Model-independent study on the structure of Λ(1405)

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  Y. Ikeda, T. Hyodo, W. Weise, Phys. Lett. B 706, 63 (2011);
  Y. Ikeda, T. Hyodo, W. Weise, Nucl. Phys. A 881 98 (2012)

- **Compositeness from weak binding relation**

  - scattering length and eigenenergy

  Y. Kamiya, T. Hyodo, Phys. Rev. C93, 035203 (2016);
  Y. Kamiya, T. Hyodo, PTEP2017, 023D02 (2017)

- **Implication from nearby CDD zero**

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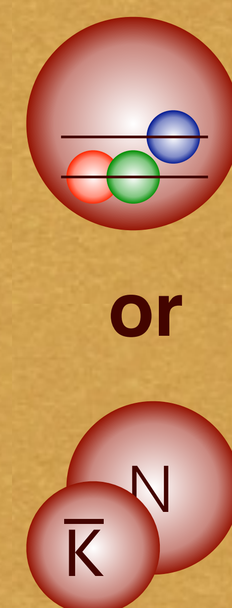
  Y. Kamiya, T. Hyodo, Phys. Rev. D97, 054019 (2018)

- **Summary**



$\bar{K}$ meson and $\bar{K}N$ interaction

Two aspects of $K(\bar{K})$ meson

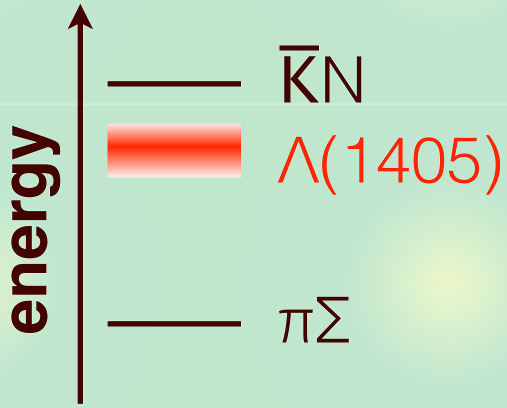
- **NG boson of chiral $SU(3)_R \otimes SU(3)_L$ —> $SU(3)_V$**
- **Massive by strange quark: $m_K \sim$ 496 MeV**

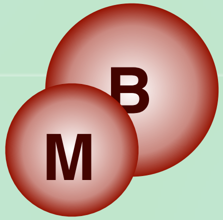
—> Spontaneous/explicit symmetry breaking

$\bar{K}N$ interaction ...

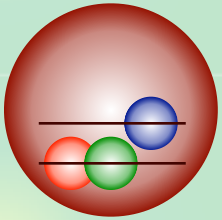
T. Hyodo, D. Jido, Prog. Part. Nucl. Phys. 67, 55 (2012)

- **is coupled with $\pi\Sigma$ channel**
- **generates $\Lambda(1405)$ below threshold**





