

# Structure of hadrons from effective field theory



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# Contents



## Introduction: exotic hadron resonances



## Compositeness of hadron resonances

S. Weinberg, Phys. Rev. 137, B672 (1965);

T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)

### - Weak binding relation from EFT

Y. Kamiya, T. Hyodo, Phys. Rev. C93, 035203 (2016);

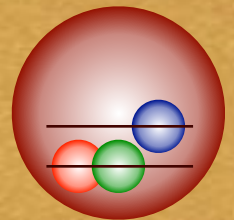
Y. Kamiya, T. Hyodo, PTEP2017, 023D02 (2017)

### - Utilizing finite volume effect

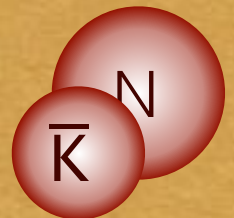
Y. Tsuchida, T. Hyodo, arXiv:1703.02675 [nucl-th]

### - Implication from nearby CDD zero

Y. Kamiya, T. Hyodo, Phys. Rev. D97, 054019 (2018)



or

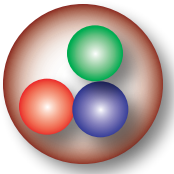


# Classification of hadrons

## Observed hadrons

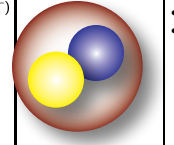
PDG2017 : <http://pdg.lbl.gov/>

$p$	1/2 <sup>+</sup> ****	$\Delta(1232)$	3/2 <sup>+</sup> ****	$\Sigma^+$	1/2 <sup>+</sup> ****	$\Xi^0$	1/2 <sup>+</sup> ****	$\Lambda_c^+$	1/2 <sup>+</sup> ****
$n$	1/2 <sup>+</sup> ****	$\Delta(1600)$	3/2 <sup>+</sup> ***	$\Sigma^0$	1/2 <sup>+</sup> ****	$\Xi^-$	1/2 <sup>+</sup> ****	$\Lambda_c(2595)^+$	1/2 <sup>-</sup> ***
$N(1440)$	1/2 <sup>+</sup> ****	$\Delta(1620)$	1/2 <sup>-</sup> ****	$\Sigma^-$	1/2 <sup>+</sup> ****	$\Xi(1530)$	3/2 <sup>+</sup> ****	$\Lambda_c(2625)^+$	3/2 <sup>-</sup> ***
$N(1520)$	3/2 <sup>-</sup> ****	$\Delta(1700)$	3/2 <sup>-</sup> ****	$\Sigma(1385)$	3/2 <sup>+</sup> ****	$\Xi(1620)$	***	$\Lambda_c(2765)^+$	*
$N(1535)$	1/2 <sup>-</sup> ****	$\Delta(1750)$	1/2 <sup>+</sup> *	$\Sigma(1480)$	***	$\Xi(1690)$	***	$\Lambda_c(2940)^+$	5/2 <sup>+</sup> ***
$N(1650)$	1/2 <sup>-</sup> ****	$\Delta(1900)$	1/2 <sup>-</sup> **	$\Sigma(1560)$	**	$\Xi(1820)$	3/2 <sup>-</sup> ***	$\Lambda_c(2455)$	1/2 <sup>+</sup> ****
$N(1675)$	5/2 <sup>-</sup> ****	$\Delta(1905)$	5/2 <sup>+</sup> ****	$\Sigma(1580)$	3/2 <sup>-</sup> **	$\Xi(1950)$	***	$\Sigma_c(2520)$	3/2 <sup>+</sup> ****
$N(1680)$	5/2 <sup>+</sup> ****	$\Delta(1910)$	1/2 <sup>+</sup> ****	$\Sigma(1620)$	1/2 <sup>-</sup> *	$\Xi(2030)$	$\geq 5/2^?$ ***	$\Sigma_c(2800)$	***
$N(1685)$	*	$\Delta(1920)$	3/2 <sup>+</sup> ***	$\Sigma(1660)$	1/2 <sup>+</sup> ***	$\Xi(2120)$	*	$\Xi_c$	1/2 <sup>+</sup> ***
$N(1700)$	3/2 <sup>-</sup> ***	$\Delta(1930)$	5/2 <sup>-</sup> ***	$\Sigma(1670)$	3/2 <sup>-</sup> ****	$\Xi(2250)$	**	$\Xi_c^0$	1/2 <sup>+</sup> ***
$N(1710)$	1/2 <sup>+</sup> ***	$\Delta(1940)$	3/2 <sup>-</sup> **	$\Sigma(1690)$	***	$\Xi(2370)$	**	$\Xi_c^+$	1/2 <sup>+</sup> ***
$N(1720)$	3/2 <sup>+</sup> ****	$\Delta(1950)$	7/2 <sup>+</sup> ****	$\Sigma(1730)$	3/2 <sup>+</sup> *	$\Xi(2500)$	*	$\Xi_c^+$	1/2 <sup>+</sup> ***
$N(1860)$	5/2 <sup>+</sup> **	$\Delta(2000)$	5/2 <sup>+</sup> **	$\Sigma(1750)$	1/2 <sup>-</sup> ***			$\Xi_c^+$	1/2 <sup>+</sup> ***
$N(1875)$	3/2 <sup>-</sup> ***	$\Delta(2150)$	1/2 <sup>-</sup> *	$\Sigma(1770)$	1/2 <sup>+</sup> **	$\Omega^-$	3/2 <sup>+</sup> ****	$\Xi_c^+$	1/2 <sup>+</sup> ***
$N(1880)$	1/2 <sup>+</sup> **	$\Delta(2200)$	7/2 <sup>-</sup> *	$\Sigma(1775)$	5/2 <sup>-</sup> ****	$\Omega(2250)^-$	***	$\Xi_c(2645)$	3/2 <sup>+</sup> ***
$N(1895)$	1/2 <sup>-</sup> **	$\Delta(2300)$	9/2 <sup>+</sup> **	$\Sigma(1840)$	3/2 <sup>+</sup> **	$\Omega(2380)^-$	**	$\Xi_c(2790)$	1/2 <sup>-</sup> ***
$N(1900)$	3/2 <sup>+</sup> ***	$\Delta(2350)$	5/2 <sup>-</sup> **	$\Sigma(1880)$	1/2 <sup>+</sup> **	$\Omega(2470)^-$	**	$\Xi_c(2815)$	3/2 <sup>-</sup> ***
$N(1990)$	7/2 <sup>+</sup> **	$\Delta(2390)$	7/2 <sup>+</sup> *	$\Sigma(1900)$	1/2 <sup>-</sup> *			$\Xi_c(2930)$	*
$N(2000)$	5/2 <sup>+</sup> **	$\Delta(2400)$	9/2 <sup>-</sup> **	$\Sigma(1915)$	5/2 <sup>+</sup> ****			$\Xi_c(2980)$	**
$N(2040)$	3/2 <sup>+</sup> *	$\Delta(2420)$	11/2 <sup>+</sup> ****	$\Sigma(1940)$	3/2 <sup>+</sup> **			$\Xi_c(3055)$	***
$N(2060)$	5/2 <sup>-</sup> **	$\Delta(2750)$	13/2 <sup>-</sup> **	$\Sigma(1940)$	3/2 <sup>-</sup> ***			$\Xi_c(3080)$	***
$N(2100)$	1/2 <sup>+</sup> *	$\Delta(2950)$	15/2 <sup>+</sup> **	$\Sigma(2000)$	1/2 <sup>-</sup> *			$\Xi_c(3123)$	*
$N(2120)$	3/2 <sup>-</sup> **			$\Sigma(2030)$	7/2 <sup>+</sup> ****			$\Omega_c^0$	1/2 <sup>+</sup> ***
$N(2190)$	7/2 <sup>-</sup> ****	$\Lambda$	1/2 <sup>+</sup> ****	$\Sigma(2070)$	5/2 <sup>+</sup> *			$\Omega_c(2770)^0$	3/2 <sup>+</sup> ***
$N(2220)$	9/2 <sup>+</sup> ****	$\Lambda(1405)$	1/2 <sup>-</sup> ****	$\Sigma(2080)$	3/2 <sup>+</sup> **			$\Xi_{cc}^+$	*
$N(2250)$	9/2 <sup>-</sup> ****	$\Lambda(1520)$	3/2 <sup>-</sup> ****	$\Sigma(2100)$	7/2 <sup>-</sup> **				
$N(2300)$	1/2 <sup>+</sup> **	$\Lambda(1600)$	1/2 <sup>+</sup> ***	$\Sigma(2250)$	***			$\Lambda_b^0$	1/2 <sup>+</sup> ***
$N(2570)$	5/2 <sup>-</sup> **	$\Lambda(1670)$	1/2 <sup>-</sup> ****	$\Sigma(2455)$	**			$\Lambda_b(5912)^0$	1/2 <sup>-</sup> ***
$N(2600)$	11/2 <sup>-</sup> ***	$\Lambda(1690)$	3/2 <sup>-</sup> ****	$\Sigma(2620)$	**			$\Lambda_b(5920)^0$	3/2 <sup>-</sup> ***
$N(2700)$	13/2 <sup>+</sup> **	$\Lambda(1710)$	1/2 <sup>+</sup> *	$\Sigma(3000)$	**			$\Sigma_b$	1/2 <sup>+</sup> ***
		$\Lambda(1800)$	1/2 <sup>-</sup> ***	$\Sigma(3170)$	*			$\Sigma_b^+$	3/2 <sup>+</sup> ****
		$\Lambda(1810)$	1/2 <sup>+</sup> ***					$\Xi_b^-$	1/2 <sup>+</sup> ***
		$\Lambda(1820)$	5/2 <sup>+</sup> ****					$\Xi_b^0$	1/2 <sup>+</sup> ***
		$\Lambda(1830)$	5/2 <sup>-</sup> ****					$\Xi_b^0(5935)^-$	1/2 <sup>+</sup> ***
		$\Lambda(1890)$	3/2 <sup>+</sup> ****					$\Xi_b^0(5945)^-$	3/2 <sup>+</sup> ***
		$\Lambda(2000)$	*					$\Xi_b^0(5955)^-$	3/2 <sup>+</sup> ***
		$\Lambda(2020)$	7/2 <sup>+</sup> *					$\Xi_b^0(5955)^-$	3/2 <sup>+</sup> ***
		$\Lambda(2050)$	3/2 <sup>-</sup> *					$\Omega_b$	1/2 <sup>+</sup> ***
		$\Lambda(2100)$	7/2 <sup>-</sup> ****						
		$\Lambda(2110)$	5/2 <sup>+</sup> ***						
		$\Lambda(2325)$	3/2 <sup>-</sup> *						
		$\Lambda(2350)$	9/2 <sup>+</sup> ***						
		$\Lambda(2585)$	**						



