

Universal physics of two neutrons with one flavored meson



U. Raha^a, Y. Kamiya^b, S.-I. Ando^c, T. Hyodo^b

^aIIT Guwahati, India, ^bYITP, Kyoto Univ. ^cSunmoon Univ., Korea

2017, Mar 13th 1

Contents



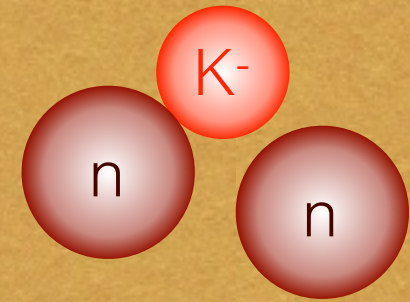
Introduction

- Mesons in few-body nuclei
- Universal physics in few-body system



Two-body meson-neutron interaction

- Experimental constraints
- Idealization



Three-body system

- Effective field theory
- RG limit cycle in the asymptotic region



Summary

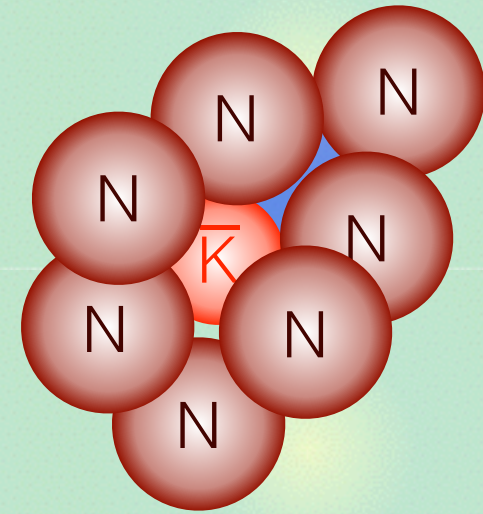
Mesons in nuclei

\bar{K} ($s\bar{u}$, $s\bar{d}$) nuclei

Y. Akaishi, T. Yamazaki, Phys. Rev. C 65, 044005 (2002), ...

- **attraction in $\bar{K}N(I=0)$: $\Lambda(1405)$**
- **quasibound states up to 7-body systems**

S. Ohnishi, W. Horiuchi, T. Hoshino, K. Miyahara, T. Hyodo, arXiv:1701.07589 [nucl-th]



D ($c\bar{u}$, $c\bar{d}$) nuclei

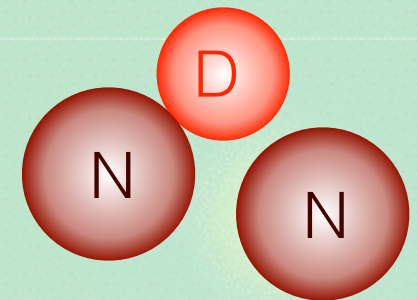
A. Hosaka, T. Hyodo, K. Sudoh, Y. Yamaguchi, S. Yasui, arXiv:1606.08685 [hep-ph]

- **hadronic molecules in XYZ \rightarrow D can be a constituent**
- **$\Lambda_c(2595) \sim DN(I=0)$ molecule? as $\Lambda(1405)$**