molecule



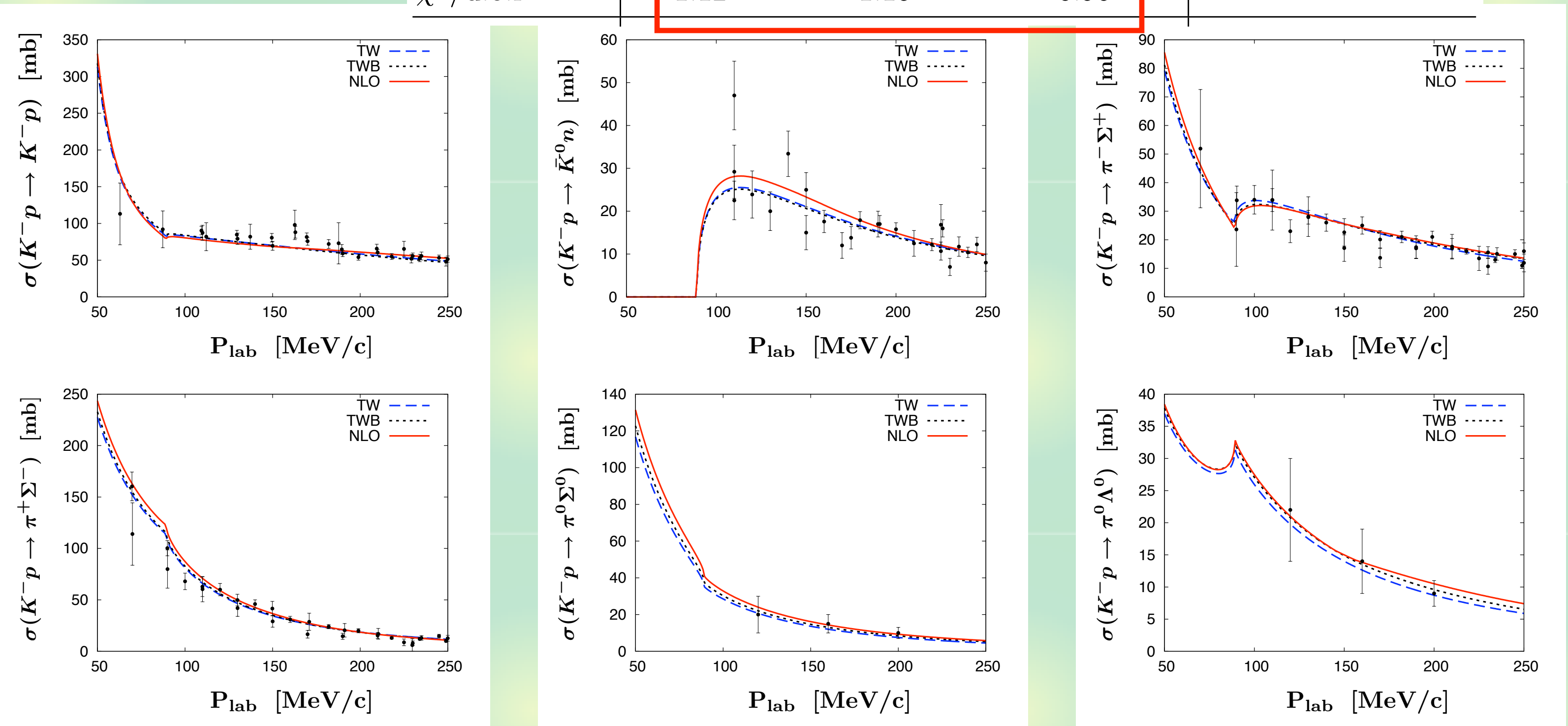
three-quark

- **is fundamental building block for $\bar{K}$-nuclei, $\bar{K}$-atoms, ...**

Fit to experiments: NLO chiral SU(3) dynamics

		TW	TWB	NLO	Experiment	
SIDDHARTA	ΔE [eV]	373	377	306	$283 \pm 36 \pm 6$	[10]
	Γ [eV]	495	514	591	$541 \pm 89 \pm 22$	[10]
Branching ratios	γ	2.36	2.36	2.37	2.36 ± 0.04	[11]
	R_n	0.20	0.19	0.19	0.189 ± 0.015	[11]
	R_c	0.66	0.66	0.66	0.664 ± 0.011	[11]
	χ^2/d.o.f	1.12	1.15	0.96		

cross sections



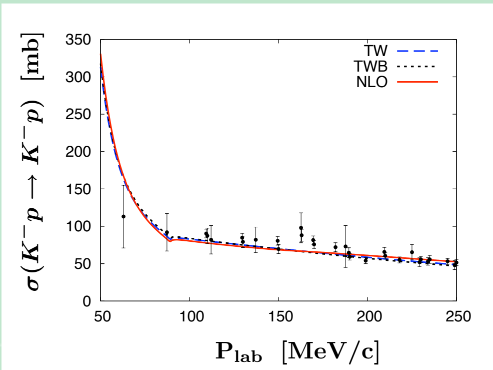
Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881 98 (2012)

Accurate description of all existing data (χ^2/d.o.f. ~ 1)

Model independent study on the structure

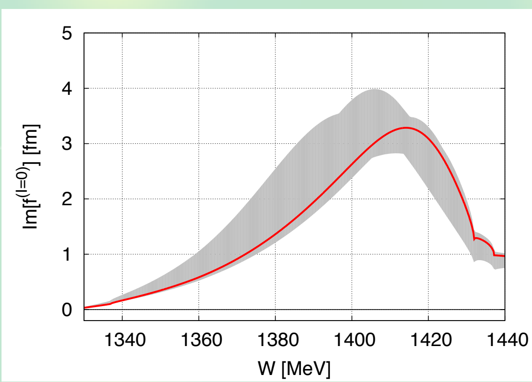
What are the model independent quantities?

