Exotic hadrons and physics of resonances

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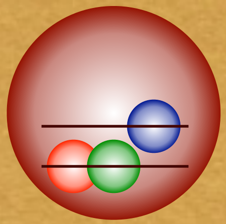
Contents

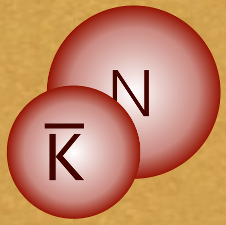
- Introduction
  - Structure of "unstable" resonance?
- Structure of $\Lambda(1405)$ resonance
  - Accurate $\bar{K}N$ scattering amplitude
  - Compositeness

    Y. Kamiya, T. Hyodo, Phys. Rev. C93, 035203 (2016);
    Y. Kamiya, T. Hyodo, PTEP2017, 023D02 (2017)
  - Implication from nearby CDD zero

    Y. Kamiya, T. Hyodo, Phys. Rev. D97, 054019 (2018)
- Summary + future



or



Classification of hadrons

Observed hadrons

PDG2018 : http://pdg.lbl.gov/

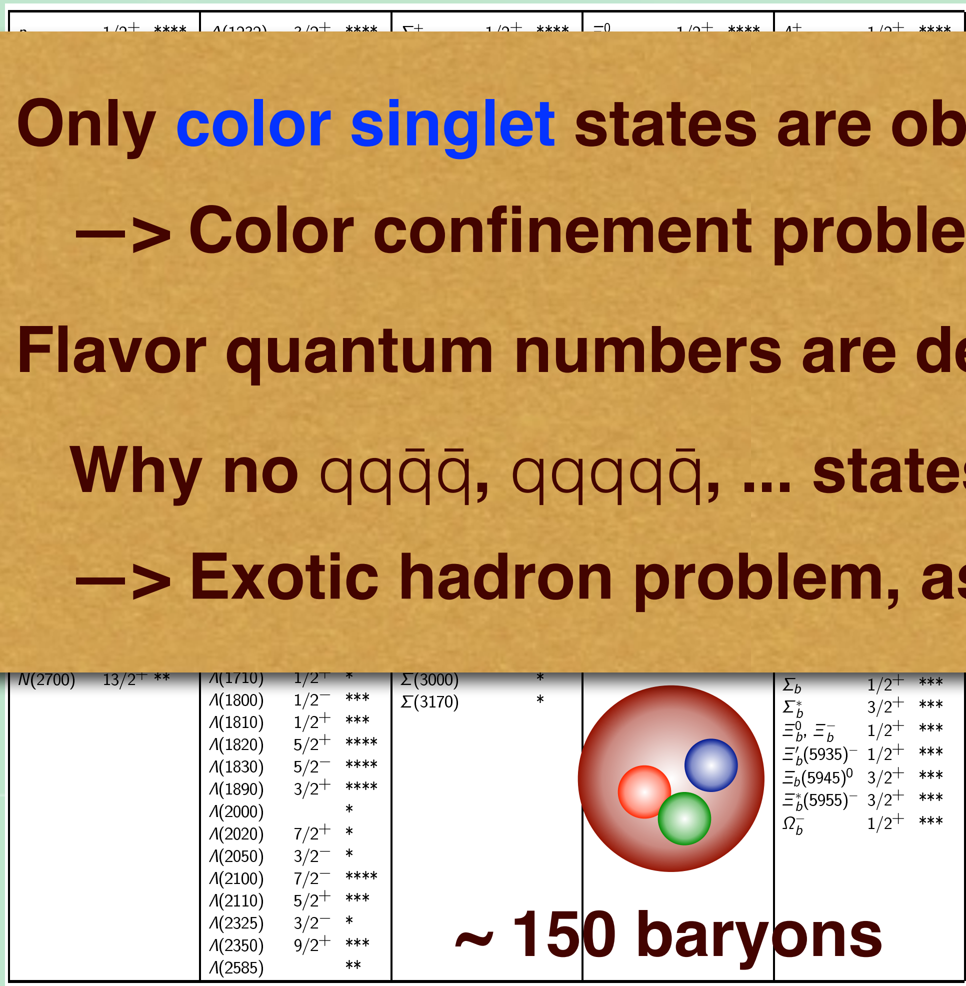
Only color singlet states are observed.

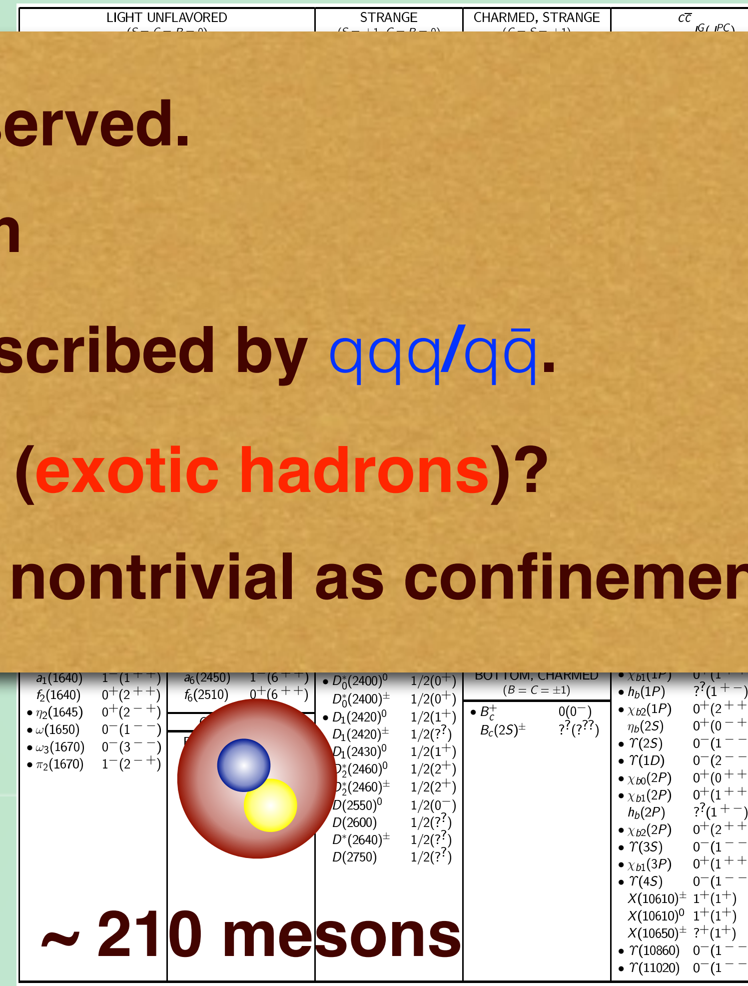
—> Color confinement problem

Flavor quantum numbers are described by $qqq/q\bar{q}$.

Why no $qq\bar{q}\bar{q}$, $qqqq\bar{q}$, ... states (exotic hadrons)?

—> Exotic hadron problem, as nontrivial as confinement!





All ~ 360 hadrons emerge from single QCD Lagrangian.

Exotic candidates beyond qqq/q$\bar{q}$

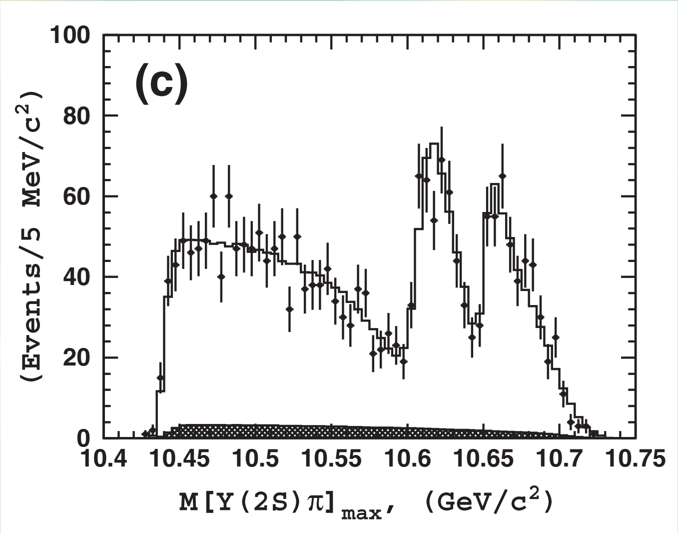
Tetraquark candidate (Belle)

: $Z_b(10610)$, $Z_b(10650)$

$\Upsilon(5S) \longrightarrow \pi^{\pm} + Z_b$

$\quad \hookrightarrow \Upsilon(nS)(b\bar{b}) + \pi^{\mp}(u\bar{d}/d\bar{u})$

A. Bondar, *et al.*, Phys. Rev. Lett. 108, 122001 (2012)



Pentaquark candidate (LHCb)

