Structure of hadrons from effective field theory

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Contents

Introduction: exotic hadron resonances

Compositeness of hadron resonances

S. Weinberg, Phys. Rev. 137, B672 (1965);
T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)

- Weak binding relation from EFT

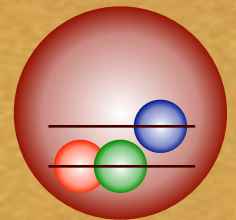Y. Kamiya, T. Hyodo, Phys. Rev. C93, 035203 (2016);
Y. Kamiya, T. Hyodo, PTEP2017, 023D02 (2017)
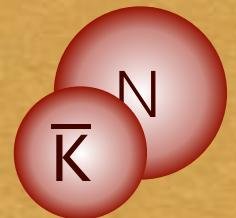
- Utilizing finite volume effect   **or**

Y. Tsuchida, T. Hyodo, Phys. Rev. C97, 0552113 (2018)

- Implication from nearby CDD zero
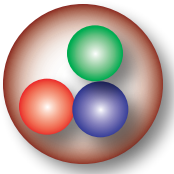
Y. Kamiya, T. Hyodo, Phys. Rev. D97, 054019 (2018)
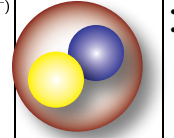
Classification of hadrons

Observed hadrons

PDG2018 : http://pdg.lbl.gov/

~ 150 baryons

	J^P			J^P			J^P			J^P			J^P	
p	$1/2^+$	****	$\Delta(1232)$	$3/2^+$	****	Σ^+	$1/2^+$	****	Ξ^0	$1/2^+$	****	Λ_c^+	$1/2^+$	****
n	$1/2^+$	****	$\Delta(1600)$	$3/2^+$	***	Σ^0	$1/2^+$	****	Ξ^-	$1/2^+$	****	$\Lambda_c(2595)^+$	$1/2^-$	***
$N(1440)$	$1/2^+$	****	$\Delta(1620)$	$1/2^-$	****	Σ^-	$1/2^+$	****	$\Xi(1530)$	$3/2^+$	****	$\Lambda_c(2625)^+$	$3/2^-$	***
$N(1520)$	$3/2^-$	****	$\Delta(1700)$	$3/2^-$	****	$\Sigma(1385)$	$3/2^+$	****	$\Xi(1620)$		*	$\Lambda_c(2765)^+$		*
$N(1535)$	$1/2^-$	****	$\Delta(1750)$	$1/2^+$	*	$\Sigma(1480)$		*	$\Xi(1690)$		***	$\Lambda_c(2880)^+$	$5/2^+$	***
$N(1650)$	$1/2^-$	****	$\Delta(1900)$	$1/2^-$	**	$\Sigma(1560)$		**	$\Xi(1820)$	$3/2^-$	***	$\Lambda_c(2940)^+$		***
$N(1675)$	$5/2^+$	****	$\Delta(1905)$	$5/2^+$	****	$\Sigma(1580)$	$3/2^-$	*	$\Xi(1950)$		***	$\Sigma_c(2455)$	$1/2^+$	****
$N(1680)$	$5/2^+$	****	$\Delta(1910)$	$1/2^+$	****	$\Sigma(1620)$	$1/2^-$	*	$\Xi(2030)$	$\geq 5/2?$	***	$\Sigma_c(2520)$	$3/2^+$	***
$N(1685)$		*	$\Delta(1920)$	$3/2^+$	***	$\Sigma(1660)$	$1/2^+$	***	$\Xi(2120)$		*	$\Sigma_c(2800)$		***
$N(1700)$	$3/2^-$	***	$\Delta(1930)$	$5/2^-$	***	$\Sigma(1670)$	$3/2^-$	****	$\Xi(2250)$		**	Ξ_c^+	$1/2^+$	***
$N(1710)$	$1/2^+$	****	$\Delta(1940)$	$3/2^-$	**	$\Sigma(1690)$		**	$\Xi(2370)$		**	Ξ_c^0	$1/2^+$	***
$N(1720)$	$3/2^+$	****	$\Delta(1950)$	$7/2^+$	****	$\Sigma(1730)$	$3/2^+$	*	$\Xi(2500)$		*	$\Xi_c'^+$	$1/2^+$	***
$N(1860)$	$5/2^+$	**	$\Delta(2000)$	$5/2^+$	**	$\Sigma(1750)$	$1/2^-$	***				$\Xi_c'^0$	$1/2^+$	***
$N(1875)$	$3/2^-$	***	$\Delta(2150)$	$1/2^-$	*	$\Sigma(1770)$	$1/2^+$	*				$\Xi_c(2645)$	$3/2^+$	***
$N(1880)$	$1/2^+$	**	$\Delta(2200)$	$7/2^-$	*	$\Sigma(1775)$	$5/2^-$	****	Ω^-	$3/2^+$	****	$\Xi_c(2790)$	$1/2^-$	***
$N(1895)$	$1/2^-$	**	$\Delta(2300)$	$9/2^+$	**	$\Sigma(1840)$	$3/2^+$	*	$\Omega(2250)^-$		***	$\Xi_c(2815)$	$3/2^-$	***
$N(1900)$	$3/2^+$	***	$\Delta(2350)$	$5/2^-$	*	$\Sigma(1880)$	$1/2^+$	**	$\Omega(2380)^-$		**	$\Xi_c(2930)$		*
$N(1990)$	$7/2^+$	**	$\Delta(2390)$	$7/2^+$	*	$\Sigma(1900)$	$1/2^-$	*	$\Omega(2470)^-$		**	$\Xi_c(2980)$		***
$N(2000)$	$5/2^+$	**	$\Delta(2400)$	$9/2^-$	**	$\Sigma(1915)$	$5/2^+$	****				$\Xi_c(3055)$		***
$N(2040)$	$3/2^+$	*	$\Delta(2420)$	$11/2^+$	****	$\Sigma(1940)$	$3/2^+$	*				$\Xi_c(3080)$		***
$N(2060)$	$5/2^-$	**	$\Delta(2750)$	$13/2^-$	**	$\Sigma(1940)$	$3/2^-$	***				$\Xi_c(3123)$		*
$N(2100)$	$1/2^+$	*	$\Delta(2950)$	$15/2^+$	**	$\Sigma(2000)$	$1/2^-$	*				Ω_c^0	$1/2^+$	***
$N(2120)$	$3/2^-$	**				$\Sigma(2030)$	$7/2^+$	****				$\Omega_c(2770)^0$	$3/2^+$	***
$N(2190)$	$7/2^-$	****	Λ	$1/2^+$	****	$\Sigma(2070)$	$5/2^+$	*						
$N(2220)$	$9/2^+$	****	$\Lambda(1405)$	$1/2^-$	****	$\Sigma(2080)$	$3/2^+$	*				Ξ_{cc}^+		*
$N(2250)$	$9/2^-$	****	$\Lambda(1520)$	$3/2^-$	****	$\Sigma(2100)$	$7/2^-$	*						
$N(2300)$	$1/2^+$	**	$\Lambda(1600)$	$1/2^+$	***	$\Sigma(2250)$		***				Λ_b^0	$1/2^+$	***
$N(2570)$	$5/2^-$	**	$\Lambda(1670)$	$1/2^-$	****	$\Sigma(2455)$		**				$\Lambda_b(5912)^0$	$1/2^-$	***
$N(2600)$	$11/2^-$	***	$\Lambda(1690)$	$3/2^-$	****	$\Sigma(2620)$		**				$\Lambda_b(5920)^0$	$3/2^-$	***
$N(2700)$	$13/2^+$	**	$\Lambda(1710)$	$1/2^+$	*	$\Sigma(3000)$		*				Σ_b	$1/2^+$	***
			$\Lambda(1800)$	$1/2^-$	***	$\Sigma(3170)$		*				Σ_b^*	$3/2^+$	***
			$\Lambda(1810)$	$1/2^+$	***							Ξ_b^0, Ξ_b^-	$1/2^+$	***
			$\Lambda(1820)$	$5/2^+$	****							$\Xi_b'(5935)^-$	$1/2^+$	***
			$\Lambda(1830)$	$5/2^-$	****							$\Xi_b(5945)^0$	$3/2^+$	***
			$\Lambda(1890)$	$3/2^+$	****							$\Xi_b^*(5955)^-$	$3/2^+$	***
			$\Lambda(2000)$		*							Ω_b^-	$1/2^+$	***
			$\Lambda(2020)$	$7/2^+$	*									
			$\Lambda(2050)$	$3/2^-$	*									
			$\Lambda(2100)$	$7/2^-$										
			$\Lambda(2110)$	$5/2^+$	***									
			$\Lambda(2325)$	$3/2^-$	*									
			$\Lambda(2350)$	$9/2^+$	***									
			$\Lambda(2585)$		**									