~ 150 baryons

LIGHT UNFLAVORED (S = C = B = 0)		STRANGE (S = ±1, C = B = 0)		CHARMED, STRANGE (C = S = ±1)		cc	
$F^0(F^C)$		$F^0(F^C)$		$F^0(F^C)$		$F^0(F^C)$	
$\pi^\pm$	1 <sup>-</sup> (0 <sup>+</sup> )	$\rho(1680)$	0 <sup>-</sup> (1 <sup>-</sup> )	$K^{*2}$	1/2(0 <sup>-</sup> )	$D_s^{*2}$	0(0 <sup>-</sup> )
$\pi^0$	1 <sup>-</sup> (0 <sup>+</sup> )	$\rho_3(1690)$	1 <sup>+</sup> (3 <sup>-</sup> )	$K^{*0}$	1/2(0 <sup>-</sup> )	$D_s^{*0}$	0(0 <sup>-</sup> )
$\eta$	0 <sup>+</sup> (0 <sup>+</sup> )	$\rho(1700)$	1 <sup>+</sup> (1 <sup>-</sup> )	$K_2^0$	1/2(0 <sup>-</sup> )	$D_{s1}^0(2317)^{\pm}$	0(0 <sup>+</sup> )
$f_0(500)$	0 <sup>+</sup> (0 <sup>+</sup> )	$a_2(1700)$	1 <sup>-</sup> (2 <sup>+</sup> )	$K_2^*$	1/2(0 <sup>-</sup> )	$D_{s1}(2460)^{\pm}$	0(1 <sup>+</sup> )
$\rho(770)$	1 <sup>+</sup> (1 <sup>+</sup> )	$f_0(1710)$	0 <sup>+</sup> (0 <sup>+</sup> )	$K_0^*(800)$	1/2(0 <sup>+</sup> )	$D_{s1}(2536)^{\pm}$	0(1 <sup>+</sup> )
$\omega(782)$	0 <sup>-</sup> (1 <sup>-</sup> )	$\eta(1760)$	0 <sup>+</sup> (0 <sup>+</sup> )	$K^*(892)$	1/2(1 <sup>-</sup> )	$D_{s2}(2573)$	0(0 <sup>+</sup> )
$\eta(958)$	0 <sup>+</sup> (0 <sup>+</sup> )	$\omega(1800)$	1 <sup>-</sup> (0 <sup>+</sup> )	$K_1^*(1270)$	1/2(1 <sup>+</sup> )	$D_{s1}(2700)^{\pm}$	0(1 <sup>-</sup> )
$f_0(980)$	0 <sup>+</sup> (0 <sup>+</sup> )	$f_0(1810)$	0 <sup>+</sup> (2 <sup>+</sup> )	$K_1^*(1400)$	1/2(1 <sup>+</sup> )	$D_{s1}^*(2860)^{\pm}$	0(0 <sup>+</sup> )
$a_0(980)$	1 <sup>-</sup> (0 <sup>+</sup> )	$X(1835)$	? <sup>?</sup> (2 <sup>-</sup> )	$K^*(1410)$	1/2(1 <sup>-</sup> )	$D_{s1}(3040)^{\pm}$	0(0 <sup>+</sup> )
$\phi(1020)$	0 <sup>-</sup> (1 <sup>-</sup> )	$X(1840)$	? <sup>?</sup> (2 <sup>?</sup> )	$K_0^*(1430)$	1/2(0 <sup>+</sup> )		
$h_1(1170)$	0 <sup>-</sup> (1 <sup>+</sup> )	$\phi_3(1850)$	0 <sup>-</sup> (3 <sup>-</sup> )	$K_2^*(1430)$	1/2(2 <sup>+</sup> )		
$b_1(1235)$	1 <sup>+</sup> (1 <sup>+</sup> )	$\eta_2(1870)$	0 <sup>+</sup> (2 <sup>+</sup> )	$K(1460)$	1/2(0 <sup>-</sup> )		
$a_1(1260)$	1 <sup>+</sup> (1 <sup>+</sup> )	$\eta_2(1880)$	1 <sup>-</sup> (2 <sup>+</sup> )	$K_0^*(1580)$	1/2(2 <sup>-</sup> )		
$f_2(1270)$	0 <sup>+</sup> (2 <sup>+</sup> )	$\rho(1900)$	1 <sup>+</sup> (1 <sup>-</sup> )	$K(1630)$	1/2(2 <sup>?</sup> )		
$f_1(1285)$	0 <sup>+</sup> (1 <sup>+</sup> )	$f_0(1910)$	0 <sup>+</sup> (2 <sup>+</sup> )	$K_1^*(1650)$	1/2(1 <sup>+</sup> )		
$\eta(1295)$	0 <sup>+</sup> (0 <sup>+</sup> )	$f_0(1950)$	0 <sup>+</sup> (2 <sup>+</sup> )	$K^*(1680)$	1/2(1 <sup>-</sup> )		
$\pi(1320)$	1 <sup>-</sup> (0 <sup>+</sup> )	$\rho_3(1990)$	1 <sup>+</sup> (3 <sup>-</sup> )	$K_0^*(1770)$	1/2(2 <sup>-</sup> )		
$a_2(1320)$	1 <sup>-</sup> (2 <sup>+</sup> )	$f_0(2010)$	0 <sup>+</sup> (2 <sup>+</sup> )	$K_3^*(1780)$	1/2(3 <sup>-</sup> )		
$h_1(1370)$	0 <sup>-</sup> (1 <sup>+</sup> )	$f_0(2020)$	0 <sup>+</sup> (0 <sup>+</sup> )	$K_0^*(1820)$	1/2(2 <sup>-</sup> )		
$h_1(1400)$	? <sup>?</sup> (1 <sup>+</sup> )	$a_1(2040)$	1 <sup>-</sup> (4 <sup>+</sup> )	$K(1830)$	1/2(0 <sup>-</sup> )		
$\pi_1(1400)$	1 <sup>-</sup> (1 <sup>+</sup> )	$f_0(2050)$	0 <sup>+</sup> (4 <sup>+</sup> )	$K_0^*(1950)$	1/2(0 <sup>+</sup> )		
$\eta(1405)$	0 <sup>+</sup> (0 <sup>+</sup> )	$\rho_2(2100)$	1 <sup>-</sup> (2 <sup>+</sup> )	$K_2^*(1980)$	1/2(2 <sup>+</sup> )		
$f_1(1420)$	0 <sup>+</sup> (1 <sup>+</sup> )	$f_0(2100)$	0 <sup>+</sup> (0 <sup>+</sup> )	$K_1^*(2045)$	1/2(4 <sup>+</sup> )		
$\omega(1420)$	0 <sup>-</sup> (1 <sup>-</sup> )	$f_2(2150)$	0 <sup>+</sup> (2 <sup>+</sup> )	$K_0^*(2250)$	1/2(2 <sup>+</sup> )		
$f_2(1430)$	0 <sup>+</sup> (2 <sup>+</sup> )	$\rho(2150)$	1 <sup>-</sup> (1 <sup>-</sup> )	$K_3^*(2380)$	1/2(5 <sup>-</sup> )		
$a_1(1450)$	1 <sup>+</sup> (1 <sup>+</sup> )	$f_0(2200)$	0 <sup>+</sup> (0 <sup>+</sup> )	$K_2^*(2400)$	1/2(2 <sup>+</sup> )		
$\eta(1475)$	0 <sup>+</sup> (0 <sup>+</sup> )	$f_1(2220)$	0 <sup>+</sup> (2 <sup>+</sup> )	$K_1^*(2500)$	1/2(3 <sup>-</sup> )		
$f_0(1500)$	0 <sup>+</sup> (0 <sup>+</sup> )	$\eta(2225)$	0 <sup>+</sup> (0 <sup>+</sup> )	$K(3100)$	? <sup>?</sup> (2 <sup>?</sup> )		
$f_1(1510)$	0 <sup>+</sup> (1 <sup>+</sup> )	$f_3(2250)$	1 <sup>+</sup> (3 <sup>-</sup> )				
$f_2(1525)$	0 <sup>+</sup> (2 <sup>+</sup> )	$f_0(2300)$	0 <sup>+</sup> (2 <sup>+</sup> )				
$f_3(1565)$	0 <sup>+</sup> (2 <sup>+</sup> )	$f_0(2300)$	0 <sup>+</sup> (4 <sup>+</sup> )				
$\omega(1570)$	1 <sup>-</sup> (1 <sup>-</sup> )	$f_0(2330)$	0 <sup>+</sup> (0 <sup>+</sup> )				
$h_1(1595)$	0 <sup>-</sup> (1 <sup>+</sup> )	$f_2(2340)$	0 <sup>+</sup> (2 <sup>+</sup> )				
$\pi_1(1600)$	1 <sup>-</sup> (1 <sup>+</sup> )	$\rho_3(2350)$	1 <sup>+</sup> (5 <sup>-</sup> )				
$a_1(1640)$	1 <sup>+</sup> (1 <sup>+</sup> )	$a_1(2450)$	1 <sup>-</sup> (6 <sup>+</sup> )				
$f_2(1640)$	0 <sup>+</sup> (2 <sup>+</sup> )	$f_0(2510)$	0 <sup>+</sup> (6 <sup>+</sup> )				
$\eta_2(1645)$	0 <sup>+</sup> (2 <sup>-</sup> )						
$\omega(1650)$	0 <sup>-</sup> (1 <sup>-</sup> )						
$\omega_3(1670)$	0 <sup>-</sup> (3 <sup>-</sup> )						
$\rho_2(1670)$	1 <sup>-</sup> (2 <sup>-</sup> )						



~ 210 mesons

All ~ 360 hadrons emerge from single QCD Lagrangian.

All flavor quantum numbers are described by  $qqq/q\bar{q}$ .

# Exotic candidates beyond $qqq/q\bar{q}$

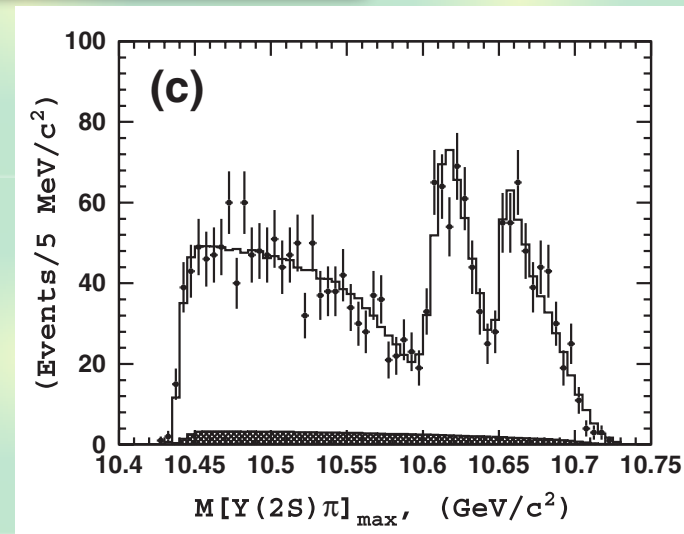
## Tetraquark candidate (Belle)

:  $Z_b(10610)$ ,  $Z_b(10650)$

$$Y(5S) \longrightarrow \pi^\pm + Z_b$$

$$\hookrightarrow Y(nS)(b\bar{b}) + \pi^\mp(u\bar{d}/d\bar{u})$$

A. Bondar, *et al.*, Phys. Rev. Lett. 108, 122001 (2012)



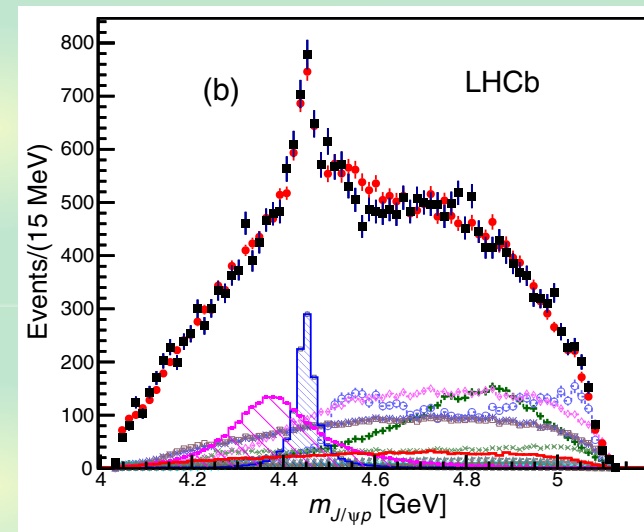
## Pentaquark candidate (LHCb)

:  $P_c(4450)$ ,  $P_c(4380)$

$$\Lambda_b \longrightarrow K^- + P_c$$

$$\hookrightarrow J/\psi(c\bar{c}) + p(uud)$$

R. Aaij, *et al.*, Phys. Rev. Lett. 115, 072001 (2015)



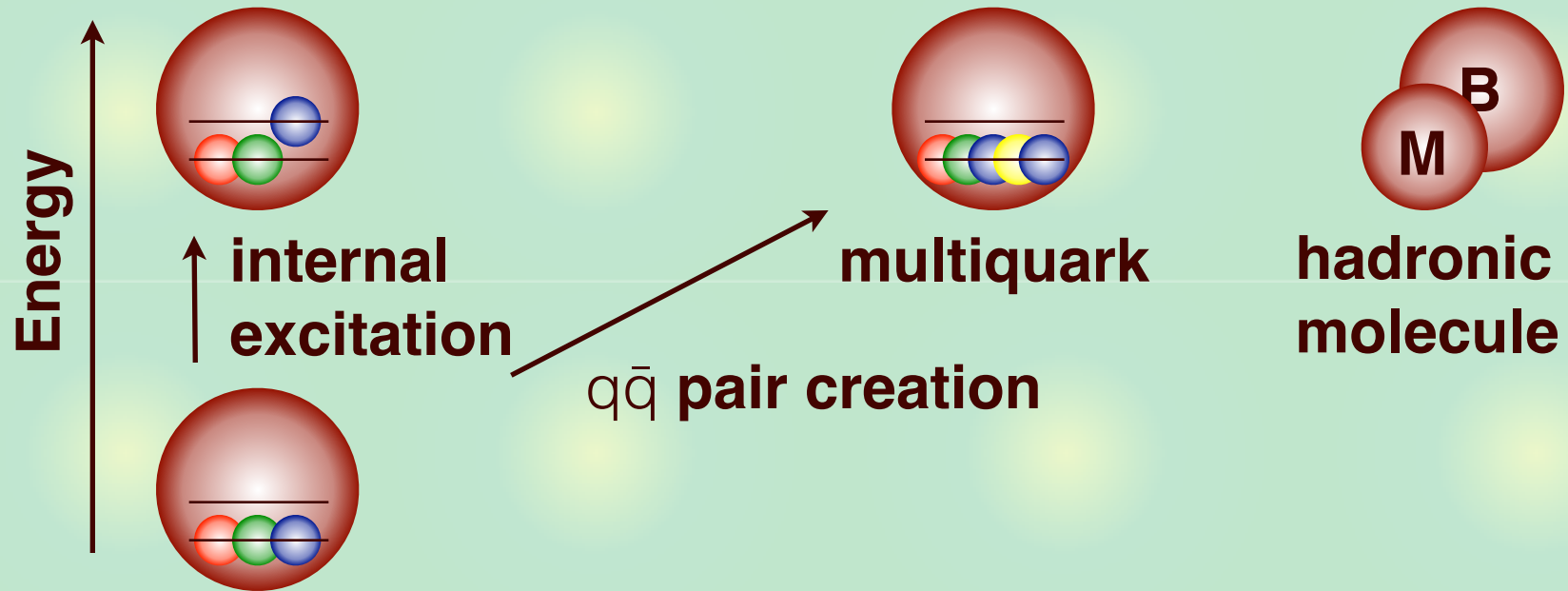
Only a few are observed. **Why only a few?**

# Various hadronic excitations

## Description of excited baryons

### Conventional structure

### Exotic structures



In QCD, non- $qqq$  structures naturally arise.

