2430)^0$	$1/2(1^+)$
• $D_2^*(2460)^0$	$1/2(2^+)$
• $D_2^*(2460)^\pm$	$1/2(2^+)$
$D(2550)^0$	$1/2(0^-)$
$D(2600)$	$1/2(?^?)$
$D^*(2640)^\pm$	$1/2(?^?)$
$D(2750)$	$1/2(?^?)$

CHARMED, STRANGE ($C = S = \pm1$)

	$I(J^P)$
• $D_s^\pm$	$0(0^-)$
• $D_s^{*\pm}$	$0(?^?)$
• $D_{s0}^*(2317)^\pm$	$0(0^+)$
• $D_{s1}(2460)^\pm$	$0(1^+)$
• $D_{s1}(2536)^\pm$	$0(1^+)$
• $D_{s2}(2573)$	$0(?^?)$
• $D_{s1}^*(2700)^\pm$	$0(1^-)$
$D_{sJ}^*(2860)^\pm$	$0(?^?)$
$D_{sJ}(3040)^\pm$	$0(?^?)$

BOTTOM ($B = \pm1$)

• $B^\pm$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^\pm/B^0$ ADMIXTURE	
• $B^\pm/B^0/B_s^0$/b-baryon ADMIXTURE	
V_{cb} and V_{ub} CKM Matrix Elements	
• B^*	$1/2(1^-)$
• $B_1(5721)^+$	$1/2(1^+)$
• $B_1(5721)^0$	$1/2(1^+)$
$B_J^*(5732)$	$?(?^?)$
• $B_2^*(5747)^+$	$1/2(2^+)$
• $B_2^*(5747)^0$	$1/2(2^+)$
• $B(5970)^+$	$?(?^?)$
• $B(5970)^0$	$?(?^?)$

BOTTOM, STRANGE ($B = \pm1$, $S = \mp1$)

• B_s^0	$0(0^-)$
• B_s^*	$0(1^-)$
• $B_{s1}(5830)^0$	$0(1^+)$
• $B_{s2}^*(5840)^0$	$0(2^+)$
$B_{sJ}^*(5850)$	$?(?^?)$

BOTTOM, CHARMED ($B = C = \pm1$)

• B_c^+	$0(0^-)$
$B_c(2S)^\pm$	$?^?(?^{??})$

$c\overline{c}$

	$I^G(J^{PC})$
• $\eta_c(1S)$	$0^+(0^{-+})$
• $J/\psi(1S)$	$0^-(1^{--})$
• $\chi_{c0}(1P)$	$0^+(0^{++})$
• $\chi_{c1}(1P)$	$0^+(1^{++})$
• $h_c(1P)$	$?^?(1^{+-})$
• $\chi_{c2}(1P)$	$0^+(2^{++})$
• $\eta_c(2S)$	$0^+(0^{-+})$
• $\psi(2S)$	$0^-(1^{--})$
• $\psi(3770)$	$0^-(1^{--})$
$X(3823)$	$?^?(?^{?-})$
• $X(3872)$	$0^+(1^{++})$
• $X(3900)^\pm$	$?(1^+)$
$X(3900)^0$	$?(?^?)$
• $\chi_{c0}(3915)$	$0^+(0^{++})$
• $\chi_{c2}(2P)$	$0^+(2^{++})$
$X(3940)$	$?^?(?^{??})$
$X(4020)^\pm$	$?(?^?)$
• $\psi(4040)$	$0^-(1^{--})$
$X(4050)^\pm$	$?(?^?)$
$X(4140)$	$0^+(?^{?+})$
• $\psi(4160)$	$0^-(1^{--})$
$X(4160)$	$?^?(?^{??})$
$X(4230)$	$?^?(1^{--})$
$X(4240)^\pm$	$?^?(0^-)$
$X(4250)^\pm$	$?(?^?)$
• $X(4260)$	$?^?(1^{--})$
$X(4350)$	$0^+(?^{?+})$
• $X(4360)$	$?^?(1^{--})$
• $\psi(4415)$	$0^-(1^{--})$
• $X(4430)^\pm$	$?(1^+)$
• $X(4660)$	$?^?(1^{--})$