~ 210 mesons

LIGHT UNFLAVORED ($S = C = B = 0$), $I^G(J^{PC})$

	$I^G(J^{PC})$		$I^G(J^{PC})$
$\pi^\pm$	$1^-(0^-)$	$\phi(1680)$	$0^-(1^{--})$
π^0	$1^-(0^{-+})$	$\rho_3(1690)$	$1^+(3^{--})$
η	$0^+(0^{-+})$	$\rho(1700)$	$1^+(1^{--})$
$f_0(500)$	$0^+(0^{++})$	$a_2(1700)$	$1^-(2^{++})$
$\rho(770)$	$1^+(1^{--})$	$f_0(1710)$	$0^+(0^{++})$
$\omega(782)$	$0^-(1^{--})$	$\eta(1760)$	$0^+(0^{-+})$
$\eta'(958)$	$0^+(0^{-+})$	$\pi(1800)$	$1^-(0^{-+})$
$f_0(980)$	$0^+(0^{++})$	$f_2(1810)$	$0^+(2^{++})$
$a_0(980)$	$1^-(0^{++})$	$X(1835)$	$?^?(?^{-+})$
$\phi(1020)$	$0^-(1^{--})$	$X(1840)$	$?^?(?^{??})$
$h_1(1170)$	$0^-(1^{+-})$	$\phi_3(1850)$	$0^-(3^{--})$
$b_1(1235)$	$1^+(1^{+-})$	$\eta_2(1870)$	$0^+(2^{-+})$
$a_1(1260)$	$1^-(1^{++})$	$\pi_2(1880)$	$1^-(2^{-+})$
$f_2(1270)$	$0^+(2^{++})$	$\rho(1900)$	$1^+(1^{--})$
$f_1(1285)$	$0^+(1^{++})$	$f_2(1910)$	$0^+(2^{++})$
$\eta(1295)$	$0^+(0^{-+})$	$f_2(1950)$	$0^+(2^{++})$
$\pi(1300)$	$1^-(0^{-+})$	$\rho_3(1990)$	$1^+(3^{--})$
$a_2(1320)$	$1^-(2^{++})$	$f_2(2010)$	$0^+(2^{++})$
$f_0(1370)$	$0^+(0^{++})$	$f_0(2020)$	$0^+(0^{++})$
$h_1(1380)$	$?^-(1^{+-})$	$a_4(2040)$	$1^-(4^{++})$
$\pi_1(1400)$	$1^-(1^{-+})$	$f_4(2050)$	$0^+(4^{++})$
$\eta(1405)$	$0^+(0^{-+})$	$\pi_2(2100)$	$1^-(2^{-+})$
$f_1(1420)$	$0^+(1^{++})$	$f_0(2100)$	$0^+(0^{++})$
$\omega(1420)$	$0^-(1^{--})$	$f_2(2150)$	$0^+(2^{++})$
$f_2(1430)$	$0^+(2^{++})$	$\rho(2150)$	$1^+(1^{--})$
$a_0(1450)$	$1^-(0^{++})$	$\phi(2170)$	$0^-(1^{--})$
$\rho(1450)$	$1^+(1^{--})$	$f_0(2200)$	$0^+(0^{++})$
$\eta(1475)$	$0^+(0^{-+})$	$f_J(2220)$	$0^+(2^{++} \text{ or } 4^{++})$
$f_0(1500)$	$0^+(0^{++})$	$\eta(2225)$	$0^+(0^{-+})$
$f'_2(1525)$	$0^+(2^{++})$	$\rho_3(2250)$	$1^+(3^{--})$
$f_2(1565)$	$0^+(2^{++})$	$f_2(2300)$	$0^+(2^{++})$
$\rho(1570)$	$1^+(1^{--})$	$f_4(2300)$	$0^+(4^{++})$
$h_1(1595)$	$0^-(1^{+-})$	$f_0(2330)$	$0^+(0^{++})$
$\pi_1(1600)$	$1^-(1^{-+})$	$f_2(2340)$	$0^+(2^{++})$
$a_1(1640)$	$1^-(1^{++})$	$\rho_5(2350)$	$1^+(5^{--})$
$f_2(1640)$	$0^+(2^{++})$	$a_6(2450)$	$1^-(6^{++})$
$\eta_2(1645)$	$0^+(2^{-+})$	$f_6(2510)$	$0^+(6^{++})$
$\omega(1650)$	$0^-(1^{--})$		
$\omega_3(1670)$	$0^-(3^{--})$		
$\pi_2(1670)$	$1^-(2^{-+})$		

STRANGE ($S = \pm 1, C = B = 0$), $I(J^P)$

	$I(J^P)$
$K^\pm$	$1/2(0^-)$
K^0	$1/2(0^-)$
K_S^0	$1/2(0^-)$
K_L^0	$1/2(0^-)$
$K_0^*(800)$	$1/2(0^+)$
$K^*(892)$	$1/2(1^-)$
$K_1(1270)$	$1/2(1^+)$
$K_1(1400)$	$1/2(1^+)$
$K^*(1410)$	$1/2(1^-)$
$K_0^*(1430)$	$1/2(0^+)$
$K_2^*(1430)$	$1/2(2^+)$
$K(1460)$	$1/2(0^-)$
$K_2(1580)$	$1/2(2^-)$
$K(1630)$	$1/2(?^?)$
$K_1(1650)$	$1/2(1^+)$
$K^*(1680)$	$1/2(1^-)$
$K_2(1770)$	$1/2(2^-)$
$K_3^*(1780)$	$1/2(3^-)$
$K_2(1820)$	$1/2(2^-)$
$K(1830)$	$1/2(0^-)$
$K_0^*(1950)$	$1/2(0^+)$
$K_2^*(1980)$	$1/2(2^+)$
$K_4^*(2045)$	$1/2(4^+)$
$K_2(2250)$	$1/2(2^-)$
$K_3(2320)$	$1/2(3^+)$
$K_5^*(2380)$	$1/2(5^-)$
$K_4(2500)$	$1/2(4^-)$
$K(3100)$	$?^?(?^{??})$

CHARMED ($C = \pm 1$)

	$I(J^P)$
$D^\pm$	$1/2(0^-)$
D^0	$1/2(0^-)$
$D^*(2007)^0$	$1/2(1^-)$
$D^*(2010)^\pm$	$1/2(1^-)$
$D_0^*(2400)^0$	$1/2(0^+)$
$D_0^*(2400)^\pm$	$1/2(0^+)$
$D_1(2420)^0$	$1/2(1^+)$
$D_1(2420)^\pm$	$1/2(?^?)$
$D_1(2430)^0$	$1/2(1^+)$
$D_2^*(2460)^0$	$1/2(2^+)$
$D_2^*(2460)^\pm$	$1/2(2^+)$
$D(2550)^0$	$1/2(0^-)$
$D(2600)$	$1/2(?^?)$
$D^*(2640)^\pm$	$1/2(?^?)$
$D(2750)$	$1/2(?^?)$

OTHER LIGHT Further States

CHARMED, STRANGE ($C = S = \pm 1$), $I(J^P)$

	$I(J^P)$
$D_s^\pm$	$0(0^-)$
$D_s^{*\pm}$	$0(?^?)$
$D_{s0}^*(2317)^\pm$	$0(0^+)$
$D_{s1}(2460)^\pm$	$0(1^+)$
$D_{s1}(2536)^\pm$	$0(1^+)$
$D_{s2}^*(2573)$	$0(?^?)$
$D_{s1}^*(2700)^\pm$	$0(1^-)$
$D_{sJ}^*(2860)^\pm$	$0(?^?)$
$D_{sJ}(3040)^\pm$	$0(?^?)$

BOTTOM ($B = \pm 1$)

	$I(J^P)$
$B^\pm$	$1/2(0^-)$
B^0	$1/2(0^-)$
$B^\pm/B^0$ ADMIXTURE	
$B^\pm/B^0/B_s^0/b$-baryon ADMIXTURE	
V_{cb} and V_{ub} CKM Matrix Elements	
B^*	$1/2(1^-)$
$B_1(5721)^+$	$1/2(1^+)$
$B_1(5721)^0$	$1/2(1^+)$
$B_J^*(5732)$	$?^?(?^?)$
$B_2^*(5747)^+$	$1/2(2^+)$
$B_2^*(5747)^0$	$1/2(2^+)$
$B(5970)^+$	$?^?(?^?)$
$B(5970)^0$	$?^?(?^?)$

BOTTOM, STRANGE ($B = \pm 1, S = \mp 1$)

	$I(J^P)$
B_s^0	$0(0^-)$
B_s^*	$0(1^-)$
$B_{s1}(5830)^0$	$0(1^+)$
$B_{s2}^*(5840)^0$	$0(2^+)$
$B_{sJ}^*(5850)$	$?^?(?^?)$

BOTTOM, CHARMED ($B = C = \pm 1$)

	$I(J^P)$
$B_c^\pm$	$0(0^-)$
$B_c(2S)^\pm$	$?^?(?^{??})$

$c\bar{c}$, $I^G(J^{PC})$

	$I^G(J^{PC})$
$\eta_c(1S)$	$0^+(0^{-+})$
$J/\psi(1S)$	$0^-(1^{--})$
$\chi_{c0}(1P)$	$0^+(0^{++})$
$\chi_{c1}(1P)$	$0^+(1^{++})$
$h_c(1P)$	$?^?(1^{+-})$
$\chi_{c2}(1P)$	$0^+(2^{++})$
$\eta_c(2S)$	$0^+(0^{-+})$
$\psi(2S)$	$0^-(1^{--})$
$\psi(3770)$	$0^-(1^{--})$
$X(3823)$	$?^?(?^{??})$
$X(3872)$	$0^+(1^{++})$
$X(3900)^\pm$	$?^?(1^+)$
$X(3900)^0$	$?^?(?^?)$
$\chi_{c0}(3915)$	$0^+(0^{++})$
$\chi_{c2}(2P)$	$0^+(2^{++})$
$X(3940)$	$?^?(?^{??})$
$X(4020)^\pm$	$?^?(?^?)$
$X(4040)$	$0^-(1^{--})$
$X(4050)^\pm$	$?^?(?^?)$
$X(4140)$	$0^+(?^{?+})$
$\psi(4160)$	$0^-(1^{--})$
$X(4160)$	$?^?(?^{??})$
$X(4230)$	$?^?(?^?)$
$X(4240)^\pm$	$?^?(0^-)$
$\psi(4250)^\pm$	$?^?(?^?)$
$X(4260)$	$?^?(?^?)$
$X(4350)$	$0^+(?^{?+})$
$X(4360)$	$?^?(1^{--})$
$\psi(4415)$	$0^-(1^{--})$
$X(4430)^\pm$	$?^?(1^+)$
$X(4660)$	$?^?(1^{--})$

$b\bar{b}$

	$I^G(J^{PC})$
$\eta_b(1S)$	$0^+(0^{-+})$
$\Upsilon(1S)$	$0^-(1^{--})$
$\chi_{b0}(1P)$	$0^+(0^{++})$
$\chi_{b1}(1P)$	$0^+(1^{++})$
$h_b(1P)$	$?^?(1^{+-})$
$\chi_{b2}(1P)$	$0^+(2^{++})$
$\eta_b(2S)$	$0^+(0^{-+})$
$\Upsilon(2S)$	$0^-(1^{--})$
$\Upsilon(1D)$	$0^-(2^{--})$
$\chi_{b0}(2P)$	$0^+(0^{++})$
$\chi_{b1}(2P)$	$0^+(1^{++})$
$h_b(2P)$	$?^?(1^{+-})$
$\chi_{b2}(2P)$	$0^+(2^{++})$
$\Upsilon(3S)$	$0^-(1^{--})$
$\chi_{b1}(3P)$	$0^+(1^{++})$
$\Upsilon(4S)$	$0^-(1^{--})$
$X(10610)^\pm$	$1^+(1^+)$
$X(10610)^0$	$1^+(1^+)$
$X(10650)^\pm$	$?^+(1^+)$
$\Upsilon(10860)$	$0^-(1^{--})$
$\Upsilon(11020)$	$0^-(1^{--})$

All ~ 360 hadrons emerge from single QCD Lagrangian.

