Exotic hadrons and emergent long range correlation in QCD



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Classification of hadrons

Observed hadrons

PDG2018 : http://pdg.lbl.gov/

	1/0+ ****	4(1000)	2/0+ ****	<u>r</u> +	1/0+ ****	-0	1/0+ ****	<u>a</u> +	1 /0+	****	1		LIGHT UN	IFLAVORED		STRA	NGE	CHARMED, ST	RANGE	С	GUPO
p	1/2 *****	$\Delta(1232)$	3/2 ****	Z '	1/2 *****		1/2 + ++++		1/2 ·	-1111			6(PC)	= <i>B</i> = 0)	P(PC)	$(3 = \pm 1, 0)$.=Б=0) (Р)	(c = 5 = :	(P)	(10)	$r^{-}(J^{-})$
<i>П</i>	1/2 ****	$\Delta(1600)$	3/2 ***	20	1/2 ****	=	1/2 ****	$\Lambda_{c}(2595)^{+}$	1/2	***		e =±	1=(0=)	a (1690)	0-(1)	• K±	1/2(0-)	• D [±]	0(0-)	• η _c (15)	$0^{-}(0^{-})$
/V(1440)	1/2 ****	<i>∆</i> (1620)	1/2 ****	Σ	1/2 ****	=(1530)	3/2 ****	$\Lambda_{c}(2625)^{+}$	3/2-	***		• π ⁰	$1^{-}(0^{-}+)$	 φ(1000) φ₃(1690) 	$1^{+}(3^{-})$	• K ⁰	$1/2(0^{-})$	• D _s • D ^{*±}	$0(0^{?})$	• $\chi_{c0}(1P)$	$0^{+}(0^{+}+)$
N(1520)	3/2 ****	$\Delta(1700)$	3/2 ****	Σ(1385)	3/2* ****	Ξ(1620)	*	$\Lambda_{c}(2765)^{+}$		*		• η	$0^{+}(0^{-}+)$	 ρ(1700) 	1+(1)	• K ⁰ ₅	1/2(0-)	• $D_{c0}^{*}(2317)^{\pm}$	0(0+)	• $\chi_{c1}(1P)$	$0^{+}(1^{+})$
N(1535)	1/2- ****	$\Delta(1750)$	$1/2^{+}$ *	$\Sigma(1480)$	*	$\Xi(1690)$	***	$\Lambda_{c}(2880)^{+}$	5/2+	***		• f ₀ (500)	0+(0++)	a2(1700)	$1^{-}(2^{++})$	• K ⁰ L	1/2(0-)	 D_{s1}(2460)[±] 	$0(1^+)$	• $h_c(1P)$?'(1+-)
N(1650)	1/2 ****	$\Delta(1900)$	1/2 **	$\Sigma(1560)$	**	$\Xi(1820)$	3/2 ***	$\Lambda_{c}(2940)^{+}$		***		 ρ(770) 	1+(1)	 f₀(1710) 	0+(0++)	K ₀ (800)	1/2(0+)	 D_{S1}(2536)[±] 	$0(1^+)$	• $\chi_{c2}(1P)$	$0^+(2^+)$
N(1675)	5/2 ****	<i>∆</i> (1905)	5/2+ ****	$\Sigma(1580)$	3/2- *	Ξ(1950)	***	$\Sigma_{c}(2455)$	$1/2^{+}$	****		• ω(782)	$0^{-}(1^{-})$	$\eta(1760)$	$0^+(0^{-+})$	• K*(892)	1/2(1-)	• D _{s2} (2573)	0(?')	• $\eta_c(2S)$	$0^{+}(0^{-+})$
N(1680)	5/2+ ****	$\Delta(1910)$	1/2+ ****	$\Sigma(1620)$	1/2- *	$\Xi(2030)$	$\geq \frac{5}{2}? ***$	$\Sigma_{c}(2520)$	3/2+	***		• 1/ (956) • fe(980)	$0^{+}(0^{+}+)$	$\bullet \pi(1000)$ $f_{0}(1810)$	$0^{+}(2^{+}+)$	• $K_1(12/0)$	$\frac{1}{2(1^+)}$	• $D_{s1}^*(2700)^+$	0(1)	• ψ(23) • ψ(3770)	$0^{-}(1^{-})$
