

Scattering of gapped Nambu-Goldstone modes



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Pions in QCD

Nambu-Goldstone (NG) modes are

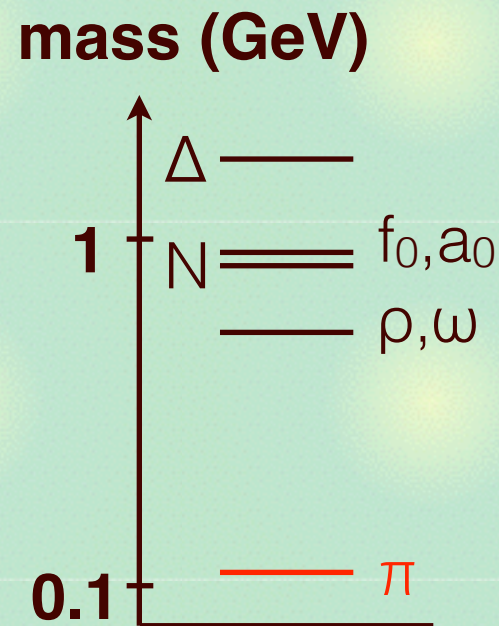
- associated with spontaneous symmetry breaking (SSB)
- massless without explicit breaking

Pions (π^+ , π^- , π^0) in QCD

- chiral symmetry $SU(2)_R \otimes SU(2)_L$
:microscopic theory
- much **lighter** than other hadrons
:experiment

(almost) massless mode \rightarrow NG mode?

- not always (gauge symmetry, edge state, ...)
- Are there any **other characteristics** of the “NG mode”?



Pion scattering

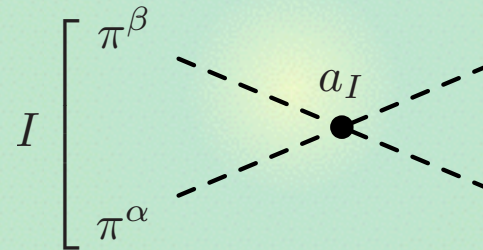
Scattering length of $\pi\pi$ system

S. Weinberg, *Phys. Rev. Lett.* **17**, 616 (1966)

- low-energy theorem : leading order ChPT

$$a_{I=0} = -\frac{7}{4} \frac{m_\pi}{8\pi f_\pi^2} \sim -0.22 \text{ fm},$$

$$a_{I=2} = \frac{1}{2} \frac{m_\pi}{8\pi f_\pi^2} \sim 0.06 \text{ fm}$$



- proportional to m_π : zero in the chiral limit

- no other constant than $f_\pi \sim \langle \bar{q}q \rangle$ (order parameter)

- recent experimental determination

R. Garcia-Martin, *et al.*, *Phys. Rev. D* **83**, 074007 (2011)

$$a_{I=0} \sim -0.31 \text{ fm}, \quad a_{I=2} \sim 0.06 \text{ fm}$$

Scattering length reflects the “NG mode” nature.

NG modes in Nonrelativistic systems

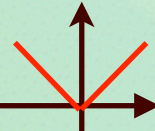
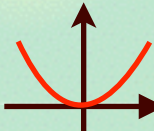
Classification of NG modes without Lorentz invariance

Y. Hidaka, Phys. Rev. Lett. 110, 091601 (2013),

H. Watanabe, H. Murayama, Phys. Rev. Lett. 108, 251602 (2012)

- number of broken symmetry \cong number of NG modes

$$N_{\text{BS}} = \text{rank} \langle 0 | [iQ^\alpha, \phi_i] | 0 \rangle, \quad \phi_i : \text{any operator}$$

- **Type-I** ($\omega \sim k$),  **Type-II** ($\omega \sim k^2$) 

$$N_{\text{BS}} = N_I + 2N_{II}$$

- **Type-II** : “special” order parameters ($\phi_i = Q^\beta$)