All flavor quantum numbers are described by qqq/q q̄.

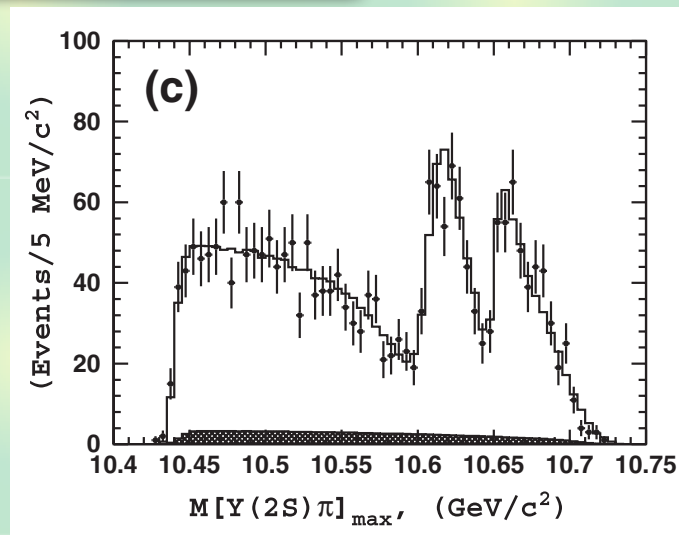## Exotic candidates beyond $qqq/q\bar{q}$

Tetraquark candidate (Belle)
: $Z_b(10610)$, $Z_b(10650)$

$$Y(5S) \longrightarrow \pi^{\pm} + Z_b$$
$$\hookrightarrow Y(nS)(b\bar{b}) + \pi^{\mp}(u\bar{d}/d\bar{u})$$

A. Bondar, *et al.*, Phys. Rev. Lett. 108, 122001 (2012)
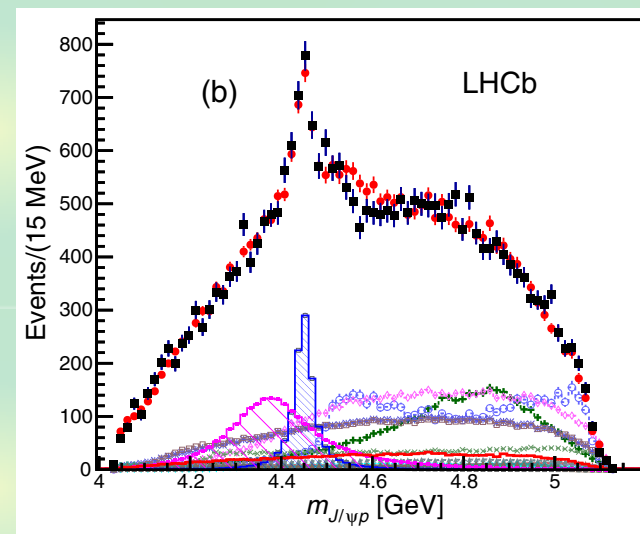
Pentaquark candidate (LHCb)
: $P_c(4450)$, $P_c(4380)$

$$\Lambda_b \longrightarrow K^- + P_c$$
$$\hookrightarrow J/\psi(c\bar{c}) + p(uud)$$
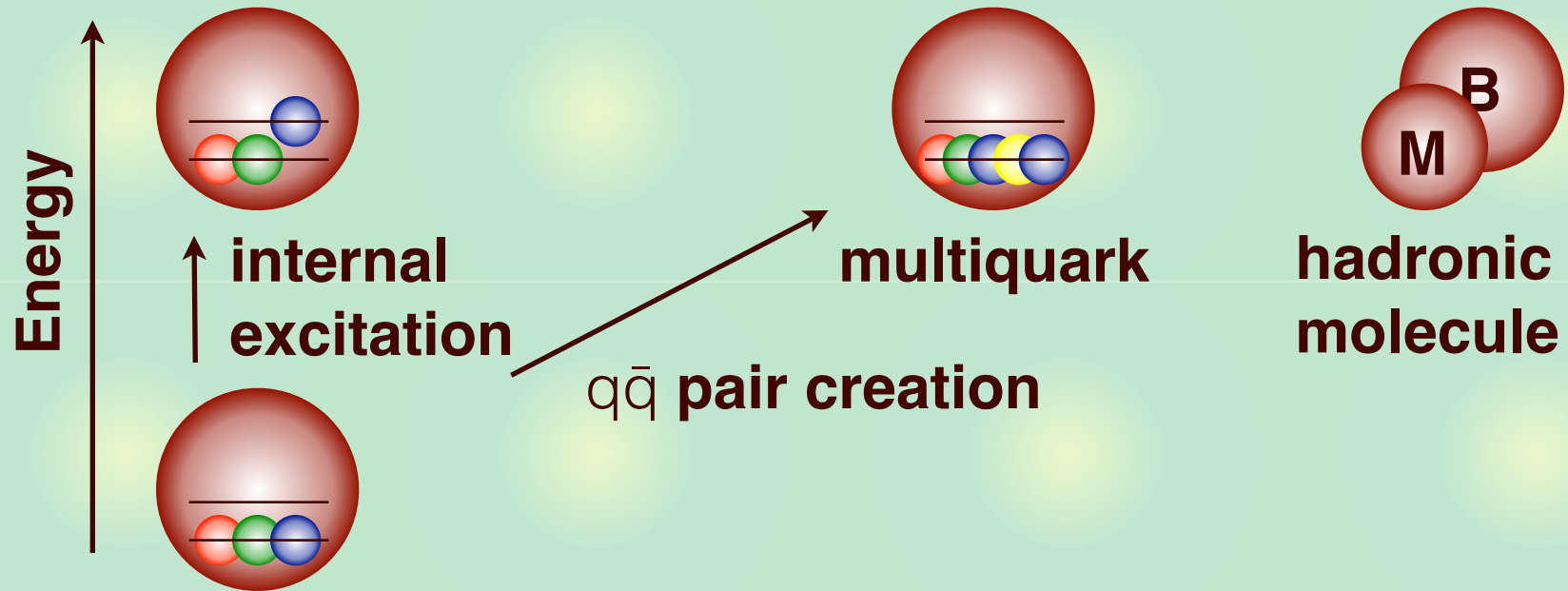
R. Aaij, *et al.*, Phys. Rev. Lett. 115, 072001 (2015)

Only a few are observed. Why only a few?

Various hadronic excitations

Description of excited baryons

Conventional structure   **Exotic structures**

Energy

↑ **internal excitation**

multiquark   **hadronic molecule**

$q\bar{q}$ **pair creation**

In QCD, non-qqq **structures naturally arise.**

- **Baryons: superposition of** qqq **+ exotic structures**

—> **How can we disentangle different components?**

5

Unstable states via strong interaction

Hadron resonances

PDG2018 : http://pdg.lbl.gov/

- **stable**/**unstable** via strong interaction

- **Excited states are mostly unstable. —> resonances**

Difficulty of resonances

Resonance as an "eigenstate" of Hamiltonian

- **complex energy**

  G. Gamow, Z. Phys. 51, 204 (1928)

  Zur Quantentheorie des Atomkernes.

  Von **G. Gamow**, z. Zt. in Göttingen.

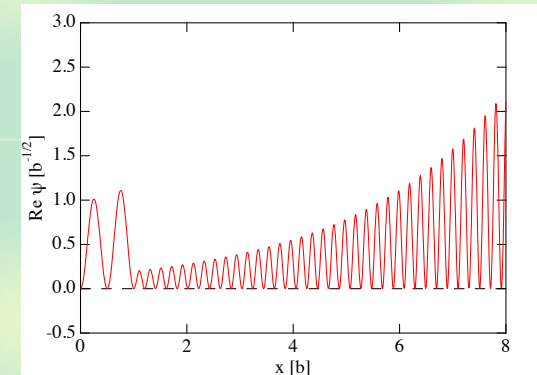 Mit 5 Abbildungen.   (Eingegangen am 2. August 1928.)

  Um diese Schwierigkeit zu überwinden, müssen wir annehmen, daß die Schwingungen gedämpft sind, und E komplex setzen:

  $$E = E_0 + i\,\frac{h\,\lambda}{4\,\pi},$$

  wo E_0 die gewöhnliche Energie ist und λ das Dämpfungsdekrement (Zerfallskonstante). Dann sehen wir aber aus den Relationen (2a) und (2b),

- **diverging wave function (**Im k < 0**)**

$$\langle R \,|\, R \rangle = \int d\boldsymbol{r} |\psi_R(\boldsymbol{r})|^2 \sim \int_0^\infty dr\, e^{-2\mathrm{Im}[k]r} \to \infty$$

Bi-orthogonal basis (Gamow vectors): normalizable!