$b\overline{b}$

$\eta_b(1S)$	$0^+(0^{-+})$
• $\Upsilon(1S)$	$0^-(1^{--})$
• $\chi_{b0}(1P)$	$0^+(0^{++})$
• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $h_b(1P)$	$?^?(1^{+-})$
• $\chi_{b2}(1P)$	$0^+(2^{++})$
$\eta_b(2S)$	$0^+(0^{-+})$
• $\Upsilon(2S)$	$0^-(1^{--})$
• $\Upsilon(1D)$	$0^-(2^{--})$
• $\chi_{b0}(2P)$	$0^+(0^{++})$
• $\chi_{b1}(2P)$	$0^+(1^{++})$
$h_b(2P)$	$?^?(1^{+-})$
• $\chi_{b2}(2P)$	$0^+(2^{++})$
• $\Upsilon(3S)$	$0^-(1^{--})$
• $\chi_{b1}(3P)$	$0^+(1^{++})$
• $\Upsilon(4S)$	$0^-(1^{--})$
$X(10610)^\pm$	$1^+(1^+)$
$X(10610)^0$	$1^+(1^+)$
$X(10650)^\pm$	$?^+(1^+)$
• $\Upsilon(10860)$	$0^-(1^{--})$
• $\Upsilon(11020)$	$0^-(1^{--})$

~ 210 mesons

Most of hadrons are unstable (above two-hadron threshold)

Nature of resonances

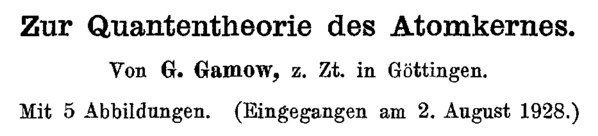
Theoretical treatment for unstable hadrons

- resonances in hadron scattering
- Above threshold, quark model does not work.
- Solve scattering equation (dynamical calculation)

Resonance as an “eigenstate” of Hamiltonian

- complex energy

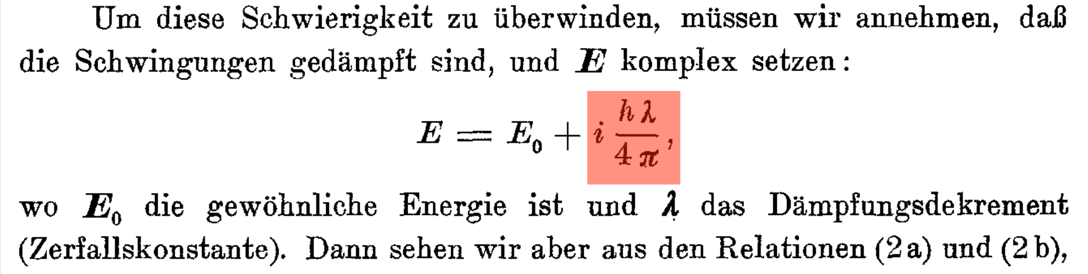
G. Gamow, Z. Phys. 51, 204 (1928)



Zur Quantentheorie des Atomkernes.

Von **G. Gamow**, z. Zt. in Göttingen.

Mit 5 Abbildungen. (Eingegangen am 2. August 1928.)

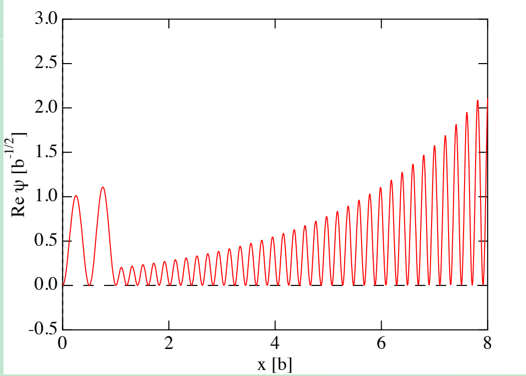


Um diese Schwierigkeit zu überwinden, müssen wir annehmen, daß die Schwingungen gedämpft sind, und E komplex setzen:

$$E = E_0 + i\frac{h\lambda}{4\pi},$$

wo E_0 die gewöhnliche Energie ist und λ das Dämpfungsdekrement (Zerfallskonstante). Dann sehen wir aber aus den Relationen (2a) und (2b),

- diverging wave function
- complex expectation value (norm, $\langle r^2 \rangle$)
- Interpretation is difficult.