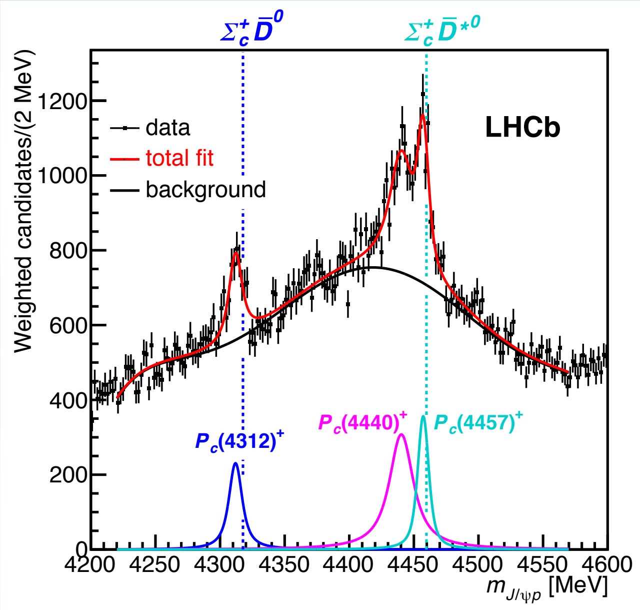
: $P_c(4450)$, $P_c(4380)$

$\Lambda_b \longrightarrow K^- + P_c$

$\quad \hookrightarrow J/\psi(c\bar{c}) + p(uud)$

R. Aaij, *et al.*, Phys. Rev. Lett. 115, 072001 (2015)

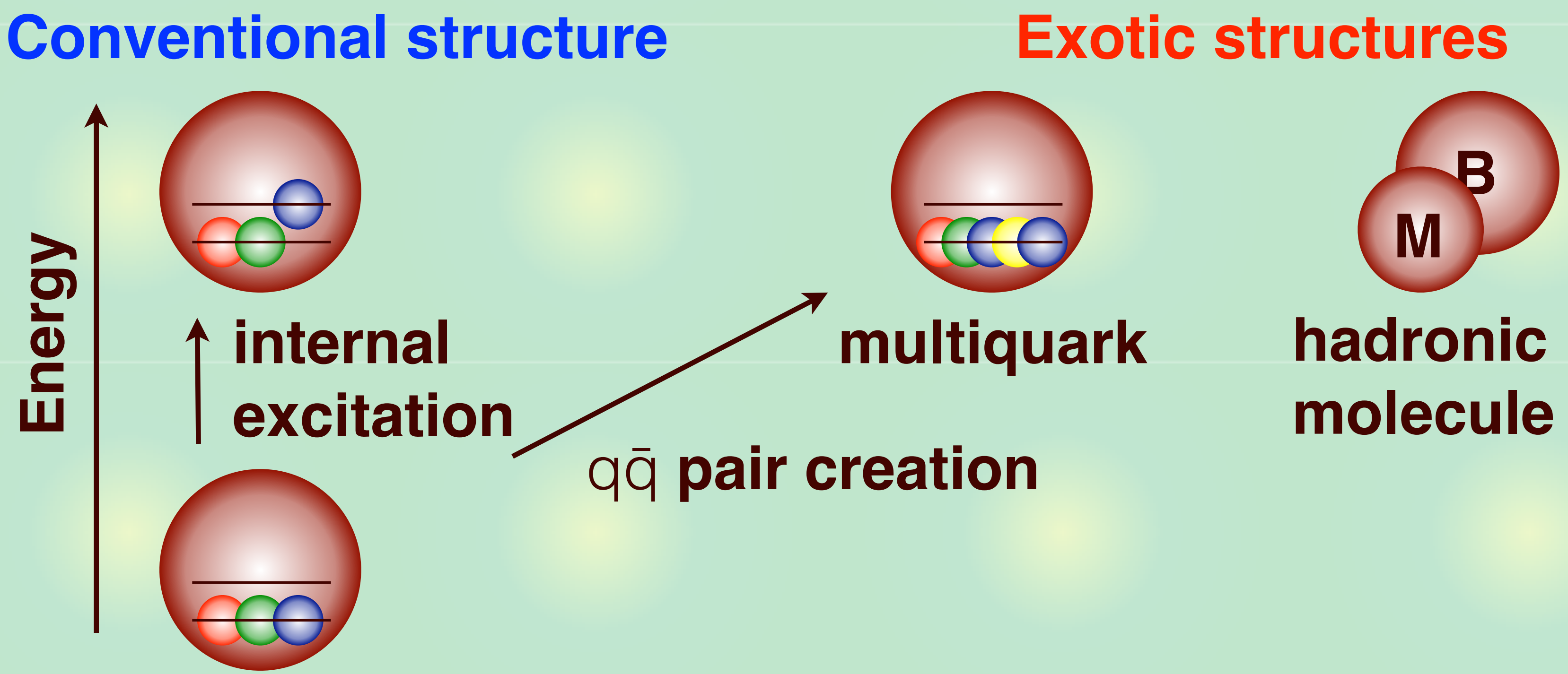
R. Aaij, *et al.*, arXiv:1904.03947[hep-ex]



Only a few are observed. Why only a few?

Various hadronic excitations

Description of excited baryons



In QCD, non-qqq structures naturally arise.

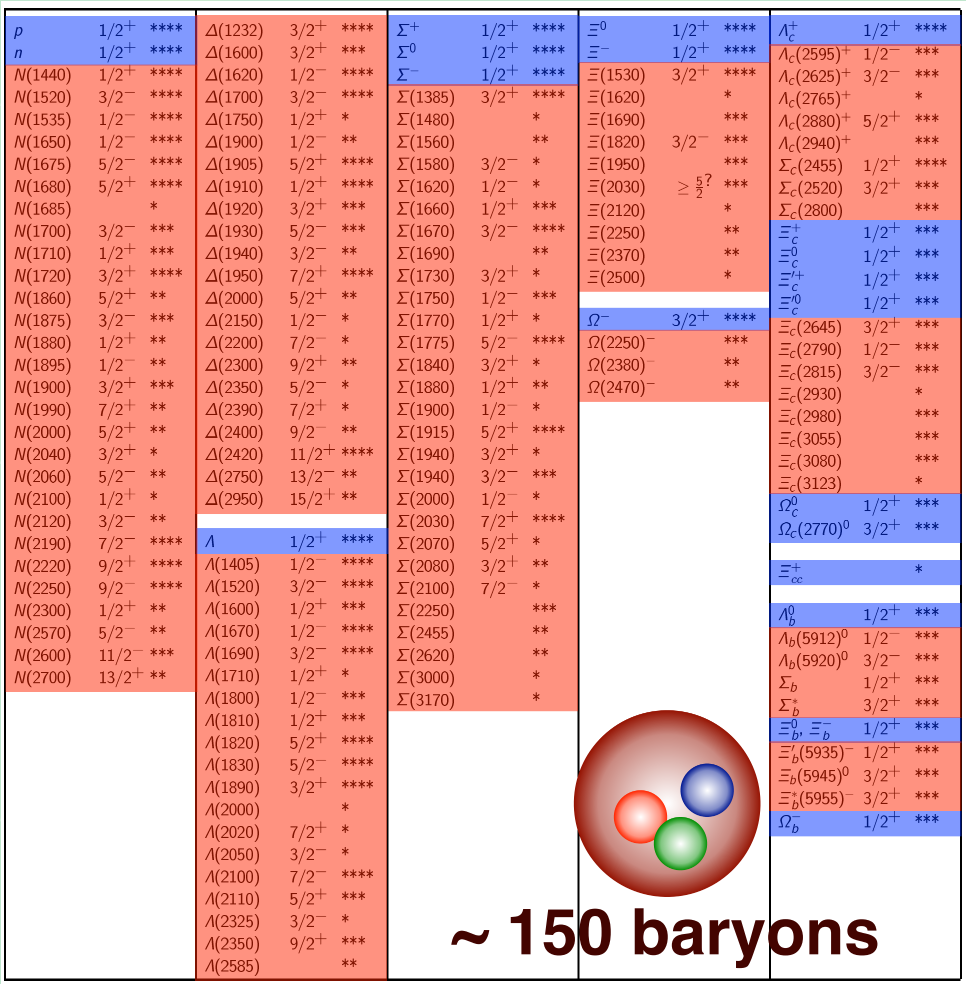
- Baryons: superposition of qqq + exotic structures

—> How can we disentangle different components?

Unstable states via strong interaction

Hadon resonances

PDG2018 : http://pdg.lbl.gov/



p	$1/2^+$	****
n	$1/2^+$	****
$N(1440)$	$1/2^+$	****
$N(1520)$	$3/2^-$	****
$N(1535)$	$1/2^-$	****
$N(1650)$	$1/2^-$	****
$N(1675)$	$5/2^-$	****
$N(1680)$	$5/2^+$	****
$N(1685)$		*
$N(1700)$	$3/2^-$	***
$N(1710)$	$1/2^+$	***
$N(1720)$	$3/2^+$	****
$N(1860)$	$5/2^+$	**
$N(1875)$	$3/2^-$	***
$N(1880)$	$1/2^+$	**
$N(1895)$	$1/2^-$	**
$N(1900)$	$3/2^+$	***
$N(1990)$	$7/2^+$	**
$N(2000)$	$5/2^+$	**
$N(2040)$	$3/2^+$	*
$N(2060)$	$5/2^-$	**
$N(2100)$	$1/2^+$	*
$N(2120)$	$3/2^-$	**
$N(2190)$	$7/2^-$	****
$N(2220)$	$9/2^+$	****
$N(2250)$	$9/2^-$	****
$N(2300)$	$1/2^+$	**
$N(2570)$	$5/2^-$	**
$N(2600)$	$11/2^-$	***
$N(2700)$	$13/2^+$	**

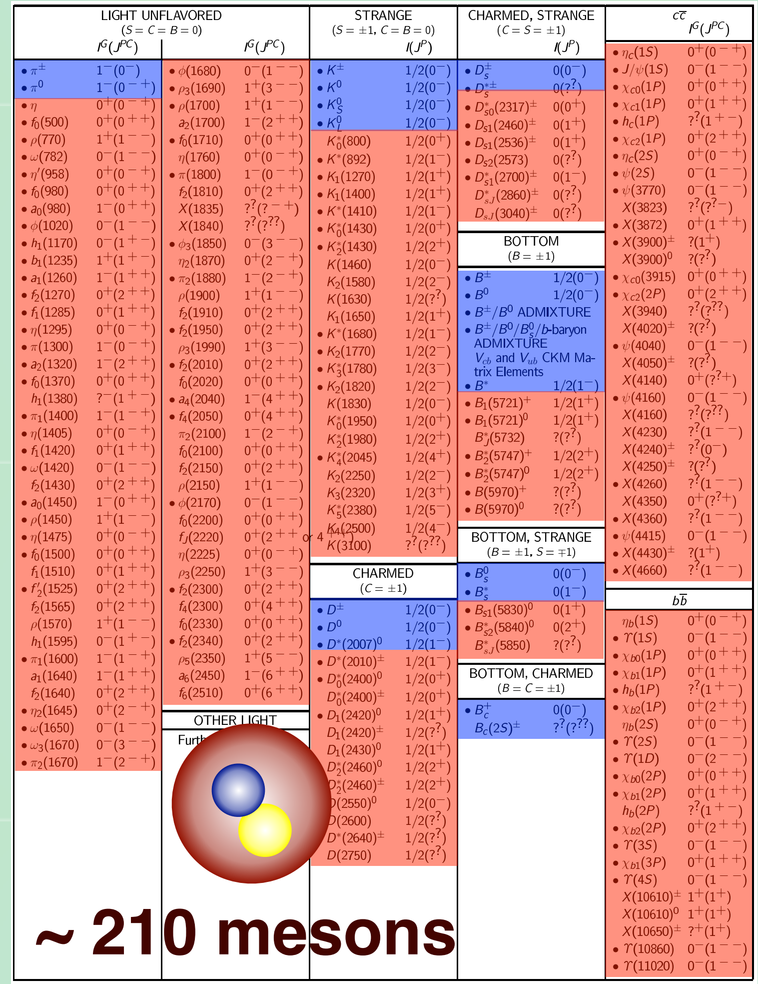
$\Delta(1232)$	$3/2^+$	****
$\Delta(1600)$	$3/2^+$	***
$\Delta(1620)$	$1/2^-$	****
$\Delta(1700)$	$3/2^-$	****
$\Delta(1750)$	$1/2^+$	*
$\Delta(1900)$	$1/2^-$	**
$\Delta(1905)$	$5/2^+$	****
$\Delta(1910)$	$1/2^+$	****
$\Delta(1920)$	$3/2^+$	***
$\Delta(1930)$	$5/2^-$	***
$\Delta(1940)$	$3/2^-$	**
$\Delta(1950)$	$7/2^+$	****
$\Delta(2000)$	$5/2^+$	**
$\Delta(2150)$	$1/2^-$	*
$\Delta(2200)$	$7/2^-$	*
$\Delta(2300)$	$9/2^+$	**
$\Delta(2350)$	$5/2^-$	*
$\Delta(2390)$	$7/2^+$	*
$\Delta(2400)$	$9/2^-$	**
$\Delta(2420)$	$11/2^+$	****
$\Delta(2750)$	$13/2^-$	**
$\Delta(2950)$	$15/2^+$	**
Λ	$1/2^+$	****
$\Lambda(1405)$	$1/2^-$	****
$\Lambda(1520)$	$3/2^-$	****
$\Lambda(1600)$	$1/2^+$	***
$\Lambda(1670)$	$1/2^-$	****
$\Lambda(1690)$	$3/2^-$	****
$\Lambda(1710)$	$1/2^+$	*
$\Lambda(1800)$	$1/2^-$	***
$\Lambda(1810)$	$1/2^+$	***
$\Lambda(1820)$	$5/2^+$	****
$\Lambda(1830)$	$5/2^-$	****
$\Lambda(1890)$	$3/2^+$	****
$\Lambda(2000)$		*
$\Lambda(2020)$	$7/2^+$	*
$\Lambda(2050)$	$3/2^-$	*
$\Lambda(2100)$	$7/2^-$	****
$\Lambda(2110)$	$5/2^+$	***
$\Lambda(2325)$	$3/2^-$	*
$\Lambda(2350)$	$9/2^+$	***
$\Lambda(2585)$		**