N. Hokkyo, Prog. Theor. Phys. 33, 1116 (1965)
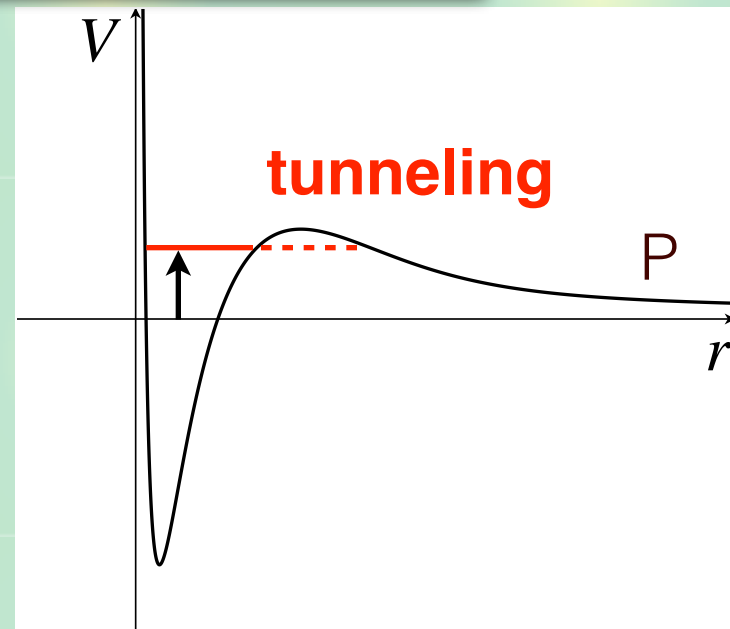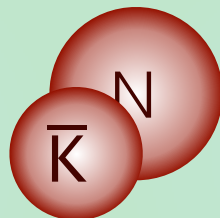T. Berggren, Nucl. Phys. A 109, 265 (1968)

$$|\tilde{R}\rangle = |R^*\rangle, \quad |\langle \tilde{R} \,|\, R \rangle| = \left| \int d\boldsymbol{r} [\psi_R(\boldsymbol{r})]^2 \right| < \infty$$

- **Complex expectation value (e.g. <r²>) —> interpretation?**
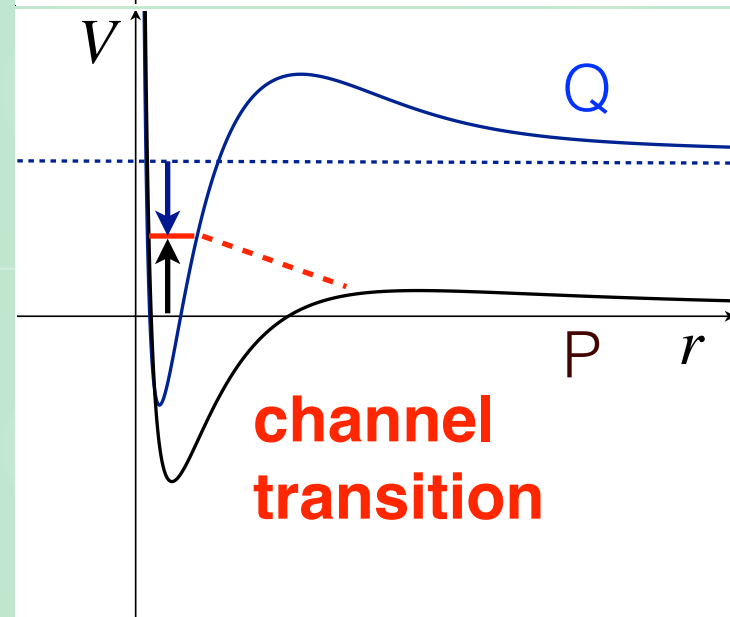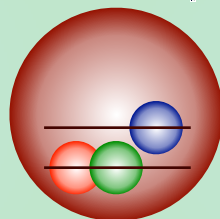
Resonances in quantum mechanics

1) Potential (shape) resonance

- 1 channel (P)

- potential barrier : $E > 0$

- unstable via tunneling

- (**composite** of P-channel)

2) Feshbach resonance

- coupled-channel ($P+Q$)

- bound state of Q: $E_Q < 0$, $E_P > 0$

- unstable via transition
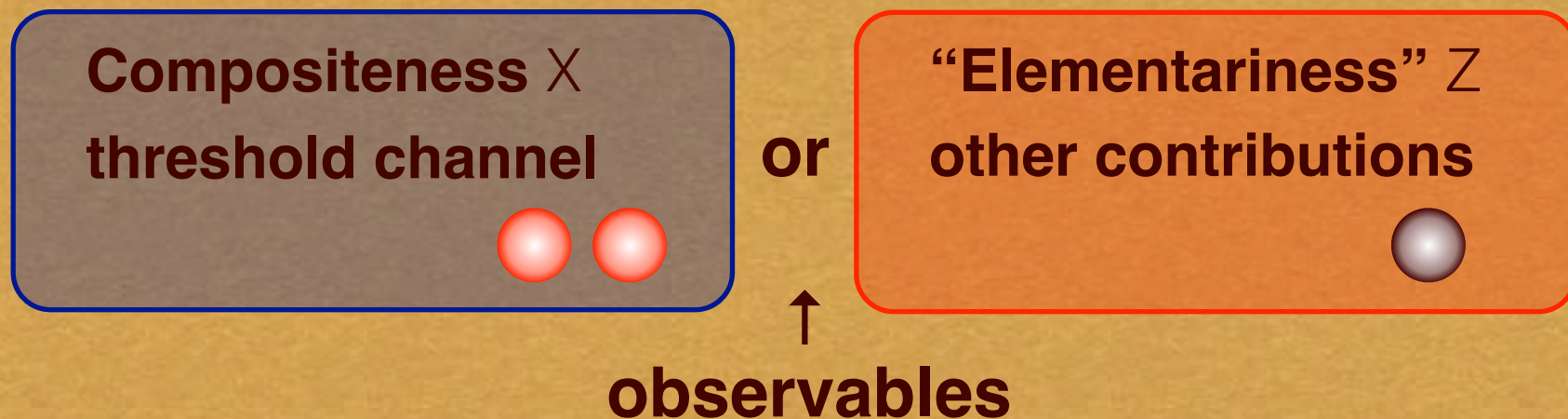
- ("elementary": other than P)

tunneling

P

channel transition

Q

P

8

Compositeness of hadrons

Internal structure of excited (exotic) hadrons

Weak binding relation for stable bound states

S. Weinberg, Phys. Rev. 137, B672 (1965)

Compositeness X **threshold channel**	**or**	**"Elementariness"** Z **other contributions**

↑
observables

Effective field theory —> description of low-energy scattering amplitude, generalization to unstable resonances

Weak binding relation for stable states

Compositeness of s-wave weakly bound state ($R \gg R_{\text{typ}}$)

S. Weinberg, Phys. Rev. 137, B672 (1965);

T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\text{typ}}}{R}\right)\right\}, \quad r_e = R\left\{\frac{X-1}{X} + \mathcal{O}\left(\frac{R_{\text{typ}}}{R}\right)\right\}$$

a_0: **scattering length,** r_e: **effective range**

$R = (2\mu B)^{-1/2}$: **radius of wave function**

R_{typ}: **length scale of interaction**

- **deuteron is** NN **composite (**$a_0 \sim R \gg r_e$**) —>** $X \sim 1$

- **internal structure from observable**

- **no nuclear force potential / wavefunction of deuteron**

Problem: applicable only for stable states.

Effective field theory

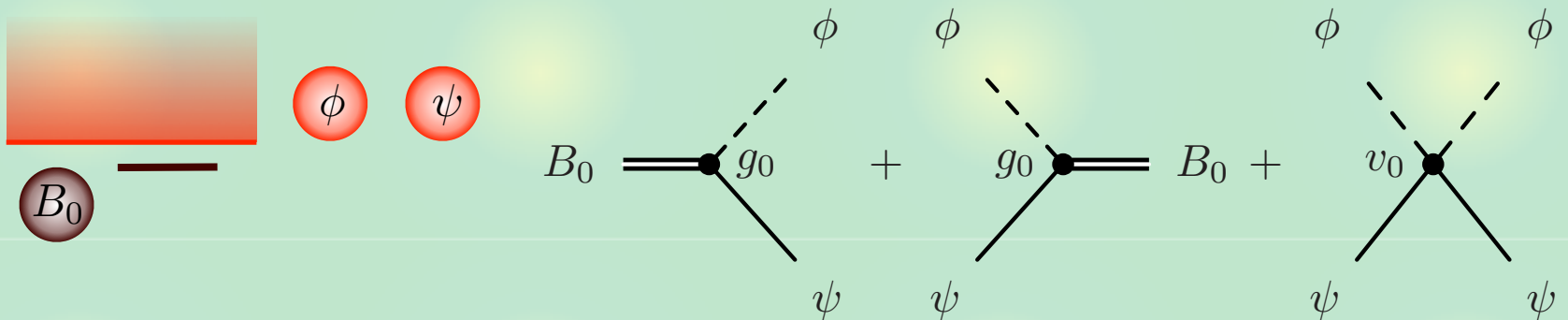## Low-energy scattering with near-threshold bound state

- **Nonrelativistic EFT with contact interaction**

D.B. Kaplan, Nucl. Phys. B494, 471 (1997)

E. Braaten, M. Kusunoki, D. Zhang, Annals Phys. 323, 1770 (2008)

$$H_{\text{free}} = \int d\boldsymbol{r} \left[\frac{1}{2M} \nabla \psi^\dagger \cdot \nabla \psi + \frac{1}{2m} \nabla \phi^\dagger \cdot \nabla \phi + \frac{1}{2M_0} \nabla B_0^\dagger \cdot \nabla B_0 + \nu_0 B_0^\dagger B_0 \right],$$

$$H_{\text{int}} = \int d\boldsymbol{r} \left[g_0 \left(B_0^\dagger \phi \psi + \psi^\dagger \phi^\dagger B_0 \right) + v_0 \psi^\dagger \phi^\dagger \phi \psi \right]$$

- **cutoff** : $\Lambda \sim 1/R_{\text{typ}}$ **(interaction range of microscopic theory)**

- **At low energy** $p \ll \Lambda$**, interaction ~ contact**

Compositeness and "elementariness"

Eigenstates

$$H_{\text{free}}|\,B_0\,\rangle = \nu_0|\,B_0\,\rangle, \quad H_{\text{free}}|\,\boldsymbol{p}\,\rangle = \frac{p^2}{2\mu}|\,\boldsymbol{p}\,\rangle$$ **free (discrete + continuum)**

$$(H_{\text{free}} + H_{\text{int}})|\,B\,\rangle = -B|\,B\,\rangle$$ **full (bound state)**

- normalization of |B> + completeness relation

$$\langle\,B\,|\,B\,\rangle = 1, \quad 1 = |\,B_0\,\rangle\langle\,B_0\,| + \int \frac{d\boldsymbol{p}}{(2\pi)^3}|\,\boldsymbol{p}\,\rangle\langle\,\boldsymbol{p}\,|$$