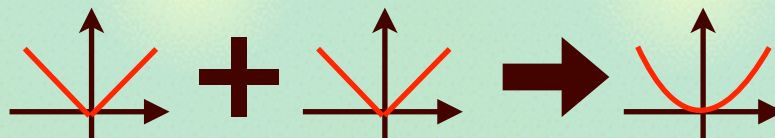
$$N_{II} = \frac{1}{2} \text{rank} \langle 0 | [iQ^\alpha, Q^\beta] | 0 \rangle, \quad \alpha, \beta = 1, \dots, N_{\text{BS}}$$

Type-II mode \leftarrow linear dependence of the NG fields

H. Nielsen, S. Chadha, Nucl. Phys. B 105, 445 (1976)

$$\langle 0 | j_0^\alpha | \pi^\alpha \rangle \neq 0, \quad \alpha = 1, \dots, N_{\text{BS}}$$

$$\sum_{\alpha} C_{\alpha} | \pi^{\alpha} \rangle = 0$$



Gapped NG modes

Gapped NG modes

S. Gongyo, S. Karasawa, Phys. Rev. D 90, 085014 (2014),

T. Hayata, Y. Hidaka, Phys. Rev. D 91, 056006 (2015),

M. Kobayashi, M. Nitta, Phys. Rev. D 92, 045028 (2015)

- pairwise mode with type II with $\partial_0\partial_0$ term in EFT



- number of the gapped NG modes (ϕ_i : operator $\neq Q^\beta$)

$$N_{\text{gapped}} = \frac{1}{2}(\text{rank}\langle 0 | [iQ^\alpha, \phi_i] | 0 \rangle - N_I)$$

- gap is **SSB origin**; no explicit breaking is needed

In what system the gapped NG modes appear?

How can we identify the gapped NG modes?

Effective Lagrangian

SO(3) \rightarrow SO(2): spin system (e.g. Heisenberg model)

$$\mathcal{L} = \frac{i\Sigma}{2} \text{Tr} [T^3 U^{-1} \partial_0 U] - \frac{F_t^2}{8} \text{Tr} [T^\alpha U^{-1} \partial_0 U] \text{Tr} [T^\alpha U^{-1} \partial_0 U] \\ + \frac{F^2}{8} \text{Tr} [T^\alpha U^{-1} \partial_i U] \text{Tr} [T^\alpha U^{-1} \partial_i U] + O(\partial_0^3, \partial_i^4),$$

- **2 broken generators T^α , 2 NG fields π^α ($\alpha = 1, 2$)**

- **representative of SO(3)/SO(2): $U = e^{i\pi^\alpha T^\alpha / F} \rightarrow gU(\pi)h(\pi, g)^{-1}$**

Quadratic terms of π field: dispersion relation

$$\mathcal{L} = -\frac{\Sigma}{2F^2} \epsilon^{\alpha\beta} \pi^\alpha \partial_0 \pi^\beta + \frac{1}{2v^2} \partial_0 \pi^\alpha \partial_0 \pi^\alpha - \frac{1}{2} \partial_i \pi^\alpha \partial_i \pi^\alpha + \mathcal{O}(\pi^4),$$

- $\Sigma \neq 0, 1/v = 0$ ($\partial_0, \partial_i \partial_i$): **type II mode**

- $\Sigma = 0, 1/v \neq 0$ ($\partial_0 \partial_0, \partial_i \partial_i$): **type I mode + type I mode**

- $\Sigma \neq 0, 1/v \neq 0$ ($\partial_0, \partial_0 \partial_0, \partial_i \partial_i$): **type II mode + gapped mode**

S. Gongyo, S. Karasawa, Phys. Rev. D 90, 085014 (2014)

Low energy constants and order parameters

Magnetization : Σ (∂_0 term)

H. Leutwyler, Phys. Rev. D 49, 3033 (1994),

C.P. Hofman, Phys. Rev. B 60, 388 (1999)

$$\Sigma = \lim_{V \rightarrow \infty} \frac{1}{V} \sum_m^N \langle 0 | S_m^3 | 0 \rangle$$