N(1685)	, *	$\Delta(1920)$	3/2+ ***	Σ(1660)	1/2+ ***	$\Xi(2120)$	*	Σ (2800)		***		• a ₀ (980)	$1^{-}(0^{++})$	X(1835)	? [?] (? ⁻ +)	• K*(1410)	$\frac{1}{2}(1^{-})$	$D_{sJ}(2000)^{-1}$	$0(?^{?})$	X(3823)	??(??-)
N(1700)	3/2 ***	$\Delta(1930)$	5/2 ***	$\Sigma(1670)$	3/2- ****	=(2250)	**	=+	$1/2^{+}$	***		 φ(1020) 	0-(1)	X(1840)	? [?] (? ^{??})	 K[*]₀(1430) 	1/2(0+)	D ₈ J(3040)	0(.)	• X(3872)	$0^{+}(1^{++})$
N(1710)	1/2+ ***	$\Lambda(1940)$	3/2 **	Σ(1690)	-/- **	=(22.50)	**	-c =0	1/2+	***		 h₁(1170) 	$0^{-}(1^{+-})$	 φ₃(1850) 	0-(3)	 K[*]₂(1430) 	1/2(2+)	BOTTO	M	• X(3900) [±]	$?(1^+)$
N(1720)	3/2+ ****	$\Delta(1950)$	7/2+ ****	$\Sigma(1730)$	3/2+ *	=(2570)	*	- <u>c</u> -/+	1/2	***		• $b_1(1235)$	$1^{+}(1^{+})$	$\eta_2(1870)$	$0^+(2^-+)$ $1^-(2^-+)$	K(1460)	1/2(0 ⁻)	(B = ± 1	1/2(0-)	X(3900) ^o	$((\cdot))$
N(1720)	5/2 ***	$\Delta(1950)$	F /2 **	$\Sigma(1750)$	1/0 ***	=(2500)		$=_{c}$	1/2	***		• $a_1(1200)$ • $f_2(1270)$	$0^{+}(2^{+})$	 π₂(1880) a(1900) 	1 (2 - 1) $1^{+}(1 1)$	$K_2(1580)$	1/2(2 ⁻)	• B ⁻	$1/2(0^{-})$	• $\chi_{c0}(3913)$	$0^{+}(2^{+})$
N(1000)	5/2 · ····	$\Delta(2000)$	5/2· · · ·	$\Sigma(1750)$	1/2 ****	0-	2/0+ ****	Ξ_c^0	$1/2^+$	***		• f ₁ (1285)	$0^{+}(1^{++})$	f ₂ (1910)	$0^{+}(2^{+}+)$	K (1650)	$\frac{1}{2(1^+)}$	• B [±] /B ⁰ ADM	XTURE	X(3940)	,?(???)
//(1875)	3/2 ***	$\Delta(2150)$	1/2	2(1770)	1/2	M	3/2 1 1000	$\Xi_{c}(2645)$	3/2+	***		 η(1295) 	0 ⁺ (0 ⁻ +)	 f₂(1950) 	$0^+(2^++)$	• K*(1680)	$1/2(1^{-})$	• B [±] /B ⁰ /B ⁰ _s /b	baryon	X(4020)±	?(??)
N(1880)	1/2+ **	$\Delta(2200)$	1/2 *	$\Sigma(1775)$	5/2 ****	$\Omega(2250)^{-1}$	***	$\Xi_{c}(2790)$	1/2	***		 π(1300) 	1-(0-+)	$\rho_3(1990)$	1+(3)	• K2(1770)	$1/2(2^{-})$	ADMIXTURE	KM M>	 ψ(4040) 	$0^{-}(1^{-})$
N(1895)	1/2 **	Δ(23 <mark>00)</mark>	9/2***	<u>Σ(1840)</u>	3/2 *	$\Omega(2380)^{-}$	**	$\Xi_{c}(2815)$	$3/2^{-}$	***		• $a_2(1320)$	$1^{-}(2^{++})$	• f ₂ (2010)	$0^+(2^{++})$	 K[*]₃(1780) 	1/2(3-)	trix Elements		X(4050)∸ X(4140)	((?))
N(1900)	3/2+ ***	<i>∆</i> (23 <mark>50)</mark>	5/2 *	Σ(1880)	1/2+ **	<i>Ω</i> (2470) [−]	**	$E_{c}(2930)$		*		• 70(1370) h.(1390)	$7^{-}(1^{+})$	$T_0(2020)$	1 - (1 + 1)	• K ₂ (1820)	1/2(2-)	• B*	$1/2(1^{-})$	×(4140)	$0^{-}(1^{-})$