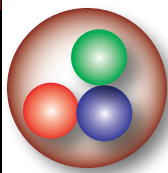
- Baryons: superposition of  $qqq$  + exotic structures
- > How can we disentangle different components?

# Unstable states via strong interaction

## Hadron resonances

PDG2017 : <http://pdg.lbl.gov/>

$p$	$1/2^+$ ****	$\Delta(1232)$	$3/2^+$ ****	$\Sigma^+$	$1/2^+$ ****	$\Xi^0$	$1/2^+$ ****	$\Lambda_c^+$	$1/2^+$ ****
$n$	$1/2^+$ ****	$\Delta(1600)$	$3/2^+$ ***	$\Sigma^0$	$1/2^+$ ****	$\Xi^-$	$1/2^+$ ****	$\Lambda_c(2595)^+$	$1/2^-$ ***
$N(1440)$	$1/2^+$ ****	$\Delta(1620)$	$1/2^-$ ****	$\Sigma^-$	$1/2^+$ ****	$\Xi(1530)$	$3/2^+$ ****	$\Lambda_c(2625)^+$	$3/2^-$ ***
$N(1520)$	$3/2^-$ ****	$\Delta(1700)$	$3/2^-$ ****	$\Xi(1385)$	$3/2^+$ ****	$\Xi(1620)$	*	$\Lambda_c(2765)^+$	*
$N(1535)$	$1/2^-$ ****	$\Delta(1750)$	$1/2^+$ *	$\Sigma(1480)$	*	$\Xi(1690)$	*	$\Lambda_c(2880)^+$	$5/2^+$ ***
$N(1650)$	$1/2^-$ ****	$\Delta(1900)$	$1/2^-$ **	$\Sigma(1560)$	**	$\Xi(1820)$	$3/2^-$ ***	$\Lambda_c(2940)^+$	***
$N(1675)$	$5/2^-$ ****	$\Delta(1905)$	$5/2^+$ ****	$\Sigma(1580)$	$3/2^-$ **	$\Xi(1950)$	*	$\Sigma_c(2455)$	$1/2^+$ ****
$N(1680)$	$5/2^+$ ****	$\Delta(1910)$	$1/2^+$ ****	$\Sigma(1620)$	$1/2^-$ *	$\Xi(2030)$	$\geq 5/2^?$ ***	$\Sigma_c(2520)$	$3/2^+$ ****
$N(1685)$	*	$\Delta(1920)$	$3/2^+$ ***	$\Sigma(1660)$	$1/2^+$ ***	$\Xi(2120)$	*	$\Sigma_c(2800)$	***
$N(1700)$	$3/2^-$ ***	$\Delta(1930)$	$5/2^-$ **	$\Sigma(1670)$	$3/2^-$ ****	$\Xi(2250)$	**	$\Xi_c$	$1/2^+$ ***
$N(1710)$	$1/2^+$ ***	$\Delta(1940)$	$3/2^-$ **	$\Sigma(1690)$	***	$\Xi(2370)$	**	$\Xi_c$	$1/2^+$ ***
$N(1720)$	$3/2^+$ ****	$\Delta(1950)$	$7/2^+$ ****	$\Sigma(1730)$	$3/2^+$ *	$\Xi(2500)$	*	$\Xi_c$	$1/2^+$ ***
$N(1860)$	$5/2^+$ **	$\Delta(2000)$	$5/2^+$ **	$\Sigma(1750)$	$1/2^-$ ***			$\Xi_c$	$1/2^+$ ***
$N(1875)$	$3/2^-$ ***	$\Delta(2150)$	$1/2^-$ *	$\Sigma(1770)$	$1/2^+$ *	$\Omega^-$	$3/2^+$ ****	$\Xi_c$	$1/2^+$ ***
$N(1880)$	$1/2^+$ **	$\Delta(2200)$	$7/2^-$ *	$\Sigma(1775)$	$5/2^-$ ****	$\Omega(2250)$	***	$\Xi_c$	$3/2^+$ ***
$N(1895)$	$1/2^-$ **	$\Delta(2300)$	$9/2^+$ **	$\Sigma(1840)$	$3/2^+$ **	$\Omega(2380)^-$	**	$\Xi_c$	$1/2^-$ ***
$N(1900)$	$3/2^+$ ***	$\Delta(2350)$	$5/2^-$ **	$\Sigma(1880)$	$1/2^+$ **	$\Omega(2470)^-$	**	$\Xi_c$	$3/2^-$ ***
$N(1990)$	$7/2^+$ **	$\Delta(2390)$	$7/2^+$ *	$\Sigma(1900)$	$1/2^-$ *			$\Xi_c$	$2/2^+$ **
$N(2000)$	$5/2^+$ **	$\Delta(2400)$	$9/2^-$ **	$\Sigma(1915)$	$5/2^+$ ****			$\Xi_c$	$3/2^+$ **
$N(2040)$	$3/2^+$ *	$\Delta(2420)$	$11/2^+$ ****	$\Sigma(1940)$	$3/2^+$ **			$\Xi_c$	$3/2^+$ **
$N(2060)$	$5/2^-$ **	$\Delta(2750)$	$13/2^-$ **	$\Sigma(1940)$	$3/2^-$ ***			$\Xi_c$	$3/2^+$ **
$N(2100)$	$1/2^+$ *	$\Delta(2950)$	$15/2^+$ **	$\Sigma(2000)$	$1/2^-$ *			$\Xi_c$	$3/2^+$ **
$N(2120)$	$3/2^-$ **			$\Sigma(2030)$	$7/2^+$ ****			$\Omega_c^0$	$1/2^+$ ***
$N(2190)$	$7/2^-$ ****	$\Lambda$	$1/2^+$ ****	$\Sigma(2070)$	$5/2^+$ *			$\Omega_c(2770)^0$	$3/2^+$ ***
$N(2220)$	$9/2^+$ ****	$\Lambda(1405)$	$1/2^-$ ****	$\Sigma(2080)$	$3/2^+$ **				
$N(2250)$	$9/2^-$ ****	$\Lambda(1520)$	$3/2^-$ ****	$\Sigma(2100)$	$7/2^-$ **			$\Xi_{cc}$	*
$N(2300)$	$1/2^+$ **	$\Lambda(1600)$	$1/2^+$ ***	$\Sigma(2250)$	***				
$N(2570)$	$5/2^-$ **	$\Lambda(1670)$	$1/2^-$ ****	$\Sigma(2455)$	**			$\Lambda_b^0$	$1/2^+$ ***
$N(2600)$	$11/2^-$ ***	$\Lambda(1690)$	$3/2^-$ ****	$\Sigma(2620)$	**			$\Lambda_b(5912)^0$	$1/2^-$ ***
$N(2700)$	$13/2^+$ **	$\Lambda(1710)$	$1/2^+$ *	$\Sigma(3000)$	*			$\Lambda_b(5920)^0$	$3/2^-$ ***
		$\Lambda(1800)$	$1/2^-$ ***	$\Sigma(3170)$	*			$\Sigma_b$	$1/2^+$ ***
		$\Lambda(1810)$	$1/2^+$ ***					$\Sigma_b^-$	$3/2^+$ ***
		$\Lambda(1820)$	$5/2^+$ ****					$\Xi_b^0, \Xi_b^-$	$1/2^+$ ***
		$\Lambda(1830)$	$5/2^-$ ****					$\Xi_b', \Xi_b$	$1/2^+$ ***
		$\Lambda(1890)$	$3/2^+$ ****					$\Xi_b(5945)^0$	$3/2^+$ ***
		$\Lambda(2000)$	*					$\Xi_b(5955)$	$3/2^+$ ***
		$\Lambda(2020)$	$7/2^+$ *					$\Xi_b(5955)$	$3/2^+$ ***
		$\Lambda(2050)$	$3/2^-$ *					$\Omega_b$	$1/2^+$ ***
		$\Lambda(2100)$	$7/2^-$ ****						
		$\Lambda(2110)$	$5/2^+$ ***						
		$\Lambda(2325)$	$3/2^-$ *						
		$\Lambda(2350)$	$9/2^+$ ***						
		$\Lambda(2585)$	**						