- projections onto free eigenstates

$$1 = Z + X, \quad Z \equiv |\langle\,B_0\,|\,B\,\rangle|^2, \quad X \equiv \int \frac{d\boldsymbol{p}}{(2\pi)^3}|\langle\,\boldsymbol{p}\,|\,B\,\rangle|^2$$
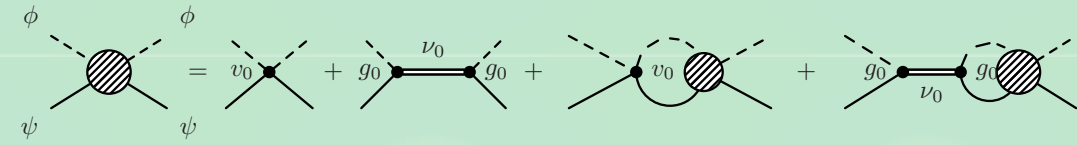
"elementariness"   **compositeness**

Z, X**: real and nonnegative —> interpreted as** **probability**

Weak binding relation

$\Psi\Phi$ scattering amplitude (exact result)

$$f(E) = -\frac{\mu}{2\pi}\frac{1}{[v(E)]^{-1} - G(E)}$$

$$v(E) = v_0 + \frac{g_0^2}{E - \nu_0}, \quad G(E) = \frac{1}{2\pi^2}\int_0^\Lambda dp\frac{p^2}{E - p^2/(2\mu) + i0^+}$$

Compositeness X <— $v(E)$, $G(E)$

$$X = \frac{G'(-B)}{G'(-B) - [1/v(-B)]'}$$

$1/R=(2\mu B)^{1/2}$ expansion: leading term <— X

$$a_0 = -f(E=0) = R\left\{\frac{2X}{1+X} + \overline{\mathcal{O}\left(\frac{R_{\text{typ}}}{R}\right)}\right\}$$ **renormalization dependent**

renormalization independent

If $R \gg R_{\text{typ}}$**, correction terms neglected:** X <— **(**B**,** a_0**)**

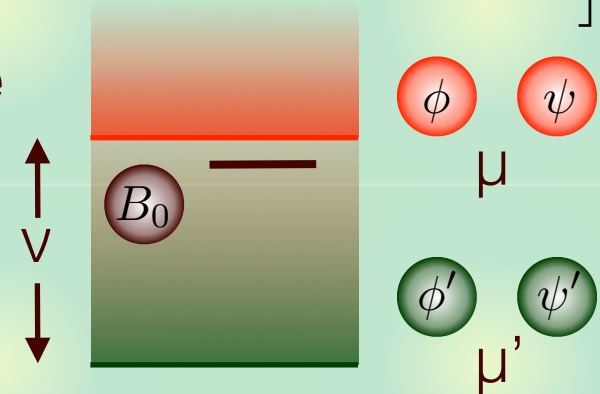# Introduction of decay channel

Introduce decay channel

$$H'_{\text{free}} = \int d\boldsymbol{r} \left[\frac{1}{2M'} \nabla \psi'^\dagger \cdot \nabla \psi' - \nu_\psi \psi'^\dagger \psi' + \frac{1}{2m'} \nabla \phi'^\dagger \cdot \nabla \phi' - \nu_\phi \phi'^\dagger \phi' \right],$$

$$H'_{\text{int}} = \int d\boldsymbol{r} \left[g'_0 \left(B_0^\dagger \phi' \psi' + \psi'^\dagger \phi'^\dagger B_0 \right) + v'_0 \psi'^\dagger \phi'^\dagger \phi' \psi' + v_0^t (\psi^\dagger \phi^\dagger \phi' \psi' + \psi'^\dagger \phi'^\dagger \phi \psi) \right],$$

Quasi-bound state: complex eigenvalue

$$H = H_{\text{free}} + H'_{\text{free}} + H_{\text{int}} + H'_{\text{int}}$$

$$H|QB\rangle = E_{QB}|QB\rangle, \quad E_{QB} \in \mathbb{C}$$

Generalized relation: correction term <— threshold difference

$$a_0 = R \left\{ \frac{2X}{1+X} + \mathcal{O}\left(\left| \frac{R_{\text{typ}}}{R} \right| \right) + \mathcal{O}\left(\left| \frac{l}{R} \right|^3 \right) \right\}, \quad R = \frac{1}{\sqrt{-2\mu E_{QB}}} \in \mathbb{C}, \quad l \equiv \frac{1}{\sqrt{2\mu\nu}}$$

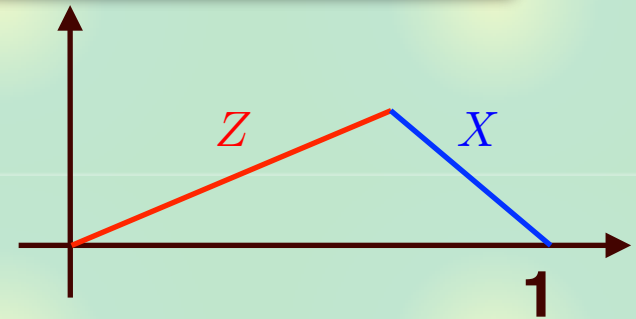**Y. Kamiya, T. Hyodo, Phys. Rev. C93, 035203 (2016)**

c.f. V. Baru, *et al.*, Phys. Lett. B586, 53 (2004),…

If $|R| \gg (R_{\text{typ}}, l)$ correction terms neglected: X <— (E_{QB}, a_0)

Complex compositeness and interpretation

Unstable states —> complex Z and X
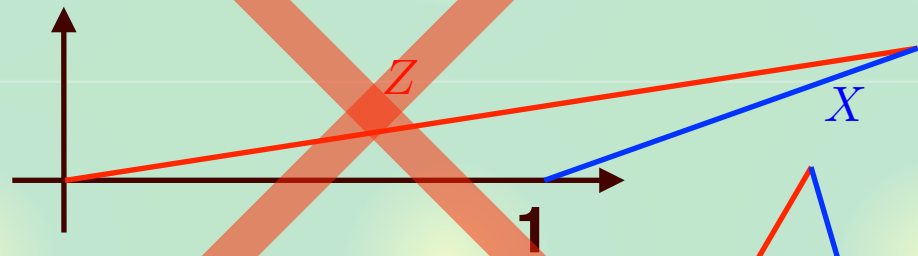
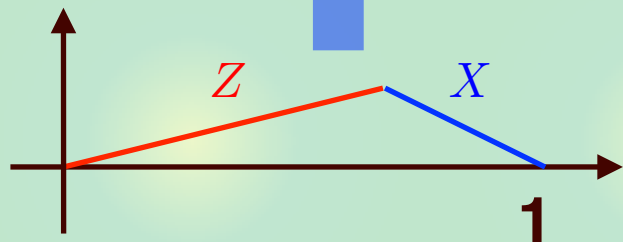$$Z + X = 1, \quad Z, X \in \mathbb{C}$$

Similarity with bound state

c.f. T. Sekihara, T. Hyodo, D. Jido, PTEP2015, 063D04 (2015)
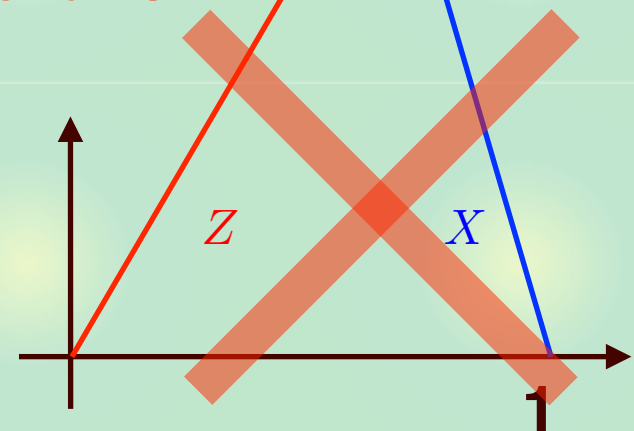
**bound state
: well defined**

$$Z + X = 1, \quad Z, X \in [0, 1]$$

small cancellation
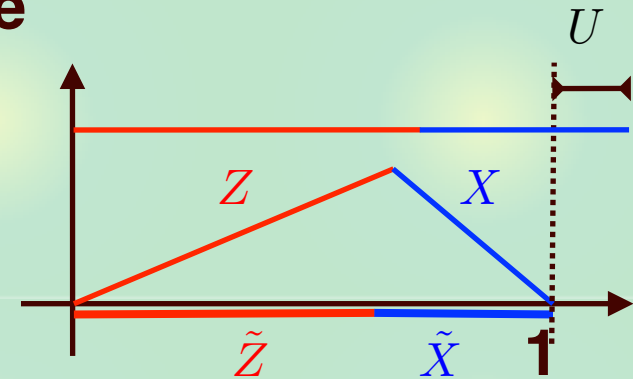
large cancellation

15

New definitions

Step 1: quantify the deviation from bound state

- 0 **for bound state**
- **becomes large when deviation is large**

$$U = |Z| + |X| - 1$$

$$\longrightarrow \cup : \textbf{ uncertainty of interpretation}$$

c.f. T. Berggren, Phys. Lett. 33B, 547 (1970)

Step 2: define new compositeness/elementariness

- **interpreted as probabilities** $\quad \tilde{Z} + \tilde{X} = 1, \quad \tilde{Z}, \tilde{X} \in [0, 1]$
- **coincide with** Z, X **for bound state if** $\cup \longrightarrow 0$

$$\tilde{Z} = \frac{1 - |X| + |Z|}{2}, \quad \tilde{X} = \frac{1 - |Z| + |X|}{2}$$

compositeness when $\cup$ is small

16

Application

Generalized weak binding relation $X \longleftarrow (E_{QB}, a_0)$

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\text{typ}}}{R}\right|\right) + \mathcal{O}\left(\left|\frac{l}{R}\right|^3\right)\right\}, \quad R = \frac{1}{\sqrt{-2\mu E_{QB}}}, \quad l \equiv \frac{1}{\sqrt{2\mu\nu}}$$

- $\Lambda(1405)$ **(higher) pole position and** $\overline{K}N$ **scattering length**

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881 98 (2012), …

- E_{QB} = -10 -26i **MeV —> |R| ~ 2 fm —> small correction term**

$$\left|\frac{R_{\text{typ}}}{R}\right| \lesssim 0.12, \quad \left|\frac{l}{R}\right|^3 \lesssim 0.16$$

energy difference from $\pi\Sigma$

vector meson exchange

Ref.	E_{QB} (MeV)	a_0 (fm)	$X_{\bar{K}N}$	$\tilde{X}_{\bar{K}N}$	U	$|r_e/a_0|$
[43]	$-10 - i26$	$1.39 - i0.85$	$1.2 + i0.1$	1.0	0.5	0.2
[44]	$-\ 4 - i\ 8$	$1.81 - i0.92$	$0.6 + i0.1$	0.6	0.0	0.7
[45]	$-13 - i20$	$1.30 - i0.85$	$0.9 - i0.2$	0.9	0.1	0.2
[46]	$2 - i10$	$1.21 - i1.47$	$0.6 + i0.0$	0.6	0.0	0.7
[46]	$-\ 3 - i12$	$1.52 - i1.85$	$1.0 + i0.5$	0.8	0.6	0.4

systematic error

$\Lambda(1405)$ **is** $\overline{K}N$ **composite <— observables**

Summary 1

Compositeness of near-threshold bound state can be determined only by observables.
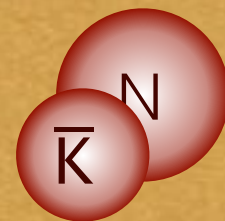
S. Weinberg, Phys. Rev. 137, B672 (1965)

Weak binding relation can be generalized to unstable states with effective field theory.