N(1990)	7/2+ **	∆(23 <mark>90)</mark>	7/2+ *	Σ(1900)	1/2 *			=(2980)		***		• π ₁ (1300)	$1^{-}(1^{-}+)$	• f ₄ (2050)	$0^{+}(4^{+}+)$	K(1830)	1/2(0-) 1/2(0+)	• $B_1(5/21)^+$	$1/2(1^+)$ $1/2(1^+)$	X(4160)	,??(???)
N(2000)	5/2+ **	<i>∆</i> (2400)	9/2 **	$\Sigma(1915)$	5/2+ ****			= (3055)		***		 η(1405) 	0 ⁺ (0 ⁻ +)	π ₂ (2100)	$1^{-}(2^{-+})$	$K_0^{(1950)}$	$1/2(0^{-1})$ $1/2(2^{+})$	B*(5732)	?(??)	X(4230)	$?^{?(1^{-'}-)}$
N(2040)	3/2+ *	<i>∆</i> (2420)	11/2 ⁺ ****	Σ(1940)	3/2+ *			=(3080)		***		• f ₁ (1420)	0+(1++)	f ₀ (2100)	0+(0++)	K*(2045)	$1/2(4^+)$	 B[*]₂(5747)⁺ 	$1/2(2^+)$	X(4240)±	? ⁽⁰⁻⁾
N(2060)	5/2 **	$\Delta(2750)$	13/2- **	Σ(1940)	3/2- ***			$=_{C}(3000)$		*		• ω(1420)	$0^{-}(1^{-})$	$f_2(2150)$	$0^+(2^+)$	K2(2250)	1/2(2-)	 B[*]₂(5747)⁰ 	$1/2(2^+)$	X(4250) [±]	?(? [:]) ??(1)
N(2100)	1/2+ *	$\Delta(2950)$	$15^{\prime}/2^{+} **$	Σ(2000)	1/2- *			$=_{C}(3123)$	1/0+	***		$I_2(1430)$ $I_2(1450)$	$1^{-}(0^{+}+)$	ρ(2150)	$0^{-}(1^{-})$	K ₃ (2320)	1/2(3+)	 B(5970)⁺ 	?(??)	• ^(4260) X(4350)	$0^{+}(7^{?+})$
N(2120)	3/2 **	_()	/-	$\Sigma(2030)$	7/2+ ****			12°C	1/2	-111-		 ρ(1450) 	1+(1)	f ₀ (2200)	$0^{+}(0^{+}+)$	K ₅ (2380)	1/2(5-)	 B(5970)⁰ 	?(?')	• X(4360)	$?(1^{-})$
N(2190)	7/2 ****	Λ	1/2+ ****	$\Sigma(2070)$	5/2+ *			$\Omega_{c}(2770)^{\circ}$	3/2	ተተተ		 η(1475) 	0+(0-+)	f_(2220)	0+(2++'	$r 4 \frac{K_4(2500)}{K(3100)}$	$\frac{1}{2(4^{\circ})}$	BOTTOM, ST	RANGE	 ψ(4415) 	$0^{-}(1^{})$
N(2230)	0/2+ ****	A(1405)	1/2 ****	$\Sigma(2000)$	3/2 2/0+ **			1 -1				 f₀(1500) 	0+(0++)	η(2225)	0+(0 - +)	A(3100)	:(:)	$(B = \pm 1, S =$	= ∓1)	• X(4430) [±]	?(1+)
N(2220)	9/2	A(1520)	2/2 ****	$\Sigma(2000)$	3/2 · · ·			\equiv_{cc}^{+}		*		$f_1(1510)$	$0^+(1^{++})$	$\rho_3(2250)$	$1^+(3^{})$	CHAR	MED	• B ⁰ _s	0(0-)	• X(4660)	?!(1)
N(2250)	9/2 1/0+ **	A(1600)	1/2+ ***	$\Sigma(2100)$	1/2 ****			.0				$\bullet T_2(1525)$ $f_6(1565)$	$0^{+}(2^{+})$	• I2(2300) fr(2300)	$0^{+}(2^{+})$	(c =	1/2(0-)	• B [*] _S	0(1)	b	Ъ
N(2300)	1/2 * ***	A(1670)	1/2 ****	Z(2250)	**			Λ_b^0	$1/2^{+}$	***		ρ(1570)	1+(1)	f ₀ (2330)	$0^{+}(0^{+}+)$	• D ⁰	$1/2(0^{-})$	 <i>B</i>[*]₂(5840)⁰ 	$0(1^+)$ $0(2^+)$	$\eta_{b}(1S)$	0+(0 - +)