LIGHT UNFLAVORED (S = C = B = 0)		STRANGE (S = ±1, C = B = 0)		CHARMED, STRANGE (C = S = ±1)		$c\bar{c}$ $F_c(F_c)$			
$F_c(F_c)$	$F_c(F_c)$	$F_c(F_c)$	$F_c(F_c)$	$F_c(F_c)$	$F_c(F_c)$	$F_c(F_c)$	$F_c(F_c)$		
$\pi^\pm$	$1^-(0^-)$	$\rho(1680)$	$0^-(1^-)$	$K^{*2}$	$1/2(0^-)$	$D_s^{*2}$	$0(0^-)$	$\eta_c(1S)$	$0^+(0^+)$
$\pi^0$	$1^-(0^+)$	$\rho(1690)$	$1^+(3^-)$	$K^0$	$1/2(0^-)$	$D_s^{*0}$	$0(0^-)$	$J/\psi(1S)$	$0^-(0^+)$
$\eta$	$0^+(0^+)$	$\rho(1700)$	$1^+(1^-)$	$K_S^0$	$1/2(0^-)$	$D_s^{*0}$	$0(0^-)$	$\chi_{c0}(1P)$	$0^+(0^+)$
$\omega(770)$	$0^+(0^+)$	$\rho(1710)$	$0^+(0^+)$	$K_L^0$	$1/2(0^-)$	$D_s^{*0}$	$0(0^+)$	$\chi_{c1}(1P)$	$0^+(1^+)$
$\omega(782)$	$0^-(1^-)$	$\rho(1760)$	$0^+(0^+)$	$K^0$	$1/2(0^-)$	$D_s^{*0}$	$0(1^+)$	$\chi_{c2}(1P)$	$0^+(2^+)$
$\eta(958)$	$0^+(0^+)$	$\rho(1800)$	$1^-(0^+)$	$K_1^0(800)$	$1/2(0^-)$	$D_{s1}(2460)^0$	$0(1^+)$	$\eta_c(2S)$	$0^+(0^+)$
$\eta(980)$	$0^+(0^+)$	$\rho(1810)$	$0^+(2^+)$	$K^*(892)$	$1/2(1^-)$	$D_{s1}(2536)^0$	$0(1^+)$	$\psi(2S)$	$0^-(1^-)$
$\omega(980)$	$0^+(0^+)$	$\rho(1820)$	$1^-(0^+)$	$K_1^*(1270)$	$1/2(1^+)$	$D_{s1}(2573)$	$0(0^+)$	$\psi(3770)$	$0^-(1^-)$
$\omega(1020)$	$0^-(1^-)$	$\rho(1840)$	$??(2^{??})$	$K_1^*(1400)$	$1/2(1^+)$	$D_{s1}(2700)^0$	$0(1^-)$	$X(3823)$	$?^?(2^-)$
$h_1(1170)$	$0^-(1^-)$	$\rho(1850)$	$0^-(3^-)$	$K^*(1410)$	$1/2(1^-)$	$D_{s1}(3040)^0$	$0(2^+)$	$X(3872)$	$0^+(1^+)$
$b_1(1235)$	$1^-(1^+)$	$\rho_2(1870)$	$0^+(2^+)$	$K_0^*(1430)$	$1/2(0^+)$			$X(3900)^0$	$?^?(1^+)$
$\omega(1260)$	$1^-(1^+)$	$\rho_2(1880)$	$1^-(2^+)$	$K(1460)$	$1/2(0^-)$	BOTTOM (B = ±1)		$X(3900)^0$	$?^?(2^+)$
$f_1(1270)$	$0^+(2^+)$	$\rho(1900)$	$1^+(1^-)$	$K_0^*(1580)$	$1/2(2^-)$	$B^{*+}$	$1/2(0^-)$	$\chi_{c0}(3915)$	$0^+(0^+)$
$f_1(1285)$	$0^+(1^+)$	$\rho_3(1910)$	$1^+(1^-)$	$K_1^*(1630)$	$1/2(2^-)$	$B^0$	$1/2(0^-)$	$\chi_{c2}(3940)$	$0^+(2^+)$
$\eta(1295)$	$0^+(0^+)$	$\rho_3(1950)$	$0^+(2^+)$	$K_1^*(1650)$	$1/2(2^+)$	$B^{*0}/B^0$	ADMIXTURE	$X(3940)$	$?^?(2^+)$
$\omega(1320)$	$1^-(0^+)$	$\rho_3(1990)$	$1^+(3^-)$	$K_1^*(1680)$	$1/2(1^-)$	$B^{*0}/B^0/B^0$	b-baryon ADMIXTURE	$X(4020)$	$?^?(2^+)$
$\omega(1330)$	$1^-(2^+)$	$\rho_3(2010)$	$0^+(2^+)$	$K_2^*(1770)$	$1/2(2^-)$	$V_{cb}$ and $V_{cb}$	KM Matrix Elements	$\psi(4040)$	$0^-(1^-)$
$h_1(1370)$	$0^+(0^+)$	$f_0(2020)$	$0^+(0^+)$	$K_3^*(1780)$	$1/2(3^-)$	$B^{*+}$	$1/2(1^-)$	$X(4050)^0$	$?^?(2^+)$
$h_1(1380)$	$1^-(1^+)$	$\omega(2040)$	$1^-(4^+)$	$K_3^*(1820)$	$1/2(2^-)$	$B^{*+}$	$1/2(1^-)$	$X(4140)$	$0^+(2^+)$
$\pi_1(1400)$	$1^-(1^+)$	$\omega(2050)$	$1^-(4^+)$	$K(1830)$	$1/2(0^-)$	$B_1(5721)^+$	$1/2(1^+)$	$\psi(4160)$	$0^-(1^-)$
$\eta(1405)$	$0^+(0^+)$	$\omega(2050)$	$1^-(4^+)$	$K_0^*(1950)$	$1/2(0^+)$	$B_1(5721)^0$	$1/2(1^+)$	$X(4160)$	$?^?(2^+)$
$f_1(1420)$	$0^+(1^+)$	$\omega(2100)$	$1^-(2^+)$	$K_2^*(1980)$	$1/2(2^+)$	$B_1(5732)$	$?^?(2^+)$	$X(4230)$	$?^?(1^-)$
$\omega(1420)$	$0^-(1^-)$	$f_2(2100)$	$0^+(0^+)$	$K_2^*(1980)$	$1/2(2^+)$	$B_2^*(5747)^+$	$1/2(2^+)$	$X(4240)^0$	$?^?(0^-)$
$f_2(1430)$	$0^+(2^+)$	$f_2(2150)$	$0^+(2^+)$	$K_4^*(2045)$	$1/2(4^+)$	$B_2^*(5747)^0$	$1/2(2^+)$	$X(4250)^0$	$?^?(2^+)$
$a_1(1450)$	$1^-(0^+)$	$\rho(2150)$	$1^-(1^-)$	$K_3^*(2250)$	$1/2(3^-)$	$B(5970)^0$	$?^?(2^+)$	$X(4260)$	$?^?(1^-)$
$\rho(1450)$	$1^+(1^-)$	$f_0(2200)$	$0^+(0^+)$	$K_3^*(2280)$	$1/2(3^-)$	$B(5970)^0$	$?^?(2^+)$	$X(4350)$	$0^+(2^+)$
$\eta(1475)$	$0^+(0^+)$	$f_2(2200)$	$0^+(2^+)$	$K_4^*(2380)$	$1/2(5^-)$	$B(5970)^0$	$?^?(2^+)$	$X(4360)$	$?^?(1^-)$
$f_0(1500)$	$0^+(0^+)$	$\eta(2225)$	$0^+(0^+)$	$K_4^*(2500)$	$1/2(4^-)$	BOTTOM, STRANGE (B = ±1, S = ±1)		$X(4415)$	$0^-(1^-)$
$f_1(1510)$	$0^+(1^+)$	$\rho_3(2250)$	$1^+(3^-)$	$K(3100)$	$?^?(2^?)$	$B_c^{*0}$	$0(0^-)$	$X(4430)$	$?^?(1^-)$
$f_2(1525)$	$0^+(2^+)$	$f_0(2300)$	$0^+(2^+)$	CHARMED (C = ±1)		$B_c^{*+}$	$0(1^-)$	$X(4660)$	$?^?(1^-)$
$f_3(1565)$	$0^+(2^+)$	$f_0(2300)$	$0^+(4^+)$	$D^{*+}$	$1/2(0^-)$	$b\bar{b}$		$\eta_b(1S)$	$0^+(0^+)$
$h_1(1570)$	$1^-(1^+)$	$f_0(2330)$	$0^+(0^+)$	$D^0$	$1/2(0^-)$	$B_{cb}(5830)^0$	$0(1^+)$	$\Upsilon(1S)$	$0^-(1^-)$
$h_1(1595)$	$0^+(1^+)$	$f_0(2340)$	$0^+(2^+)$	$D^{*0}$	$1/2(0^-)$	$B_{cb}(5840)^0$	$0(2^+)$	$\chi_{b0}(1P)$	$0^+(0^+)$
$\pi_1(1600)$	$1^-(1^+)$	$f_0(2340)$	$0^+(2^+)$	$D^{*0}$	$1/2(0^-)$	$B_{cb}^*(5850)$	$?^?(2^+)$	$\chi_{b1}(1P)$	$0^+(1^+)$
$a_1(1640)$	$1^-(1^+)$	$\rho_3(2350)$	$1^+(5^-)$	$D(2010)^+$	$1/2(1^-)$	BOTTOM, CHARMED (B = C = 0)		$\chi_{b2}(1P)$	$?^?(1^+)$
$f_2(1640)$	$0^+(2^+)$	$a_1(2450)$	$1^-(6^+)$	$D_3^*(2400)^0$	$1/2(0^+)$	$B_c^{*+}$	$0(0^-)$	$h_b(1P)$	$?^?(1^+)$
$\rho_2(1645)$	$0^+(2^+)$	$f_0(2510)$	$0^+(6^+)$	$D_3^*(2400)^0$	$1/2(0^+)$	$B_c(2S)^+$	$?^?(2^?)$	$\chi_{b1}(2P)$	$0^+(1^+)$
$\omega(1650)$	$0^-(1^-)$			$D_1(2430)^0$	$1/2(1^+)$			$h_b(2P)$	$?^?(1^+)$
$\omega_3(1670)$	$0^-(3^-)$			$D_1^*(2460)^0$	$1/2(1^+)$			$\chi_{b2}(2P)$	$0^+(2^+)$
$\rho_2(1670)$	$1^-(2^+)$			$D_2^*(2460)^0$	$1/2(2^+)$			$\Upsilon(3S)$	$0^-(1^-)$
				$D_2^*(2460)^0$	$1/2(2^+)$			$\chi_{b1}(3P)$	$0^+(1^+)$
				$D(2550)^0$	$1/2(0^-)$			$\Upsilon(4S)$	$0^-(1^-)$
				$D(2550)^0$	$1/2(0^-)$			$X(10610)^0$	$1^+(1^+)$
				$D(2600)$	$1/2(2^?)$			$X(10610)^0$	$1^+(1^+)$
				$D(2640)^+$	$1/2(2^?)$			$X(10650)^0$	$?^+(1^+)$
				$D(2750)$	$1/$				

# Difficulty of resonances

## Resonance as an “eigenstate” of Hamiltonian

### - complex energy

G. Gamow, Z. Phys. 51, 204 (1928)

Zur Quantentheorie des Atomkernes.

Von G. Gamow, z. Zt. in Göttingen.

Mit 5 Abbildungen. (Eingegangen am 2. August 1928.)

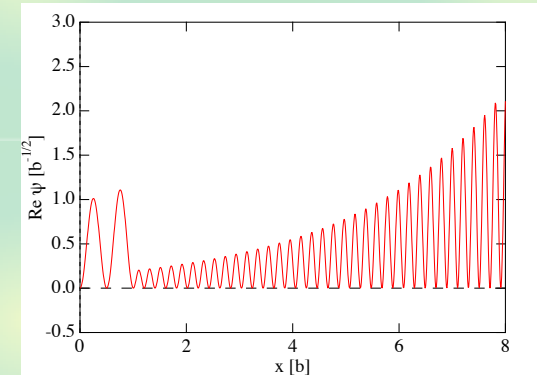
Um diese Schwierigkeit zu überwinden, müssen wir annehmen, daß die Schwingungen gedämpft sind, und  $E$  komplex setzen :

$$E = E_0 + i \frac{\hbar \lambda}{4\pi},$$

wo  $E_0$  die gewöhnliche Energie ist und  $\lambda$  das Dämpfungsdekrement (Zerfallskonstante). Dann sehen wir aber aus den Relationen (2a) und (2b),

### - diverging wave function ( $\text{Im } k < 0$ )

$$\langle R | R \rangle = \int d\mathbf{r} |\psi_R(\mathbf{r})|^2 \sim \int_0^\infty dr e^{-2\text{Im}[k]r} \rightarrow \infty$$



## Bi-orthogonal basis (Gamow vectors): normalizable!

N. Hokkyo, Prog. Theor. Phys. 33, 1116 (1965)

T. Berggren, Nucl. Phys. A 109, 265 (1968)

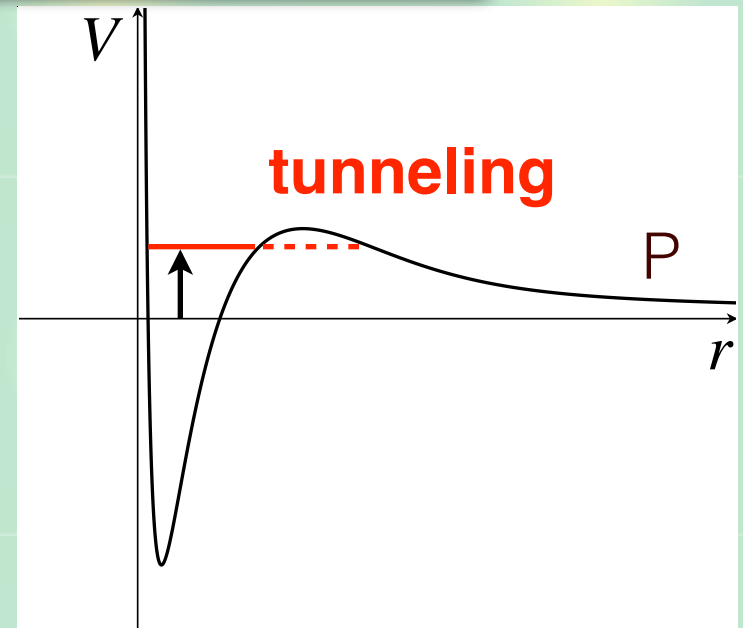
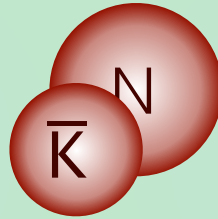
$$|\tilde{R}\rangle = |R^*\rangle, \quad |\langle \tilde{R} | R \rangle| = \left| \int d\mathbf{r} [\psi_R(\mathbf{r})]^2 \right| < \infty$$