- observables



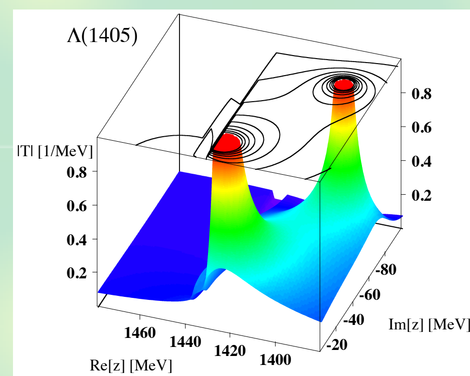
- wave function
- off-shell amplitude

- on-shell scattering amplitude



scattering length

- its analytic continuation



pole, zero

We use these to study the structure of Λ(1405).

Weak binding relation for stable states

Compositeness X **of s-wave** **weakly bound** **state (**$R \gg R_{typ}$**)**

S. Weinberg, Phys. Rev. 137, B672 (1965);
T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)

$$|d\rangle = \sqrt{X}\,|NN\rangle + \sqrt{1-X}\,|\text{others}\rangle$$

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\text{typ}}}{R}\right)\right\}, \quad r_e = R\left\{\frac{X-1}{X} + \mathcal{O}\left(\frac{R_{\text{typ}}}{R}\right)\right\}$$

a_0: **scattering length,** r_e: **effective range**
$R = (2\mu B)^{-1/2}$: **length scale of binding energy**
R_{typ}: **length scale of interaction**

- **Deuteron is** NN **composite (**$a_0 \sim R \gg r_e$**) —>** $X \sim 1$
- **Internal structure from** **model-independent quantities**

Problem: applicable only for stable states.

Effective field theory

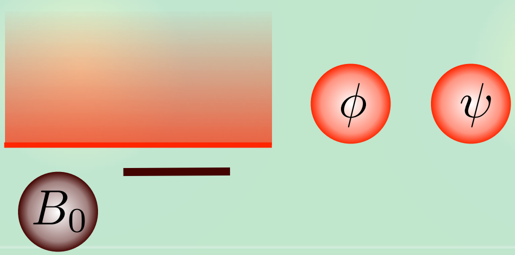
Low-energy scattering with near-threshold bound state

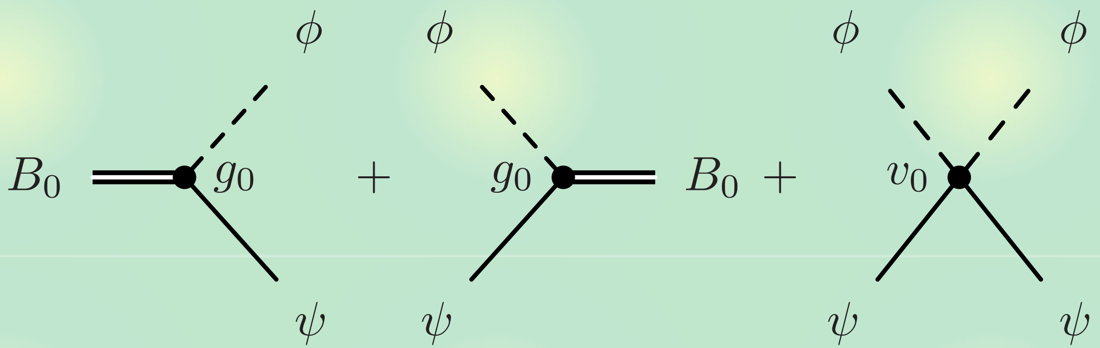
- **Nonrelativistic EFT with contact interaction**