Σ^+	$1/2^+$	****
Σ^0	$1/2^+$	****
Σ^-	$1/2^+$	****
$\Sigma(1385)$	$3/2^+$	****
$\Sigma(1480)$		*
$\Sigma(1560)$		**
$\Sigma(1580)$	$3/2^-$	*
$\Sigma(1620)$	$1/2^-$	*
$\Sigma(1660)$	$1/2^+$	***
$\Sigma(1670)$	$3/2^-$	****
$\Sigma(1690)$		**
$\Sigma(1730)$	$3/2^+$	*
$\Sigma(1750)$	$1/2^-$	***
$\Sigma(1770)$	$1/2^+$	*
$\Sigma(1775)$	$5/2^-$	****
$\Sigma(1840)$	$3/2^+$	*
$\Sigma(1880)$	$1/2^+$	**
$\Sigma(1900)$	$1/2^-$	*
$\Sigma(1915)$	$5/2^+$	****
$\Sigma(1940)$	$3/2^+$	*
$\Sigma(1940)$	$3/2^-$	***
$\Sigma(2000)$	$1/2^-$	*
$\Sigma(2030)$	$7/2^+$	****
$\Sigma(2070)$	$5/2^+$	*
$\Sigma(2080)$	$3/2^+$	**
$\Sigma(2100)$	$7/2^-$	*
$\Sigma(2250)$		***
$\Sigma(2455)$		**
$\Sigma(2620)$		**
$\Sigma(3000)$		*
$\Sigma(3170)$		*

Ξ^0	$1/2^+$	****
Ξ^-	$1/2^+$	****
$\Xi(1530)$	$3/2^+$	****
$\Xi(1620)$		*
$\Xi(1690)$		***
$\Xi(1820)$	$3/2^-$	***
$\Xi(1950)$		***
$\Xi(2030)$	$\geq \frac{5}{2}^?$	***
$\Xi(2120)$		*
$\Xi(2250)$		**
$\Xi(2370)$		**
$\Xi(2500)$		*
Ω^-	$3/2^+$	****
$\Omega(2250)^-$		***
$\Omega(2380)^-$		**
$\Omega(2470)^-$		**

Λ_c^+	$1/2^+$	****
$\Lambda_c(2595)^+$	$1/2^-$	***
$\Lambda_c(2625)^+$	$3/2^-$	***
$\Lambda_c(2765)^+$		*
$\Lambda_c(2880)^+$	$5/2^+$	***
$\Lambda_c(2940)^+$		***
$\Sigma_c(2455)$	$1/2^+$	****
$\Sigma_c(2520)$	$3/2^+$	***
$\Sigma_c(2800)$		***
Ξ_c^+	$1/2^+$	***
Ξ_c^0	$1/2^+$	***
$\Xi_c'^+$	$1/2^+$	***
$\Xi_c'^0$	$1/2^+$	***
$\Xi_c(2645)$	$3/2^+$	***
$\Xi_c(2790)$	$1/2^-$	***
$\Xi_c(2815)$	$3/2^-$	***
$\Xi_c(2930)$		*
$\Xi_c(2980)$		***
$\Xi_c(3055)$		***
$\Xi_c(3080)$		***
$\Xi_c(3123)$		*
Ω_c^0	$1/2^+$	***
$\Omega_c(2770)^0$	$3/2^+$	***
Ξ_{cc}^+		*
Λ_b^0	$1/2^+$	***
$\Lambda_b(5912)^0$	$1/2^-$	***
$\Lambda_b(5920)^0$	$3/2^-$	***
Σ_b	$1/2^+$	***
Σ_b^*	$3/2^+$	***
Ξ_b^0, Ξ_b^-	$1/2^+$	***
$\Xi_b'(5935)^-$	$1/2^+$	***
$\Xi_b(5945)^0$	$3/2^+$	***
$\Xi_b^*(5955)^-$	$3/2^+$	***
Ω_b^-	$1/2^+$	***

~ 150 baryons



LIGHT UNFLAVORED ($S = C = B = 0$)

	$I^G(J^{PC})$
• $\pi^\pm$	$1^-(0^-)$
• π^0	$1^-(0^{-+})$
• η	$0^+(0^{-+})$
• $f_0(500)$	$0^+(0^{++})$
• $\rho(770)$	$1^+(1^{--})$
• $\omega(782)$	$0^-(1^{--})$
• $\eta'(958)$	$0^+(0^{-+})$
• $f_0(980)$	$0^+(0^{++})$
• $a_0(980)$	$1^-(0^{++})$
• $\phi(1020)$	$0^-(1^{--})$
• $h_1(1170)$	$0^-(1^{+-})$
• $b_1(1235)$	$1^+(1^{+-})$
• $a_1(1260)$	$1^-(1^{++})$
• $f_2(1270)$	$0^+(2^{++})$
• $f_1(1285)$	$0^+(1^{++})$
• $\eta(1295)$	$0^+(0^{-+})$
• $\pi(1300)$	$1^-(0^{-+})$
• $a_2(1320)$	$1^-(2^{++})$
• $f_0(1370)$	$0^+(0^{++})$
$h_1(1380)$	$?^-(1^{+-})$
• $\pi_1(1400)$	$1^-(1^{-+})$
• $\eta(1405)$	$0^+(0^{-+})$
• $f_1(1420)$	$0^+(1^{++})$
• $\omega(1420)$	$0^-(1^{--})$
$f_2(1430)$	$0^+(2^{++})$
• $a_0(1450)$	$1^-(0^{++})$
• $\rho(1450)$	$1^+(1^{--})$
• $\eta(1475)$	$0^+(0^{-+})$
• $f_0(1500)$	$0^+(0^{++})$
$f_1(1510)$	$0^+(1^{++})$
• $f_2'(1525)$	$0^+(2^{++})$
$f_2(1565)$	$0^+(2^{++})$
$\rho(1570)$	$1^+(1^{--})$
$h_1(1595)$	$0^-(1^{+-})$
• $\pi_1(1600)$	$1^-(1^{-+})$
$a_1(1640)$	$1^-(1^{++})$
$f_2(1640)$	$0^+(2^{++})$
• $\eta_2(1645)$	$0^+(2^{-+})$
• $\omega(1650)$	$0^-(1^{--})$
• $\omega_3(1670)$	$0^-(3^{--})$
• $\pi_2(1670)$	$1^-(2^{-+})$

	$I^G(J^{PC})$
• $\phi(1680)$	$0^-(1^{--})$
• $\rho_3(1690)$	$1^+(3^{--})$
• $\rho(1700)$	$1^+(1^{--})$
$a_2(1700)$	$1^-(2^{++})$
• $f_0(1710)$	$0^+(0^{++})$
$\eta(1760)$	$0^+(0^{-+})$
• $\pi(1800)$	$1^-(0^{-+})$
$f_2(1810)$	$0^+(2^{++})$
$X(1835)$	$?^?(?^{-+})$
$X(1840)$	$?^?(?^{??})$
• $\phi_3(1850)$	$0^-(3^{--})$
$\eta_2(1870)$	$0^+(2^{-+})$
• $\pi_2(1880)$	$1^-(2^{-+})$
$\rho(1900)$	$1^+(1^{--})$
$f_2(1910)$	$0^+(2^{++})$
• $f_2(1950)$	$0^+(2^{++})$
$\rho_3(1990)$	$1^+(3^{--})$
• $f_2(2010)$	$0^+(2^{++})$
$f_0(2020)$	$0^+(0^{++})$
• $a_4(2040)$	$1^-(4^{++})$
• $f_4(2050)$	$0^+(4^{++})$
$\pi_2(2100)$	$1^-(2^{-+})$
$f_0(2100)$	$0^+(0^{++})$
$f_2(2150)$	$0^+(2^{++})$
$\rho(2150)$	$1^+(1^{--})$
• $\phi(2170)$	$0^-(1^{--})$
$f_0(2200)$	$0^+(0^{++})$
$f_J(2220)$	$0^+(2^{++}$ or $4^{++})$
$\eta(2225)$	$0^+(0^{-+})$
$\rho_3(2250)$	$1^+(3^{--})$
• $f_2(2300)$	$0^+(2^{++})$
$f_4(2300)$	$0^+(4^{++})$
$f_0(2330)$	$0^+(0^{++})$
• $f_2(2340)$	$0^+(2^{++})$
$\rho_5(2350)$	$1^+(5^{--})$
$a_6(2450)$	$1^-(6^{++})$
$f_6(2510)$	$0^+(6^{++})$

OTHER LIGHT

Furth

STRANGE ($S = \pm1$, $C = B = 0$)

	$I(J^P)$
• $K^\pm$	$1/2(0^-)$
• K^0	$1/2(0^-)$
• K_S^0	$1/2(0^-)$
• K_L^0	$1/2(0^-)$
$K_0^*(800)$	$1/2(0^+)$
• $K^*(892)$	$1/2(1^-)$
• $K_1(1270)$	$1/2(1^+)$
• $K_1(1400)$	$1/2(1^+)$
• $K^*(1410)$	$1/2(1^-)$
• $K_0^*(1430)$	$1/2(0^+)$
• $K_2^*(1430)$	$1/2(2^+)$
$K(1460)$	$1/2(0^-)$
$K_2(1580)$	$1/2(2^-)$
$K(1630)$	$1/2(?^?)$
$K_1(1650)$	$1/2(1^+)$
• $K^*(1680)$	$1/2(1^-)$
• $K_2(1770)$	$1/2(2^-)$
• $K_3^*(1780)$	$1/2(3^-)$
• $K_2(1820)$	$1/2(2^-)$
$K(1830)$	$1/2(0^-)$
$K_0^*(1950)$	$1/2(0^+)$
$K_2^*(1980)$	$1/2(2^+)$
• $K_4^*(2045)$	$1/2(4^+)$
$K_2(2250)$	$1/2(2^-)$
$K_3(2320)$	$1/2(3^+)$
$K_5^*(2380)$	$1/2(5^-)$
$K_4(2500)$	$1/2(4^-)$
$K(3100)$	$?^?(?^{??})$

CHARMED ($C = \pm1$)

	$I(J^P)$
• $D^\pm$	$1/2(0^-)$
• D^0	$1/2(0^-)$
• $D^*(2007)^0$	$1/2(1^-)$
• $D^*(2010)^\pm$	$1/2(1^-)$
• $D_0^*(2400)^0$	$1/2(0^+)$
$D_0^*(2400)^\pm$	$1/2(0^+)$
• $D_1(2420)^0$	$1/2(1^+)$
$D_1(2420)^\pm$	$1/2(?^?)$
$D_1(2430)^0$	$1/2(1^+)$
$D_2^*(2460)^0$	$1/2(2^+)$
$D_2^*(2460)^\pm$	$1/2(2^+)$
$D(2550)^0$	$1/2(0^-)$
$D(2600)$	$1/2(?^?)$
$D^*(2640)^\pm$	$1/2(?^?)$
$D(2750)$	$1/2(?^?)$

CHARMED, STRANGE ($C = S = \pm1$)

	$I(J^P)$
• $D_s^\pm$	$0(0^-)$
• $D_s^{*\pm}$	$0(?^?)$
• $D_{s0}^*(2317)^\pm$	$0(0^+)$
• $D_{s1}(2460)^\pm$	$0(1^+)$
• $D_{s1}(2536)^\pm$	$0(1^+)$
• $D_{s2}(2573)$	$0(?^?)$
• $D_{s1}^*(2700)^\pm$	$0(1^-)$
$D_{sJ}^*(2860)^\pm$	$0(?^?)$
$D_{sJ}(3040)^\pm$	$0(?^?)$