### - Complex expectation value (e.g. $\langle r^2 \rangle$ ) $\rightarrow$ interpretation?

# Resonances in quantum mechanics

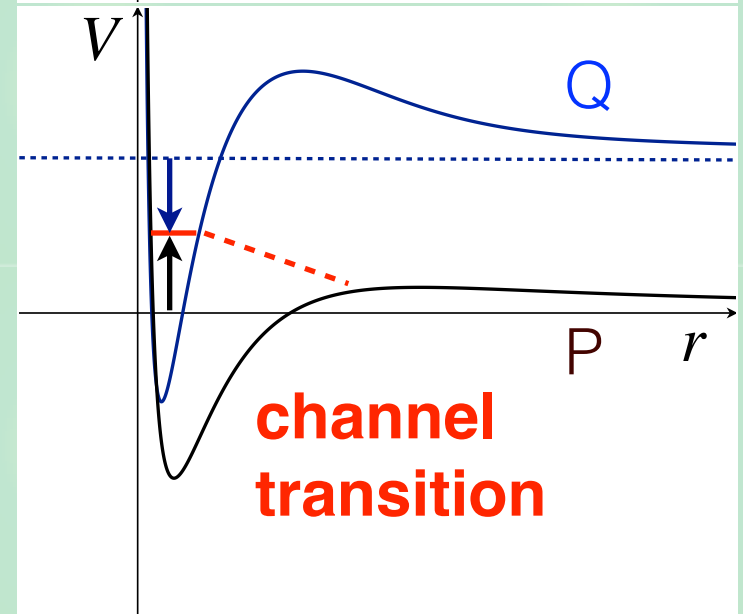
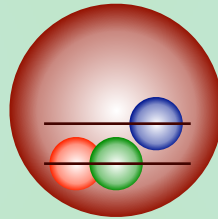
## 1) Potential (shape) resonance

- 1 channel (P)
- potential barrier :  $E > 0$
- unstable via tunneling
- (**composite** of P-channel)



## 2) Feshbach resonance

- coupled-channel (P+Q)
- bound state of Q:  $E_Q < 0$ ,  $E_P > 0$
- unstable via transition
- (**“elementary”**: other than P)

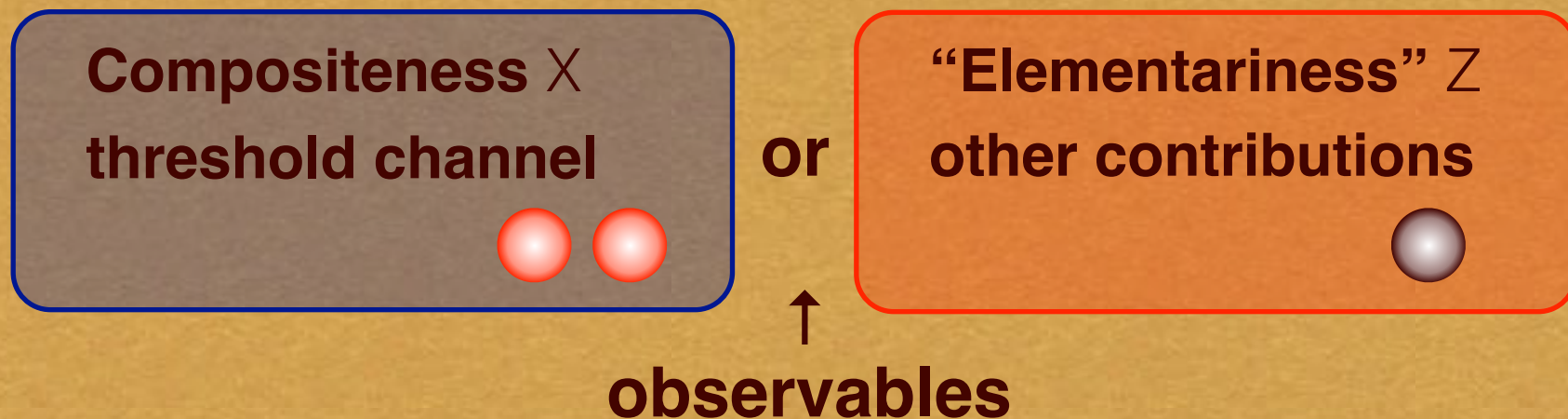




# Compositeness of hadrons

- Internal structure of excited (**exotic**) hadrons
- Weak binding relation for stable bound states

*S. Weinberg, Phys. Rev. 137, B672 (1965)*



- Effective field theory  $\rightarrow$  description of low-energy scattering amplitude, generalization to **unstable** resonances

## Weak binding relation for stable states

Compositeness of s-wave **weakly bound** state ( $R \gg R_{\text{typ}}$ )

S. Weinberg, Phys. Rev. 137, B672 (1965);

T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)

$$a_0 = R \left\{ \frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\text{typ}}}{R}\right) \right\}, \quad r_e = R \left\{ \frac{X-1}{X} + \mathcal{O}\left(\frac{R_{\text{typ}}}{R}\right) \right\}$$

$a_0$ : **scattering length**,  $r_e$ : **effective range**

$R = (2\mu B)^{-1/2}$ : **radius of wave function**

$R_{\text{typ}}$ : **length scale of interaction**

- **deuteron is NN composite** ( $a_0 \sim R \gg r_e$ )  $\rightarrow X \sim 1$
- **internal structure from observable**
- **no nuclear force potential / wavefunction of deuteron**

**Problem: applicable only for stable states.**

# Effective field theory

## Low-energy scattering with near-threshold bound state

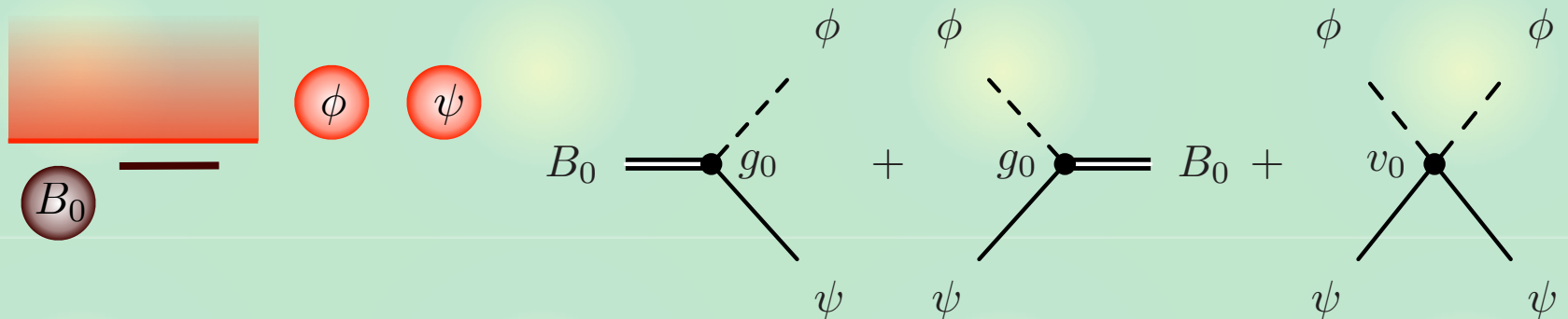
### - Nonrelativistic EFT with **contact interaction**

D.B. Kaplan, Nucl. Phys. B494, 471 (1997)

E. Braaten, M. Kusunoki, D. Zhang, Annals Phys. 323, 1770 (2008)

$$H_{\text{free}} = \int d\mathbf{r} \left[ \frac{1}{2M} \nabla \psi^\dagger \cdot \nabla \psi + \frac{1}{2m} \nabla \phi^\dagger \cdot \nabla \phi + \frac{1}{2M_0} \nabla B_0^\dagger \cdot \nabla B_0 + \nu_0 B_0^\dagger B_0 \right],$$

$$H_{\text{int}} = \int d\mathbf{r} \left[ g_0 \left( B_0^\dagger \phi \psi + \psi^\dagger \phi^\dagger B_0 \right) + v_0 \psi^\dagger \phi^\dagger \phi \psi \right]$$



- **cutoff** :  $\Lambda \sim 1/R_{\text{typ}}$  (interaction range of microscopic theory)

- At low energy  $p \ll \Lambda$ , interaction  $\sim$  contact

# Compositeness and “elementariness”

## Eigenstates

$$H_{\text{free}}|B_0\rangle = \nu_0|B_0\rangle, \quad H_{\text{free}}|\mathbf{p}\rangle = \frac{p^2}{2\mu}|\mathbf{p}\rangle \quad \text{free (discrete + continuum)}$$

$$(H_{\text{free}} + H_{\text{int}})|B\rangle = -B|B\rangle \quad \text{full (bound state)}$$

### - normalization of $|B\rangle$ + completeness relation

$$\langle B|B\rangle = 1, \quad 1 = |B_0\rangle\langle B_0| + \int \frac{d\mathbf{p}}{(2\pi)^3} |\mathbf{p}\rangle\langle \mathbf{p}|$$

### - projections onto free eigenstates

$$1 = Z + X, \quad \underbrace{Z \equiv |\langle B_0|B\rangle|^2}_{\text{“elementariness”}}, \quad \underbrace{X \equiv \int \frac{d\mathbf{p}}{(2\pi)^3} |\langle \mathbf{p}|B\rangle|^2}_{\text{compositeness}}$$

“elementariness”      compositeness



$Z, X$ : real and nonnegative  $\rightarrow$  interpreted as **probability**

# Weak binding relation

## $\Psi\Phi$ scattering amplitude (exact result)

$$f(E) = -\frac{\mu}{2\pi} \frac{1}{[v(E)]^{-1} - G(E)}$$

$$v(E) = v_0 + \frac{g_0^2}{E - \nu_0}, \quad G(E) = \frac{1}{2\pi^2} \int_0^\Lambda dp \frac{p^2}{E - p^2/(2\mu) + i0^+}$$

## Compositeness $X \leftarrow -v(E), G(E)$

$$X = \frac{G'(-B)}{G'(-B) - [1/v(-B)]'}$$

$1/R = (2\mu B)^{1/2}$  expansion: leading term  $\leftarrow X$

$$a_0 = -f(E=0) = R \left\{ \frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\text{typ}}}{R}\right) \right\}$$

renormalization independent

If  $R \gg R_{\text{typ}}$ , correction terms neglected:  $X \leftarrow (B, a_0)$

# Introduction of decay channel

## Introduce decay channel

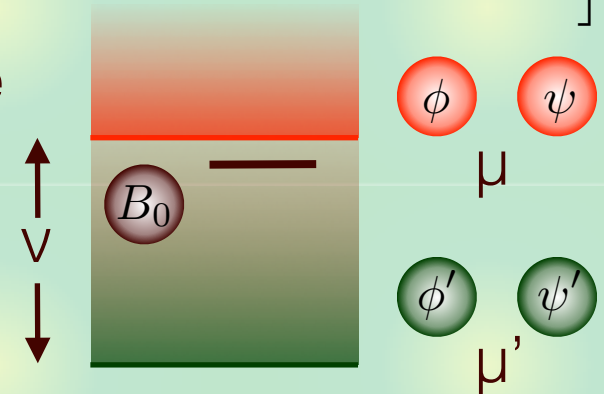
$$H'_{\text{free}} = \int d\mathbf{r} \left[ \frac{1}{2M'} \nabla \psi'^{\dagger} \cdot \nabla \psi' - \nu_{\psi} \psi'^{\dagger} \psi' + \frac{1}{2m'} \nabla \phi'^{\dagger} \cdot \nabla \phi' - \nu_{\phi} \phi'^{\dagger} \phi' \right],$$

$$H'_{\text{int}} = \int d\mathbf{r} \left[ g'_0 \left( B_0^{\dagger} \phi' \psi' + \psi'^{\dagger} \phi'^{\dagger} B_0 \right) + v'_0 \psi'^{\dagger} \phi'^{\dagger} \phi' \psi' + v_0^t (\psi^{\dagger} \phi^{\dagger} \phi' \psi' + \psi'^{\dagger} \phi'^{\dagger} \phi \psi) \right],$$

## Quasi-bound state: complex eigenvalue

$$H = H_{\text{free}} + H'_{\text{free}} + H_{\text{int}} + H'_{\text{int}}$$

$$H |QB\rangle = E_{QB} |QB\rangle, \quad E_{QB} \in \mathbb{C}$$



## Generalized relation: **correction term** ← threshold difference

$$a_0 = R \left\{ \frac{2X}{1+X} + \mathcal{O} \left( \left| \frac{R_{\text{typ}}}{R} \right| \right) + \mathcal{O} \left( \left| \frac{l}{R} \right|^3 \right) \right\}, \quad R = \frac{1}{\sqrt{-2\mu E_{QB}}} \in \mathbb{C}, \quad l \equiv \frac{1}{\sqrt{2\mu\nu}}$$

Y. Kamiya, T. Hyodo, Phys. Rev. C93, 035203 (2016)

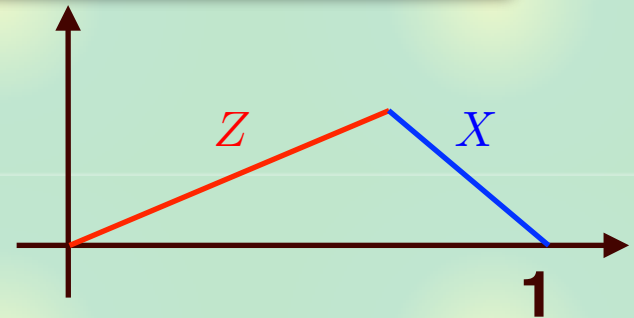
c.f. V. Baru, *et al.*, Phys. Lett. B586, 53 (2004),...

