Quark mass dependence of H-dibaryon in $\Lambda\Lambda$ scattering

Yasuhiro Yamaguchi[a], **Tetsuo Hyodo**[b]

[a]*Istituto Nazionale di Fisica Nucleare (INFN), Sezione di Genova*
[b]*Yukawa Institute for Theoretical Physics, Kyoto Univ.*

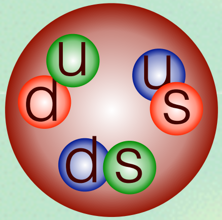
2017, Jan 27th

Contents

H-dibaryon in ΛΛ scattering

H-dibaryon: uuddss **bound state predicted in a quark model**

R.L. Jaffe, Phys. Rev. Lett. 38, 195 (1977)



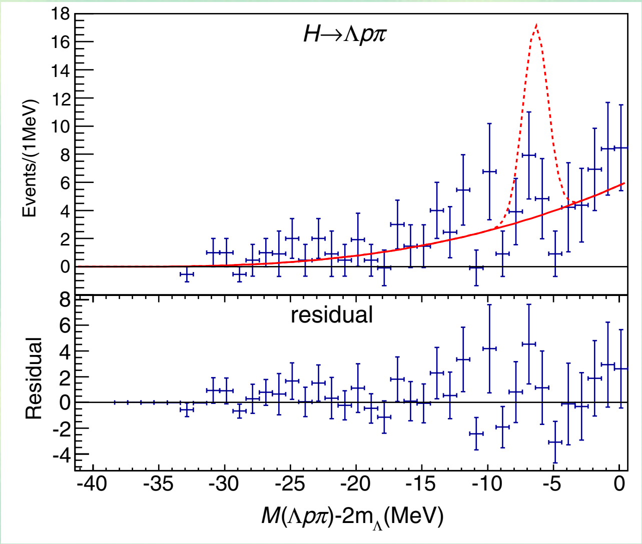
Experiments: Negative

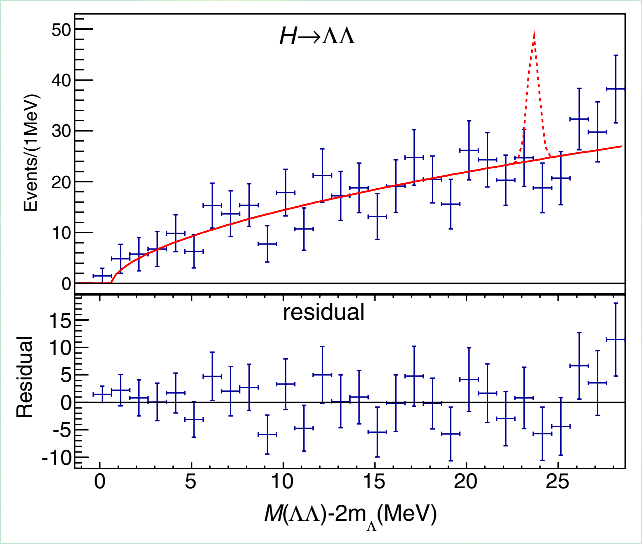
- Nagara event: double Λ hyper nuclei -> no deeply bound H

H. Takahashi, *et al.*, Phys. Rev. Lett. 87, 212502 (2001)

- Belle: Υ(1S), Υ(2S) **decay -> no signal (<< deuteron)**

B.H. Kim, *et al.*, [Belle collaboration] Phys. Rev. Lett. 114, 022301 (2015)

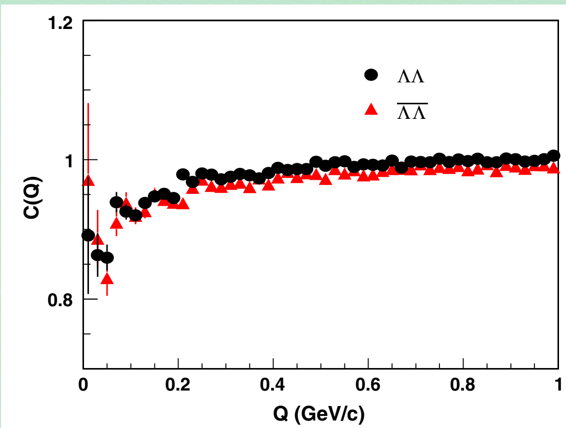


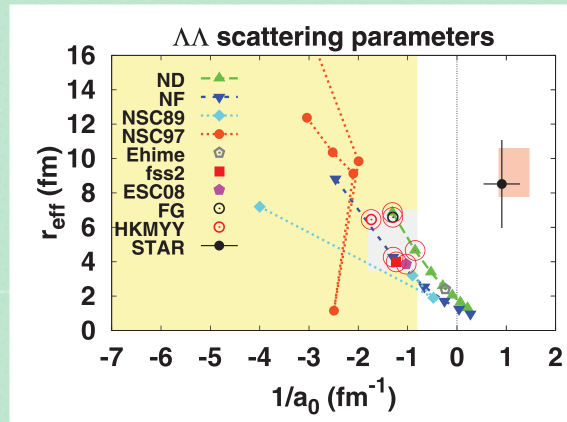


Recent activities

RHIC-STAR: ΛΛ correlation —> scattering length

L. Adamczyk, *et al.*, [STAR collaboration] Phys. Rev. Lett. 114, 022301 (2015);
K. Morita, T. Furumoto, A. Ohnishi, Phys. Rev. C 91, 024916 (2015)

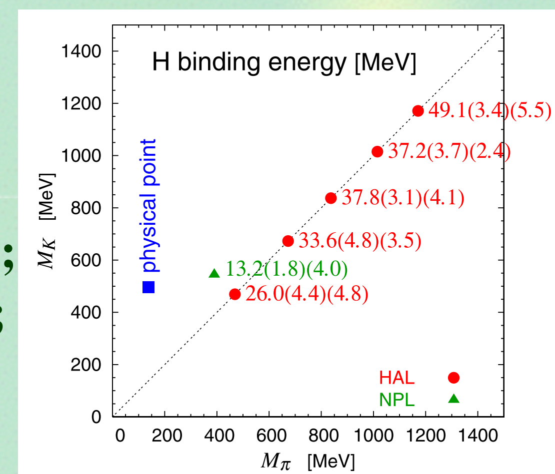




H-dibaryon in lattice QCD

- **Bound** at unphysical quark masses

HAL QCD, T. Inoue *et al.*, Phys. Rev. Lett. 106, 162002 (2011);
NPLQCD, S. Beane *et al.*, Phys. Rev. Lett. 106, 162001 (2011);
HAL QCD, T. Inoue *et al.*, Nucl. Phys. A881, 28 (2012); …



- Physical point simulation is ongoing.