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\text{typ}}}{R}\right|\right) + \mathcal{O}\left(\left|\frac{l}{R}\right|^3\right)\right\}, \quad R = \frac{1}{\sqrt{-2\mu E_{QB}}}, \quad l \equiv \frac{1}{\sqrt{2\mu\nu}}$$

Precise determination of the pole position and scattering length shows that $\Lambda(1405)$ is dominated by $\overline{K}N$ composite component.

Y. Kamiya, T. Hyodo, Phys. Rev. C93, 035203 (2016);
Y. Kamiya, T. Hyodo, PTEP2017, 023D02 (2017)

N

$\overline{K}$

Use of finite volume eigenstates?

Wavefunction of resonance
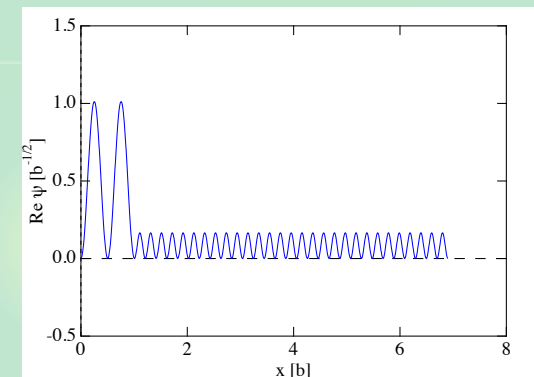
- **outgoing boundary condition (c.f.** $\exp\{-\kappa r\}$**)**

$$\psi(r) \sim \exp[ipr]$$
$$= \exp\{i[\mathrm{Re}\ p]r\}\exp\{-[\mathrm{Im}\ p]r\}$$
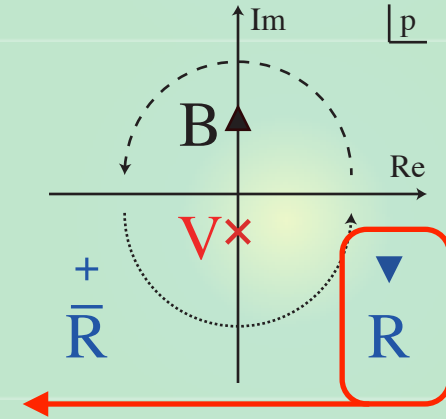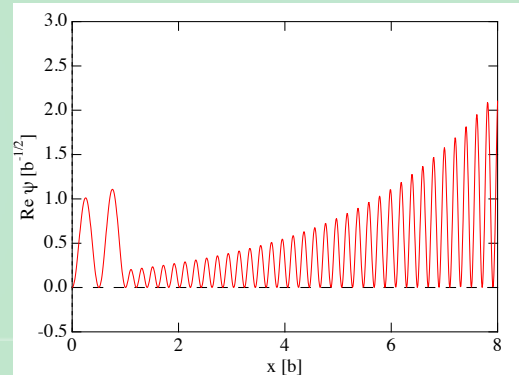
- **If** $\mathrm{Im}\ p < 0$**,** ψ **is not square integrable.**

- **complex eigenvalues (energy,** X, $<r^2>$**, …)**

Finite-volume **system with size** L

- ψ **is square integrable on** $[0,L]^3$**.**

- **real eigenvalues (energy,** X**)**

—> Probabilistic interpretation!

19

Compositeness in finite volume

Effective field theory in finite box of size L

- **discrete real eigenenergies in finite volume (FV)**

$$H|\Psi^{(m)}\rangle = E^{(m)}|\Psi^{(m)}\rangle, \quad E^{(m+1)} > E^{(m)}, \quad \langle\Psi^{(m)}|\Psi^{(l)}\rangle = \delta_{ml}$$

- **Compositeness**

$$X^{(m)} = \langle\Psi^{(m)}|\hat{P}_{\text{two-body}}|\Psi^{(m)}\rangle, \quad \boxed{\hat{P}_{\text{two-body}} = \frac{1}{L^3}\sum_{n}|\boldsymbol{p_n}\rangle\langle\boldsymbol{p_n}|}$$

$$= \frac{I'_{\text{FV}}(E^{(m)})}{I'_{\text{FV}}(E^{(m)}) - [1/v(E^{(m)})]'}, \quad 1 - I_{\text{FV}}(E^{(m)})v(E^{(m)}) = 0$$

c.f.) infinite volume: $\mathsf{I_{FV}(E;L)} \longrightarrow \mathsf{G(E)}$

<u>Y. Kamiya, T. Hyodo, Phys. Rev. C 93, 035203 (2016); PTEP 023D02 (2017).</u>

- **Compositeness** $\mathsf{X^{(m)}}$ **is defined for each FV eigenstate.**

- $\mathsf{X^{(m)}}$ **can be interpreted as a probability.**

- $\mathsf{X^{(m)}}$ **has** L **dependence through** $\mathsf{I_{FV}}$ **and** $\mathsf{E^{(m)}}$**.**

Compositeness of resonances

Which is the eigenstate representing the resonance?

- **choose first excited state** $E^{(1)}(L)$

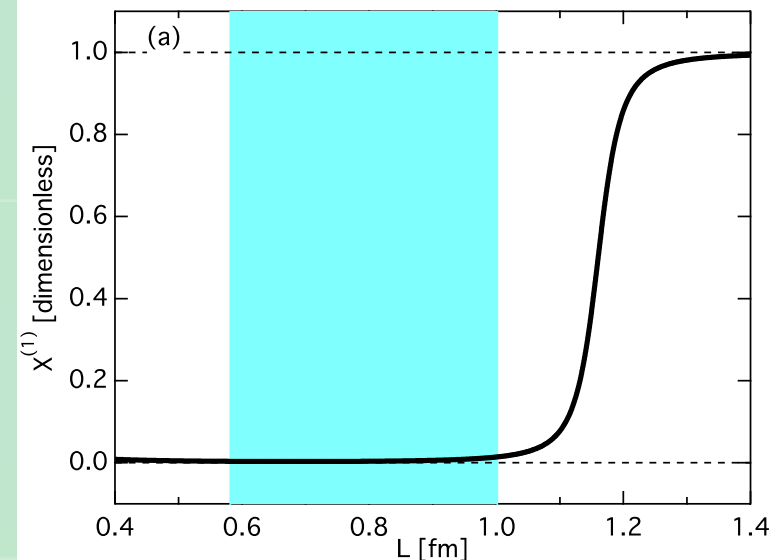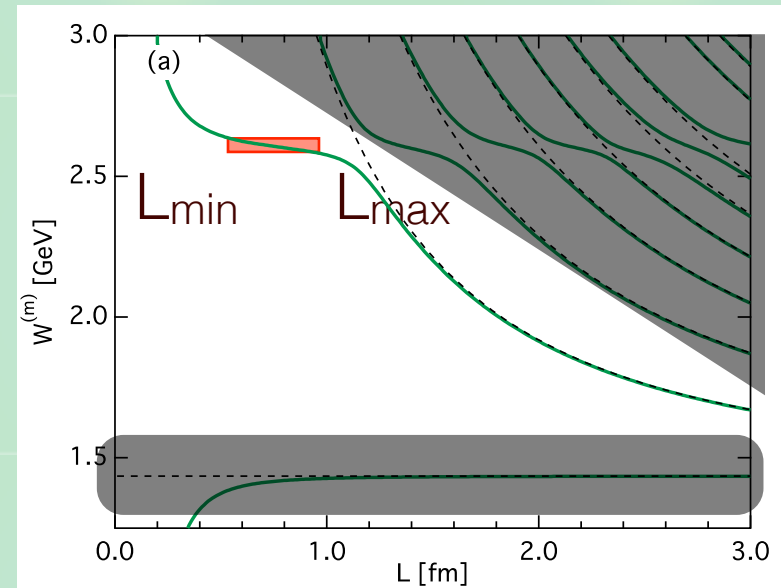- **energy region** $\longrightarrow (L_{min}, L_{max})$