If  $|R| \gg (R_{\text{typ}}, l)$  correction terms neglected:  $X \leftarrow (E_{QB}, a_0)$

# Complex compositeness and interpretation

Unstable states  $\rightarrow$  complex  $Z$  and  $X$

$$Z + X = 1, \quad Z, X \in \mathbb{C}$$

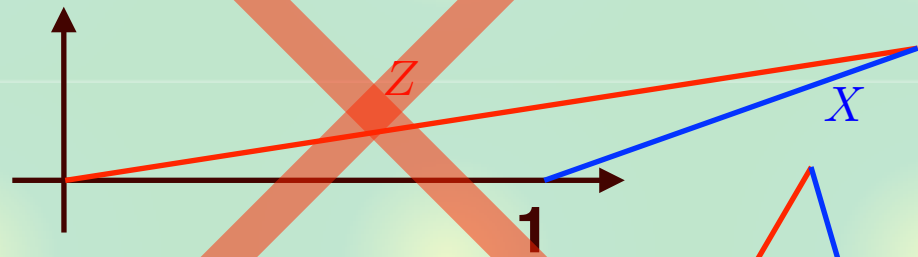
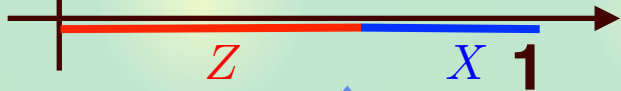


Similarity with bound state

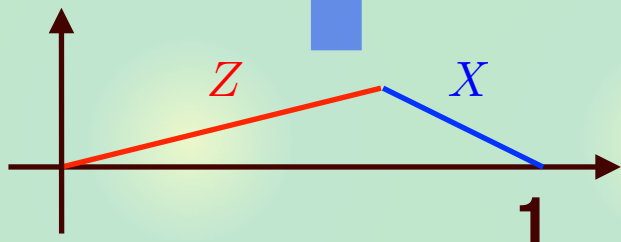
c.f. [T. Sekihara, T. Hyodo, D. Jido, PTEP2015, 063D04 \(2015\)](#)

bound state  
: well defined

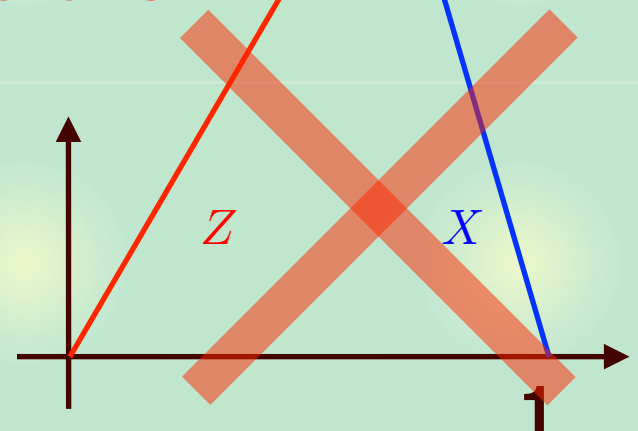
$$Z + X = 1, \quad Z, X \in [0, 1]$$



large cancellation



small cancellation



# New definitions

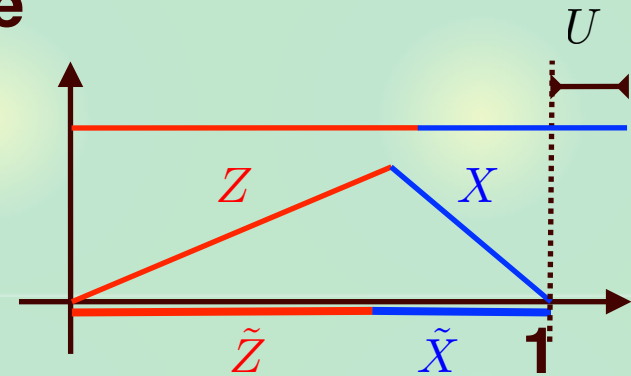
## Step 1: quantify the deviation from bound state

- 0 for bound state
- becomes large when deviation is large

$$U = |Z| + |X| - 1$$

→  $U$ : uncertainty of interpretation

c.f. T. Berggren, Phys. Lett. 33B, 547 (1970)



## Step 2: define new compositeness/elementariness

- interpreted as probabilities  $\tilde{Z} + \tilde{X} = 1$ ,  $\tilde{Z}, \tilde{X} \in [0, 1]$
- coincide with  $Z, X$  for bound state if  $U \rightarrow 0$

$$\tilde{Z} = \frac{1 - |X| + |Z|}{2}, \quad \tilde{X} = \frac{1 - |Z| + |X|}{2}$$

**compositeness when  $U$  is small**



# Application

Generalized weak binding relation  $X \leftarrow (E_{QB}, a_0)$

$$a_0 = R \left\{ \frac{2X}{1+X} + \mathcal{O} \left( \left| \frac{R_{\text{typ}}}{R} \right| \right) + \mathcal{O} \left( \left| \frac{l}{R} \right|^3 \right) \right\}, \quad R = \frac{1}{\sqrt{-2\mu E_{QB}}}, \quad l \equiv \frac{1}{\sqrt{2\mu\nu}}$$

-  $\Lambda(1405)$  (higher) pole position and  $\bar{K}N$  scattering length

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881 98 (2012), ...

-  $E_{QB} = -10 - 26i$  MeV  $\rightarrow |R| \sim 2$  fm  $\rightarrow$  small correction term

$\left| \frac{R_{\text{typ}}}{R} \right| \lesssim 0.12, \quad \left| \frac{l}{R} \right|^3 \lesssim 0.16$  **energy difference from  $\pi\Sigma$**

**vector meson exchange**

Ref.	$E_{QB}$ (MeV)	$a_0$ (fm)	$X_{\bar{K}N}$	$\tilde{X}_{\bar{K}N}$	$U$	$ r_e/a_0 $
[43]	$-10 - i26$	$1.39 - i0.85$	$1.2 + i0.1$	1.0	0.5	0.2
[44]	$-4 - i8$	$1.81 - i0.92$	$0.6 + i0.1$	0.6	0.0	0.7
[45]	$-13 - i20$	$1.30 - i0.85$	$0.9 - i0.2$	0.9	0.1	0.2
[46]	$2 - i10$	$1.21 - i1.47$	$0.6 + i0.0$	0.6	0.0	0.7
[46]	$-3 - i12$	$1.52 - i1.85$	$1.0 + i0.5$	0.8	0.6	0.4

**systematic error**

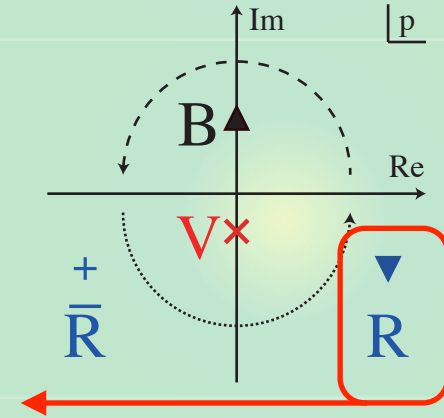
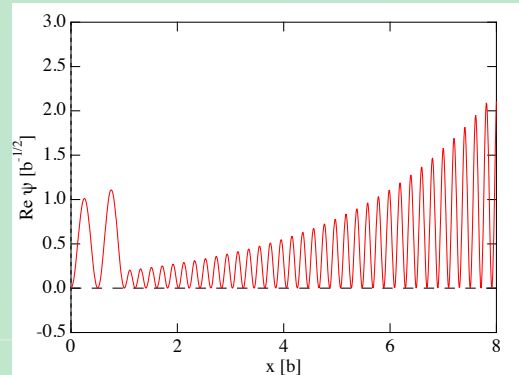
$\Lambda(1405)$  is  $\bar{K}N$  composite  $\leftarrow$  observables

# Use of finite volume eigenstates?

## Wavefunction of resonance

- outgoing boundary condition (c.f.  $\exp\{-kr\}$ )

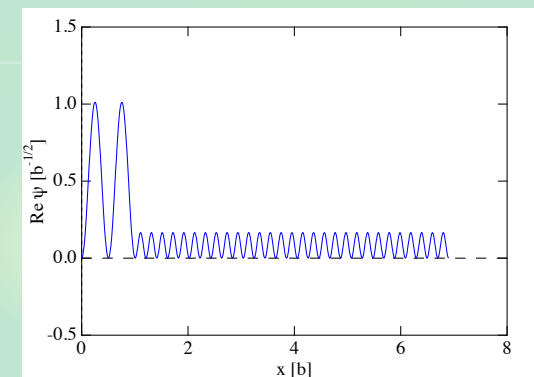
$$\begin{aligned}\psi(r) &\sim \exp[ipr] \\ &= \exp\{i[\operatorname{Re} p]r\} \exp\{-[\operatorname{Im} p]r\}\end{aligned}$$



- If  $\operatorname{Im} p < 0$ ,  $\psi$  is not square integrable.
- complex eigenvalues (energy,  $X$ ,  $\langle r^2 \rangle$ , ...)

## Finite-volume system with size $L$

- $\psi$  is square integrable on  $[0, L]^3$ .
- real eigenvalues (energy,  $X$ )
- > Probabilistic interpretation!



# Compositeness in finite volume

Effective field theory in finite box of size  $L$

- discrete real eigenenergies in finite volume (FV)

$$H|\Psi^{(m)}\rangle = E^{(m)}|\Psi^{(m)}\rangle, \quad E^{(m+1)} > E^{(m)}, \quad \langle \Psi^{(m)} | \Psi^{(l)} \rangle = \delta_{ml}$$

- **Compositeness**

$$X^{(m)} = \langle \Psi^{(m)} | \hat{P}_{\text{two-body}} | \Psi^{(m)} \rangle, \quad \hat{P}_{\text{two-body}} = \frac{1}{L^3} \sum_n |p_n\rangle \langle p_n|$$

$$= \frac{I'_{\text{FV}}(E^{(m)})}{I'_{\text{FV}}(E^{(m)}) - [1/v(E^{(m)})]'}, \quad 1 - I_{\text{FV}}(E^{(m)})v(E^{(m)}) = 0$$

**c.f.) infinite volume:**  $I_{\text{FV}}(E;L) \rightarrow G(E)$

Y. Kamiya, T. Hyodo, Phys. Rev. C 93, 035203 (2016); PTEP 023D02 (2017).

- **Compositeness**  $X^{(m)}$  is defined for **each** FV eigenstate.
- $X^{(m)}$  can be interpreted as a **probability**.
- $X^{(m)}$  has  **$L$  dependence** through  $I_{\text{FV}}$  and  $E^{(m)}$ .

# Compositeness of resonances

Which is the eigenstate representing the resonance?