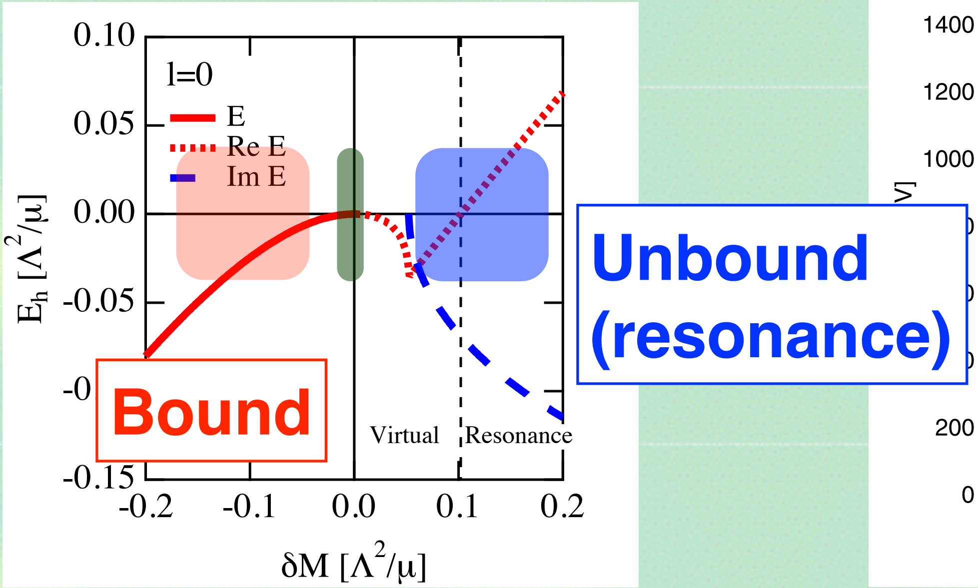
Near-threshold scaling

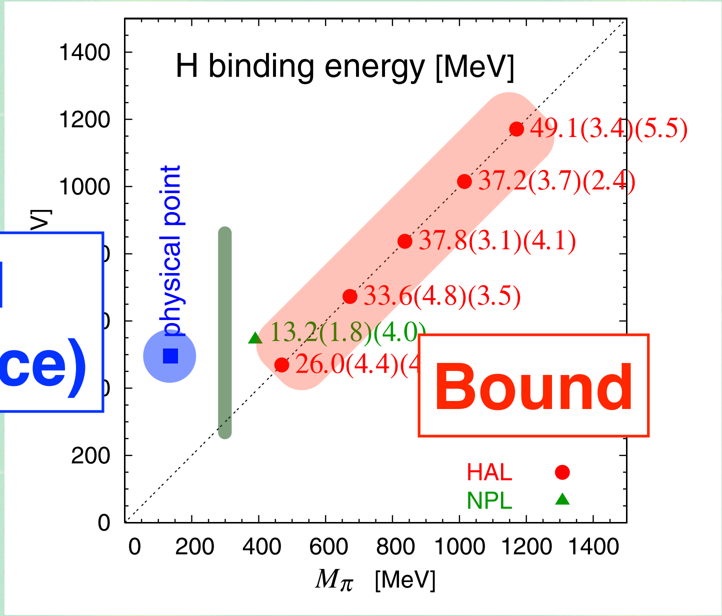
Extraplation: unbound at physical point

S. Shanahan, A. Thomas, R. Young, Phys. Rev. Lett. 107, 092004 (2011);
J. Haidenbauer, U.G. Meissner, Phys. Lett. B 706, 100 (2011)

Near-threshold scaling in s-wave (bound -> unbound)

T. Hyodo, Phys. Rev. C90, 055208 (2014)





- unitary limit (infinitely large scattering length)

Unitary limit at unphysical quark masses?

Purpose of this talk

- How does the H-dibaryon bound state in the $\Lambda\Lambda$ scattering change along with the **variation of the quark masses**?
- Input: three lightest lattice data in **SU(3) limit**.
- Effective framework which describes the $\Lambda\Lambda$ scattering in a relatively wide range of quark masses.

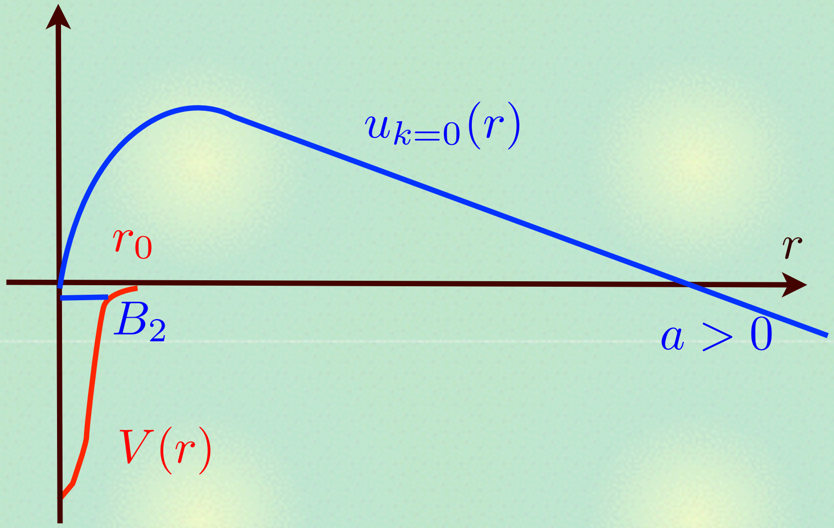
(Quantitative prediction at physical point may be given by lattice QCD / systematic ChPT.)