$$E_{min} \leq E^{(1)}(L) \leq E_{max}$$

- L_{min}**: finite-volume effect on wavefunction**

- L_{max}**: mixing of scattering state**

Compositeness of resonance

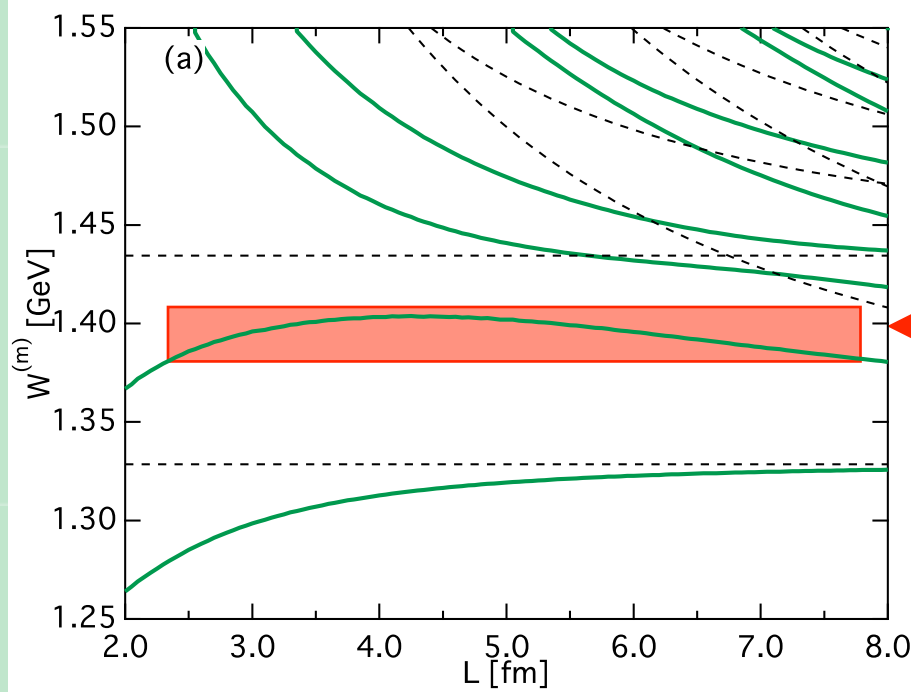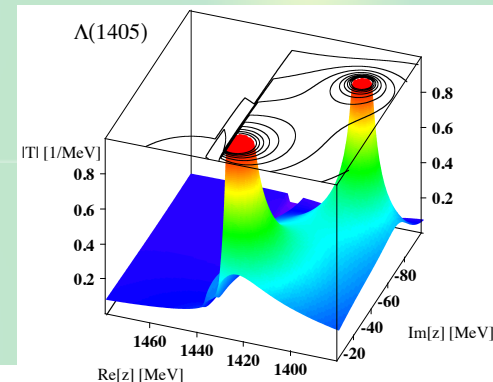$$X_{res} = \frac{1}{L_{max} - L_{min}} \int_{L_{min}}^{L_{max}} X^{(1)}(L) dL$$

- **interpreted as a probability**

Eigenenergies of $\Lambda(1405)$

ETW model ($\overline{K}N$-$\pi\Sigma$ 2channel, WT interaction)

Y. Ikeda, T. Hyodo, W. Weise, Nucl. Phys. A881, 98 (2012)

- two poles, consistent with SIDDHARTA
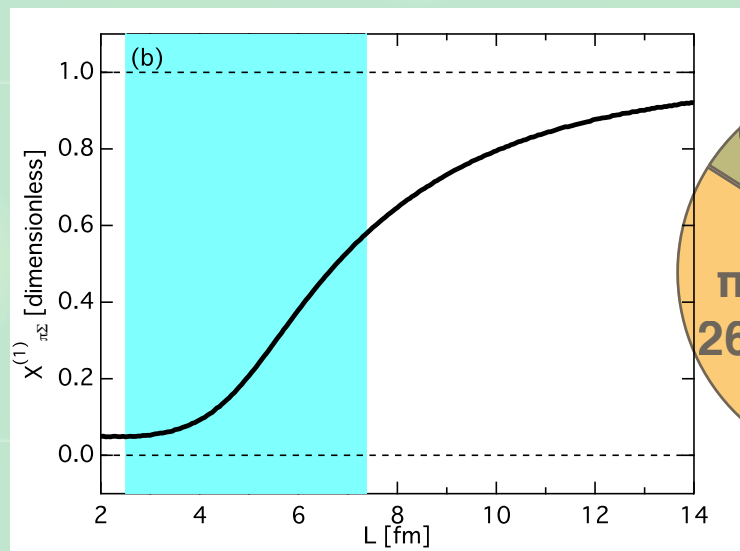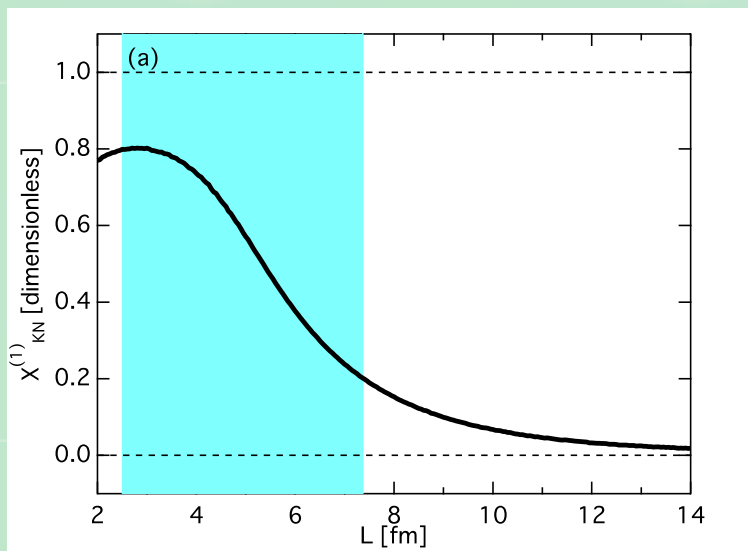
Finite volume eigenenergies

$\overline{K}N$ **threshold**

$\Lambda(1405)$

$\pi\Sigma$ **threshold**

$\Lambda(1405)$ **is represented by a single FV eigenstate.**
(# of FV eigenstates <—> # of $\pi/2$ crossings of phase shift)

22

Compositeness of $\Lambda(1405)$

Compositeness $X_{res,\bar{K}N}$, $X_{res,\pi\Sigma}$
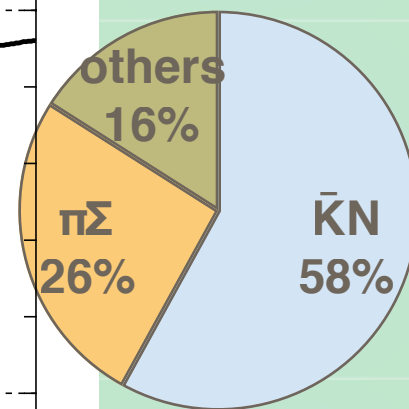
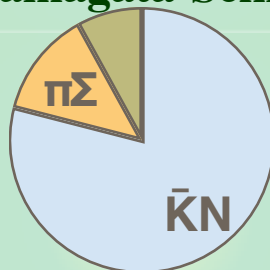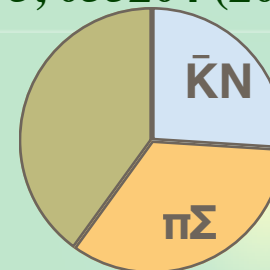## Complex compositeness at each pole —> real-valued

Y. Kamiya, T. Hyodo, Phys. Rev. C 93, 035203 (2016); PTEP 023D02 (2017),
T. Sekihara, T. Arai, J. Yamagata-Sekihara, S. Yasui, Phys. Rev. C 93, 035204 (2016)

- High-mass pole   - Low-mass pole

X_{res} **represents the contributions from both poles**
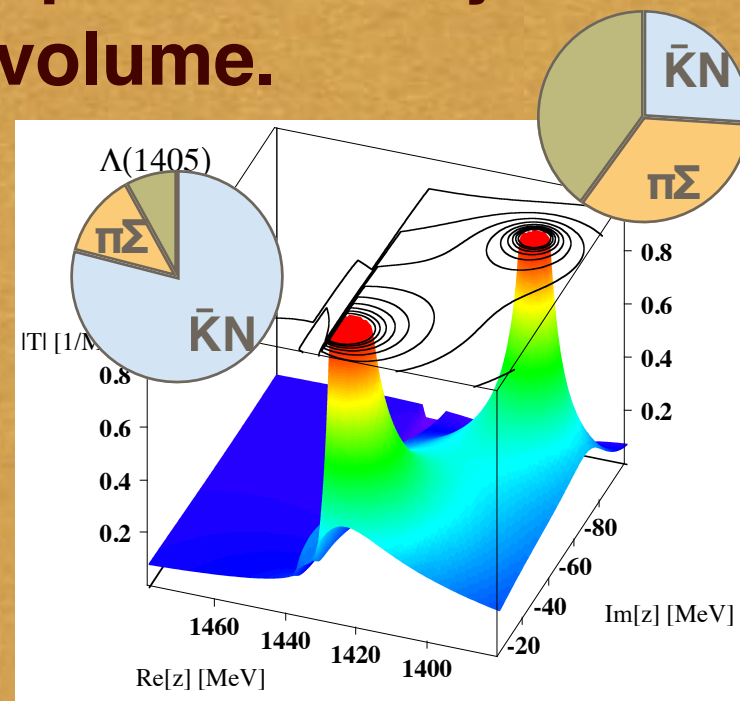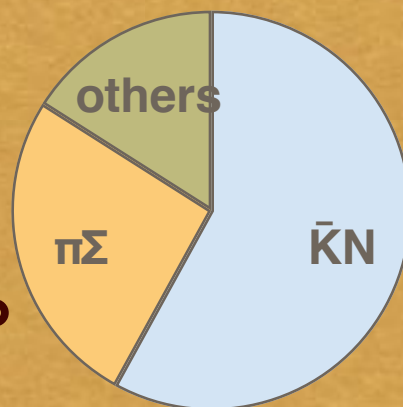
23

Summary 2

- We propose a new definition of compositeness of resonances using finite-volume eigenstates.

- Two poles of Λ(1405) are represented by **a single eigenstate** in finite volume.