T. Mizutani, A. Ramos, Phys. Rev. C 74, 065201 (2006)

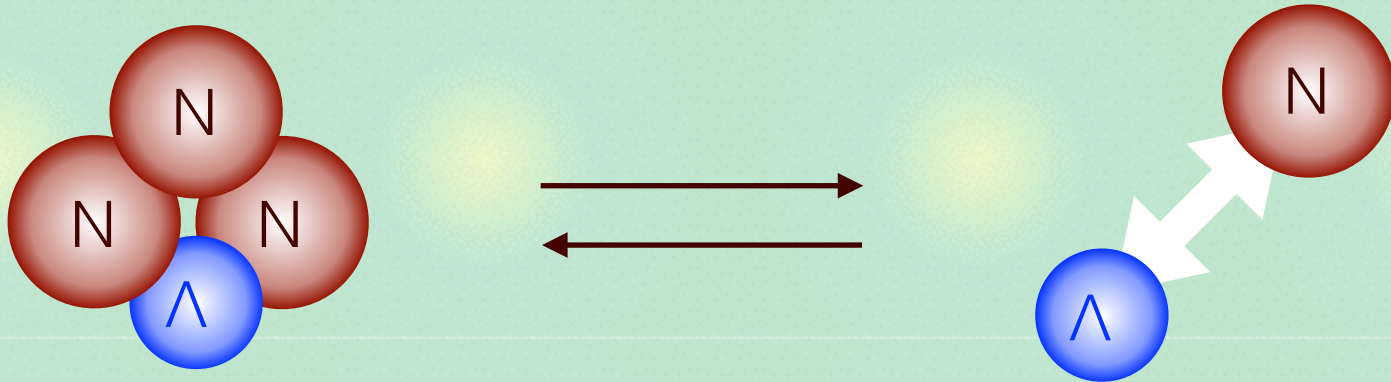
- **DNN quasibound state**

M. Bayar, C.W. Xiao, T. Hyodo, A. Dote, M. Oka, E. Oset, Phys. Rev. C86, 044004 (2012)

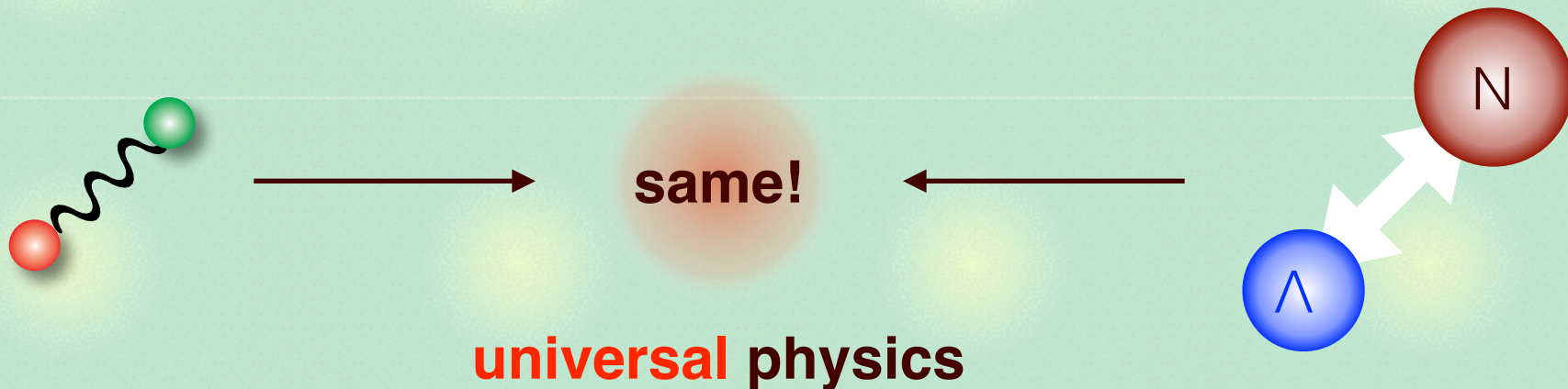


Study of few-body systems

Properties of few-body systems \leftrightarrow two-body interaction
- c.f. hypernuclei



In some cases, different interactions give the same physics.



Two-body universal physics

Universal two-body physics

E. Braaten, H.-W. Hammer, Phys. Rept. 428, 259 (2006)

1) s-wave short range interaction

2) scattering length : $|a| \gg r_s$: interaction range

- system is scale invariant

- a shallow bound state exists if $a > 0$

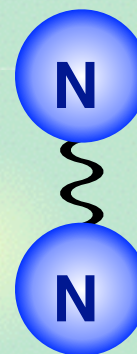
$$B_2 = \frac{1}{ma^2} \left[1 + \mathcal{O}\left(\frac{r_s}{a}\right) \right]$$

vdW

strong

Examples: nucleons and ^4He atoms

| | N [MeV] | ^4He [mK] |
|----------|-------------|--------------------|
| B_2 | 2.22 | 1.31 |
| $1/ma^2$ | 1.41 | 1.12 |