Low-energy baryon-baryon scattering

Length scales in the SU(3) limit

HAL QCD, T. Inoue *et al.*, Nucl. Phys. A881, 28 (2012)

- **Interaction range by NG boson exchange:** r^0 ~ 0.24-0.42 **fm**
- **large scattering length:** a ~ 1.2-1.7 **fm**
- **large radius <— small binding energy:** 0.77-1.14 **fm**



At low energy, the interaction can be treated as point like.

Effective Lagrangian

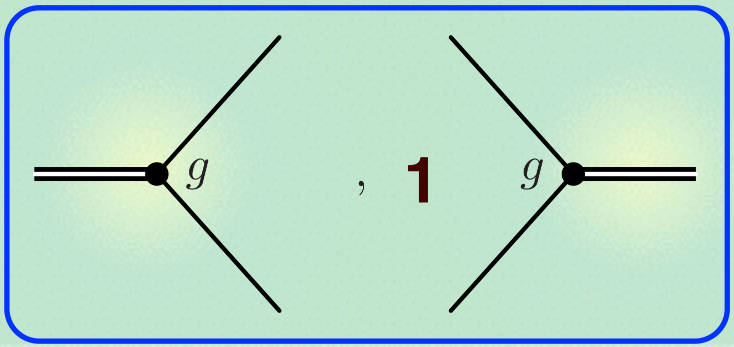
Low-energy effective Lagrangian with contact interactions

c.f. D.B. Kaplan, Nucl. Phys. B494, 471 (1997)

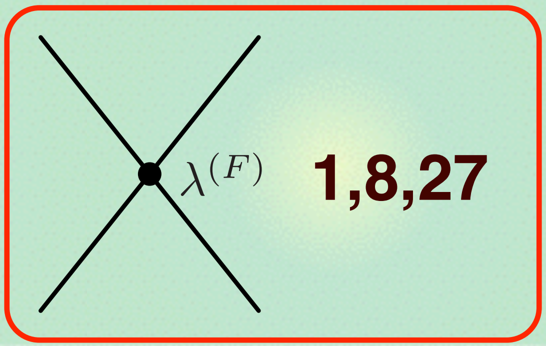
$$\mathcal{L}_{\text{free}} = \sum_{a=1}^{4} \sum_{\sigma=\uparrow,\downarrow} B^{\dagger}_{a,\sigma} \left(i\frac{\partial}{\partial t} + \frac{\nabla^2}{2M_a} + \delta_a \right) B_{a,\sigma} + H^{\dagger} \left(i\frac{\partial}{\partial t} + \frac{\nabla^2}{2M_H} + \nu \right) H$$

$$\mathcal{L}_{\text{int}} = \underline{-g[D^{(1)\dagger} H + H^{\dagger} D^{(1)}]} - \underline{\lambda^{(1)} D^{(1)\dagger} D^{(1)} - \lambda^{(8)} D^{(8)\dagger} D^{(8)} - \lambda^{(27)} D^{(27)\dagger} D^{(27)}}$$

$$D^{(F)} = [BB]^{(F)}_{J=0,S=-2,I=0}$$



,



Length scales at the physical point

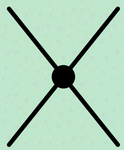
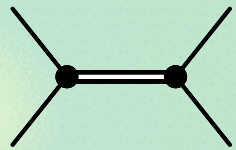
- No π exchange in ΛΛ. π exchange in NΞ (ΛΛ + 25 MeV)

—> safely applicable below NΞ threshold

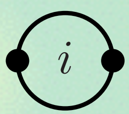
Low-energy scattering amplitude

Coupled-channel scattering amplitude (i=ΛΛ, NΞ, ΣΣ)

$$f_{ii}(E) = \frac{\mu_i}{2\pi}[(\mathcal{A}^{\text{tree}}(E))^{-1} + I(E)]^{-1}_{ii}$$

$\mathcal{A}^{\text{tree}}_{ij}(E) =$ i  j $+$ i  j

$$= -\left(V_{ij} + \frac{g^2 d_i^\dagger d_j}{E - \nu + i0^+}\right), \quad V = U^{-1}\begin{pmatrix} \lambda^{(1)} & & \\ & \lambda^{(8)} & \\ & & \lambda^{(27)} \end{pmatrix} U, \quad d = \begin{pmatrix} -\sqrt{\frac{1}{8}} \\ -\sqrt{\frac{1}{2}} \\ \sqrt{\frac{3}{8}} \end{pmatrix}$$

$I_i(E) =$ 

$$= \frac{\mu_i}{\pi^2}\left(-\Lambda + k_i \text{artanh}\frac{\Lambda}{k_i}\right), \quad k_i = \sqrt{2\mu_i(E - \Delta_i)}$$

EFT describes the low-energy scattering for a given (m_l, m_s).

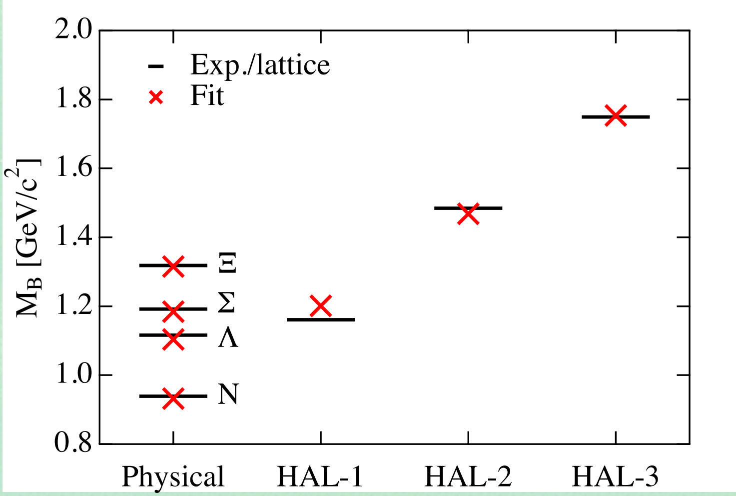
- **scattering length, bound state pole, …**
- **quark mass dep. —> baryon masses and couplings λ**