- choose first excited state  $E^{(1)}(L)$
- energy region  $\rightarrow (L_{\min}, L_{\max})$

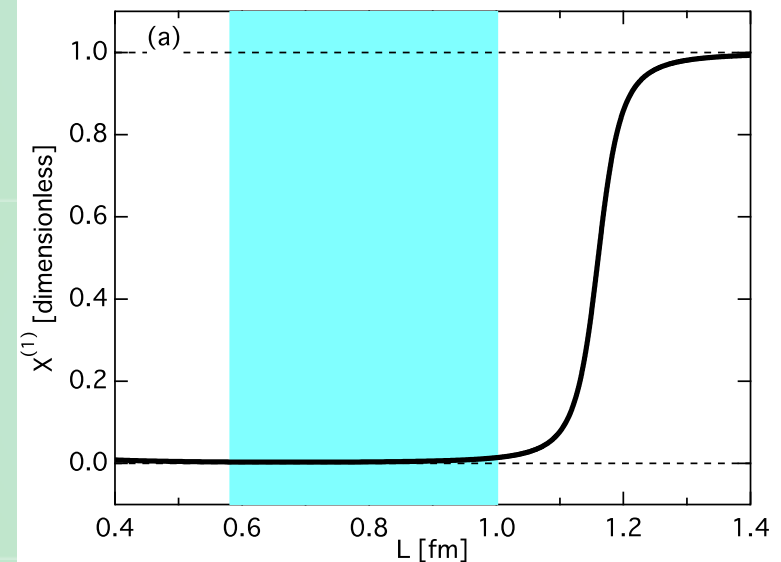
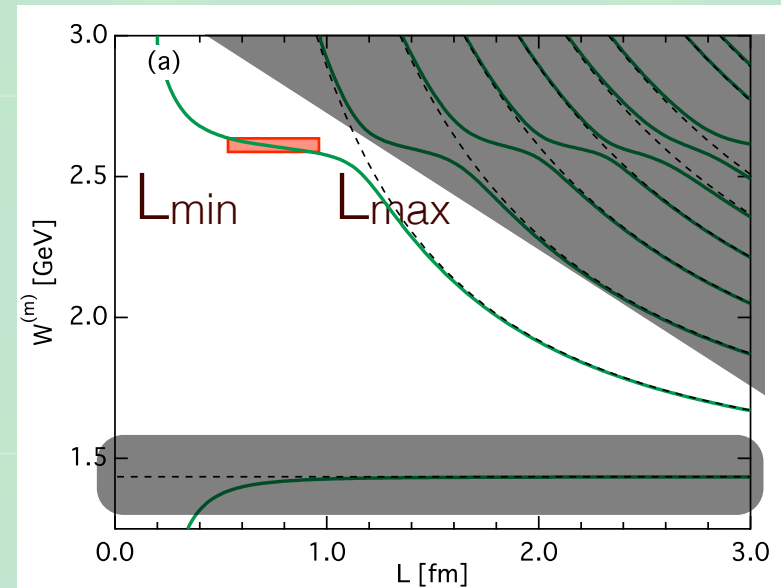
$$E_{\min} \leq E^{(1)}(L) \leq E_{\max}$$

- $L_{\min}$ : finite-volume effect on wavefunction
- $L_{\max}$ : mixing of scattering state

## Compositeness of resonance

$$X_{\text{res}} = \frac{1}{L_{\max} - L_{\min}} \int_{L_{\min}}^{L_{\max}} X^{(1)}(L) dL$$

- interpreted as a probability



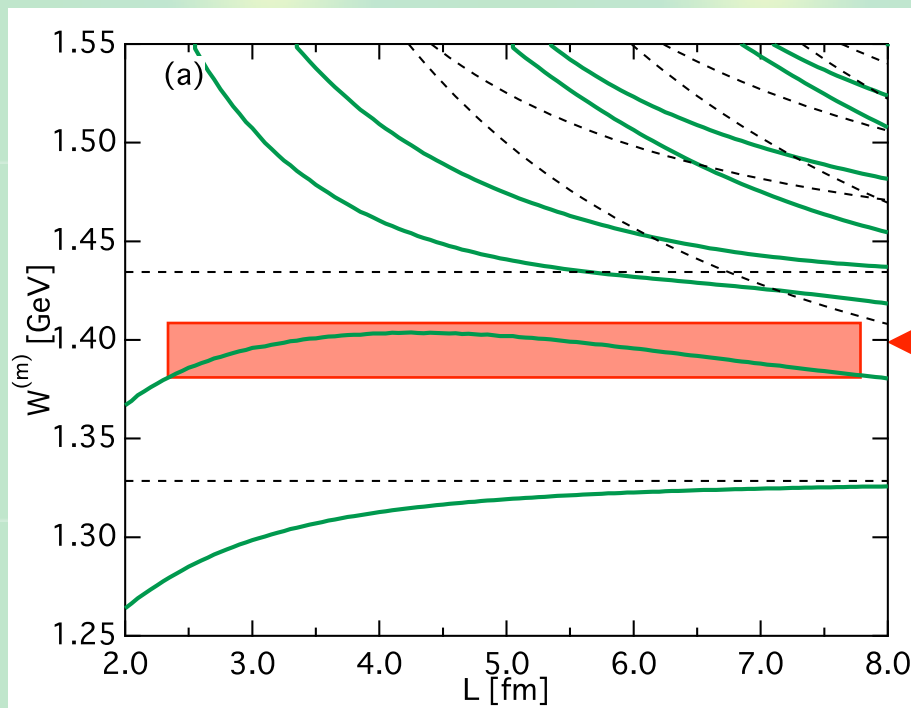
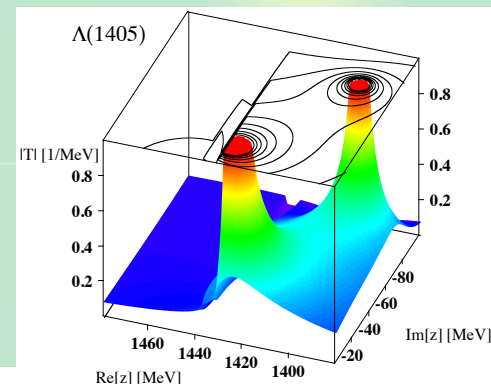
# Eigenenergies of $\Lambda(1405)$

ETW model ( $\bar{K}N$ - $\pi\Sigma$  2channel, WT interaction)

Y. Ikeda, T. Hyodo, W. Weise, Nucl. Phys. A881, 98 (2012)

- two poles, consistent with SIDDHARTA

Finite volume eigenenergies



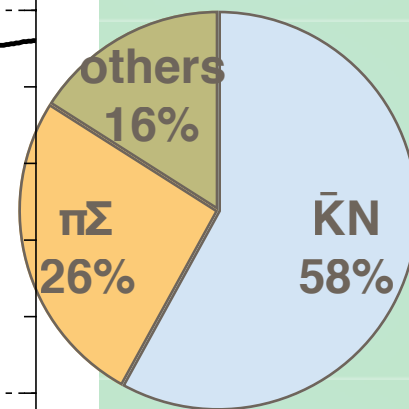
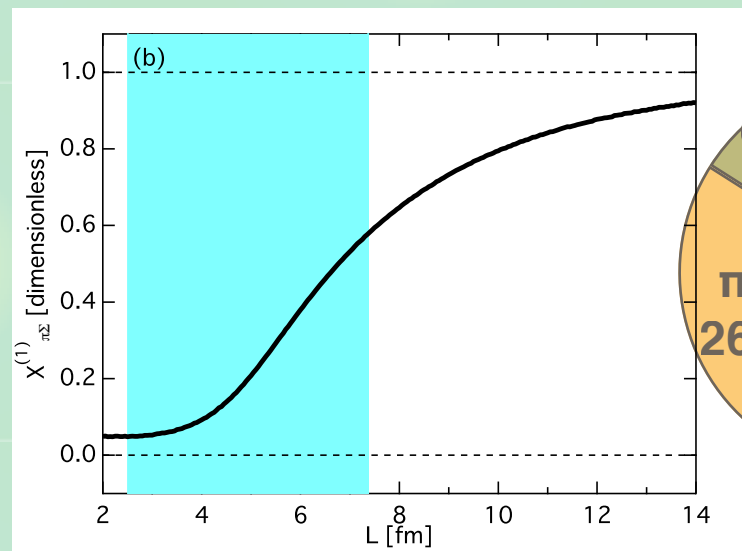
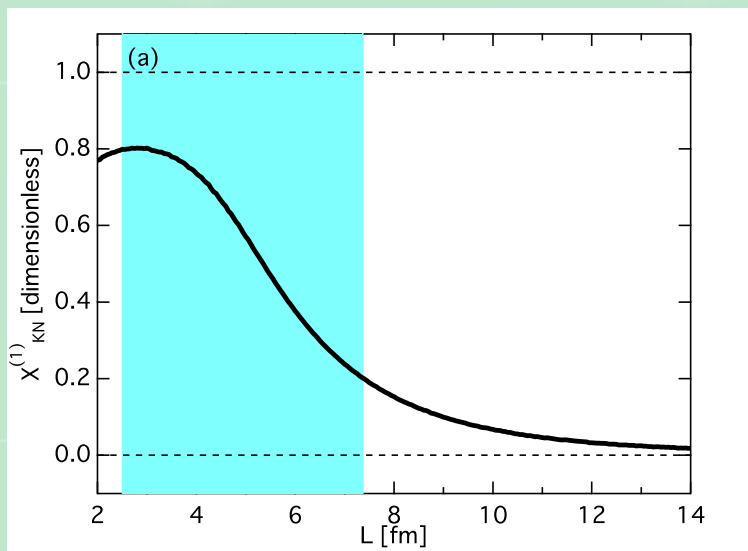
$\bar{K}N$  threshold  
 $\Lambda(1405)$   
 $\pi\Sigma$  threshold

$\Lambda(1405)$  is represented by a single FV eigenstate.

(# of FV eigenstates  $\leftrightarrow$  # of  $\pi/2$  crossings of phase shift)

# Compositeness of $\Lambda(1405)$

Compositeness  $X_{\text{res}, \bar{K}N}$ ,  $X_{\text{res}, \pi\Sigma}$

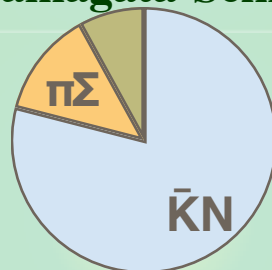


Complex compositeness at each pole  $\rightarrow$  real-valued

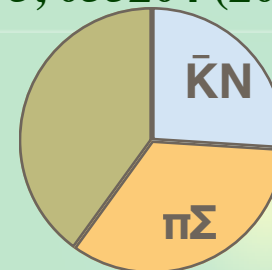
Y. Kamiya, T. Hyodo, Phys. Rev. C 93, 035203 (2016); PTEP 023D02 (2017),

T. Sekihara, T. Arai, J. Yamagata-Sekihara, S. Yasui, Phys. Rev. C 93, 035204 (2016)

- High-mass pole



- Low-mass pole



$X_{\text{res}}$  represents the contributions from **both poles**

## Analytic structure of scattering amplitude

**Pole** of scattering amplitude  $f(E_{\text{pole}})=\infty$

J.R. Taylor, *Scattering theory* (Wiley, New York, 1972)

- represents (complex) eigenvalue of Hamiltonian
- unique through analytic continuation

**CDD (Castillejo-Dalitz-Dyson) zero**

L. Castillejo, R.H. Dalitz, F.J. Dyson, *Phys. Rev.* 124, 264 (1961)

- pole of inverse amplitude, zero of amplitude  $f(E_{\text{CDD}})=0$
- unique through analytic continuation
- role of CDD zero in hadron scattering, resonances, etc.