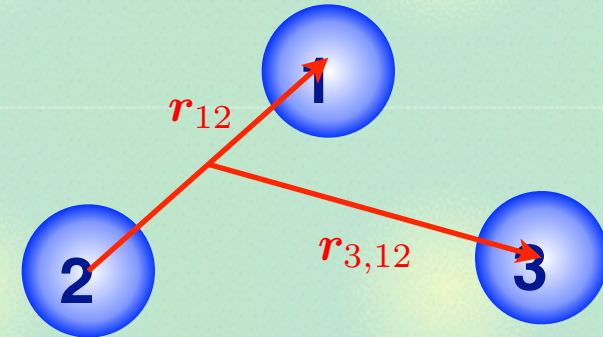
^4He

Three-body universal physics

Three-body system in hyperspherical coordinates

$$(\mathbf{r}_{12}, \mathbf{r}_{3,12}) \leftrightarrow (\underline{R}, \alpha_3, \hat{\mathbf{r}}_{12}, \hat{\mathbf{r}}_{3,12})$$

hyperradius hyperangular variables Ω
(dimensionless)



If $|a| \rightarrow \infty$, system is scale invariant.

$$V(R, \Omega) \propto \frac{1}{R^2}$$

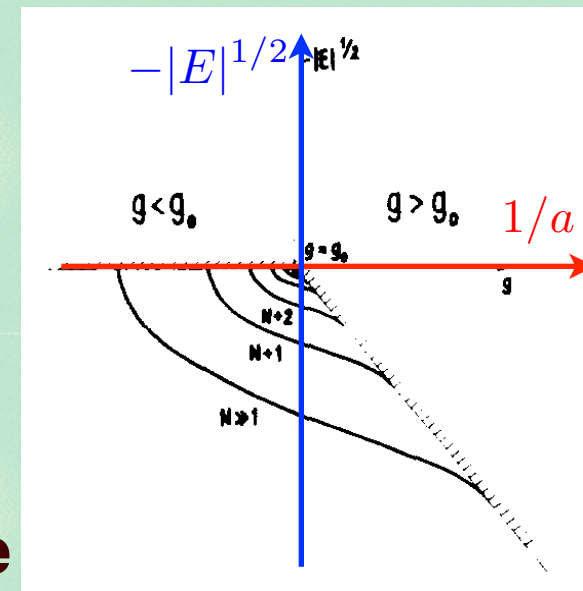
Efimov effect: attractive $1/R^2$

V. Efimov, Phys. Lett. B 33, 563-564 (1970)

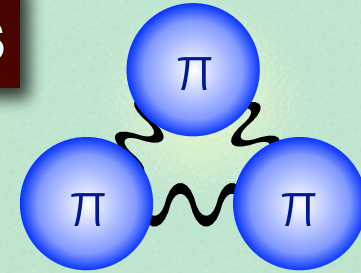
$$B_3^n / B_3^{n+1} \approx 22.7^2$$

- infinitely many bound states
- discrete scale invariance: RG limit cycle

P.F. Bedaque, H.-W. Hammer, U. van Kolck, Phys. Rev. Lett. 82, 463-437 (1999)