Modeling quark mass dependence

“Quark masses” via GMOR relation

$$B_0 m_l = \frac{m_\pi^2}{2}, \quad B_0 m_s = m_K^2 - \frac{m_\pi^2}{2}$$

$$B_0 = -\frac{\langle \bar{q}q \rangle}{3F_0^2}$$



Baryon masses: linear in m_q

$$M_N(m_l, m_s) = M_0 - (2\alpha + 2\beta + 4\sigma) B_0 m_l - 2\sigma B_0 m_s,$$

$$M_\Lambda(m_l, m_s) = M_0 - (\alpha + 2\beta + 4\sigma) B_0 m_l - (\alpha + 2\sigma) B_0 m_s,$$

$$M_\Sigma(m_l, m_s) = M_0 - \left(\frac{5}{3}\alpha + \frac{2}{3}\beta + 4\sigma\right) B_0 m_l - \left(\frac{1}{3}\alpha + \frac{4}{3}\beta + 2\sigma\right) B_0 m_s,$$

$$M_\Xi(m_l, m_s) = M_0 - \left(\frac{1}{3}\alpha + \frac{4}{3}\beta + 4\sigma\right) B_0 m_l - \left(\frac{5}{3}\alpha + \frac{2}{3}\beta + 2\sigma\right) B_0 m_s$$

- three mass difference by (α, β) —> GMO relation

- fit to experiment/lattice —> reasonable

HAL QCD, T. Inoue *et al.*, Nucl. Phys. A881, 28 (2012)

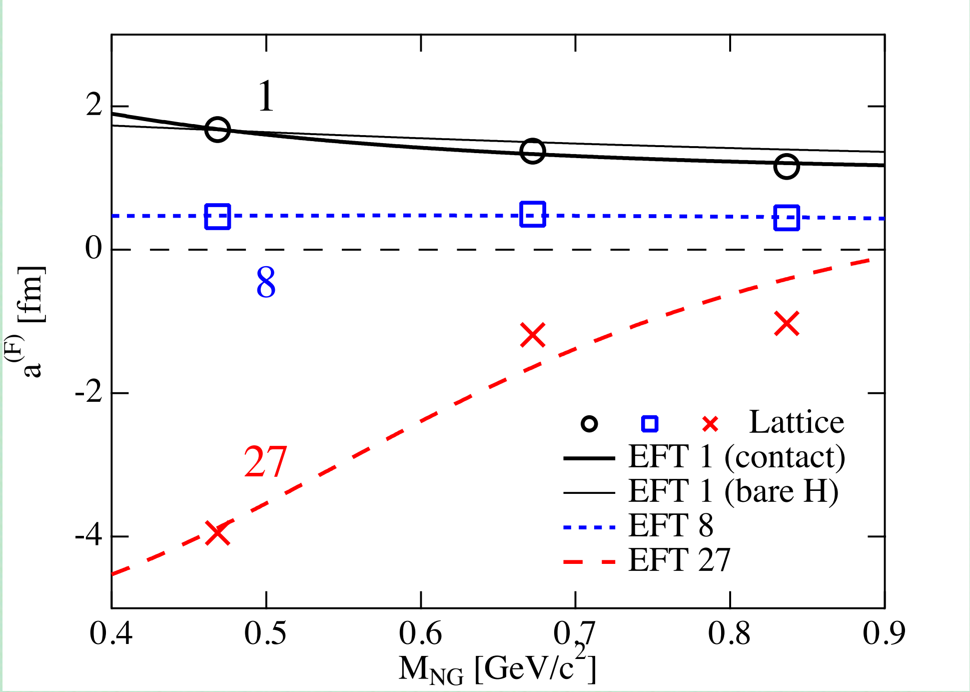
Modeling quark mass dependence

Coupling constants <— scattering length in SU(3) limit

T. Inoue, private communication.

- a = -f(E=0) **1: bound, 8: repulsive, 27: attractive**
- **linear in** m_q: $\lambda^{(F)}(m_l, m_s) = \lambda_0^{(F)} + \lambda_1^{(F)} B_0 (2m_l + m_s)$
- **singlet channel:** g=0 **(contact)**, g≠0 **(bare H)**

this talk



repulsive

attractive

Pole of the scattering amplitude

Scattering amplitude and S-matrix (for each partial wave)

J.R. Taylor, *Scattering theory* (Wiley, New York, 1972)

$$f(p) = \frac{\mathscr{f}(-p) - \mathscr{f}(p)}{2ip\mathscr{f}(p)}, \quad s(p) = \frac{\mathscr{f}(-p)}{\mathscr{f}(p)}$$

- Jost function <— asymptotic form of wave function

$$\psi_p(r) \sim \underbrace{\mathscr{f}(p)e^{-ipr}}_{\text{incoming}} \underbrace{- \mathscr{f}(-p)e^{ipr}}_{\text{outgoing}}$$