- Structure of Λ(1405):

  - $\bar{K}N$: 58%

  - $\pi\Sigma$: 26%

  - others: 16%

Y. Tsuchida, T. Hyodo, Phys. Rev. C97, 0552113 (2018)

Analytic structure of scattering amplitude

Pole of scattering amplitude $f(E_{pole})=\infty$

J.R. Taylor, *Scattering theory* (Wiley, New York, 1972)

- represents (complex) eigenvalue of Hamiltonian
- unique through analytic continuation

CDD (Castillejo-Dalitz-Dyson) zero

L. Castillejo, R.H. Dalitz, F.J. Dyson, Phys. Rev. 124, 264 (1961)

- pole of inverse amplitude, zero of amplitude $f(E_{CDD})=0$
- unique through analytic continuation
- role of CDD zero in hadron scattering, resonances, etc.
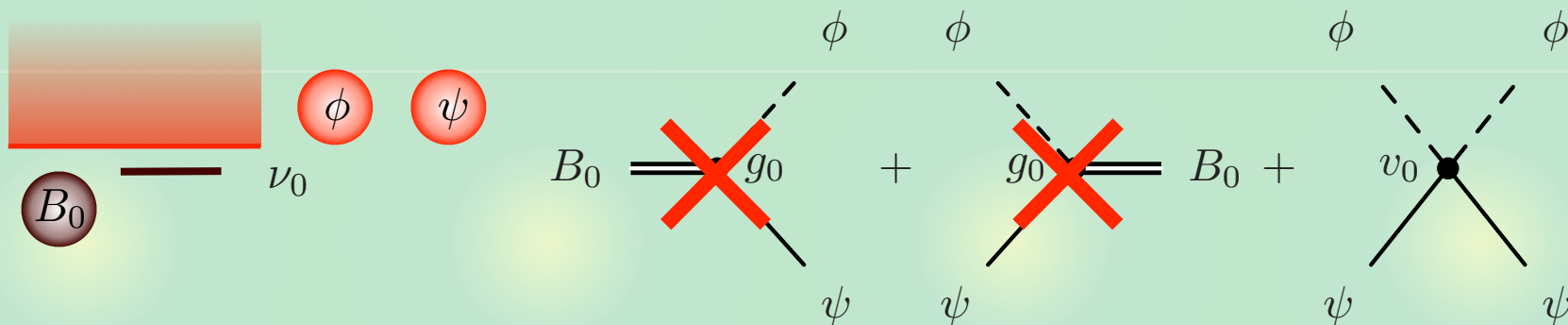
V. Baru, *et al.*, Eur. Phys. J. A 44, 93 (2010),
C. Hanhart, *et al.*, Eur. Phys. J. A 47, 101 (2011),
Z.H. Guo, J.A. Oller, Phys. Rev. D93, 054014 (2016)

CDD zero <—> elementary/composite?

Fate of pole in zero coupling limit

Contact interaction EFT

- **Consider a pole of amplitude**

$$f(E_\text{pole}) = \infty$$

Zero coupling limit (ZCL) $g_0 \longrightarrow 0$ **: only 4-point interaction**

- **Composite (~potential): 4-point interaction origin**
  —> pole remains in the amplitude

- **Elementary (~Feshbach):** B_0 **origin**
  —> pole moves toward v_0 **and finally decouples**

Pole and CDD zero in ZCL
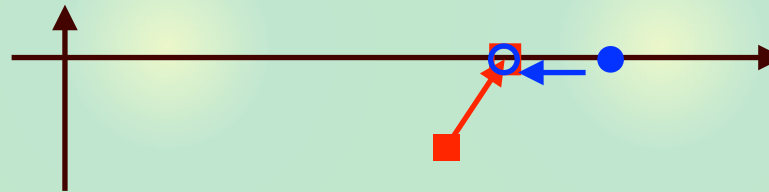
Scattering amplitude

$$t(E) = \frac{v_0(E - \nu_0) + g_0^2}{(E - \nu_0)[1 - v_0 G(E)] - g_0^2 G(E)}$$

- **Pole** : $(E_{\text{pole}} - \nu_0)[1 - v_0 G(E_{\text{pole}})] - g_0^2 G(E_{\text{pole}}) = 0$
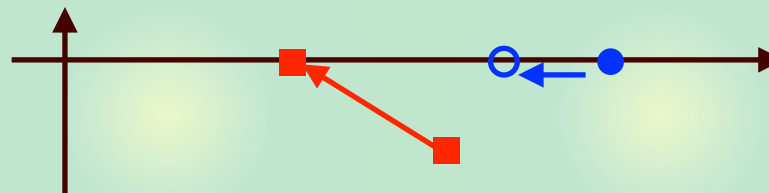
- **CDD zero** : $E_{\text{CDD}} = \nu_0 - \dfrac{g_0^2}{v_0} \xrightarrow{\text{ZCL}} \nu_0$

ZCL $(g_0 \longrightarrow 0)$

- **Pole (case I) :** $E_{\text{pole}} \to \nu_0$ **vanishing residue —> elementary**

- **Pole (case II) :** $1 - v_0 G(E_{\text{pole}}) = 0$ **finite residue —> composite**

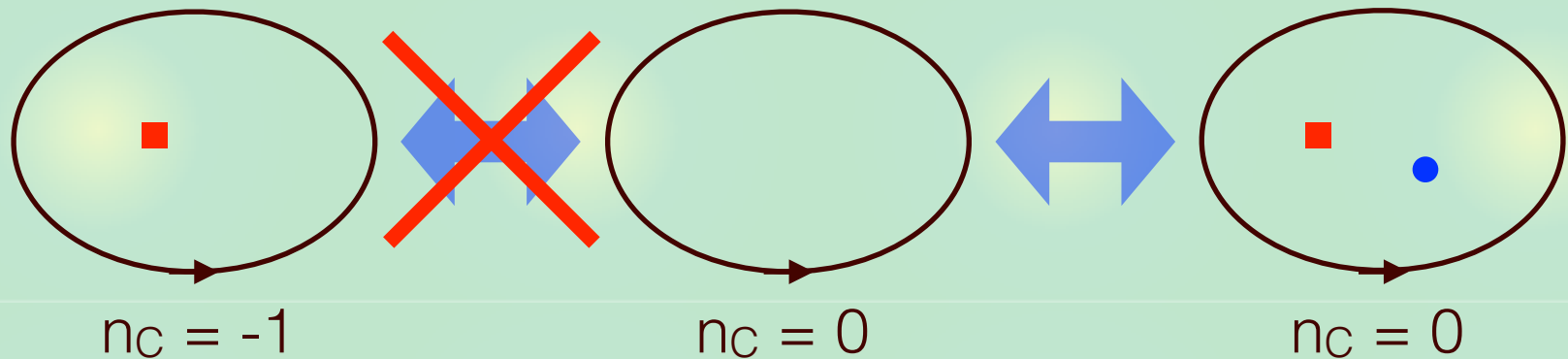**Elementary —> pole encounters with CDD zero**

General discussion

Scattering amplitude $f(E)$ **is meromorphic in energy**

Y. Kamiya, T. Hyodo, Phys. Rev. D97, 054019 (2018)

$$\frac{1}{2\pi} \oint_C dz \frac{d \arg f(z)}{dz} = n_Z - n_P \equiv n_C$$

- n_Z (n_P) : **number of zeros (poles) in contour C**
- **Topological invariant of** $\pi_1(\mathrm{U}(1)) \cong \mathbb{Z}$

$n_C = -1$ $n_C = 0$ $n_C = 0$

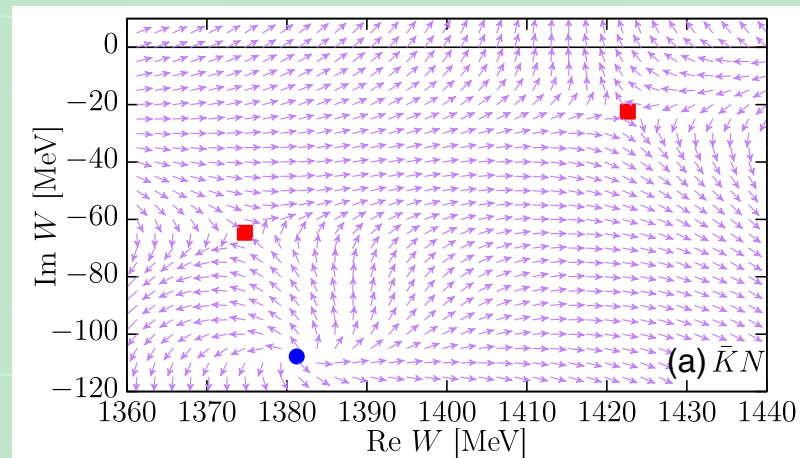**Pole cannot decouple without merging with CDD zero**

—> existence of nearby CDD zero indicates "elementary" (i.e. origin is not in this channel).

Example: $\Lambda(1405)$
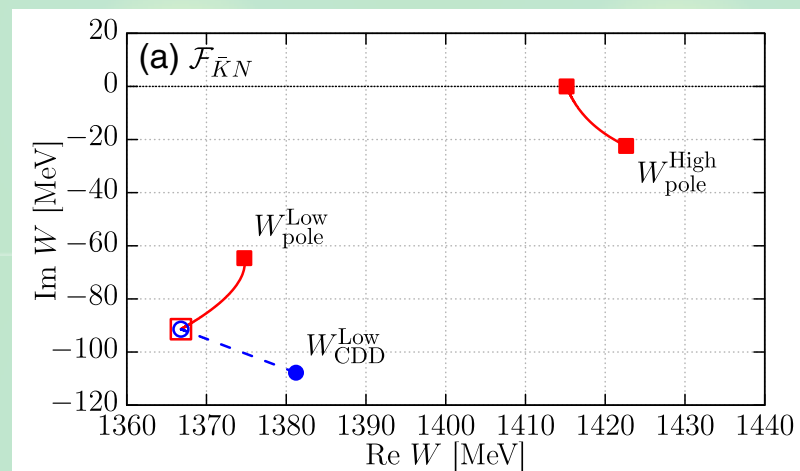
Poles and zero positions in the $\overline{K}N$ amplitude

Y. Ikeda, T. Hyodo, W. Weise, Nucl. Phys. A881, 98 (2012)

- **Two poles for Λ(1405)**

- **CDD zero exists near the Low-mass pole**

(a) $\bar{K}N$

Trajectories toward ZCL

- **High mass pole remains in the $\overline{K}N$ amplitude.**

- **Low-mass pole decouples by merging with CDD zero**

(a) $\mathcal{F}_{\bar{K}N}$

$W_{\text{pole}}^{\text{High}}$

$W_{\text{pole}}^{\text{Low}}$

$W_{\text{CDD}}^{\text{Low}}$

High-mass pole can be $\overline{K}N$ composite, but lower one is not.

Summary 3

📌 **"Elementary" pole decouples from the amplitude in the zero coupling limit.**

📌 **For a pole to decouple from the amplitude, there must be a nearby CDD zero.**

$$\frac{1}{2\pi} \oint_C dz \frac{d \arg f(z)}{dz} = n_Z - n_P$$

📌 **The dynamical (composite) component of the eigenstate is small if a CDD zero exists near the eigenstate pole.**

Y. Kamiya, T. Hyodo, Phys. Rev. D97, 054019 (2018)