BOTTOM ($B = \pm1$)

	$I(J^P)$
• $B^\pm$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^\pm/B^0$ ADMIXTURE	
• $B^\pm/B^0/B_s^0/b$-baryon ADMIXTURE	
V_{cb} and V_{ub} CKM Matrix Elements	
• B^*	$1/2(1^-)$
• $B_1(5721)^+$	$1/2(1^+)$
• $B_1(5721)^0$	$1/2(1^+)$
$B_J^*(5732)$	$?(?^?)$
• $B_2^*(5747)^+$	$1/2(2^+)$
• $B_2^*(5747)^0$	$1/2(2^+)$
• $B(5970)^+$	$?(?^?)$
• $B(5970)^0$	$?(?^?)$

BOTTOM, STRANGE ($B = \pm1$, $S = \mp1$)

• B_s^0	$0(0^-)$
• B_s^*	$0(1^-)$
• $B_{s1}(5830)^0$	$0(1^+)$
• $B_{s2}^*(5840)^0$	$0(2^+)$
$B_{sJ}^*(5850)$	$?(?^?)$

BOTTOM, CHARMED ($B = C = \pm1$)

• B_c^+	$0(0^-)$
$B_c(2S)^\pm$	$?^?(?^{??})$

$c\overline{c}$

	$I^G(J^{PC})$
• $\eta_c(1S)$	$0^+(0^{-+})$
• $J/\psi(1S)$	$0^-(1^{--})$
• $\chi_{c0}(1P)$	$0^+(0^{++})$
• $\chi_{c1}(1P)$	$0^+(1^{++})$
• $h_c(1P)$	$?^?(1^{+-})$
• $\chi_{c2}(1P)$	$0^+(2^{++})$
• $\eta_c(2S)$	$0^+(0^{-+})$
• $\psi(2S)$	$0^-(1^{--})$
• $\psi(3770)$	$0^-(1^{--})$
$X(3823)$	$?^?(?^{?-})$
• $X(3872)$	$0^+(1^{++})$
• $X(3900)^\pm$	$?(1^+)$
$X(3900)^0$	$?(?^?)$
• $\chi_{c0}(3915)$	$0^+(0^{++})$
• $\chi_{c2}(2P)$	$0^+(2^{++})$
$X(3940)$	$?^?(?^{??})$
$X(4020)^\pm$	$?(?^?)$
• $\psi(4040)$	$0^-(1^{--})$
$X(4050)^\pm$	$?(?^?)$
$X(4140)$	$0^+(?^{?+})$
• $\psi(4160)$	$0^-(1^{--})$
$X(4160)$	$?^?(?^{??})$
$X(4230)$	$?^?(1^{--})$
$X(4240)^\pm$	$?^?(0^-)$
$X(4250)^\pm$	$?(?^?)$
• $X(4260)$	$?^?(1^{--})$
$X(4350)$	$0^+(?^{?+})$
• $X(4360)$	$?^?(1^{--})$
• $\psi(4415)$	$0^-(1^{--})$
• $X(4430)^\pm$	$?(1^+)$
• $X(4660)$	$?^?(1^{--})$

$b\overline{b}$

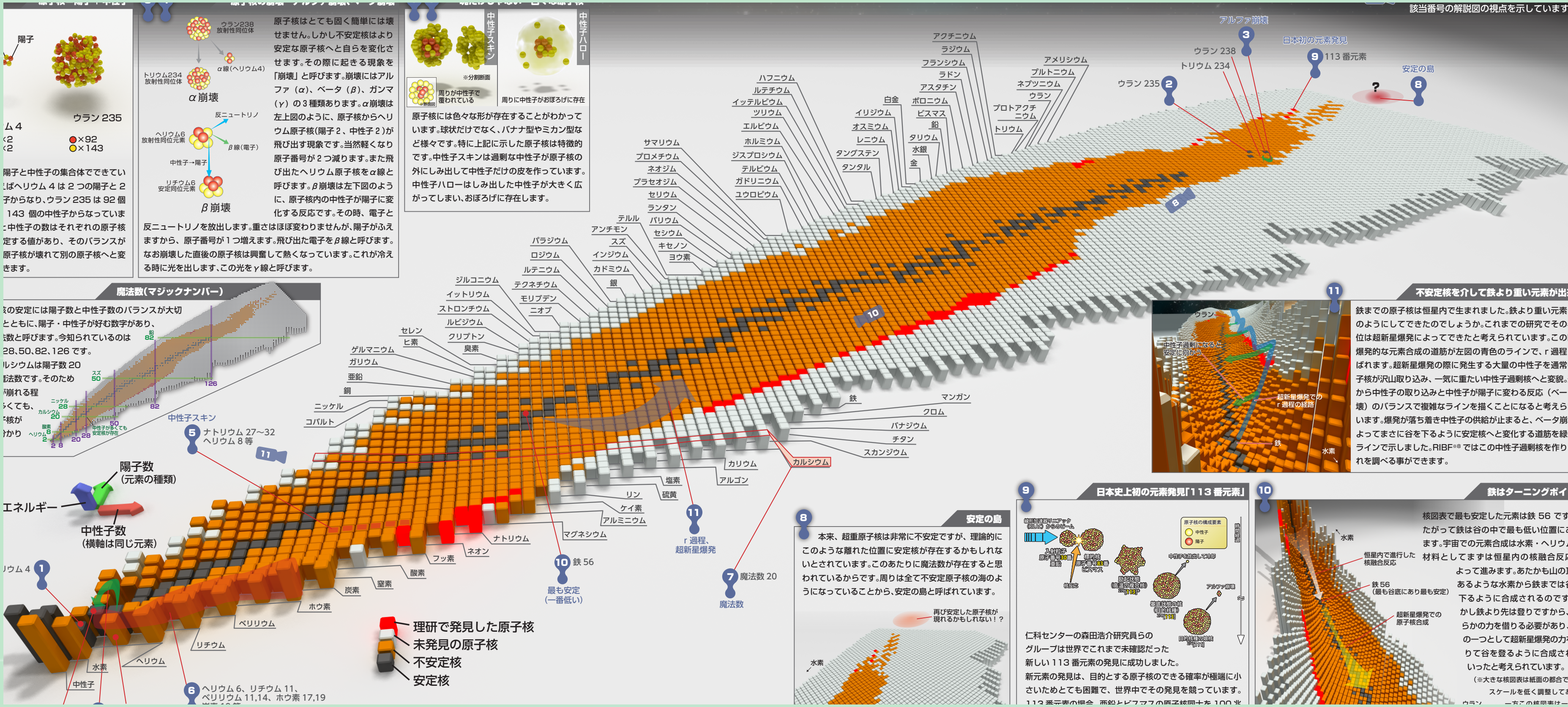
$\eta_b(1S)$	$0^+(0^{-+})$
• $\Upsilon(1S)$	$0^-(1^{--})$
• $\chi_{b0}(1P)$	$0^+(0^{++})$
• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $h_b(1P)$	$?^?(1^{+-})$
• $\chi_{b2}(1P)$	$0^+(2^{++})$
$\eta_b(2S)$	$0^+(0^{-+})$
• $\Upsilon(2S)$	$0^-(1^{--})$
• $\Upsilon(1D)$	$0^-(2^{--})$
• $\chi_{b0}(2P)$	$0^+(0^{++})$
• $\chi_{b1}(2P)$	$0^+(1^{++})$
$h_b(2P)$	$?^?(1^{+-})$
• $\chi_{b2}(2P)$	$0^+(2^{++})$
• $\Upsilon(3S)$	$0^-(1^{--})$
• $\chi_{b1}(3P)$	$0^+(1^{++})$
• $\Upsilon(4S)$	$0^-(1^{--})$
$X(10610)^\pm$	$1^+(1^+)$
$X(10610)^0$	$1^+(1^+)$
$X(10650)^\pm$	$?^+(1^+)$
• $\Upsilon(10860)$	$0^-(1^{--})$
• $\Upsilon(11020)$	$0^-(1^{--})$

~ 210 mesons

- stable/unstable via strong interaction
- Excited states are mostly unstable. —> resonances

Relation to unstable nuclei

Stable nuclei (~300), unstable nuclei (~2000)



https://www.nishina.riken.jp/enjoy/kakuzu/index.html

Structure of unstable nuclei

- clustering, halo nuclei, Efimov effect, ...

Difficulty of resonances

Resonance as an “eigenstate” of Hamiltonian

- complex energy

G. Gamow, Z. Phys. 51, 204 (1928)

Zur Quantentheorie des Atomkernes.

Von **G. Gamow**, z. Zt. in Göttingen.

Mit 5 Abbildungen. (Eingegangen am 2. August 1928.)

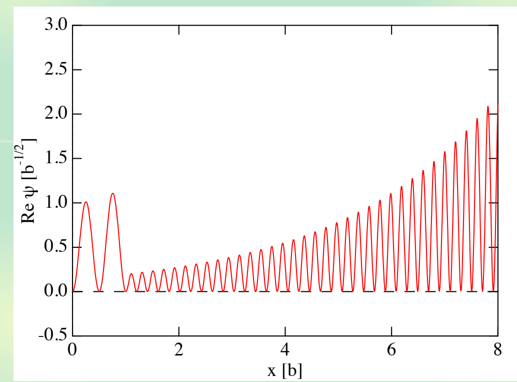
Um diese Schwierigkeit zu überwinden, müssen wir annehmen, daß die Schwingungen gedämpft sind, und $\boldsymbol{E}$ komplex setzen:

$$E = E_0 + i\frac{h\lambda}{4\pi},$$

wo $\boldsymbol{E}_0$ die gewöhnliche Energie ist und $\boldsymbol{\lambda}$ das Dämpfungsdekrement (Zerfallskonstante). Dann sehen wir aber aus den Relationen (2a) und (2b),

- diverging wave function (Im k < 0)

$$\langle\, R \,|\, R \,\rangle = \int d\boldsymbol{r} |\psi_R(\boldsymbol{r})|^2 \sim \int_0^\infty dr e^{-2\mathrm{Im}[k]r} \to \infty$$



Bi-orthogonal basis (Gamow vectors): normalizable!

N. Hokkyo, Prog. Theor. Phys. 33, 1116 (1965)

T. Berggren, Nucl. Phys. A 109, 265 (1968)

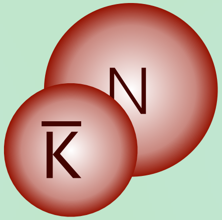
$$|\, \tilde{R} \,\rangle = |\, R^* \,\rangle, \quad |\langle\, \tilde{R} \,|\, R \,\rangle| = \left| \int d\boldsymbol{r} [\psi_R(\boldsymbol{r})]^2 \right| < \infty$$

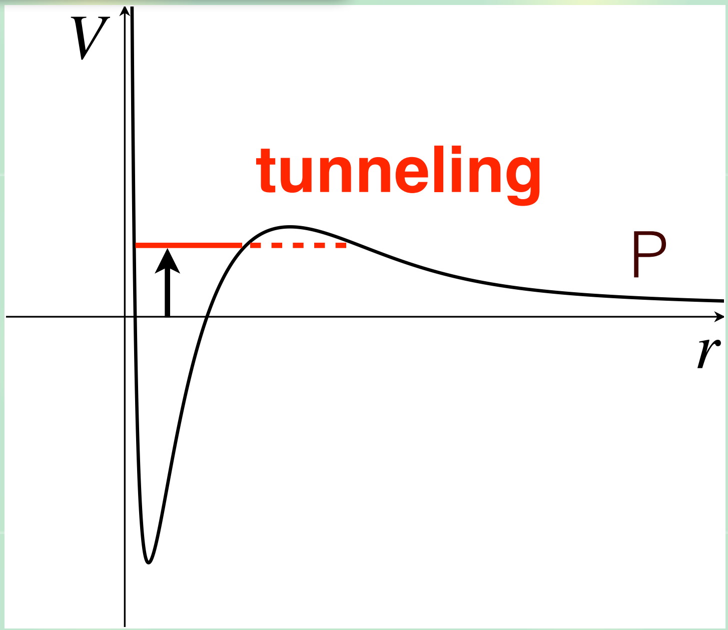
- Complex expectation value (norm, <r^2>) —> interpretation?