N(2570)	5/2 **	/(1670)	1/2	2(2455)	**			$\Lambda_b(5912)^0$	$1/2^{-}$	***		h1(1595)	0-(1+-)	 f₂(2340) 	0+(2++)	 D*(2007)⁰ 	1/2(1-)	$B_{s_1}^*(5850)$?(? [?])	• 7(1S)	$0^{-}(1^{})$
N(2600)	11/2 ***	/(1690)	3/2 ****	$\Sigma(2620)$	**			$\Lambda_b(5920)^0$	$3/2^{-}$	***		 π₁(1600) 	$1^{-}(1^{-+})$	$\rho_5(2350)$	1+(5)	 D[*](2010)[±] 	$1/2(1^{-})$	DOTTOM CU		• χ _{b0} (1P)	$0^+(0^++)$
N(2700)	13/2+ **	Λ(1710)	1/2⊤ *	Σ(3000)	*			Σ_b	$1/2^{+}$	***		$a_1(1640)$	$1^{-}(1^{++})$	a ₆ (2450) f (2510)	$1^{-}(6^{+}^{+})$	 D[*]₀(2400)⁰ 	1/2(0+)	(B = C = i)	ARIVIED ±1)	• $\chi_{b1}(1P)$ • $h_{*}(1P)$	$7^{?}(1+-)$
		A(1800)	1/2 ***	$\Sigma(3170)$	*			Σ_{h}^{*}	$3/2^{+}$	***		• m(1645)	$0^{+}(2^{-}+)$	16(2510)	0.(0)	$D_0^*(2400)^{\pm}$	1/2(0+)	• B ⁺	0(0-)	• $\chi_{lp}(1P)$	$0^{+}(2^{+}+)$
		<i>Л</i> (1810)	1/2+ ***					Ξĕ.Ξ-	$1/2^{+}$	***		 ω(1650) 	$0^{-}(1^{-})$	OTHE	r light	• D ₁ (2420)* D ₂ (2420)±	$\frac{1}{2(1^{+})}$ $\frac{1}{2(7^{2})}$	$B_c(2S)^{\pm}$??(???)	$\eta_b(2S)$	0+(0 - +)
		A(1820)	5/2+ ****					$= \frac{1}{2}, \frac{1}{2}, \frac{1}{2}$	$-\frac{1}{1/2+}$	***		 ω₃(1670) 	0-(3)	Further S	tates	$D_1(2420)$ $D_1(2430)^0$	$1/2(1^+)$,	. ,	• Υ(2S)	0-(1)
		A(1830)	5/2 ****					= 6(5)(5)	2/0+	***		 π₂(1670) 	$1^{-}(2^{-+})$			 D[*]₂(2460)⁰ 	1/2(2+)			• T(1D)	$0^{-}(2^{-})$
		A(1890)	3/2+ ****					=b(5945)-	- 2/2+	***						 D₂[−](2460)[±] 	1/2(2+)			• XE0(2F)	$0^{+}(0^{+})$
		A(2000)	*					$=_{b}(5955)$	3/2	***						$D(2550)^{0}$	1/2(0-)			$h_b(2P)$	$?(1^{+})$
		1(2020)	7/2+ *					Ω_b^-	$1/2^{+}$	***						D(2600)	1/2(?*)			• χ _{b2} (2P)	$0^{+}(2^{++})$
		$\Lambda(2050)$	3/2- *					1								D(2750)	1/2(:*)			• 7(3S)	0-(1)
		A(2100)	~/~ 7/2 [—] ****													0(2133)	1/2(.)			• $\chi_{b1}(3P)$	$0^+(1^+)$
		A(2110)	5/2+ ***																	 I (45) X(10610)³ 	• (1) • 1 ⁺ (1 ⁺)
		A(222E)	3/∠ 2/2 [—] *						_				64	0						X(10610) ^C	1+(1+)
		/1(2323) A(23E0)	3/∠ 0/2+ ***		しょう	\mathbf{n}	arv	nn	C			\sim	21		ne	SO	nc			$X(10650)^{\pm}$	· ?+(1+)
		/1(2350)	9/2' *** **	1		VN	vui y		J							00		·		• 7(10860)	0-(1)
		/1(2585)	ተተ]									• /(11020)	∪ (1 -)