incoming **outgoing**

Pole of the amplitude $f(p) \to \infty$

- zero of Jost function $\mathscr{f}(p) = 0$

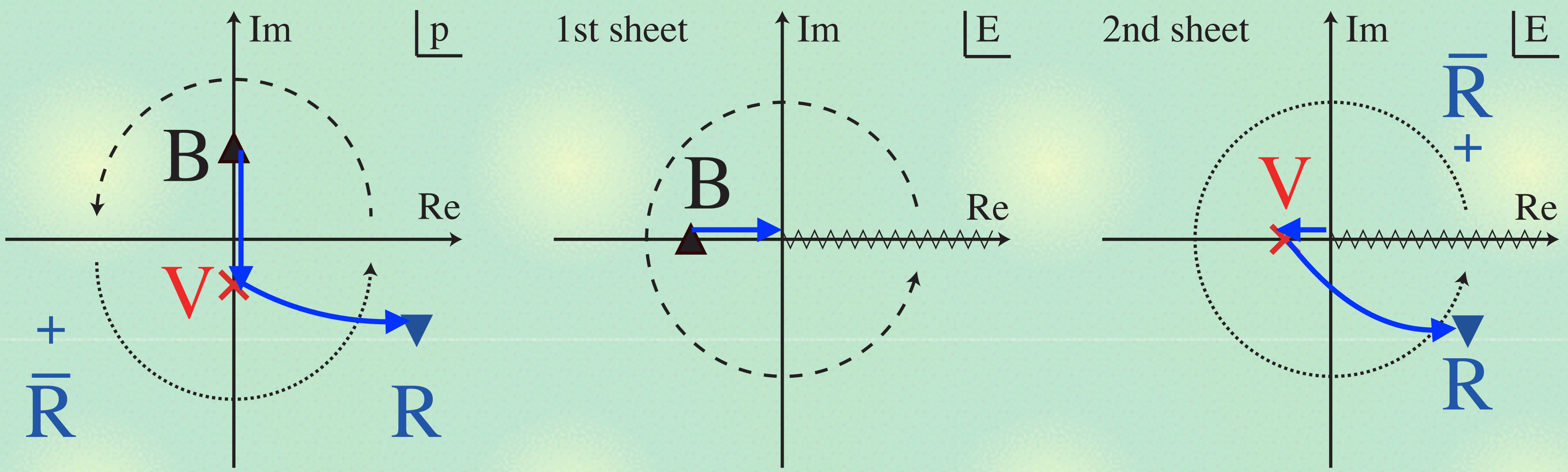
- outgoing wave only $\psi_p(r)|_{\mathscr{f}(p)=0} \sim e^{ipr}$

- eigenstate of Hamiltonian <— bound state $\psi_{i\kappa}(r)|_{\mathscr{f}(i\kappa)=0} \sim e^{-\kappa r}$

Pole of the scattering amplitude

Analytic continuation of f(p) —> pole in complex p plane

T. Hyodo, Genshikaku Kenkyu Vol. 61 No. 1, 15 (2016)



- **Resonance: 4th quadrant of complex p plane**
- **Virtual state: negative imaginary p axis (s-wave only)**
- **natural generalization of bound state**

Pole trajectory from bound state to resonance (s-wave)

- **Pole goes through virtual state.**

T. Hyodo, Phys. Rev. C90, 055208 (2014)

CDD pole of the scattering amplitude

CDD pole: pole of the inverse amplitude

L. Castillejo, R.H. Dalitz, F.J. Dyson, Phys. Rev. 101, 453 (1956)

- zero of the scattering amplitude

$$f(p_c) = 0, \quad 1/f(p_c) \to \infty$$

- related to independent particle (but $p_c \neq$ bare pole)

G.F. Chew, S.C. Frautschi, Phys. Rev. 124, 264 (1961)

- effective range expansion converges only in $|p|<|p_c|$

$$1/f(p) = p\cot\delta(p) - ip$$

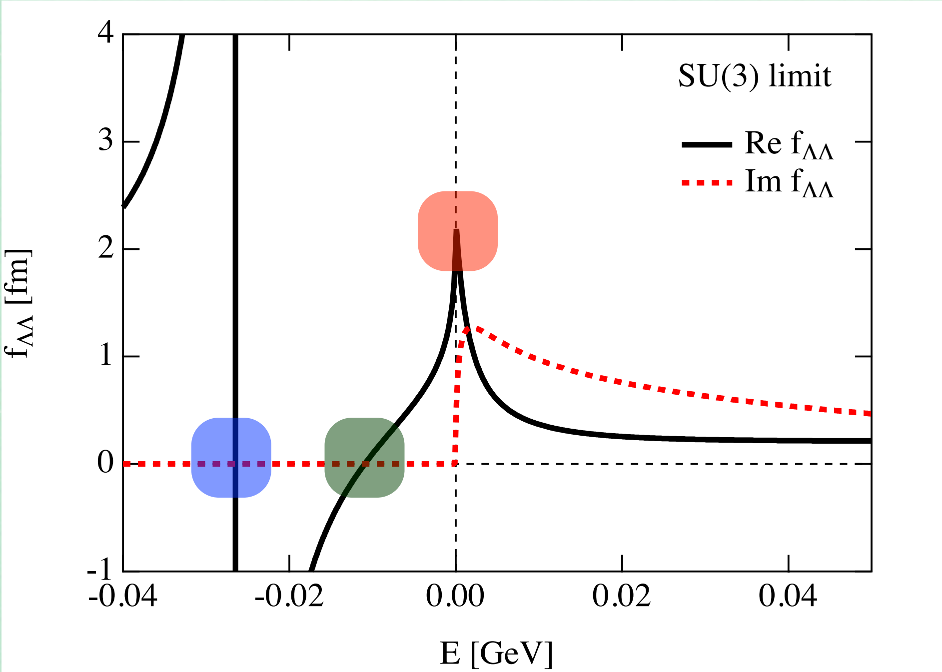
Ramsauer-Townsend effect: CDD pole above threshold

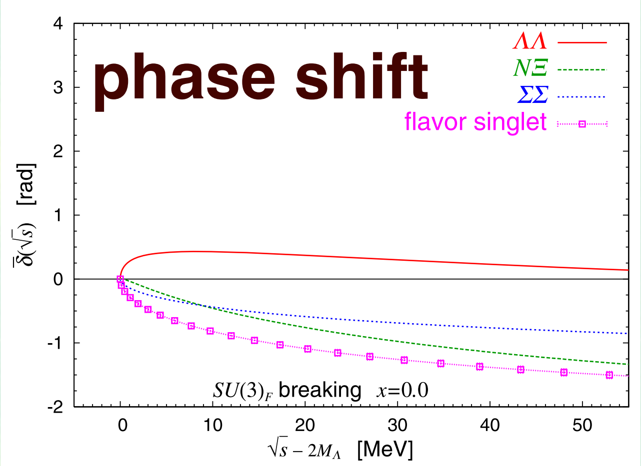
J.R. Taylor, *Scattering theory* (Wiley, New York, 1972)

- $f(p_c)=0$ —> phase shift $\delta(p_c)=\pi$: no scattering
- $s(p_c)=1$: incoming = outgoing, perfect transmission

ΛΛ scattering : SU(3) limit

ΛΛ scattering amplitude in the SU(3) limit





HAL QCD, T. Inoue *et al.*,
Nucl. Phys. A881, 28 (2012)