Classification of resonances

1) Potential (shape) resonance

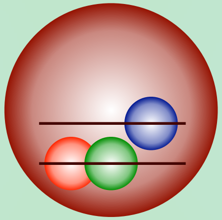
- 1 channel (P)
- potential barrier : E>0
- unstable via tunneling
- (composite of P-channel)

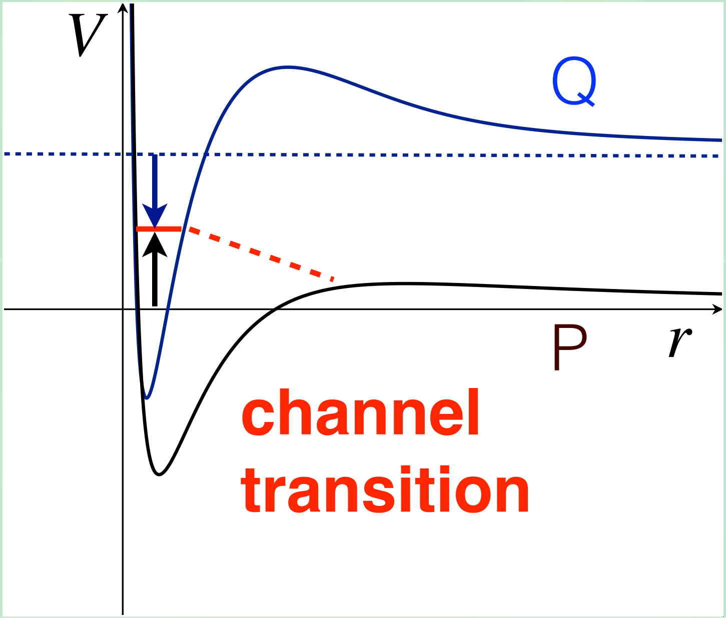




2) Feshbach resonance

- coupled-channel (P+Q)
- bound state of Q: $E_Q<0$, $E_P>0$
- unstable via transition
- ("elementary": other than P)





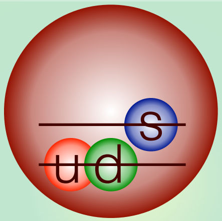
Strategy

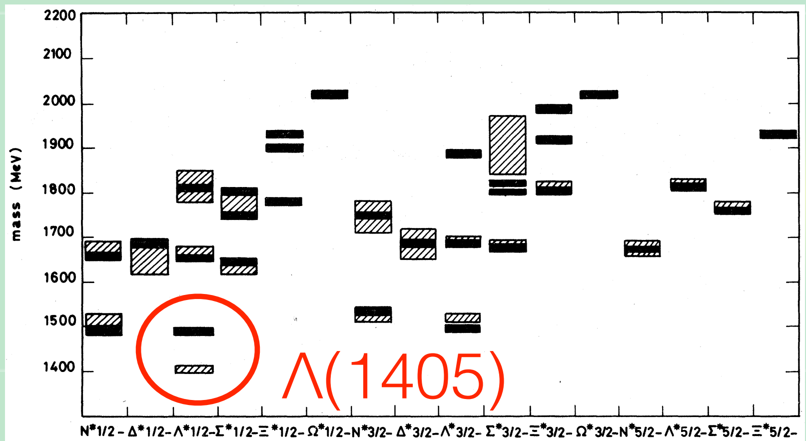
- Structure of **unstable** resonances
- Methods to distinguish the structure by
  - observables (cross section, ...)
  - on-shell scattering amplitude (a_0, r_e, ...)
  - its analytic continuation (pole, zero, ...)
  - wave function
  - off-shell amplitude
- Accurate **scattering amplitude** is needed.

Λ(1405) and $\bar{K}N$ scattering

Λ(1405) does not fit in standard picture —> exotic candidate

N. Isgur and G. Karl, Phys. Rev. D18, 4187 (1978)



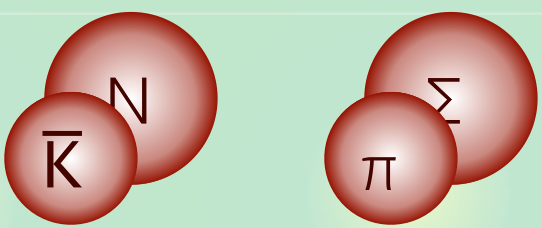


: theory

: experiment

Resonance in coupled-channel scattering

- coupling to MB states

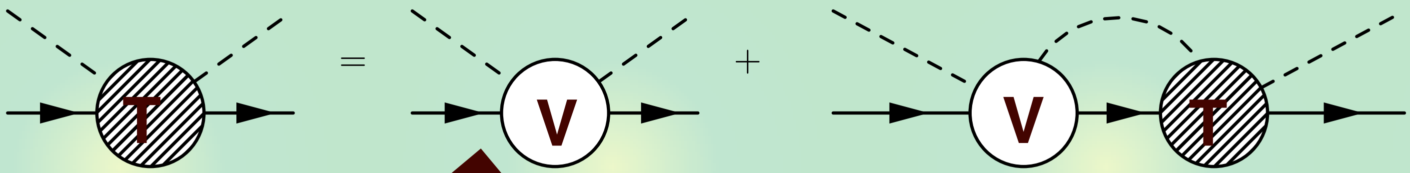


Detailed analysis of $\bar{K}N$-$\pi\Sigma$ scattering is necessary.

Construction of the realistic amplitude

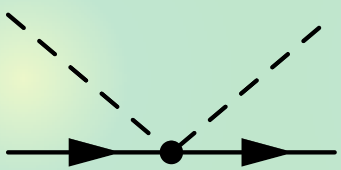
Chiral coupled-channel approach with systematic χ^2 fitting

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881 98 (2012)



Chiral perturbation theory

1) TW term

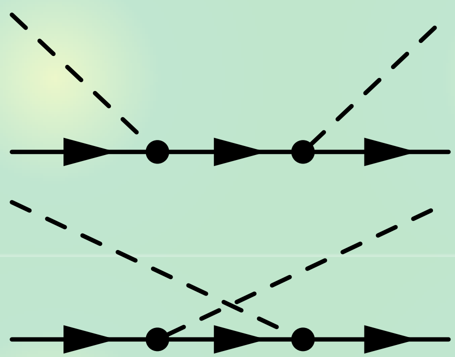


$\mathcal{O}(p)$

6 cutoffs

TW model

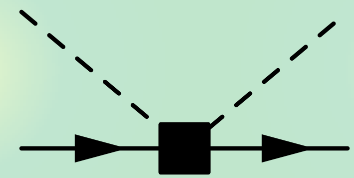
2) Born terms



$\mathcal{O}(p)$

TWB model

3) NLO terms



$\mathcal{O}(p^2)$

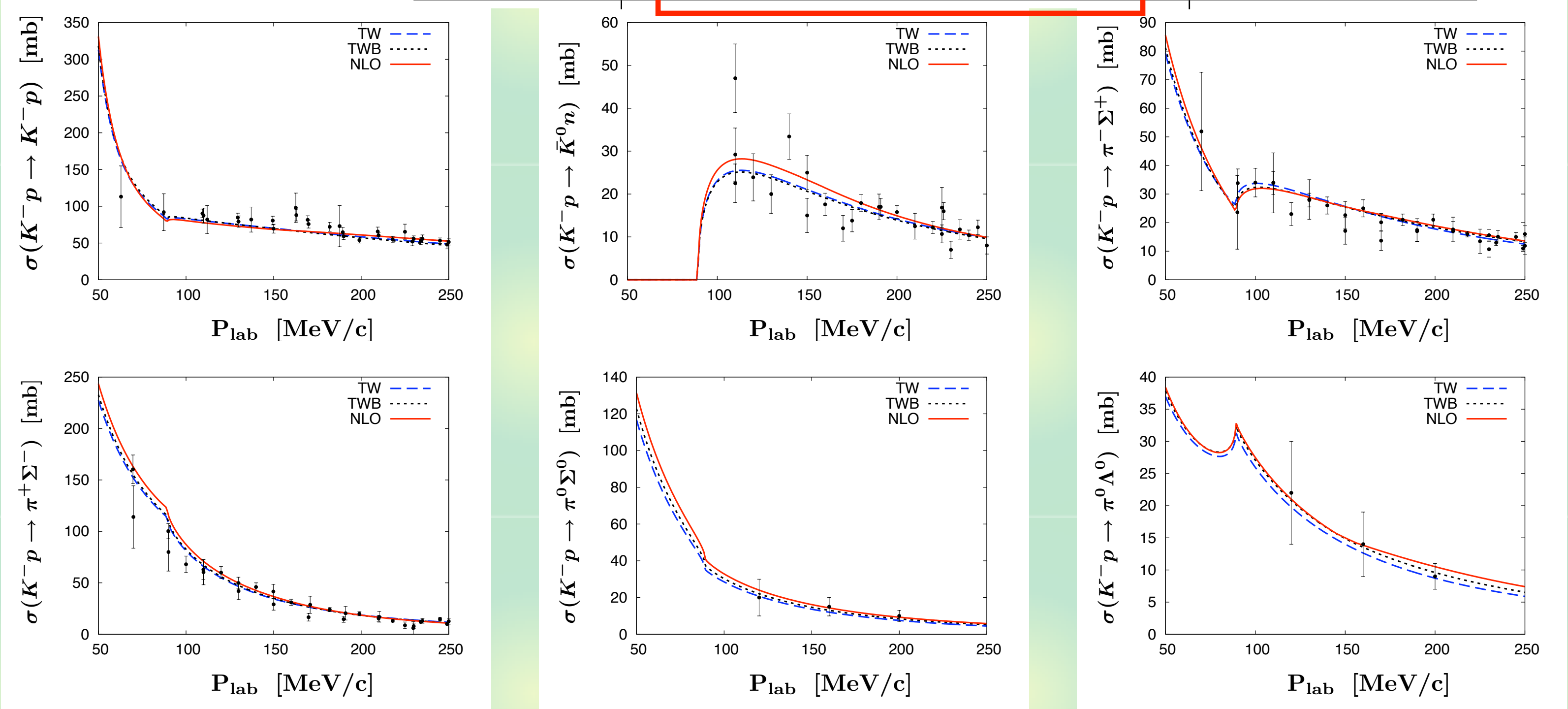
7 LECs

NLO model

Fit to experiments: NLO chiral SU(3) dynamics

		TW	TWB	NLO	Experiment	
SIDDHARTA	ΔE [eV]	373	377	306	$283 \pm 36 \pm 6$	[10]
	Γ [eV]	495	514	591	$541 \pm 89 \pm 22$	[10]
Branching ratios	γ	2.36	2.36	2.37	2.36 ± 0.04	[11]
	R_n	0.20	0.19	0.19	0.189 ± 0.015	[11]
	R_c	0.66	0.66	0.66	0.664 ± 0.011	[11]
	χ^2/d.o.f	1.12	1.15	0.96		