All ~ 360 hadrons emerge from single QCD Lagrangian.

Threshold rules in hadron spectroscopy

Hadrons near an s-wave two-body threshold



"hadronic molecule" (various flavors, baryon numbers, ...)

Two-body universal physics

Near-threshold s-wave state: universal physics

- E. Braaten, H.-W. Hammer, Phys. Rept. 428, 259 (2006); P. Naidon, S. Endo, Rept. Prog. Phys. 80, 056001 (2017)
- scattering length $|a| \gg$ interaction range r_e
- size of (quasi-)bound state $\sim |a|$: loosely bound
- relation with eigenenergy $\boldsymbol{\Xi}$

$$a(E) \sim \frac{i}{k} = \frac{i}{\sqrt{2\mu E}}$$

^k $\sqrt{2\mu E}$ Examples: d, $\Lambda(1405)$, ⁴He dimer

strong

vdW





⁴He

Classification of hadrons

1/0+ ****

1/0+ **** 1+

Observed hadrons

1/0+ **** 1/0+ **** -0

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STRANGE

CHARMED, STRANGE

LIGHT UNFLAVORED

CC GLEC Only color singlet states are observed. —> Color confinement problem Flavor quantum numbers are described by $qqq/q\bar{q}$. Why no qqqqq, qqqqq, ... states (exotic hadrons)? —> Exotic hadron problem, as notrivial as confinement!