D.B. Kaplan, Nucl. Phys. B494, 471 (1997)
E. Braaten, M. Kusunoki, D. Zhang, Annals Phys. 323, 1770 (2008)

$$H_{\text{free}} = \int d\boldsymbol{r}\left[\frac{1}{2M}\nabla\psi^{\dagger}\cdot\nabla\psi + \frac{1}{2m}\nabla\phi^{\dagger}\cdot\nabla\phi + \frac{1}{2M_0}\nabla B_0^{\dagger}\cdot\nabla B_0 + \nu_0 B_0^{\dagger}B_0\right],$$

$$H_{\text{int}} = \int d\boldsymbol{r}\left[g_0\left(B_0^{\dagger}\phi\psi + \psi^{\dagger}\phi^{\dagger}B_0\right) + v_0\psi^{\dagger}\phi^{\dagger}\phi\psi\right]$$





- **cutoff** : Λ ~ 1/R_{typ} **(interaction range of microscopic theory)**
- **At low energy** p ≪ Λ**, interaction ~ contact**

Compositeness and "elementariness"

Eigenstates

$$H_{\rm free}|\,B_0\,\rangle = \nu_0|\,B_0\,\rangle, \quad H_{\rm free}|\,\boldsymbol{p}\,\rangle = \frac{p^2}{2\mu}|\,\boldsymbol{p}\,\rangle$$

free (discrete + continuum)

$$(H_{\rm free}+H_{\rm int})|\,B\,\rangle = -B|\,B\,\rangle$$

full (bound state)

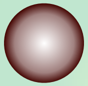
- normalization of |B> + completeness relation

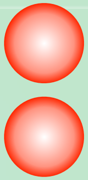
$$\langle\,B\,|\,B\,\rangle = 1, \quad 1 = |\,B_0\,\rangle\langle\,B_0\,| + \int \frac{d\boldsymbol{p}}{(2\pi)^3}|\,\boldsymbol{p}\,\rangle\langle\,\boldsymbol{p}\,|$$

- projections onto free eigenstates

$$1 = Z + X, \quad \underline{Z \equiv |\langle\,B_0\,|\,B\,\rangle|^2}, \quad \underline{X \equiv \int \frac{d\boldsymbol{p}}{(2\pi)^3}|\langle\,\boldsymbol{p}\,|\,B\,\rangle|^2}$$

"elementariness" **compositeness**



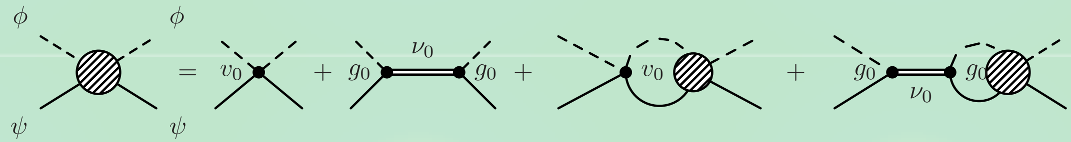


Z, X**: real and nonnegative —> interpreted as probability**

Weak binding relation

ΨΦ scattering amplitude (exact result)

$$f(E)=-\frac{\mu}{2\pi}\frac{1}{[v(E)]^{-1}-G(E)}$$



$$v(E)=v_0+\frac{g_0^2}{E-\nu_0},\quad G(E)=\frac{1}{2\pi^2}\int_0^{\Lambda}dp\frac{p^2}{E-p^2/(2\mu)+i0^+}$$

Compositeness X <— v(E), G(E)

$$X=\frac{G'(-B)}{G'(-B)-[1/v(-B)]'}$$

$1/R=(2\mu B)^{1/2}$ expansion: leading term <— X

$$a_0=-f(E=0)=R\left\{\frac{2X}{1+X}+\mathcal{O}\left(\frac{R_{\rm typ}}{R}\right)\right\}$$

model (cutoff) dependent

model (cutoff) independent

If $R \gg R_{\rm typ}$, correction terms neglected: X <— (B, a_0)