cross sections

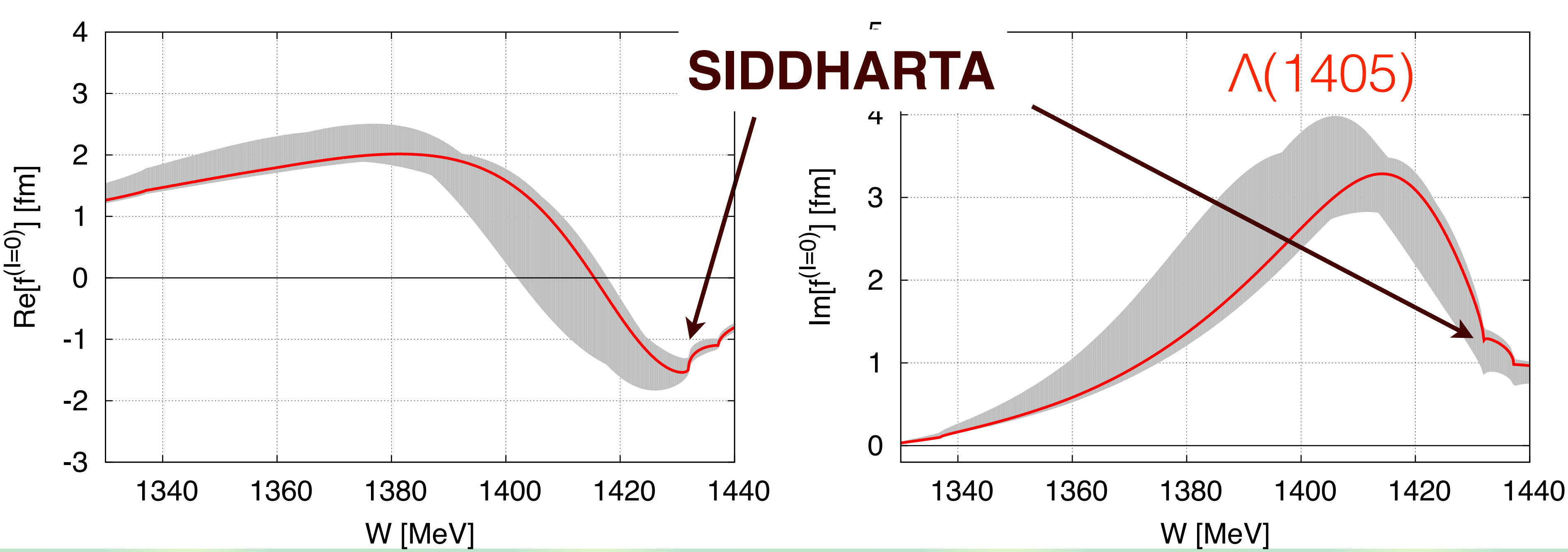


Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881 98 (2012)

Accurate description of all existing data (χ^2/d.o.f. ~ 1)

Subthreshold extrapolation

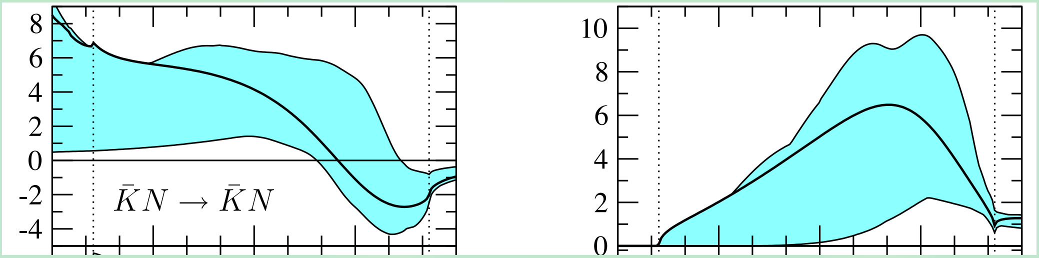
Uncertainty of $\bar{K}N \longrightarrow \bar{K}N$ (I=0) amplitude below threshold



Y. Kamiya, K. Miyahara, S. Ohnishi, Y. Ikeda, T. Hyodo, E. Oset, W. Weise, NPA 954, 41 (2016)

- c.f. without SIDDHARTA

R. Nissler, Doctoral Thesis (2007)

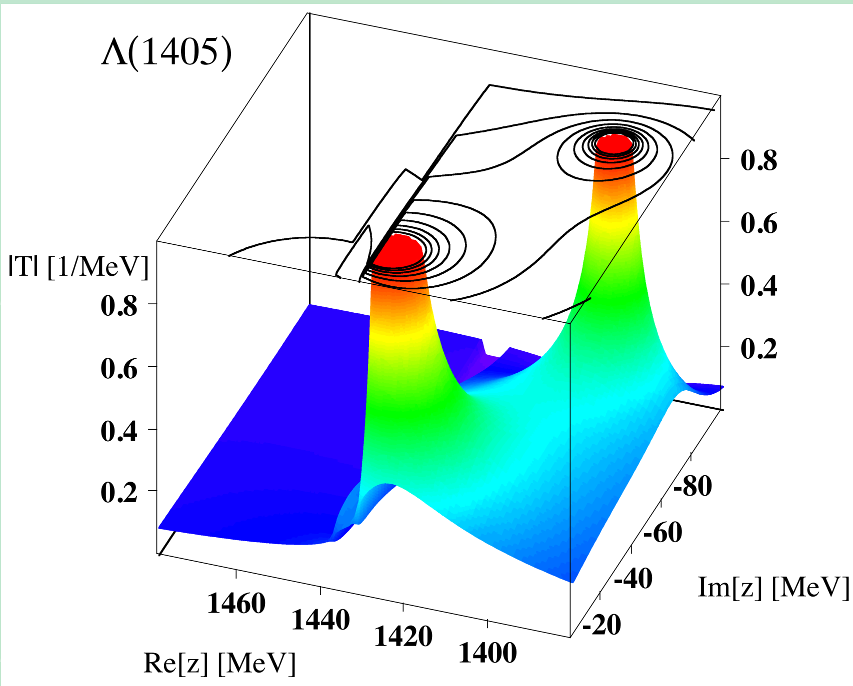


SIDDHARTA is essential for subthreshold extrapolation.

Extrapolation to complex energy: two poles

Two poles: superposition of two eigenstates

J.A. Oller, U.G. Meissner, PLB 500, 263 (2001);
D. Jido, J.A. Oller, E. Oset, A. Ramos, U.G. Meissner, NPA 723, 205 (2003);



Λ(1405) 1/2⁻ $I(J^P) = 0(\frac{1}{2}^-)$ Status: ∗∗∗∗

In the 1998 Note on the $\Lambda(1405)$ in PDG 98, R.H. Dalitz discussed the S-shaped cusp behavior of the intensity at the N-$\overline{K}$ threshold observed in THOMAS 73 and HEMINGWAY 85. He commented that this behavior "is characteristic of S-wave coupling; the other below threshold hyperon, the $\Sigma(1385)$, has no such threshold distortion because its N-$\overline{K}$ coupling is P-wave. For $\Lambda(1405)$ this asymmetry is the sole direct evidence that $J^P = 1/2^-$."

A recent measurement by the CLAS collaboration, MORIYA 14, definitively established the long-assumed $J^P = 1/2^-$ spin-parity assignment of the $\Lambda(1405)$. The experiment produced the $\Lambda(1405)$ spin-polarized in the photoproduction process $\gamma p \to K^+ \Lambda(1405)$ and measured the decay of the $\Lambda(1405)$(polarized) $\to \Sigma^+$(polarized)π^-. The observed isotropic decay of $\Lambda(1405)$ is consistent with spin $J = 1/2$. The polarization transfer to the Σ^+(polarized) direction revealed negative parity, and thus established $J^P = 1/2^-$.

See the related review(s):
Pole Structure of the $\Lambda(1405)$ Region

$\Lambda(1405)$ REGION POLE POSITIONS

REAL PART

VALUE (MeV)	DOCUMENT ID		TECN
• • • We do not use the following data for averages, fits, limits, etc. • • •			
1429^{+8}_{-7}	[1] MAI	15	DPWA
1325^{+15}_{-15}	[2] MAI	15	DPWA
1434^{+2}_{-2}	[3] MAI	15	DPWA
1330^{+4}_{-5}	[4] MAI	15	DPWA
1421^{+3}_{-2}	[5] GUO	13	DPWA
1388 ± 9	[6] GUO	13	DPWA
1424^{+7}_{-23}	[7] IKEDA	12	DPWA
1381^{+18}_{-6}	[8] IKEDA	12	DPWA

NLO analysis confirms the two-po

Now tabulated in PDG

M. Tanabashi, *et al.*, PRD 98, 030001 (2018), `http://pdg.lbl.gov/`

Compositeness of hadrons

- **Find a measure to distinguish the structure**
- **Weak binding relation for stable bound states**

  S. Weinberg, Phys. Rev. 137, B672 (1965)

Compositeness X
threshold channel

or

“Elementariness” Z
other contributions

↑
observables (a_0, B)

- **Effective field theory —> description of low-energy scattering amplitude, generalization to unstable resonances**

Weak binding relation for stable states

Compositeness X **of s-wave weakly bound state** ($R \gg R_{typ}$)

S. Weinberg, Phys. Rev. 137, B672 (1965);
T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)

$$|d\rangle = \sqrt{X}\,|NN\rangle + \sqrt{1-X}\,|\text{others}\rangle$$

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\text{typ}}}{R}\right)\right\}, \quad r_e = R\left\{\frac{X-1}{X} + \mathcal{O}\left(\frac{R_{\text{typ}}}{R}\right)\right\}$$

a_0: **scattering length,** r_e: **effective range**
$R = (2\mu B)^{-1/2}$: **radius of wave function**
R_{typ}: **length scale of interaction**

- **Deuteron is** NN **composite** ($a_0 \sim R \gg r_e$) **—>** $X \sim 1$
- **Internal structure from observable**

Problem: applicable only for stable states.