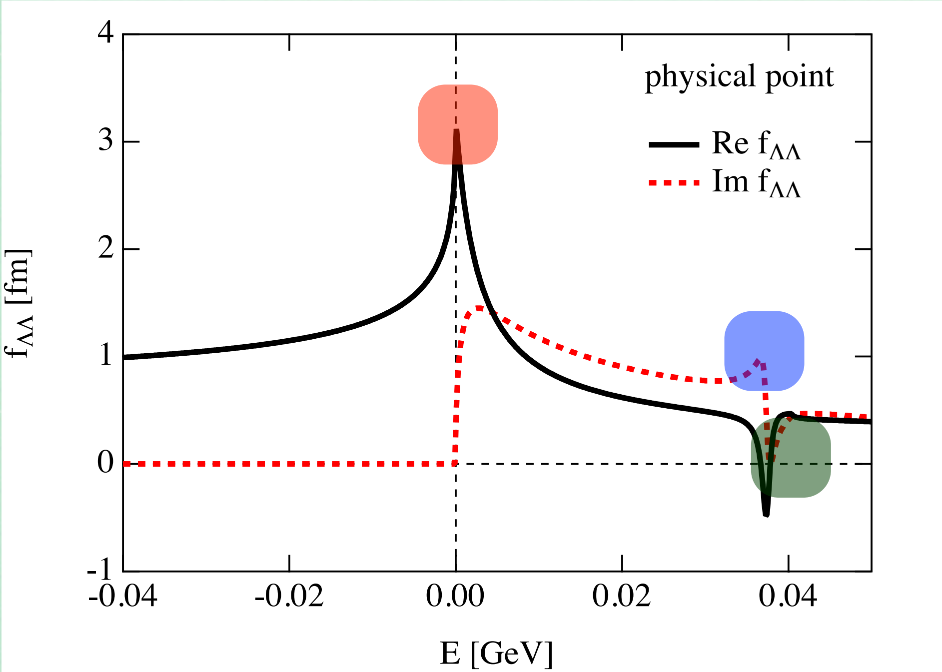
- **bound H <— bound state in 1**
- **attractive scattering length** a = -f(E=0) **<— attraction in 27**

$$f_{\Lambda\Lambda}(E) = \frac{1}{8} f^{(1)}(E) + \frac{1}{5} f^{(8)}(E) + \frac{27}{40} f^{(27)}(E)$$

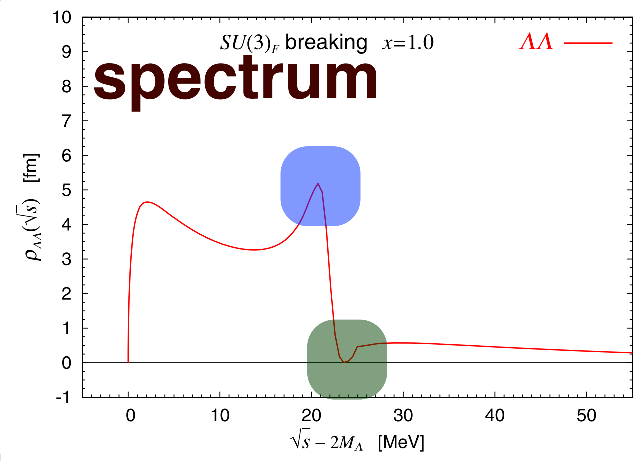
- **CDD pole below threshold:** f(E)=0 **—> ERE breaks down.**