Universal physics of three pions



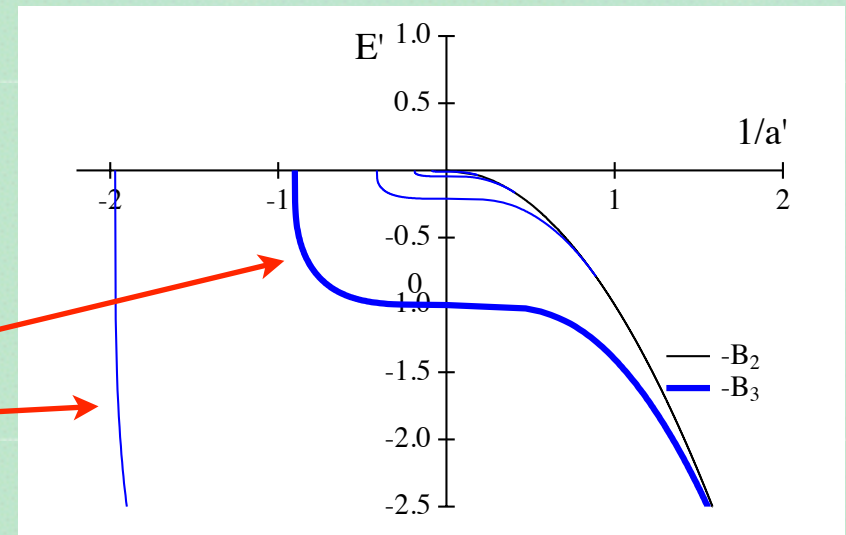
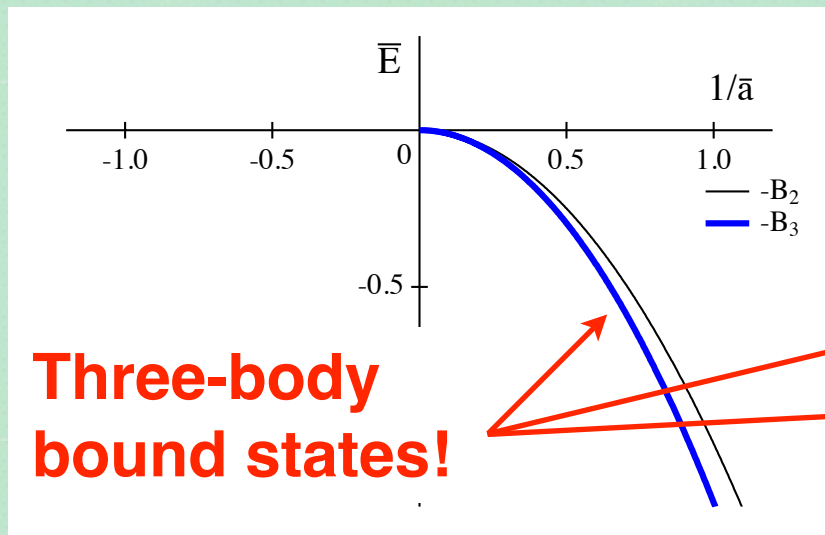
Universal physics \leftarrow large scattering length a

T. Hyodo, T. Hatsuda, Y. Nishida, Phys. Rev. C89, 032201(R) (2014)

- $l=0$ $\pi\pi$ scattering length can be increased by **changing** m_q

isospin sym. ($\pi^0\pi^0\pi^0$ - $\pi^0\pi^-\pi^+$)

isospin breaking ($\pi^0\pi^0\pi^0$)



- Efimov effect of $\pi^0\pi^0\pi^0$ @ unphysical m_q
- Coupled-channel effect reduces the attraction.

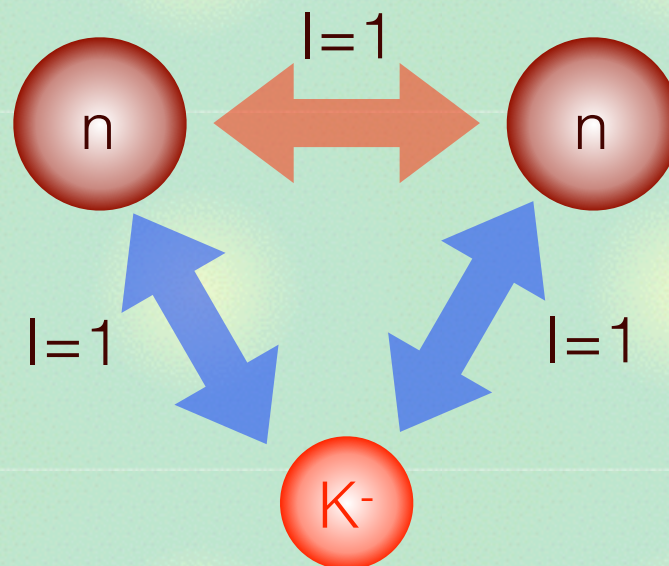
Two neutrons and one flavored meson

K-nn/D⁰nn **system with** $J=0$, $l=3/2$, $l_3=-3/2$

- **different from** $J=0$, $l=1/2$ (**so-called** K-pp- \bar{K}^0 np)
- **all interactions: isospin** $l=1$ (**no** $\Lambda(1405)$)

Desirable features for Efimov effect

- **no coupled channels**
- **no Coulomb interaction**
- $a_{nn} \sim -20 \text{ fm} \gg r_s \sim O(1) \text{ fm}$



Two-body K-n: unitary? \rightarrow yes, with suitable idealization.