Introduction of decay channel

Introduce decay channel

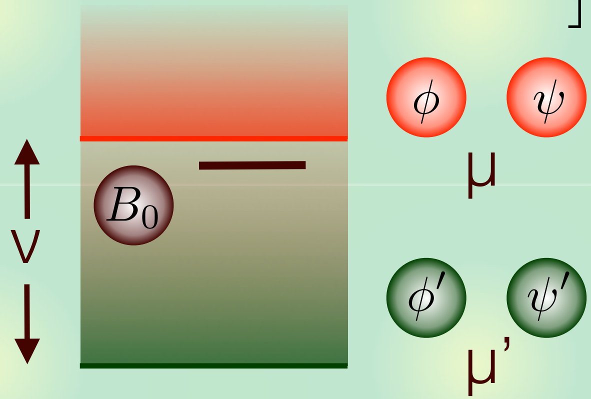
$$H'_{\rm free} = \int d\boldsymbol{r}\left[\frac{1}{2M'}\nabla\psi'^{\dagger}\cdot\nabla\psi' - \nu_{\psi}\psi'^{\dagger}\psi' + \frac{1}{2m'}\nabla\phi'^{\dagger}\cdot\nabla\phi' - \nu_{\phi}\phi'^{\dagger}\phi'\right],$$

$$H'_{\rm int} = \int d\boldsymbol{r}\left[g'_0\left(B_0^{\dagger}\phi'\psi' + \psi'^{\dagger}\phi'^{\dagger}B_0\right) + v'_0\psi'^{\dagger}\phi'^{\dagger}\phi'\psi' + v_0^{t}(\psi^{\dagger}\phi^{\dagger}\phi'\psi' + \psi'^{\dagger}\phi'^{\dagger}\phi\psi)\right],$$

Quasi-bound state: complex eigenvalue

$$H = H_{\rm free} + H'_{\rm free} + H_{\rm int} + H'_{\rm int}$$

$$H|\,QB\,\rangle = E_{QB}|\,QB\,\rangle, \quad E_{QB}\in\mathbb{C}$$



Generalized relation: correction term <— threshold difference

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\rm typ}}{R}\right|\right) + \mathcal{O}\left(\left|\frac{l}{R}\right|^3\right)\right\}, \quad R = \frac{1}{\sqrt{-2\mu E_{QB}}}\in\mathbb{C}, \quad l \equiv \frac{1}{\sqrt{2\mu\nu}}$$

Y. Kamiya, T. Hyodo, Phys. Rev. C93, 035203 (2016)

c.f. V. Baru, *et al.*, Phys. Lett. B586, 53 (2004),…

If $|R| \gg (R_{\rm typ}, l)$ correction terms neglected: X <— (E_{QB}, a_0)

Application to Λ(1405)

Generalized weak binding relation

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\rm typ}}{R}\right|\right) + \mathcal{O}\left(\left|\frac{l}{R}\right|^3\right)\right\}, \quad R = \frac{1}{\sqrt{-2\mu E_{QB}}}, \quad l \equiv \frac{1}{\sqrt{2\mu\nu}}$$

- **Consider** Λ(1405) **in** $\bar{\rm K}$N **scattering**
- **To determine** X**, we need (**E_{QB}**,** a_0**)**

From $\bar{\rm K}$N scattering amplitude

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881 98 (2012)

$$E_{QB} = -10 - 26i \text{ MeV}, \quad a_0 = 1.39 - 0.85i \text{ fm}$$

Neglecting the correction terms, we obtain

$$X_{\bar{K}N} = 1.2 + i0.1, \quad 1 - X_{\bar{K}N} = -0.2 - i0.1$$

Λ(1405) **is** $\bar{\rm K}$N **composite dominance**

Analytic structure of scattering amplitude

Pole of scattering amplitude $f(E_{pole})=\infty$

J.R. Taylor, *Scattering theory* (Wiley, New York, 1972)

- **represents (complex) eigenvalue of Hamiltonian**
- **unique through analytic continuation**

CDD (Castillejo-Dalitz-Dyson) zero

L. Castillejo, R.H. Dalitz, F.J. Dyson, Phys. Rev. 124, 264 (1961)

- **pole of inverse amplitude, zero of amplitude** $f(E_{CDD})=0$
- **unique through analytic continuation**
- **role of CDD zero in hadron scattering, resonances, etc.**

V. Baru, *et al.*, Eur. Phys. J. A 44, 93 (2010),
C. Hanhart, *et al.*, Eur. Phys. J. A 47, 101 (2011),
Z.H. Guo, J.A. Oller, Phys. Rev. D93, 054014 (2016)