ΛΛ scattering : Physical point

ΛΛ scattering amplitude at the physical point



SU(3) pot. + phys. mass

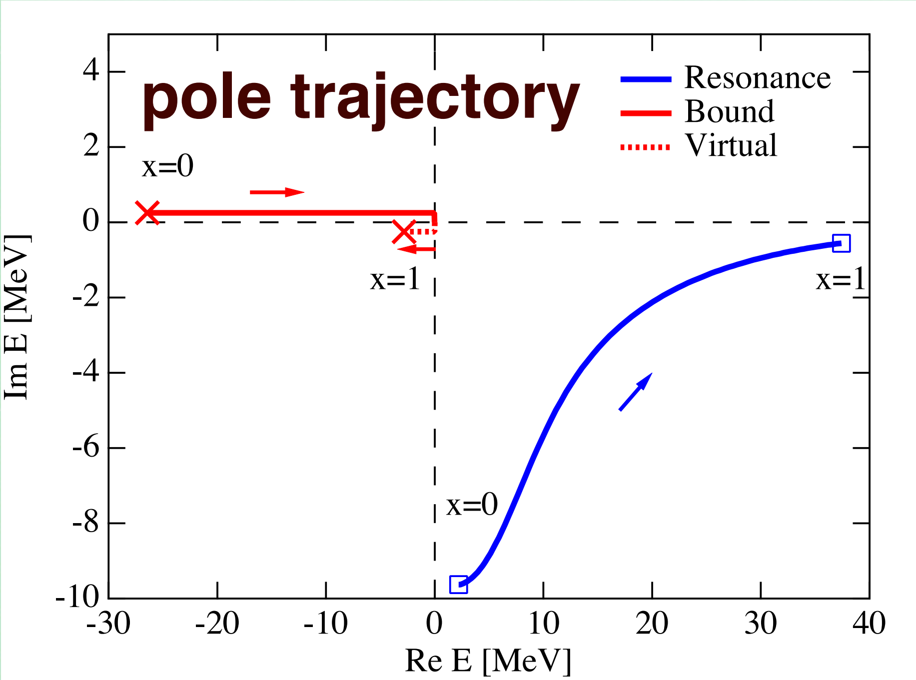


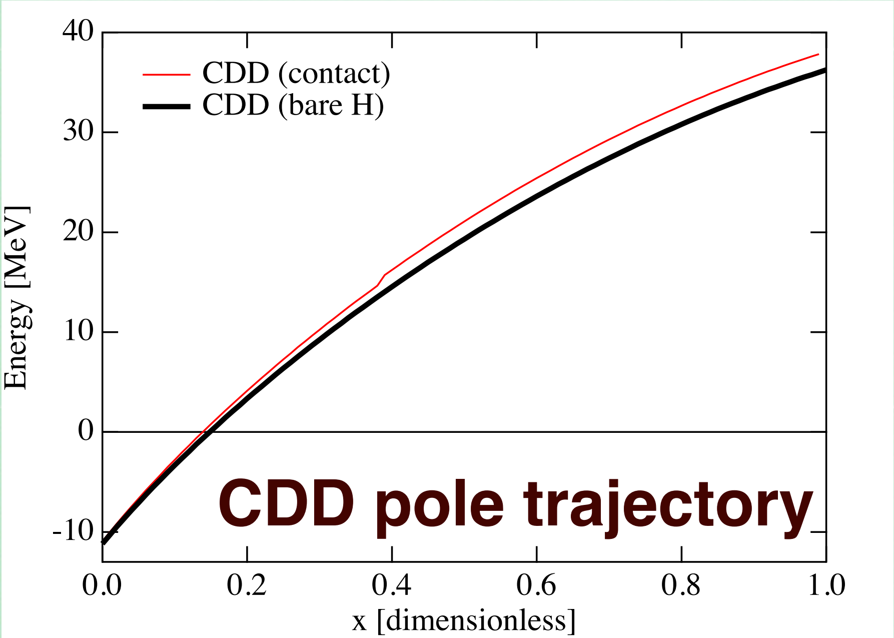
HAL QCD, T. Inoue *et al.*, Nucl. Phys. A881, 28 (2012)

- **no bound H, but a resonance below NΞ threshold**
- **attractive scattering length:** $a_{\Lambda\Lambda}$ = -3.2 fm
- **Ramsauer-Townsend effect near resonance:** f(E)=0
  <— remnant of the CDD pole

Pole trajectories from SU(3) limit to physical point

Pole trajectory with x (x=0: SU(3) limit, x=1: phys. point)





- pole is not continuously connected
  - bound state —> virtual state
  - shadow pole —> resonance

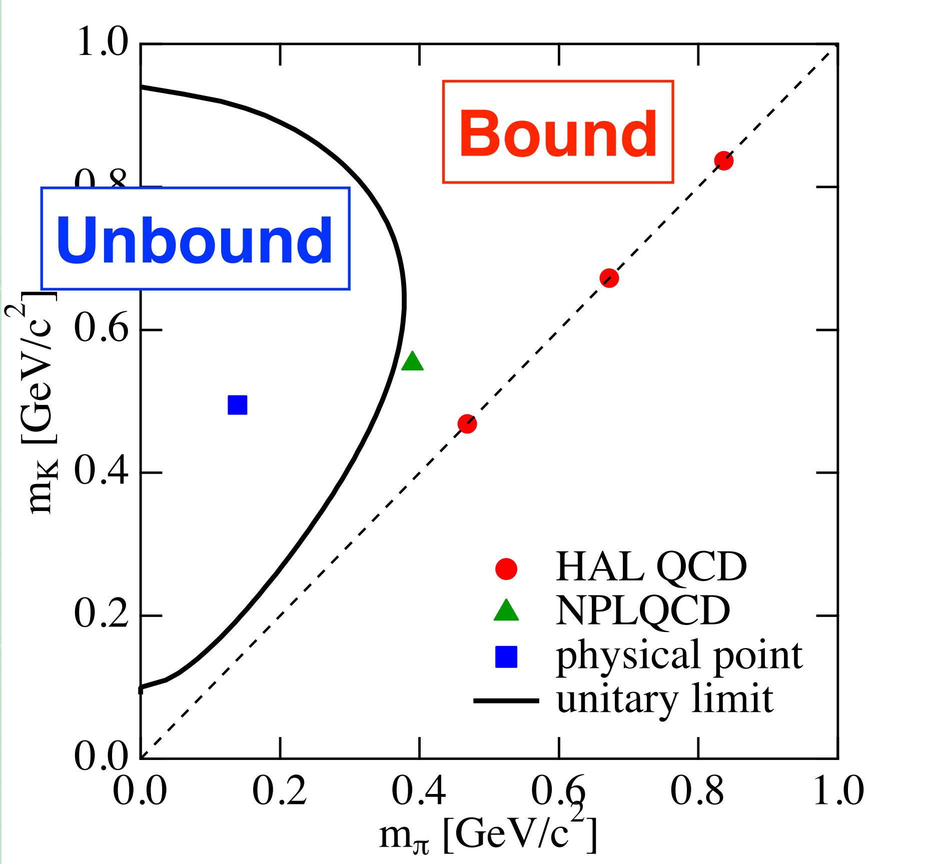
  R. Eden, J. Taylor, Phys. Rev. 133, B1575 (1964)