Three-body K-nn: Efimov? \rightarrow yes, if two-body K-n is unitary.

Kaon-neutron interaction

Experimental database in $\bar{K}N$ sector

- cross sections of K^-p scattering (elastic, inelastic)
- threshold branching ratios
- kaonic hydrogen by SIDDHARTA $\rightarrow a_{K^-p}$

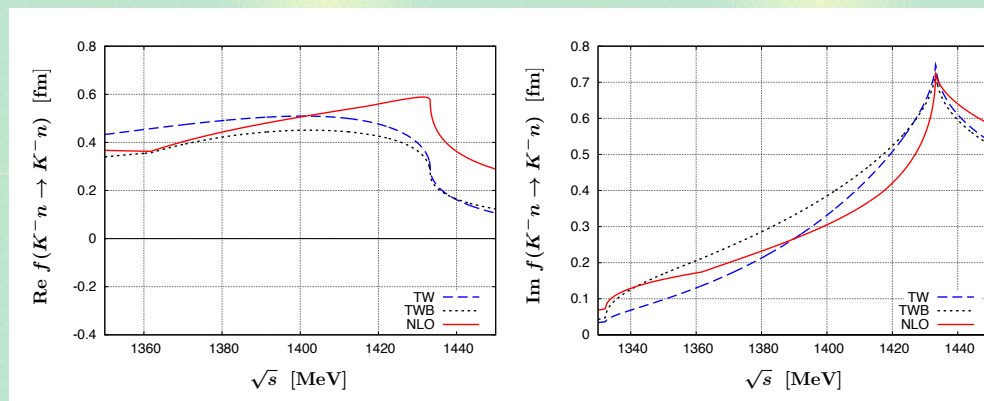
M. Bazzi, *et al.*, Phys. Lett. B704, 113 (2011); Nucl. Phys. A881, 88 (2012),
 U.G. Meissner, U. Raha, A. Rusetsuky, Eur. Phys. J. C35, 349 (2004).

Analysis in NLO chiral SU(3) dynamics

Y. Ikeda, T. Hyodo, W. Weise, Phys. Lett. B (2012); Nucl. Phys. A881, 98 (2012)

- data fitted as $\chi^2/\text{d.o.f.} \sim 1$

$$a_{0,K^-n} = -0.57^{+0.21}_{-0.04} - i0.72^{+0.41}_{-0.26} \text{ fm}$$



K^-p scattering length is **attractive** (no quasi-bound state).

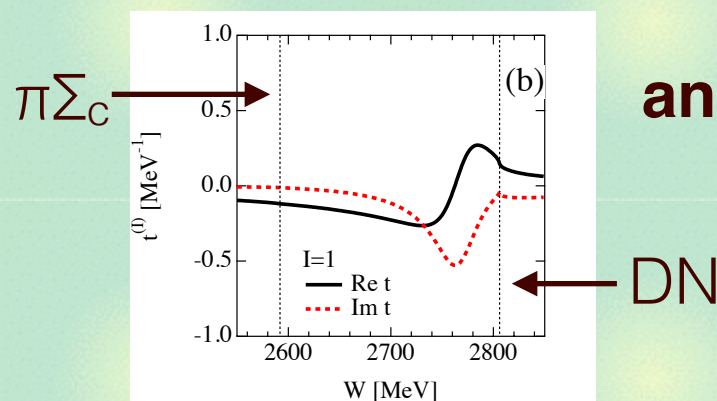
D-neutron interaction

No experimental information of scattering length

- **SU(4) contact interaction model ($\Lambda_c(2595)$ fixed)**

T. Mizutani, A. Ramos, *Phys. Rev. C* **74**, 065201 (2006),

M. Bayar, C.W. Xiao, T. Hyodo, A. Dote, M. Oka, E. Oset, *Phys. Rev. C* **86**, 044004 (2012)



- $\Sigma_c(2800)$ as a quasi-bound state (DN threshold ~ 2804 MeV)

C. Patrignani *et al.*, (Particle Data Group) *Chin. Phys.* **C40**, 100001 (2016)

$$M_{\Sigma_c^0(2800)} = 2806_{-7}^{+5} \text{ MeV}, \quad \Gamma_{\Sigma_c^0(2800)} = 72_{-15}^{+22} \text{ MeV}$$

D^0n scattering length is **repulsive** (with quasi-bound state).