Distance between pole and zero <—> origin of the state

Fate of pole in zero coupling limit

Zero coupling limit (ZCL): switching off the channel coupling

R.J. Eden, J.R. Taylor, Phys. Rev. 133, B1575 (1964)

$$H = \lim_{x\to 0}\begin{pmatrix} T_{11}+V_{11} & xV_{12} & \cdots \\ xV_{21} & T_{22}+V_{22} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} = \begin{pmatrix} T_{11}+V_{11} & 0 & \cdots \\ 0 & T_{22}+V_{22} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

- pole exists in all components at the same position for $x \neq 0$
- pole exists only in channel i with V_{ii} origin at $x=0$

Pole behavior in 11 amplitude toward ZCL ($x \to 0$)

- channel 1 origin : pole remains in 11 amplitude
- channel 2, … origin : pole decouples from 11 amplitude

How can a pole decouple from an amplitude?

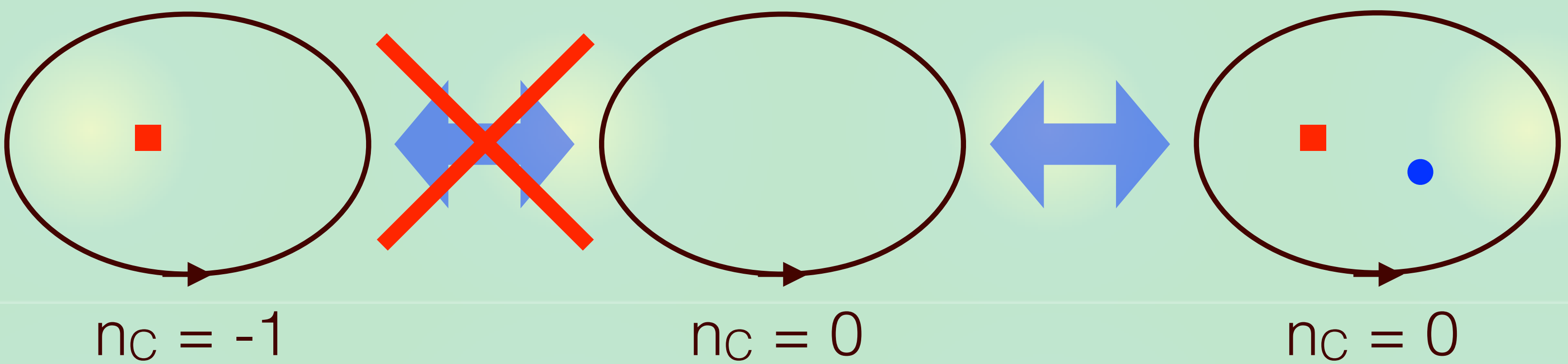
General discussion

Scattering amplitude f(E) **is meromorphic in energy**

Y. Kamiya, T. Hyodo, Phys. Rev. D97, 054019 (2018)

$$\frac{1}{2\pi}\oint_C dz \frac{d \arg f(z)}{dz} = n_Z - n_P \equiv n_C$$

- n_Z (n_P) : **number of zeros (poles) in contour C**
- **Topological invariant of** $\pi_1(U(1)) \cong \mathbb{Z}$