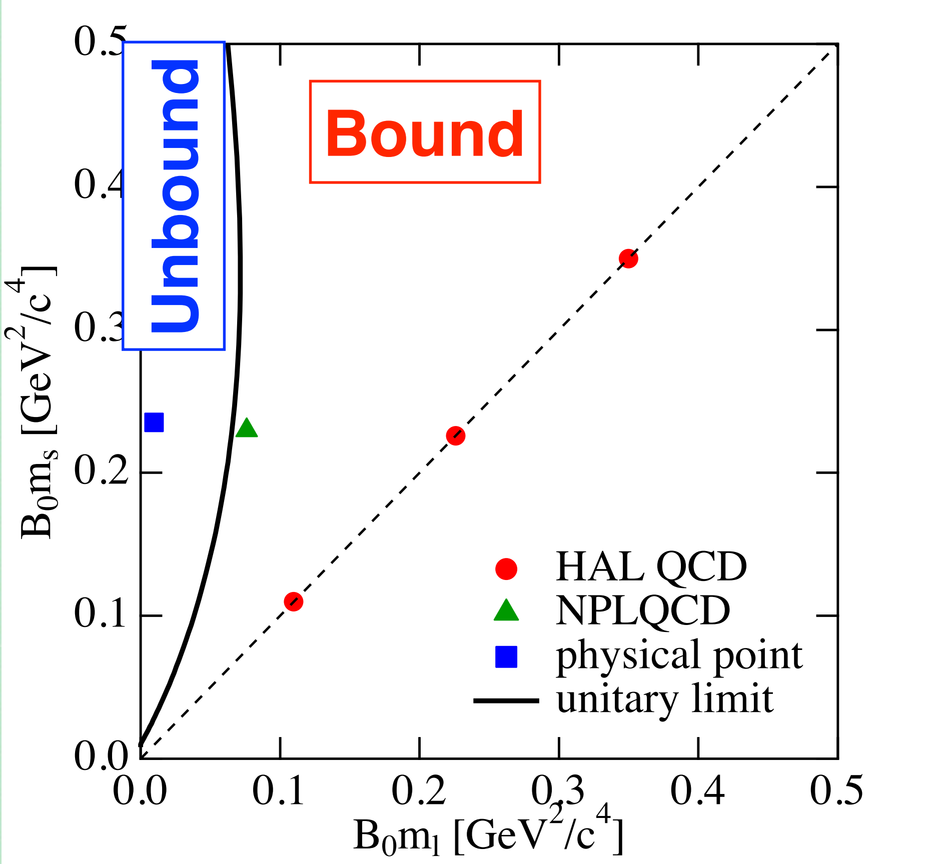
- CDD pole is continuously connected

Extrapolation and unitary limit

Extrapolation in the NGboson/quark mass plane

$$B_0 m_l = \frac{m_\pi^2}{2}, \quad B_0 m_s = m_K^2 - \frac{m_\pi^2}{2}$$



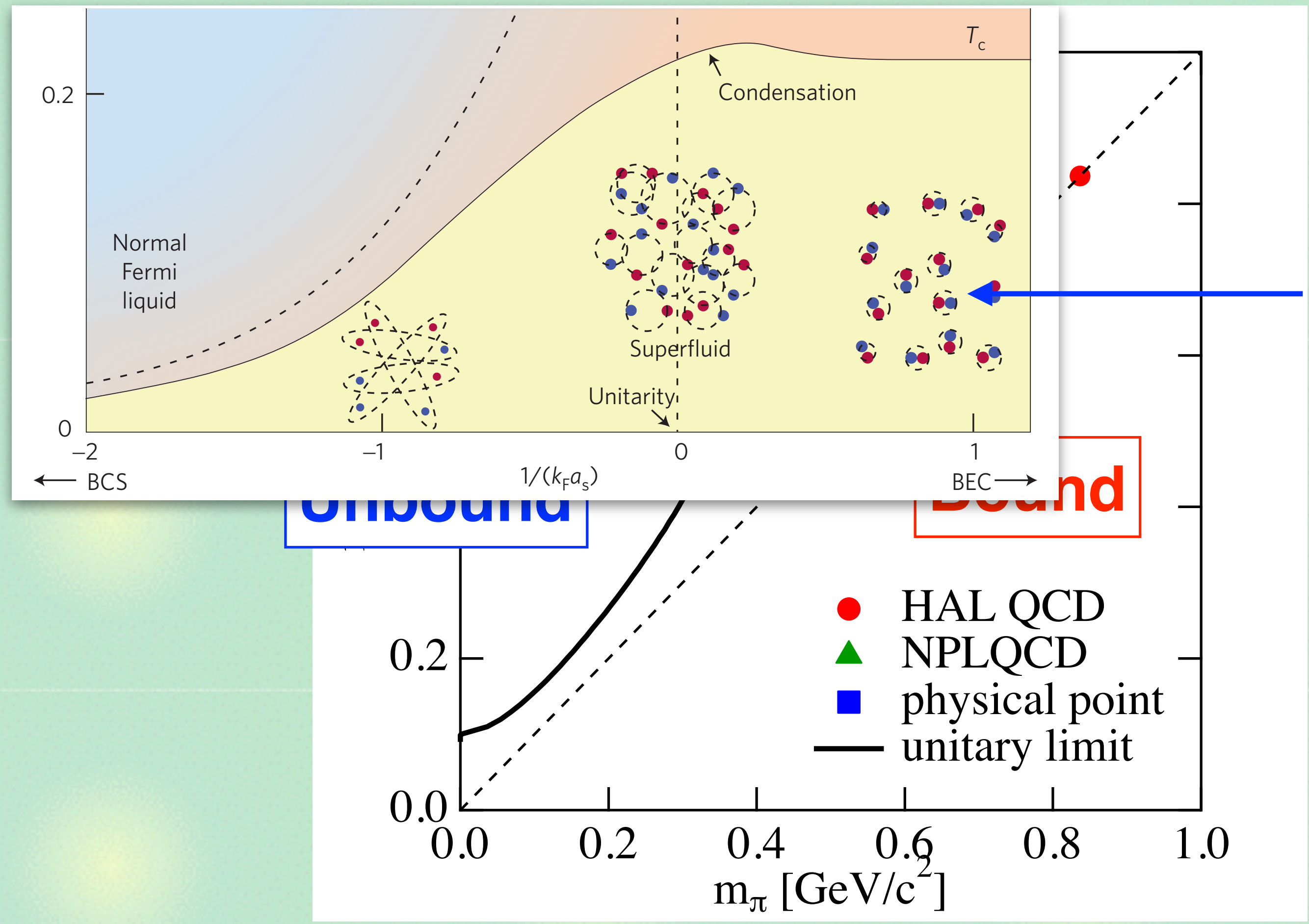


- unitary limit between SU(3) limit and physical point

Implication to many-body system

Many-body system of Λ baryons: BEC-BCS crossover

W. Zwerger, Lect. Notes Phys. 836, 1 (2012); M. Randeria, Nature Phys. 6, 561 (2010)



Superfluid of H-dibaryons = “H-matter”

R. Tamagaki, Prog. Theor. Phys. 85, 321 (1991)

- “H-matter” may be realized with unphysical quark masses.

Summary

- We study the quark mass dependence of the H-dibaryon and the ΛΛ interaction using EFT.

- SU(3) limit: bound H with attractive scattering length <— CDD pole below the threshold.

- Physical point: Ramsauer-Townsend effect near resonance H <— remnant of the CDD pole.

- The ΛΛ scattering undergoes the unitary limit between SU(3) limit and physical point.

Y. Yamaguchi, T. Hyodo, Phys. Rev. C94, 065207 (2016)