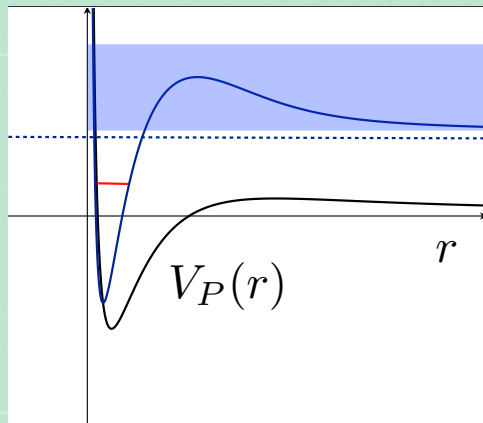
Idealization

When decay channels ($\pi\Sigma$ - $\pi\Lambda$, $\pi\Sigma_c$ - $\pi\Lambda_c$) are neglected:

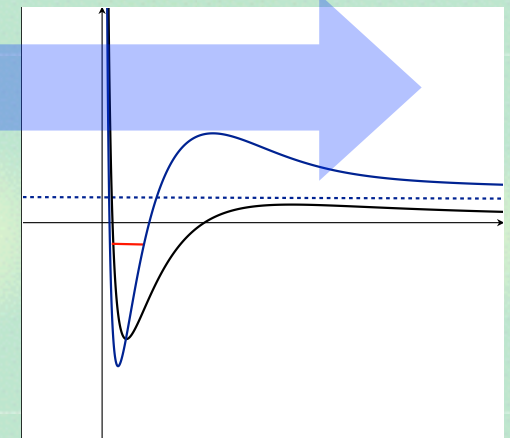
- $a_{K-n} < 0$ (attractive), $a_{D0n} > 0$ (repulsive, with bound state)

Extrapolation in quark mass

strange, unbound



charm, bound



$$|a| \longrightarrow \infty$$

unitary limit

Unitary limit: neglecting decay channels and tuning $m_{s/c}$

Contact interaction model

Model for $\bar{K}N/DN$ scattering

T. Mizutani, A. Ramos, *Phys. Rev. C* **74**, 065201 (2006),

Y. Ikeda, T. Hyodo, W. Weise, *Phys. Lett. B* (2012); *Nucl. Phys. A* **881**, 98 (2012)

$$T(W) = [V^{-1}(W) - G(W)]^{-1}$$

- **four channels:** $K-n-\pi-\Lambda-\pi^0\Sigma^--\pi-\Sigma^0$, $D^0n-\pi-\Lambda_c-\pi^0\Sigma_c^0-\pi-\Sigma_c^+$
- $V(W)$: **flavor symmetric contact interaction**

$$V_{ij}(W) = -\frac{C_{ij}}{f_i f_j} (2W - M_i - M_j) \sqrt{\frac{M_i + E_i}{2M_i}} \sqrt{\frac{M_j + E_j}{2M_j}}$$

- **cutoff in $G(W) \leftarrow$ Experiments ($\bar{K}N$), $\Sigma_c(2800)$ (DN)**