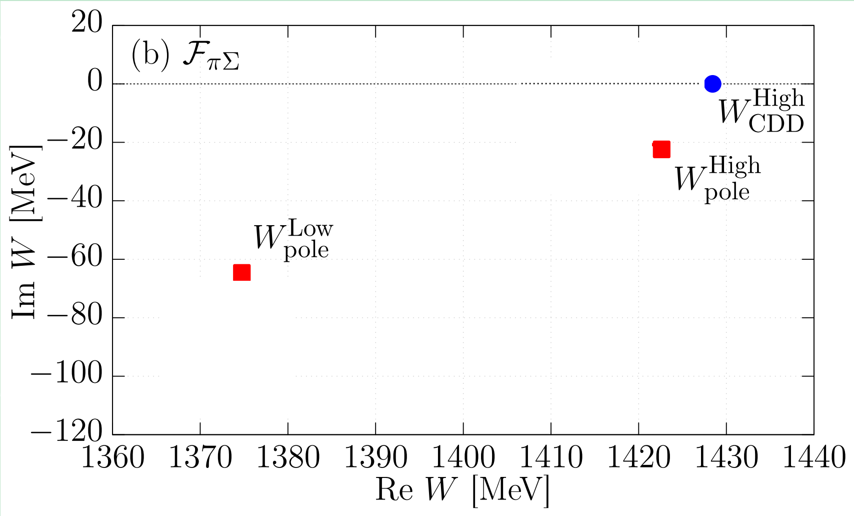


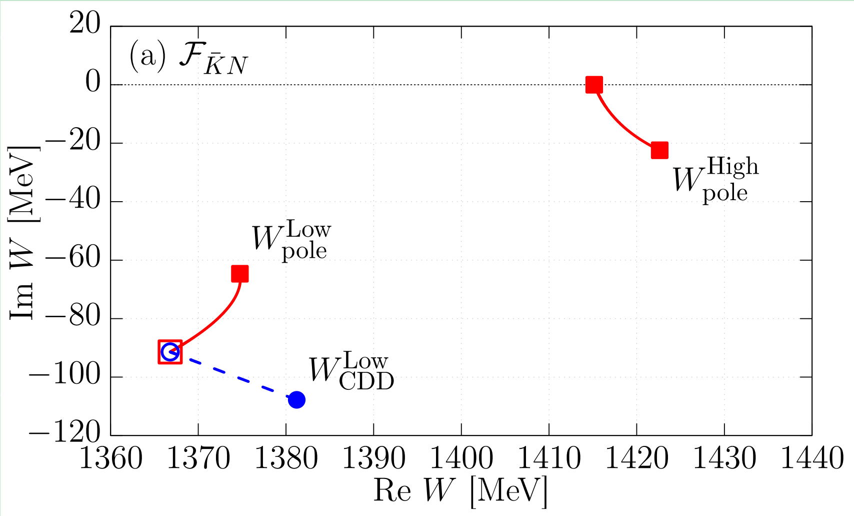
Pole cannot decouple without merging with CDD zero

—> existence of nearby CDD zero indicates "elementary" (origin is in other channel).

Example: Λ(1405)

Poles and zeros in the $\bar{K}N$ and πΣ amplitudes





- In πΣ amplitude, CDD zero exists near the high-mass pole, and merges with it to decouple in the ZCL.

- In $\bar{K}N$ amplitude, CDD zero exists near the low-mass pole, and merges with it to decouple in the ZCL.

Low- (high-)mass pole is not $\bar{K}N$ (πΣ) composite.

Summary

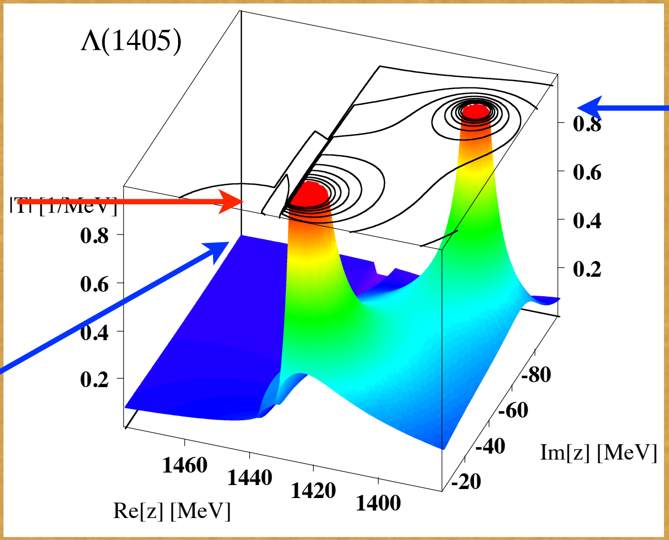
- **We study the structure of Λ(1405) from model independent quantities.**

  - **Compositeness from weak binding relation**

    Y. Kamiya, T. Hyodo, Phys. Rev. C93, 035203 (2016);
    Y. Kamiya, T. Hyodo, PTEP2017, 023D02 (2017)

$\bar{K}N$ dominant

$\bar{K}N$ origin



πΣ origin

  - **CDD zero analysis**

    Y. Kamiya, T. Hyodo, Phys. Rev. D97, 054019 (2018)