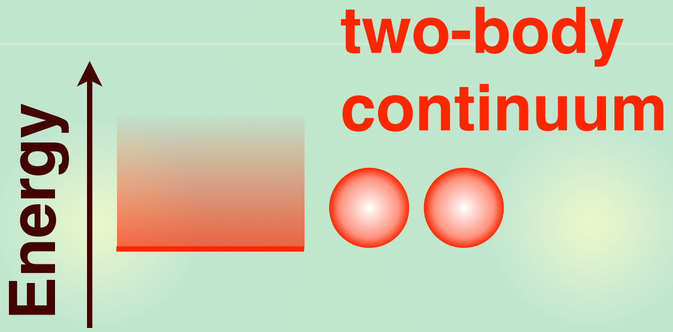


Dynamically generated states

Dynamical calculation of two-hadron scattering

model space

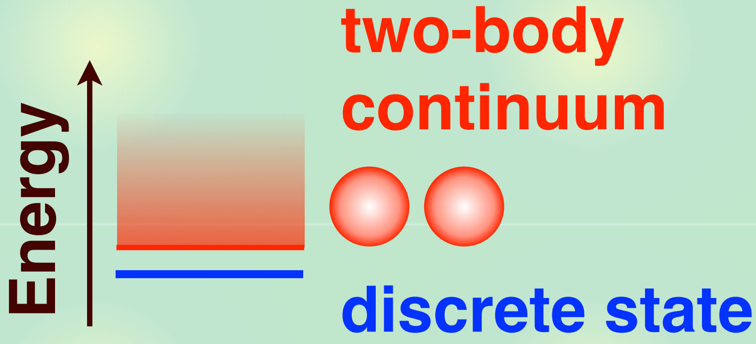
- Eigenstates of H_0 (and V)
- Bare fields (and interaction)