Extrapolation parameter x

$$m(x) = m_K(1 - x) + m_D x, \quad \text{etc.}$$

- $x=0$ ($x=1$) **corresponds to the $\bar{K}N$ (DN) scattering.**

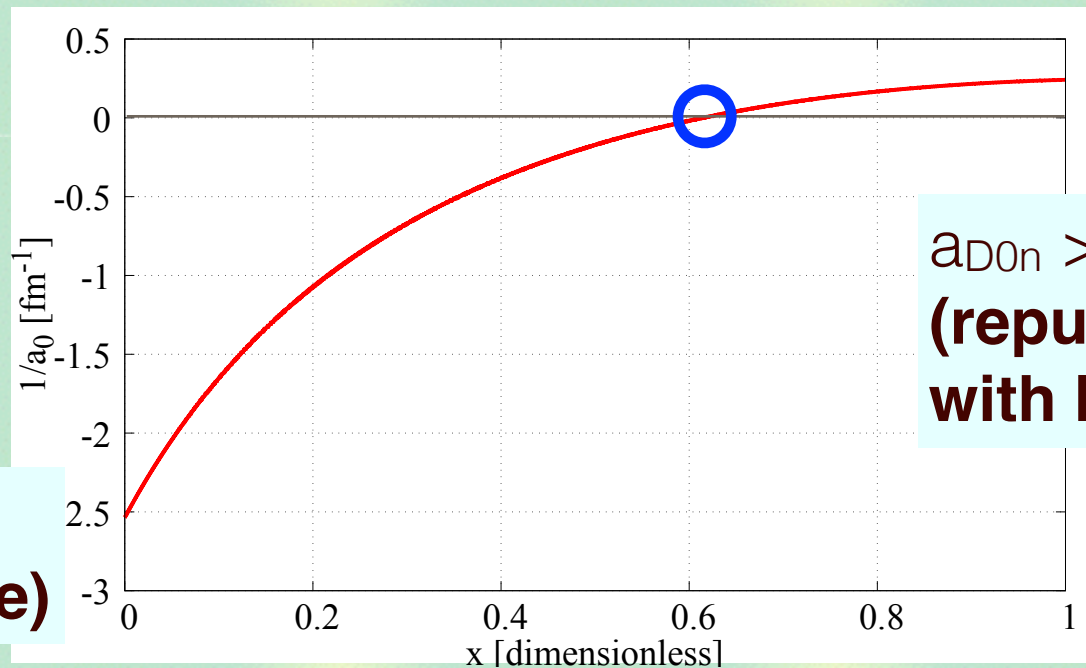
Model extrapolation

Extrapolation

- eliminate couplings to decay channels

$$C_{1i} = C_{i1} = 0 \quad \text{for } i = 2, 3, 4$$

- vary extrapolation parameter x to calculate $1/a_{K-n}$



$a_{D^0n} > 0$
**(repulsive,
 with bound state)**

$a_{K-n} < 0$
(attractive)

Unitary limit at $m_K = 1337 \text{ MeV}$ ($x=0.62$)

Effective field theory

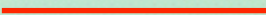
Effective field theory with dihadron fields

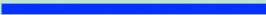
P.F. Bedaque, H.-W. Hammer, U. van Kolck, *Phys. Rev. Lett.* **82**, 463-437 (1999)

S.-I. Ando, U. Raha, Y. Oh, *Phys. Rev. C* **92**, 024325 (2015)

$$\mathcal{L} = \mathcal{L}_n + \mathcal{L}_K + \mathcal{L}_{s(nn)} + \mathcal{L}_{d(nK)} + \mathcal{L}_{3\text{-body}}$$

- hadron fields

n 

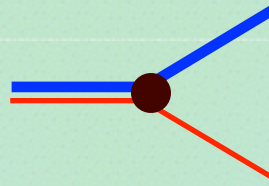
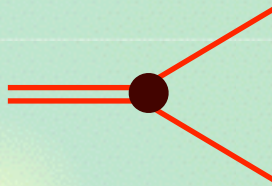
K^- 