N(2700) 13/2 ⁺ **	\Lambda(1/10) \Lambda(1800) \Lambda(1810) \Lambda(1820) \Lambda(1830) \Lambda(1830) \Lambda(1890) \Lambda(2000) \Lambda(2020) \Lambda(2050) \Lambda(2100)	$1/2^{+}$ $1/2^{-}$ $5/2^{+}$ $5/2^{-}$ $3/2^{+}$ $7/2^{+}$ $3/2^{-}$ $7/2^{-}$	* *** *** * * * * * * * * * * *	$\Sigma(3000)$ * $\Sigma(3170)$ *	8	$\begin{array}{ccccccc} \Sigma_b & 1/2^+ & *** \\ \Sigma_b^* & 3/2^+ & *** \\ \Xi_b^0, \Xi_b^- & 1/2^+ & *** \\ \Xi_b^0(5935)^- & 1/2^+ & *** \\ \Xi_b^0(5945)^0 & 3/2^+ & *** \\ \Omega_b^- & 1/2^+ & *** \end{array}$	$\begin{array}{c} \mathbf{a}_1(1640) & 1 & (1^{}) \\ 5_2(1640) & 0^+(2^{-+}) \\ \mathbf{*}_{12}(1645) & 0^+(2^{-+}) \\ \mathbf{*}_{1}(1650) & 0^-(1^{}) \\ \mathbf{*}_{2}(1670) & 0^-(3^{}) \\ \mathbf{*}_{2}(1670) & 1^-(2^{-+}) \end{array}$	$\begin{array}{c} a_{6}(2450) & 1 & (6^{}) \\ f_{6}(2510) & 0^{+}(6^{++}) \\ \hline \\ $	$\begin{array}{l} \bullet \mathcal{D}_{12}(2400)^{2} & 1/2(0^{+})\\ \mathcal{D}_{13}(2400)^{2} & 1/2(0^{+})\\ \bullet \mathcal{D}_{14}(2420)^{6} & 1/2(1^{+})\\ \mathcal{D}_{14}(2420)^{6} & 1/2(1^{+})\\ \mathcal{D}_{14}(2420)^{6} & 1/2(1^{+})\\ \bullet \mathcal{D}_{2}(2460)^{6} & 1/2(2^{+})\\ \bullet \mathcal{D}_{2}(2460)^{6} & 1/2(2^{+})\\ \mathcal{D}_{2}(2560)^{6} & 1/2(2^{+})\\ \mathcal{D}_{2}(2560)^{6} & 1/2(2^{+})\\ \mathcal{D}_{2}(2560)^{4} & 1/2(2^{+})\\ \mathcal{D}_{2}(2560$	$\begin{array}{c} \text{Bot IOM, CINNED}\\ (B = C = \pm 1) \\ \bullet B_{c}^{+} & 0(0^{-}) \\ B_{c}(25)^{\pm} & ?^{?}(?^{?}) \end{array}$	$\begin{array}{c} \bullet_{01}(1^{\prime\prime}) & \circ_{1}^{\prime}(1^{\prime}-) \\ \bullet_{bb}(DP) & ?^{\prime}(1^{\prime}-) \\ \bullet_{bb}(DP) & 0^{\dagger}(2^{\prime}+) \\ \eta_{b}(25) & 0^{\prime}(0^{-}+) \\ \circ_{1}(25) & 0^{\prime}(0^{-}+) \\ \circ_{1}(25) & 0^{\prime}(0^{-}+) \\ \bullet_{1}(2P) & 0^{\prime}(0^{-}+) \\ \bullet_{bb}(2P) & 0^{\prime}(0^{+}+) \\ \bullet_{bb}(2P) & 0^{\prime}(2^{+}+) \\ \bullet_{bb}(2P) & 0^{\prime}(2^{+}+) \\ \bullet_{1}(3S) & 0^{\prime}(2^{-}-) \\ \bullet_{1}(3S) & 0^{\prime}(1^{-}-) \\ \bullet_{1}(3S) & 0^{\prime}(1^{-}-) \\ \bullet_{1}(4S) & 0^{\prime}(1^{-}-) \end{array}$
	Λ(2110) Λ(2325) Λ(2350) Λ(2585)	5/2+ 3/2- 9/2+	*** * *** **	~ 15	i0 bary	ons	~ 21	0 me	sons)	$\begin{array}{c} X(10610)^{\pm} \ 1^{+}(1^{+}) \\ X(10610)^{0} \ 1^{+}(1^{+}) \\ X(10650)^{\pm} \ ?^{+}(1^{+}) \\ \bullet \ \gamma(10860) \ 0^{-}(1^{-}) \\ \bullet \ \gamma(11020) \ 0^{-}(1^{-}) \end{array}$

All ~ 360 hadrons emerge from single QCD Lagrangian.

V

Long range correlation in QCD?

Two-body potential

$$r) \propto rac{1}{r}$$
 : long (infinite) range

$$V(r) \propto \frac{e^{-mr}}{r}$$
 : finite (~1/m) range



Hadron-hadron interaction is considered to be finite range.

- Longest interaction range
 - < exchange of lightest particle (π) ~ 1 fm
- Absence of the long range force is the basis for the (standard) scattering theory, Lüscher/HAL method, etc.

There can be (quasi) long range force beyond 1 fm.

M. Sanchez Sanchez, L.S. Geng, J. Lu, T. Hyodo, M.P. Valderrama, PRD98, 054001 (2018)

NN potential

Low energy NN interaction : π exchange



- Static approx. $p^{\mu} = (M_N, p), \quad p'^{\mu} = (M_N, p'), \quad q^{\mu} = p'^{\mu} p^{\mu} = (0, q)$
- Coupling $g\bar{N}i\gamma_5\pi N \sim g\chi^{\dagger}\sigma \cdot q\chi$ (isospin ignored)



NN* potential (exchange)

 $N^*(k)$

N(p)



Mass difference = energy transfer

 $N^*(k')$

N(p')