nonperturbative (Schrödinger/LS) equation

physical states

- Eigenstates of $H_0 + V$
- Spectral function

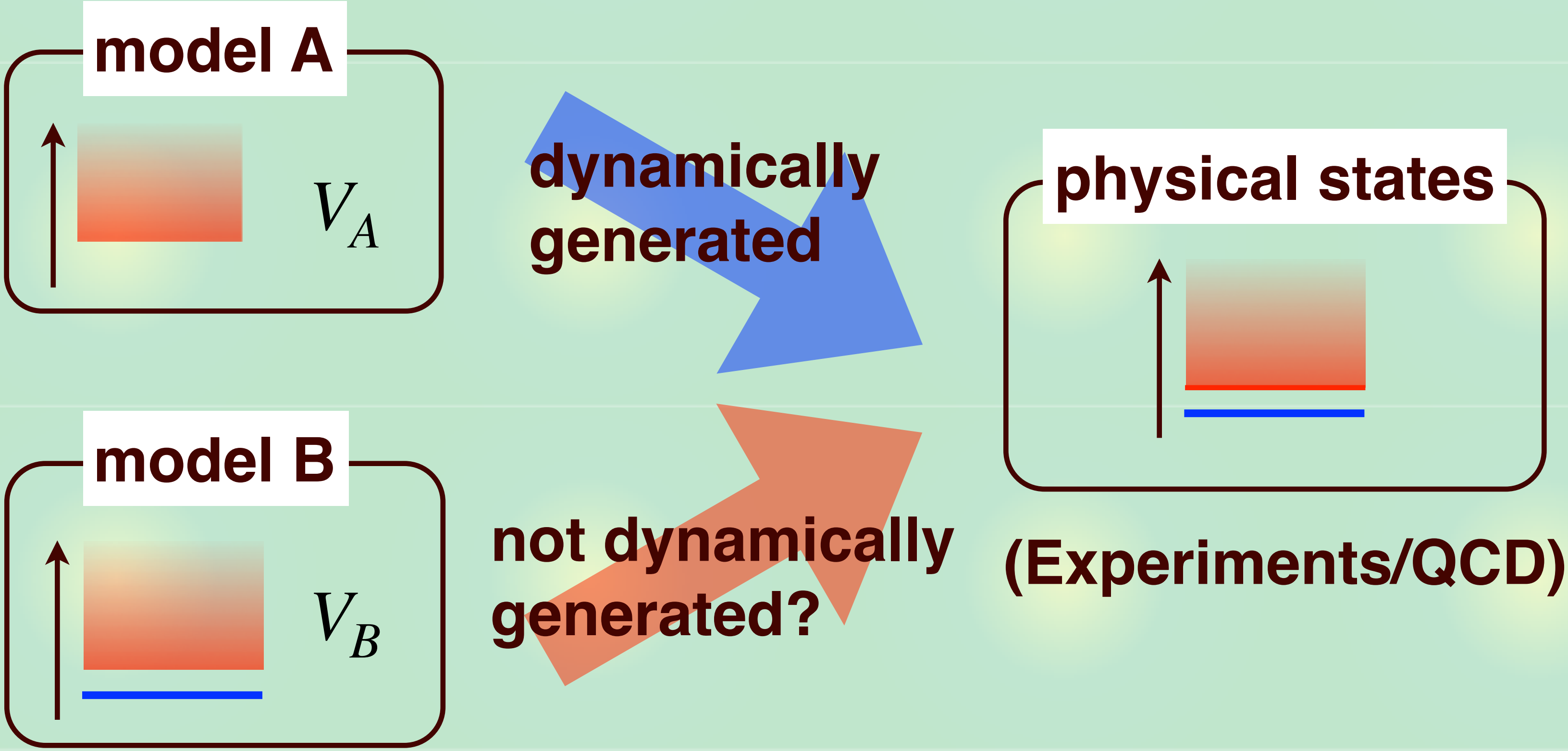


Strong attraction can give additional states (e.g. NN and d)

Additional discrete state is “dynamically generated.”

Dynamically generated states?

Q: Which hadron is dynamically generated?



- Model B equivalent to model A can always be constructed.

S. Weinberg, Phys. Rev. 130, 776 (1963)

Comparison with data alone is not conclusive.

Introduction to compositeness

One way to quantify it: compositeness X

- weak-binding relation

S. Weinberg, Phys. Rev. 137, B672 (1965) d

- integration of spectral density

V. Baru, *et al.*, Phys. Lett. B586, 53 (2004) $f_0(980), a_0(980)$

- evaluation of compositeness at pole (complex number)

T. Hyodo, D. Jido, A. Hosaka Phys. Rev. C 85, 015201 (2012)

F. Aceti, E. Oset, Phys. Rev. D 86, 014012 (2012) ρ

T. Sekihara, T. Hyodo, Phys. Rev. C 87, 045202 (2013) $\Lambda(1405), \sigma, f_0(980), a_0(980)$

C. W. Xiao, F. Aceti, M. Bayar, Eur. Phys. J. A 49, 22 (2013) K^*

T. Hyodo, Phys. Rev. Lett. 111, 2132002 (2013) $\Lambda_c(2595)$

F. Aceti, L. Dai, L. Geng, E. Oset, Y. Zhang, Eur. Phys. J. A 50, 57 (2014) $\Delta, \Sigma^*, \Xi^*, \Omega$

+ many others

Relation to observable (on-shell quantity)?