- ferromagnet ($\Sigma \neq 0$, $1/v = 0$): type II mode

- n.b. $S^3 \propto [S^1, S^2]$, $\Sigma \sim \langle 0 | [iQ^1, Q^2] | 0 \rangle$

Staggered Magnetization : $1/v$ ($\partial_0 \partial_0$ term)

S. Gongyo, Y. Kikuchi, T. Hyodo, T. Kunihiro, PTEP 2016, 083B01 (2016)

$$\Sigma_h = \lim_{V \rightarrow \infty} \frac{1}{V} \sum_m^N \langle 0 | (-1)^m S_m^3 | 0 \rangle$$

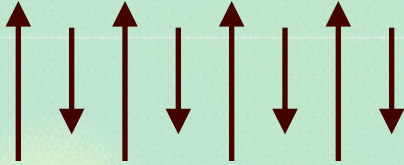


- antiferromagnet ($\Sigma = 0$, $1/v \neq 0$): type I mode + type I mode

- n.b. $1/v \sim \langle 0 | [iQ^\alpha, \pi^\beta] | 0 \rangle$

Realization of the gapped mode

Ferrimagnet



- magnetization + staggered magnetization
- $\Sigma \neq 0$, $1/v \neq 0$: type II mode + **gapped mode**

$$\mathcal{L} = -\frac{\Sigma}{2F^2} \epsilon^{\alpha\beta} \pi^\alpha \partial_0 \pi^\beta + \frac{1}{2v^2} \partial_0 \pi^\alpha \partial_0 \pi^\alpha - \frac{1}{2} \partial_i \pi^\alpha \partial_i \pi^\alpha + \mathcal{O}(\pi^4),$$

- gap is determined by the order parameters

$$\nu_M = \frac{\Sigma v^2}{F^2}$$

- consistent with Holstein-Primakoff transformation

S. Brehmer, H.J. Mikeska, S. Yamamoto, J. Phys.: Cond. Matt. **9**, 3921 (1997),
 S.K. Pati, S. Ramasesha, D. Sen, J. Phys.: Cond. Matt. **9**, 8707 (1997).

Scattering lengths

Scattering lengths: π^4 terms in effective Lagrangian

$$\mathcal{L}^4 = \frac{\Sigma}{24F^4} \epsilon^{\alpha\beta} \pi^\alpha \partial_0 \pi^\beta \pi^\gamma \pi^\gamma - \frac{1}{6v^2 F^2} [\partial_0 \pi^\alpha \partial_0 \pi^\alpha \pi^\beta \pi^\beta - \pi^\alpha \partial_0 \pi^\alpha \pi^\beta \partial_0 \pi^\beta] \\ + \frac{1}{6F^2} [\partial_i \pi^\alpha \partial_i \pi^\alpha \pi^\beta \pi^\beta - \pi^\alpha \partial_i \pi^\alpha \pi^\beta \partial_i \pi^\beta] + \dots$$

- vanish among Type I / Type II modes (c.f. chiral limit)

Scattering lengths including **gapped modes**

$$a^{II+M \rightarrow II+M} = \frac{\nu_M}{12F^2}, \quad a^{M+M \rightarrow M+M} = \frac{\nu_M}{6F^2}$$

- finite and proportional to the gap
- no other constant than the order parameters (c.f. Weinberg's result)

Scattering lengths \leftarrow NG boson nature of the gapped mode


Summary

 We construct EFT for $SO(3) \rightarrow SO(2)$

 Ferrimagnet 

- magnetization + staggered magnetization

- type II mode + **gapped NG mode** $\nu_M = \frac{\sum v^2}{F^2}$

 Scattering length of the gapped NG modes

- finite and **proportional to the gap**

$$a^{II+M \rightarrow II+M} = \frac{\nu_M}{12F^2}, \quad a^{M+M \rightarrow M+M} = \frac{\nu_M}{6F^2}$$

S. Gongyo, Y. Kikuchi, T. Hyodo, T. Kunihiro, PTEP 2016, 083B01 (2016)