Effective field theory

Low-energy scattering with near-threshold bound state

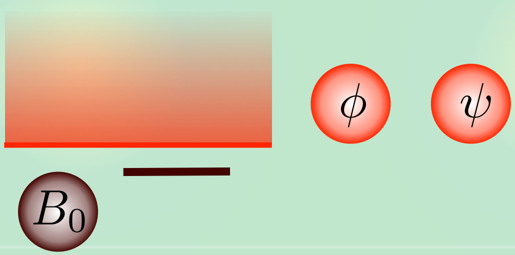
- Nonrelativistic EFT with contact interaction

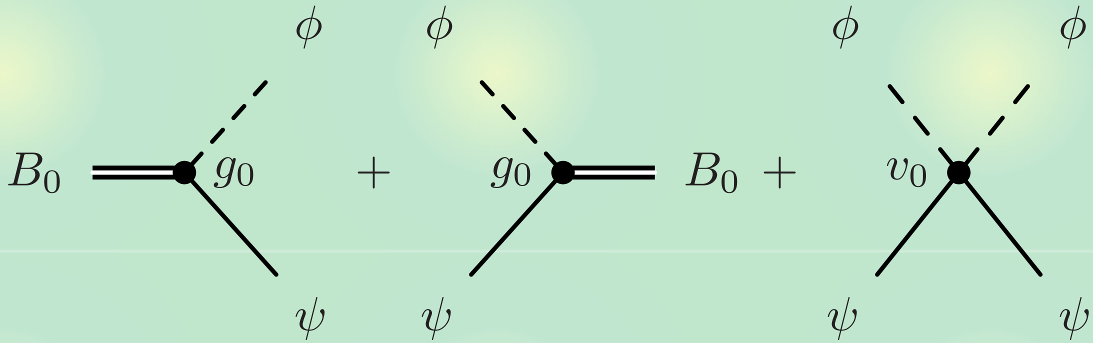
D.B. Kaplan, Nucl. Phys. B494, 471 (1997)

E. Braaten, M. Kusunoki, D. Zhang, Annals Phys. 323, 1770 (2008)

$$H_{\text{free}} = \int d\boldsymbol{r} \left[\frac{1}{2M} \nabla\psi^\dagger \cdot \nabla\psi + \frac{1}{2m} \nabla\phi^\dagger \cdot \nabla\phi + \frac{1}{2M_0} \nabla B_0^\dagger \cdot \nabla B_0 + \nu_0 B_0^\dagger B_0 \right],$$

$$H_{\text{int}} = \int d\boldsymbol{r} \left[g_0 \left(B_0^\dagger \phi\psi + \psi^\dagger \phi^\dagger B_0 \right) + v_0 \psi^\dagger \phi^\dagger \phi\psi \right]$$





- **cutoff** : $\Lambda \sim 1/R_{\text{typ}}$ **(interaction range of microscopic theory)**

- **At low energy** $p \ll \Lambda$, **interaction ~ contact**

Compositeness and "elementariness"

Eigenstates

$$H_{\rm free}|\,B_0\,\rangle = \nu_0|\,B_0\,\rangle, \quad H_{\rm free}|\,\boldsymbol{p}\,\rangle = \frac{p^2}{2\mu}|\,\boldsymbol{p}\,\rangle$$ **free (discrete + continuum)**

$$(H_{\rm free} + H_{\rm int})|\,B\,\rangle = -B|\,B\,\rangle$$ **full (bound state)**

- normalization of |B> + completeness relation

$$\langle\,B\,|\,B\,\rangle = 1, \quad 1 = |\,B_0\,\rangle\langle\,B_0\,| + \int \frac{d\boldsymbol{p}}{(2\pi)^3}|\,\boldsymbol{p}\,\rangle\langle\,\boldsymbol{p}\,|$$

- projections onto free eigenstates

$$1 = Z + X, \quad Z \equiv |\langle\,B_0\,|\,B\,\rangle|^2, \quad X \equiv \int \frac{d\boldsymbol{p}}{(2\pi)^3}|\langle\,\boldsymbol{p}\,|\,B\,\rangle|^2$$

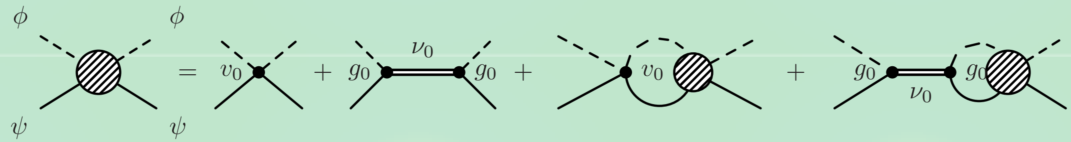
"elementariness" **compositeness**

Z, X**: real and nonnegative —> interpreted as probability**

Weak binding relation

ΨΦ scattering amplitude (exact result)

$$f(E) = -\frac{\mu}{2\pi}\frac{1}{[v(E)]^{-1} - G(E)}$$



$$v(E) = v_0 + \frac{g_0^2}{E - \nu_0}, \quad G(E) = \frac{1}{2\pi^2}\int_0^{\Lambda} dp \frac{p^2}{E - p^2/(2\mu) + i0^+}$$

Compositeness X <— v(E), G(E)

$$X = \frac{G'(-B)}{G'(-B) - [1/v(-B)]'}$$

1/R=(2μB)$^{1/2}$ expansion: leading term <— X

$$a_0 = -f(E=0) = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\mathrm{typ}}}{R}\right)\right\}$$

renormalization dependent

renormalization independent

If R ≫ R_{typ}, correction terms neglected: X <— (B, a_0)

Introduction of decay channel

Introduce decay channel

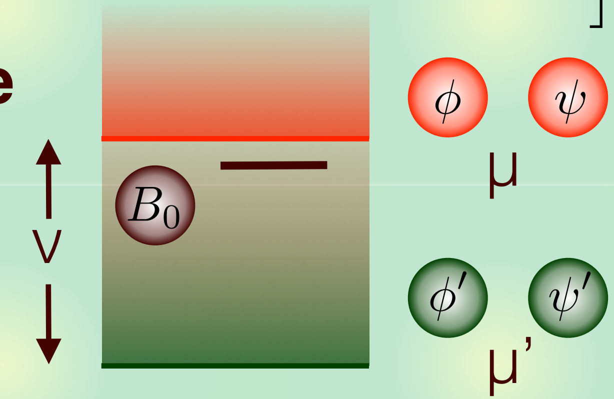
$$H'_{\rm free} = \int d\boldsymbol{r}\left[\frac{1}{2M'}\nabla\psi'^{\dagger}\cdot\nabla\psi' - \nu_{\psi}\psi'^{\dagger}\psi' + \frac{1}{2m'}\nabla\phi'^{\dagger}\cdot\nabla\phi' - \nu_{\phi}\phi'^{\dagger}\phi'\right],$$

$$H'_{\rm int} = \int d\boldsymbol{r}\left[g'_0\left(B_0^{\dagger}\phi'\psi' + \psi'^{\dagger}\phi'^{\dagger}B_0\right) + v'_0\psi'^{\dagger}\phi'^{\dagger}\phi'\psi' + v_0^t(\psi^{\dagger}\phi^{\dagger}\phi'\psi' + \psi'^{\dagger}\phi'^{\dagger}\phi\psi)\right],$$

Quasi-bound state: complex eigenvalue

$$H = H_{\rm free} + H'_{\rm free} + H_{\rm int} + H'_{\rm int}$$

$$H|QB\rangle = E_{QB}|QB\rangle, \quad E_{QB}\in\mathbb{C}$$



Generalized relation: correction term <— threshold difference

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\rm typ}}{R}\right|\right) + \mathcal{O}\left(\left|\frac{l}{R}\right|^3\right)\right\}, \quad R = \frac{1}{\sqrt{-2\mu E_{QB}}}\in\mathbb{C}, \quad l \equiv \frac{1}{\sqrt{2\mu\nu}}$$

Y. Kamiya, T. Hyodo, Phys. Rev. C93, 035203 (2016)

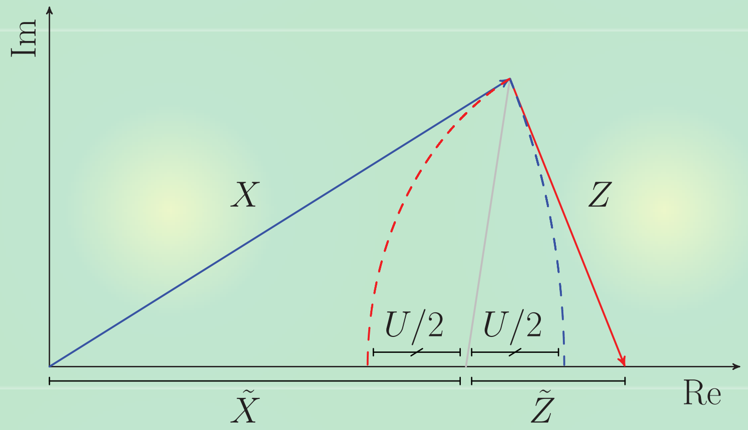
c.f. V. Baru, *et al.*, Phys. Lett. B586, 53 (2004),…

If |R| ≫ ($R_{\rm typ}$, l) correction terms neglected: X <— (E_{QB}, a_0)

Complex compositeness

Unstable states —> complex Z and X

$$Z + X = 1, \quad Z, X \in \mathbb{C}$$



- **Probabilistic interpretation?**

New definition

$$\tilde{Z} = \frac{1 - |X| + |Z|}{2}, \quad \tilde{X} = \frac{1 - |Z| + |X|}{2}$$

- **interpreted as probabilities** $\tilde{Z} + \tilde{X} = 1, \quad \tilde{Z}, \tilde{X} \in [0, 1]$
- **reduces to Z and X in the bound state limit**

U/2: uncertainty of interpretation

$$U = |Z| + |X| - 1$$

c.f. T. Berggren, Phys. Lett. 33B, 547 (1970)

- **Sensible interpretation only for small U/2 case**

Evaluation of compositeness

Generalized weak-binding relation

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\rm typ}}{R}\right|\right) + \mathcal{O}\left(\left|\frac{l}{R}\right|^3\right)\right\}, \quad R = \frac{1}{\sqrt{-2\mu E_{QB}}}, \quad l \equiv \frac{1}{\sqrt{2\mu\nu}}$$

(E_{QB}, a_0) determinations by several groups

- neglecting correction terms:

	E_h [MeV]	a_0 [fm]	$X_{\bar{K}N}$	$\tilde{X}_{\bar{K}N}$	$U/2$
Set 1 [35]	$-10 - i26$	$1.39 - i0.85$	$1.2 + i0.1$	1.0	0.3
Set 2 [36]	$-4 - i8$	$1.81 - i0.92$	$0.6 + i0.1$	0.6	0.0
Set 3 [37]	$-13 - i20$	$1.30 - i0.85$	$0.9 - i0.2$	0.9	0.1
Set 4 [38]	$2 - i10$	$1.21 - i1.47$	$0.6 + i0.0$	0.6	0.0
Set 5 [38]	$-3 - i12$	$1.52 - i1.85$	$1.0 + i0.5$	0.8	0.3

- In all cases, X~1 **with small** U/2 **(complex nature)**

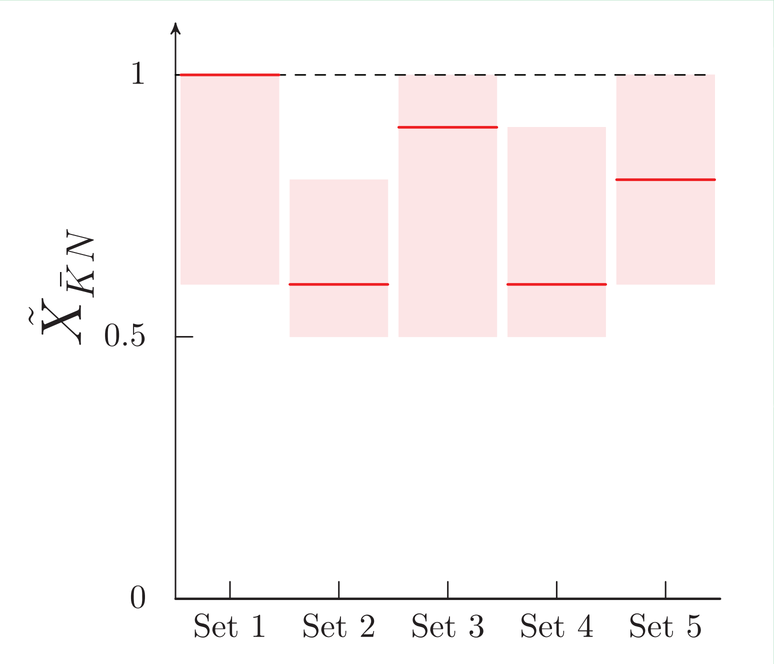
Λ(1405) : $\bar{K}$N **composite dominance <— observables**

Uncertainty estimation

Estimation of correction terms : $|R| \sim 2$ fm

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\text{typ}}}{R}\right|\right) + \mathcal{O}\left(\left|\frac{l}{R}\right|^3\right)\right\}, \quad R = \frac{1}{\sqrt{-2\mu E_{QB}}}, \quad l \equiv \frac{1}{\sqrt{2\mu\nu}}$$

- **ρ meson exchange picture :** $R_{\text{typ}} \sim 0.25$ fm
- **energy difference from πΣ :** $l \sim 1.08$ fm



K̄N composite dominance holds even with correction terms.