- dihadron fields

$s_{(nn)}$ 

$d_{(nK)}$ 

- interactions



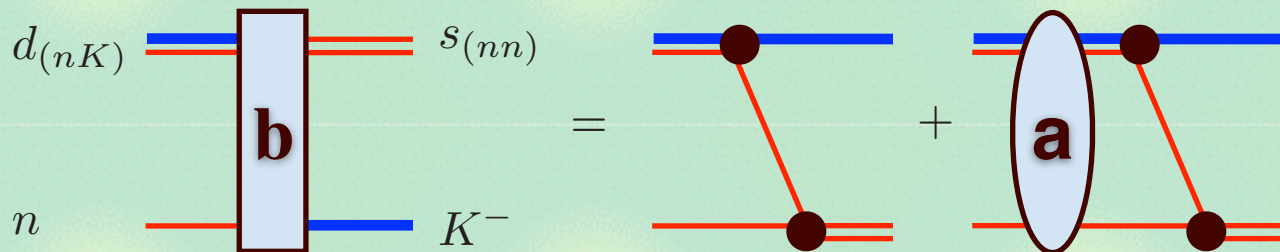
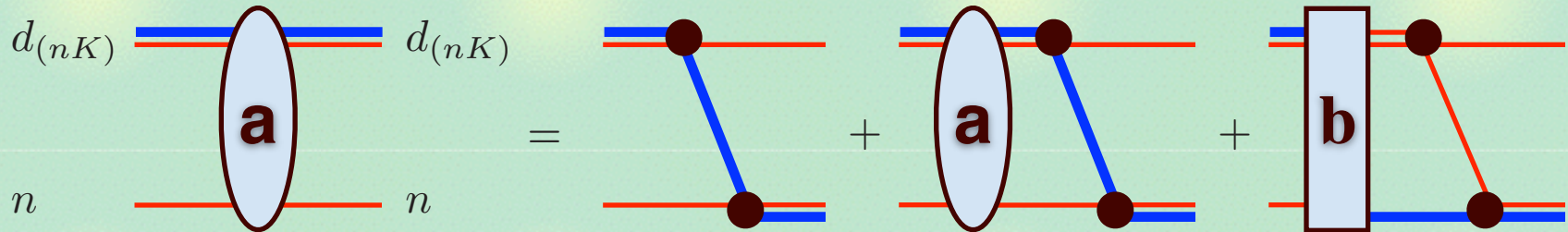
Renormalization in two-body sector (dressing $s_{(nn)}$, $d_{(nK)}$)

- system is determined by $m_K, M_n, a_{s(nn)}, a_{d(nK)}$.

Three-body equation

Three-body equation for K - nn system

- coupled integral equations (quasi two-body)
- dressed dihadron propagators



If Efimov effect occurs, we need three body contact term.

Asymptotic expressions

$|a| \rightarrow \infty$ **limit: check whether Efimov effect occurs**

- **transcendental equation for three identical bosons**

P.F. Bedaque, H.-W. Hammer, U. van Kolck, Phys. Rev. Lett. 82, 463-437 (1999)

$$1 = \frac{8}{\sqrt{3}s} \frac{\sin(\pi s/6)}{\cos(\pi s/2)} \Rightarrow s = \pm i s_0, \quad s_0 = 1.00624$$

- **imaginary s : RG limit cycle with factor $\exp[\pi/s_0] \sim 22.7$**

For K-nn/D⁰nn system at $x=0.62$:

- $a_{s(nn)} \rightarrow \infty$ **and** $a_{d(nK)} \rightarrow \infty$

$$1 = C_1 \frac{2\pi}{s} \frac{\sin[s \arcsin(a/2)]}{\cos(\pi s/2)} + C_2 \frac{4\pi^2}{s^2} \frac{\sin^2[s \operatorname{arccot}(\sqrt{4b-1})]}{\cos^2(\pi s/2)} \Rightarrow s_0 = 1.01156$$

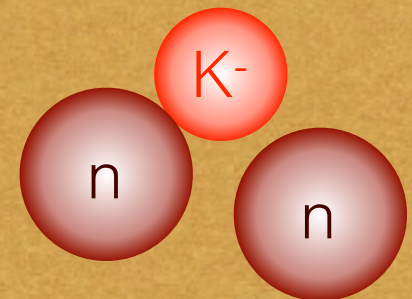
- $a_{s(nn)}$ **fixed and** $a_{d(nK)} \rightarrow \infty$

$$1 = C_1 \frac{2\pi}{s} \frac{\sin[s \arcsin(a/2)]}{\cos(\pi s/2)} \Rightarrow s_0 = 0.327675$$

RG limit cycle in the asymptotic expressions: Efimov effect

Summary

- 📌 **Universal physics of the K^-nn/D^0nn system.**
- 📌 **Two-body sector: meson-neutron scattering length can be **infinitely large** with suitable idealization (tune m_q , neglect decay channels).**
- 📌 **Three-body sector: **Efimov effect** occurs when two-body interaction is unitary.**
- 📌 **Implication at physical point: three-body resonance?**



[U. Raha, Y. Kamiya, S.-I. Ando, T. Hyodo, in preparation](#)