Compositeness of hadrons

- Structure of unstable state is nontrivial.
- Compositeness $0 \leq X \leq 1$

  - weight of dynamically generated component

  $$|\,\mathrm{state}\rangle = \sqrt{X}\,|\,\mathrm{dynamically\ \ generated}\rangle + \sqrt{1-X}\,|\,\mathrm{others}\rangle$$

  - fully dynamically generated: $X = 1$

  - weak-binding relation: $X \leftarrow$ observables

  S. Weinberg, Phys. Rev. 137, B672 (1965)

- generalization to unstable resonances

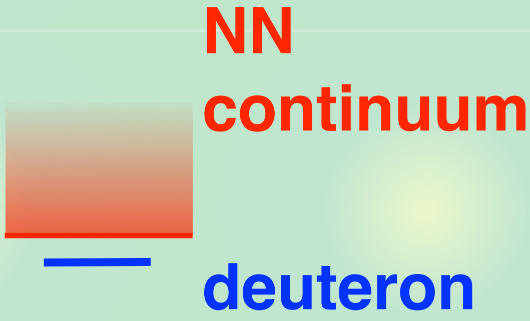
  Y. Kamiya, T. Hyodo, Phys. Rev. C93, 035203 (2016);
  Y. Kamiya, T. Hyodo, PTEP2017, 023D02 (2017)

Weak-binding relation for stable states

Compositeness X of s-wave weakly bound state $(R \gg R_{\rm typ})$

S. Weinberg, Phys. Rev. 137, B672 (1965);
T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)

$$|d\rangle = \sqrt{X}\,|NN\rangle + \sqrt{1-X}\,|\text{others}\rangle$$


NN continuum

deuteron

range of interaction

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right)\right\}, \quad R = \frac{1}{\sqrt{2\mu B}}$$

scattering length

radius of state

- Deuteron is NN composite: $a_0 \sim R \Rightarrow X \sim 1$
- Internal structure from observable (a_0, B)

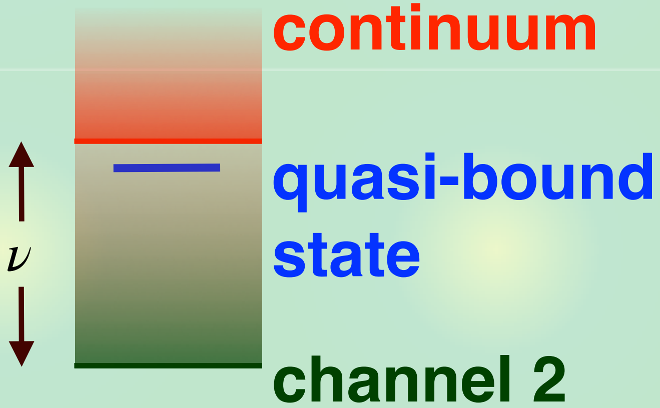
Problem: applicable only for stable states

Generalization to unstable hadron resonance

Generalized weak-binding relation

Y. Kamiya, T. Hyodo, Phys. Rev. C93, 035203 (2016);
Y. Kamiya, T. Hyodo, PTEP2017, 023D02 (2017)

$$|h\rangle = \sqrt{X}\,|\text{channel 1}\rangle + \sqrt{1-X}\,|\text{others}\rangle$$



range of interaction

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\text{typ}}}{R}\right|\right) + \underline{\mathcal{O}\left(\left|\frac{\ell}{R}\right|^3\right)}\right\}, \quad R = \frac{1}{\sqrt{-2\mu E_h}}$$

scattering length (complex) **radius of state (complex)**

- new correction term: scale of threshold difference

$$\ell \equiv \frac{1}{\sqrt{2\mu\nu}}$$

Compositeness $X \leftarrow$ **observables** (a_0, E_h) **when** $|R| \gg (R_{\text{typ}}, \ell)$

Evaluation of compositeness

Application: $\bar{K}N$ component in $\Lambda(1405)$

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\text{typ}}}{R}\right|\right) + \mathcal{O}\left(\left|\frac{\ell}{R}\right|^3\right)\right\}, \quad R = \frac{1}{\sqrt{-2\mu E_h}}, \quad \ell \equiv \frac{1}{\sqrt{2\mu\nu}}$$