V. Baru, *et al.*, *Eur. Phys. J. A* 44, 93 (2010),

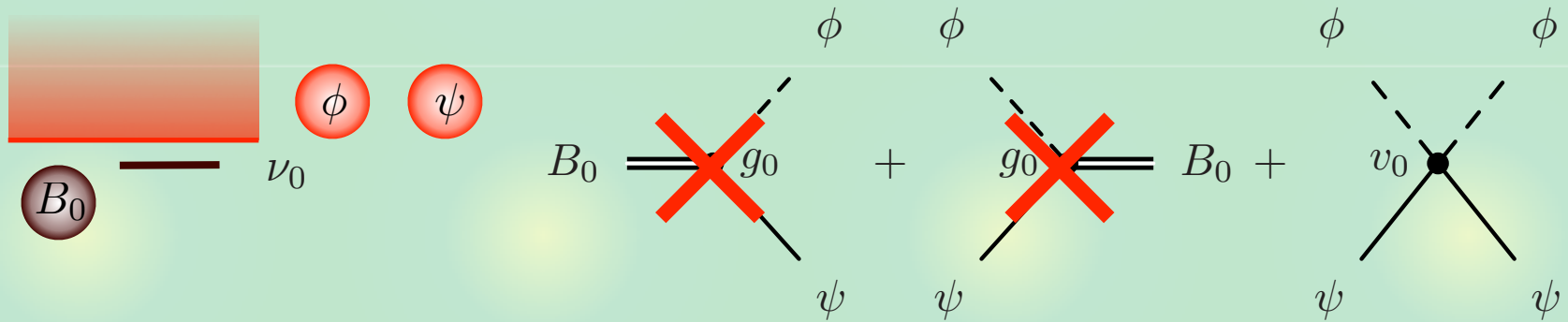
C. Hanhart, *et al.*, *Eur. Phys. J. A* 47, 101 (2011),

Z.H. Guo, J.A. Oller, *Phys. Rev. D* 93, 054014 (2016)

**CDD zero  $\leftrightarrow$  elementary/composite?**

# Fate of pole in zero coupling limit

## Contact interaction EFT



- Consider a pole of amplitude

$$f(E_{\text{pole}}) = \infty$$

Zero coupling limit (ZCL)  $g_0 \rightarrow 0$  : only 4-point interaction

- Composite ( $\sim$ potential): 4-point interaction origin  $\rightarrow$  pole **remains** in the amplitude



- Elementary ( $\sim$ Feshbach):  $B_0$  origin

$\rightarrow$  pole moves toward  $\nu_0$  and finally **decouples**





# Pole and CDD zero in ZCL

## Scattering amplitude

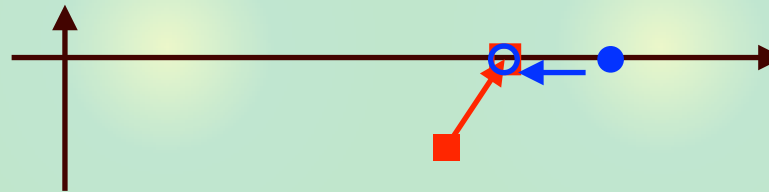
$$t(E) = \frac{v_0(E - \nu_0) + g_0^2}{(E - \nu_0)[1 - v_0G(E)] - g_0^2G(E)}$$

- **Pole** :  $(E_{\text{pole}} - \nu_0)[1 - v_0G(E_{\text{pole}})] - g_0^2G(E_{\text{pole}}) = 0$

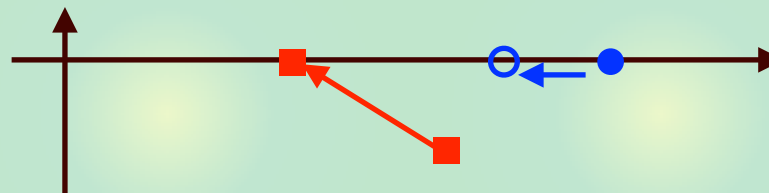
- **CDD zero** :  $E_{\text{CDD}} = \nu_0 - \frac{g_0^2}{v_0} \xrightarrow{\text{ZCL}} \nu_0$

## ZCL ( $g_0 \rightarrow 0$ )

- **Pole (case I)** :  $E_{\text{pole}} \rightarrow \nu_0$  **vanishing residue**  $\rightarrow$  elementary



- **Pole (case II)** :  $1 - v_0G(E_{\text{pole}}) = 0$  **finite residue**  $\rightarrow$  composite



Elementary  $\rightarrow$  **pole** encounters with CDD **zero**

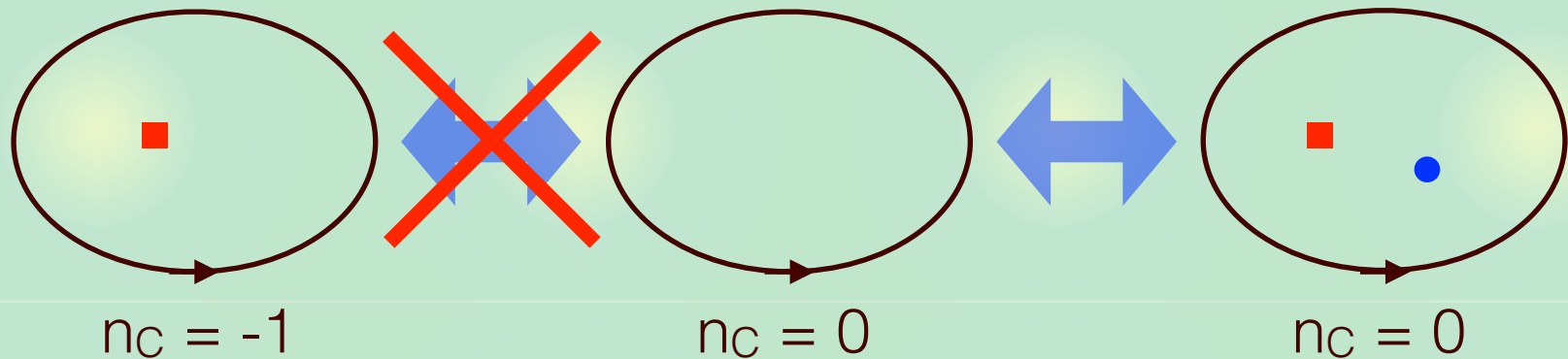
# General discussion

Scattering amplitude  $f(E)$  is meromorphic in energy

Y. Kamiya, T. Hyodo, Phys. Rev. D97, 054019 (2018)

$$\frac{1}{2\pi} \oint_C dz \frac{d \arg f(z)}{dz} = n_Z - n_P \equiv n_C$$

- $n_Z$  ( $n_P$ ) : number of **zeros** (**poles**) in contour  $C$
- Topological invariant of  $\pi_1(U(1)) \cong \mathbb{Z}$



Pole cannot decouple without merging with CDD zero

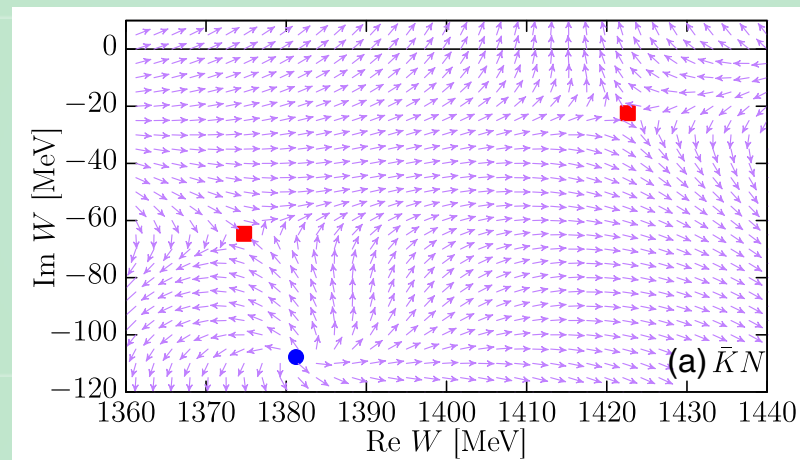
—> existence of nearby CDD zero indicates “elementary” (i.e. origin is not in this channel).

# Example: $\Lambda(1405)$

## Poles and zero positions in the $\bar{K}N$ amplitude

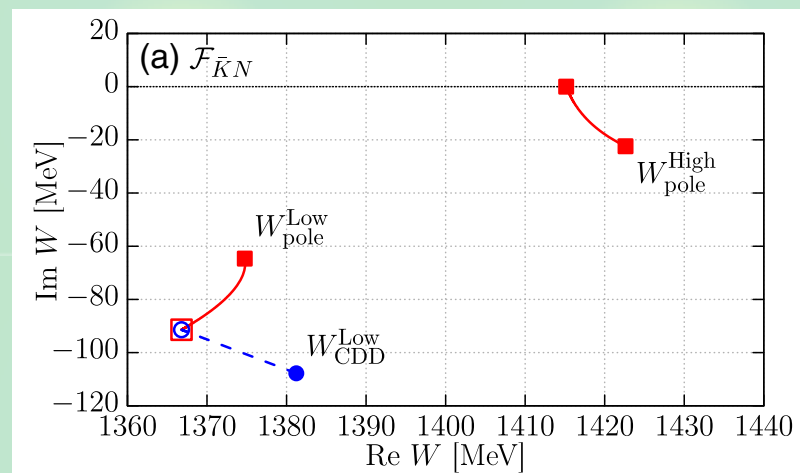
Y. Ikeda, T. Hyodo, W. Weise, Nucl. Phys. A881, 98 (2012)

- Two poles for  $\Lambda(1405)$
- **CDD zero** exists near the Low-mass pole



## Trajectories toward ZCL

- High mass pole remains in the  $\bar{K}N$  amplitude.
- Low-mass pole decouples by merging with CDD zero



High-mass pole can be  $\bar{K}N$  composite, but lower one is not.

# Summary 1

- Compositeness of **near-threshold** bound state can be determined only by **observables**.

S. Weinberg, Phys. Rev. 137, B672 (1965)

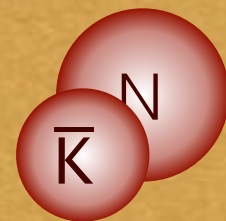
- Weak binding relation can be **generalized to unstable states** with effective field theory.

$$a_0 = R \left\{ \frac{2X}{1+X} + \mathcal{O} \left( \left| \frac{R_{\text{typ}}}{R} \right| \right) + \mathcal{O} \left( \left| \frac{l}{R} \right|^3 \right) \right\}, \quad R = \frac{1}{\sqrt{-2\mu E_{QB}}}, \quad l \equiv \frac{1}{\sqrt{2\mu\nu}}$$

- Precise determination of the pole position and scattering length shows that  $\Lambda(1405)$  is dominated by  **$\bar{K}N$  composite component**.

Y. Kamiya, T. Hyodo, Phys. Rev. C93, 035203 (2016);

Y. Kamiya, T. Hyodo, PTEP2017, 023D02 (2017)



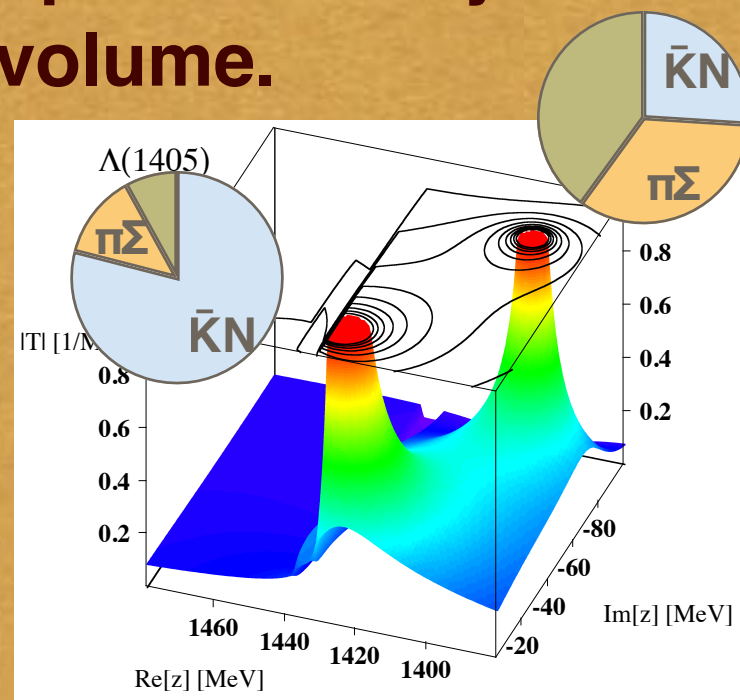
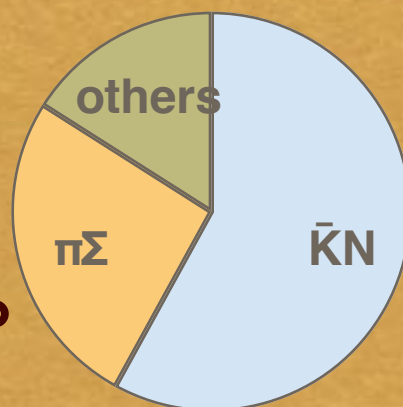
## Summary 2

We propose a new definition of compositeness of resonances using finite-volume eigenstates.

Two poles of  $\Lambda(1405)$  are represented by a **single eigenstate** in finite volume.


Structure of  $\Lambda(1405)$ :


- $\bar{K}N$ : 58%
- $\pi\Sigma$ : 26%
- others: 16%




[Y. Tsuchida, T. Hyodo, arXiv:1703.02675 \[nucl-th\]](https://arxiv.org/abs/1703.02675)

## Summary 3

 “Elementary” pole decouples from the amplitude in the zero coupling limit.

 For a pole to decouple from the amplitude, there must be a **nearby CDD zero**.

$$\frac{1}{2\pi} \oint_C dz \frac{d \arg f(z)}{dz} = n_Z - n_P$$

 The dynamical (composite) component of the eigenstate is small if a CDD zero exists near the eigenstate pole.

[Y. Kamiya, T. Hyodo, Phys. Rev. D97, 054019 \(2018\)](#)