 $\pi(q)$

 $\Delta = M_{N^*} - M_N$

- Static approx. $p^{\mu} = (M_N, p), \quad p'^{\mu} = (M_{N^*}, p'), \quad q^{\mu} = (\Delta, q)$
- Coupling $\tilde{g} \ \bar{N}^* \pi N + \text{h.c.} \sim \tilde{g} \ \chi^\dagger \mathbf{1} \chi$

Potential (P_o: spin exchange factor) $\mu = \sqrt{m_{\pi}^2 - \Delta^2}$ $V(r) \sim \text{F.T.} \left\{ \tilde{g}^2 \frac{1}{\Delta^2 - q^2 - m_{\pi}^2} \right\} P_{\sigma} = \text{F.T.} \left\{ \tilde{g}^2 \frac{-1}{q^2 + \mu^2} \right\} P_{\sigma} \sim \tilde{g}^2 P_{\sigma} \frac{e^{-\mu r}}{r}$

- Sign of V(r) is fixed and attractive (c.f. σ exchange in NN)
- Effective mass $\mu=0$ —> long range force (Coulomb like)

Unitary limit and zero-energy resonance

What does $\mu = (m_{\pi^2} - \Delta^2)^{1/2} = 0 <=> \Delta = m_{\pi}$ mean?



- $\Delta = m_{\pi}$: N* lies on top of the πN threshold —> $a_{\pi N} = \infty$



Remarks and toward physical realization

N*N ~ πNN : effective description of three-body system



Similarity with the Efimov effect

- spatially large three-body system via unitary two-body int.

Realization in physical hadron systems

- No system with exact μ =0 (N*: Δ ~595 MeV / m_{π}~140 MeV)
- Is there any system with small μ ?

Hadronic molecules by (quasi)long range correlation

Strange dibaryon

 $\Lambda(1405)=\Lambda^*$: $\overline{K}N$ quasibound state near the threshold

T. Hyodo, D. Jido, Prog. Part. Nucl. Phys. 67, 55 (2012)

- \overline{K} exchange between \wedge^* and N



- $\mu \sim 91$ MeV: \overline{K} exchange has longer tail than expected

- attractive in spin singlet channel —> KNN as A*N system <u>T. Uchino, T. Hyodo, M. Oka, Nucl. Phys. A, 868-869, 53 (2011)</u>

Hadronic molecules by (quasi)long range correlation

Doubly charmed exotic meson

- We consider $D_{s0}(c\bar{s}, 0^+)D(c\bar{q}, 0^-)$ system via K exchange
 - Charm C=2: manifestly exotic (ccqs)

D_{s0}(2317), KD threshold $\begin{array}{c} \int \mathbf{D}_{s0}(c\bar{s}, 0^{+}) & KD & D_{s0} & D \\ \int \Delta \sim 450 & m_{K} \sim 495 & D \\ D(c\bar{q}, 0^{-}) & D(c\bar{q}, 0^{-}) & D_{s0} & D_{s0} \\ \end{array}$

- K exchange gives quasi-long range (μ ~200 MeV) attraction

Can the attraction generate a bound state?

Hadronic molecules by (quasi)long range correlation

Prediction of binding energy

- Effective Lagrangian for $D_{s0}DK$ (and HQ partners) coupling $\mathcal{L} = \frac{h}{2} \operatorname{Tr}[\bar{H}_a S_b \mathcal{A}_{ab} \gamma_5] + C.C.$
 - coupling constant $h : D_0 \longrightarrow D\pi$ decay + SU(3) symmetry
 - Short range cutoff R_c < hadron size



- $R_c \sim 0.5 \text{ fm}$ —> $\sim 6 \text{ MeV}$ binding

Summary

Long range correlation among hadrons emerges when the mass difference \triangle matches with the mass of the exchange particle m.

D

Κ



 $V(r) \sim \frac{e^{-\mu r}}{r}, \quad \mu = \sqrt{m^2 - \Delta^2}$

K exchange in $D_{s0}(0^+)D(0^-)$ system: $\mu \sim 200$ MeV —> prediction of exotic charmed tetraquark

M. Sanchez Sanchez, L.S. Geng, J. Lu, T. Hyodo, M.P. Valderrama, Phys. Rev. D98, 054001 (2018)