(a_0, E_h) : determinations by several groups adopted in PDG

- neglecting correction terms:

	E_h [MeV]	a_0 [fm]	$X_{\bar{K}N}$	$\tilde{X}_{\bar{K}N}$	$U/2$
Set 1 [35]	$-10-i26$	$1.39-i0.85$	$1.2+i0.1$	1.0	0.3
Set 2 [36]	$-\ \ 4-i\ \ 8$	$1.81-i0.92$	$0.6+i0.1$	0.6	0.0
Set 3 [37]	$-13-i20$	$1.30-i0.85$	$0.9-i0.2$	0.9	0.1
Set 4 [38]	$2-i10$	$1.21-i1.47$	$0.6+i0.0$	0.6	0.0
Set 5 [38]	$-\ \ 3-i12$	$1.52-i1.85$	$1.0+i0.5$	0.8	0.3

- In all cases, $X \sim 1$

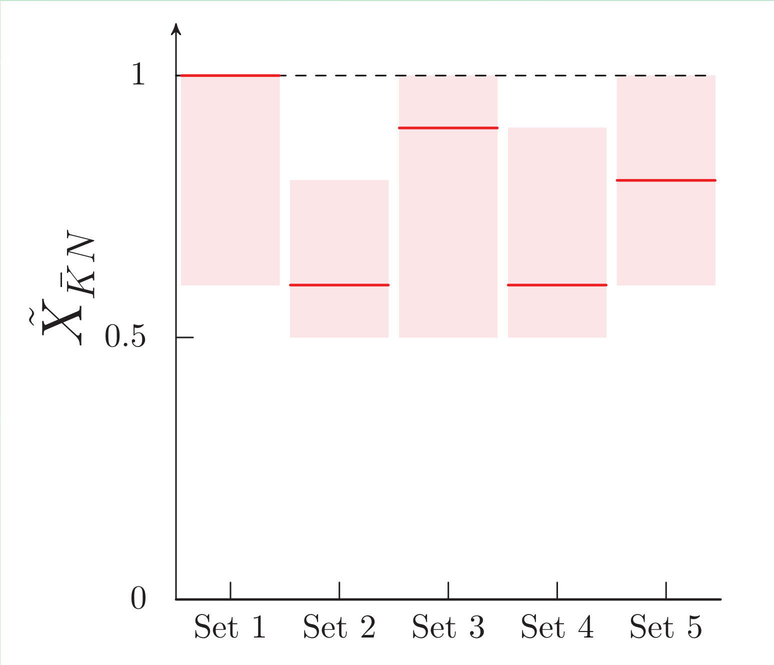
$\Lambda(1405)$: dominated by dynamically generated $\bar{K}N$

Uncertainty estimation

Estimation of correction terms: $|R| \sim 2\ \mathrm{fm}$

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\mathrm{typ}}}{R}\right|\right) + \mathcal{O}\left(\left|\frac{\ell}{R}\right|^3\right)\right\}, \quad R = \frac{1}{\sqrt{-2\mu E_h}}, \quad \ell \equiv \frac{1}{\sqrt{2\mu\nu}}$$

- **ρ meson exchange picture:** $R_{\mathrm{typ}} \sim 0.25\ \mathrm{fm}$
- **energy difference from $\pi\Sigma$:** $\ell \sim 1.08\ \mathrm{fm}$



$\bar{K}N$ composite dominance holds even with correction terms.

Summary

- **Nonperturbtative calculation can dynamically generate hadrons in addition to those in the model space.**
- **Compositeness: quantitative measure of "dynamically generated" nature**

  S. Weinberg, Phys. Rev. 137, B672 (1965);
  T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)
- **Generalized weak-binding relation shows that high-mass pole of $\Lambda(1405)$ is dominated by dynamically generated $\bar{K}N$ component.**

  Y. Kamiya, T. Hyodo, Phys. Rev. C93, 035203 (2016);
  Y. Kamiya, T. Hyodo, PTEP2017, 023D02 (2017)