Analytic structure of scattering amplitude

Pole of scattering amplitude $f(E_{pole})=\infty$

J.R. Taylor, *Scattering theory* (Wiley, New York, 1972)

- represents (complex) eigenvalue of Hamiltonian

CDD (Castillejo-Dalitz-Dyson) zero

L. Castillejo, R.H. Dalitz, F.J. Dyson, Phys. Rev. 124, 264 (1961)

- pole of inverse amplitude, zero of amplitude $f(E_{CDD})=0$
- role of CDD zero in hadron scattering, resonances, etc.

V. Baru, *et al.*, Eur. Phys. J. A 44, 93 (2010),
C. Hanhart, *et al.*, Eur. Phys. J. A 47, 101 (2011),
Z.H. Guo, J.A. Oller, Phys. Rev. D93, 054014 (2016)

Distance between pole and zero <—> origin of the state

Fate of pole in zero coupling limit

Zero coupling limit (ZCL): switching off the channel coupling

R.J. Eden, J.R. Taylor, Phys. Rev. 133, B1575 (1964)

$$H = \lim_{x\to 0}\begin{pmatrix} T_{11}+V_{11} & xV_{12} & \cdots \\ xV_{21} & T_{22}+V_{22} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} = \begin{pmatrix} T_{11}+V_{11} & 0 & \cdots \\ 0 & T_{22}+V_{22} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

- pole exists in all components at the same position for $x \neq 0$
- pole exists only in channel i with V_{ii} origin at $x=0$

Pole behavior in 11 amplitude toward ZCL ($x \to 0$)

- channel 1 origin : pole remains in 11 amplitude
- channel 2, … origin : pole decouples from 11 amplitude

How can a pole decouple from an amplitude?

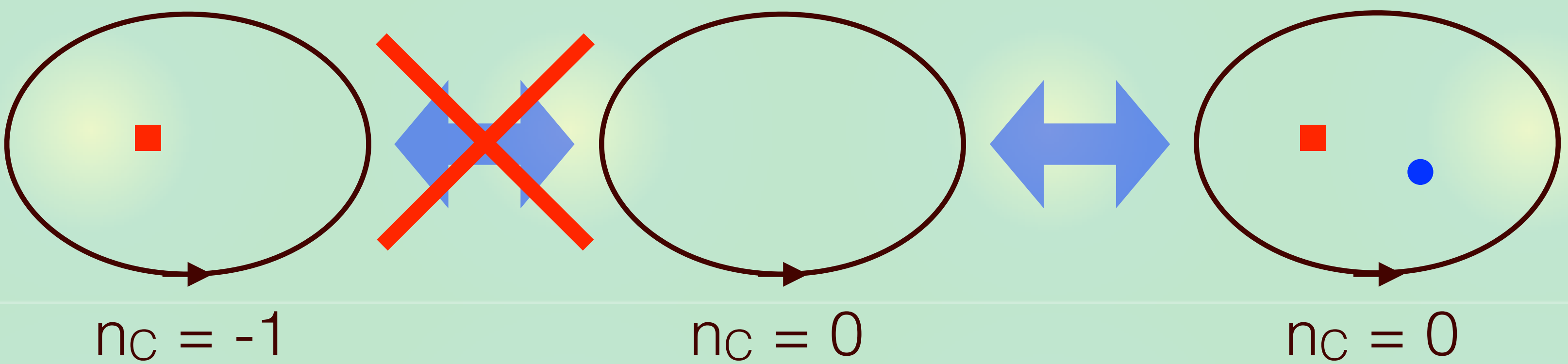
General discussion

Scattering amplitude f(E) is meromorphic in energy

Y. Kamiya, T. Hyodo, Phys. Rev. D97, 054019 (2018)

$$\frac{1}{2\pi}\oint_C dz \frac{d \arg f(z)}{dz} = n_Z - n_P \equiv n_C$$

- n_Z (n_P) : **number of zeros (poles) in contour C**
- **Topological invariant of** $\pi_1(U(1)) \cong \mathbb{Z}$

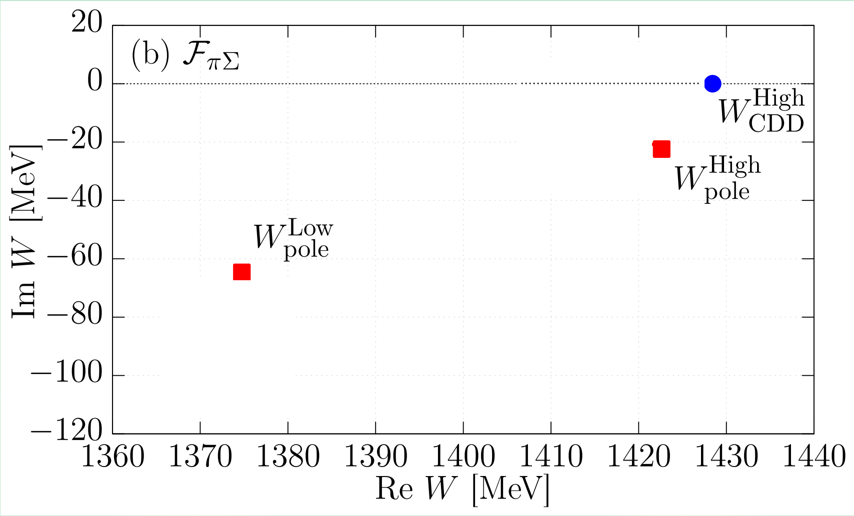


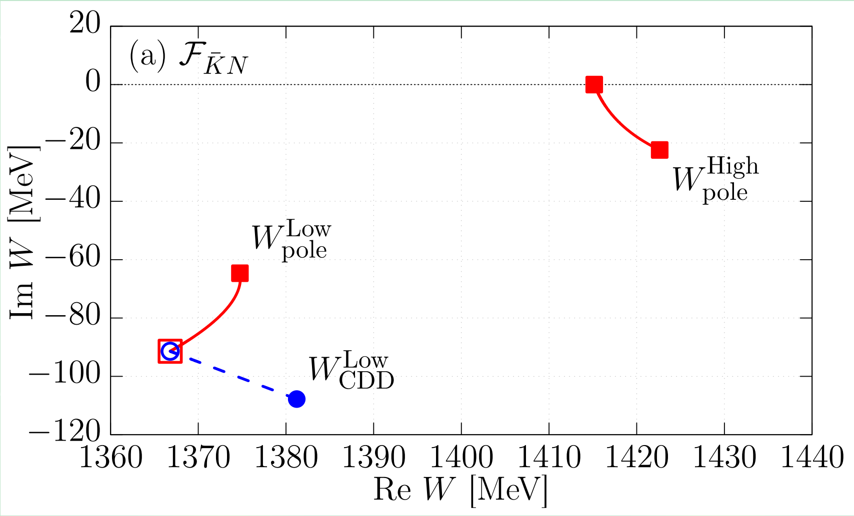
Pole cannot decouple without merging with CDD zero

—> existence of nearby CDD zero indicates "elementary" (origin is in other channel).

Example: Λ(1405)

Poles and zeros in the $\bar{K}N$ and $\pi\Sigma$ amplitudes





- In $\pi\Sigma$ amplitude, CDD zero exists near the high-mass pole, and merges with it to decouple in the ZCL.

- In $\bar{K}N$ amplitude, CDD zero exists near the low-mass pole, and merges with it to decouple in the ZCL.

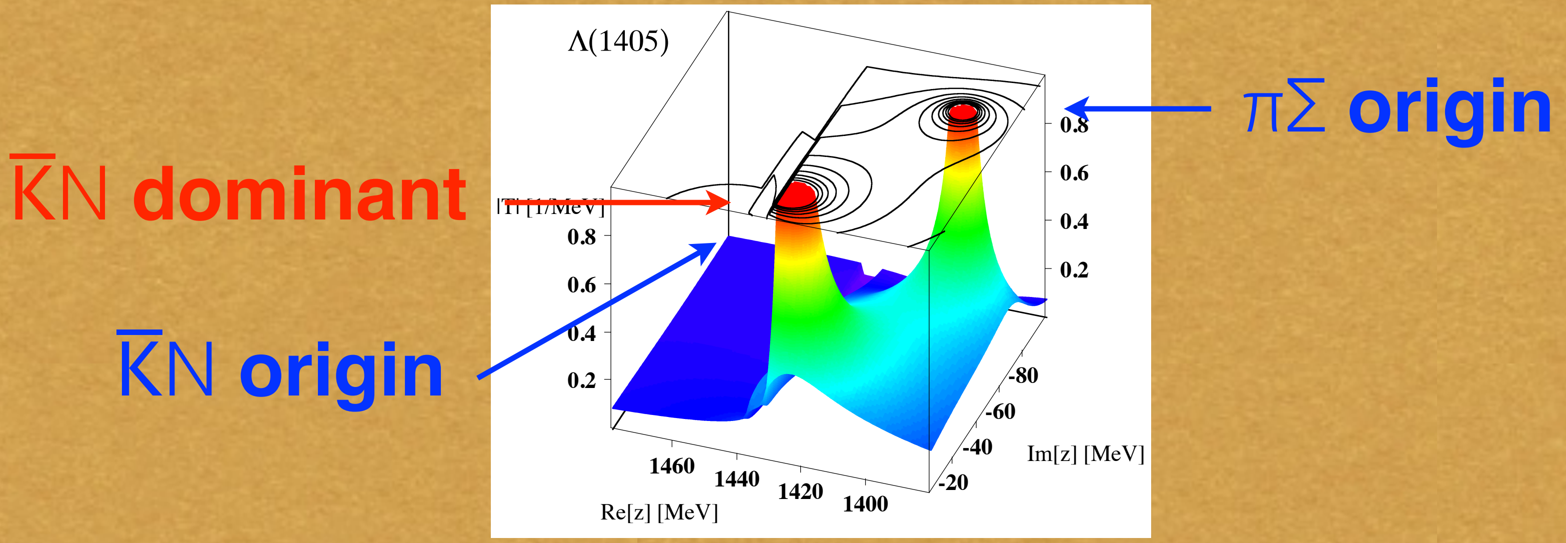
Low- (high-)mass pole is not $\bar{K}N$ ($\pi\Sigma$) composite.

Summary

Structure of Λ(1405) from scattering amplitude

- **Compositeness** from weak binding relation

Y. Kamiya, T. Hyodo, Phys. Rev. C93, 035203 (2016);
Y. Kamiya, T. Hyodo, PTEP2017, 023D02 (2017)



- **CDD zero** analysis

Y. Kamiya, T. Hyodo, Phys. Rev. D97, 054019 (2018)

Future

Resonances (unstable quantum states)

- many unknowns
  - structure?
  - complex expectation values?
  - breaking of time reversal symmetry?
- theoretical foundation
  - non-hermitian quantum mechanics
- important for future hadron-nuclear physics