ギャップを持つ南部ゴールドストーンモードの散乱

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2016, Sep. 22nd

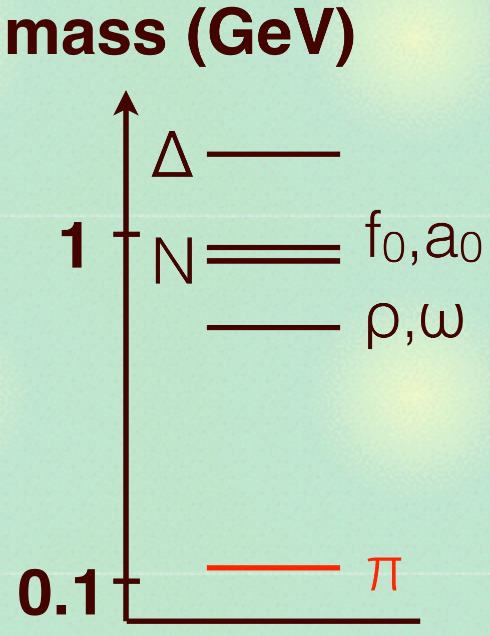
Pions in QCD

Nambu-Goldstone (NG) modes are

- **associated with spontaneous symmetry breaking (SSB)**
- **massless without explicit breaking**

Pions (π^+, π^-, π^0) in QCD

- **chiral symmetry $SU(2)_R \otimes SU(2)_L$**
  :microscopic theory
- **much lighter than other hadrons**
  :experiment



(almost) massless mode -> NG mode?

- **not always (gauge symmetry, edge state, ...)**
- **Are there any other characteristics of the "NG mode"?**

Pion scattering

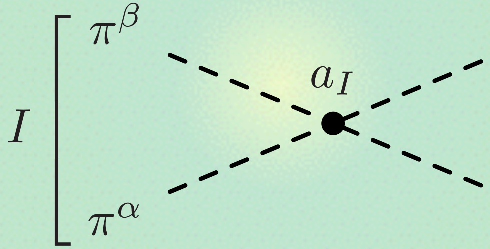
Scattering length of ππ system

S. Weinberg, Phys. Rev. Lett. 17, 616 (1966)

- low-energy theorem : leading order ChPT

$$a_{I=0} = -\frac{7}{4}\frac{m_\pi}{8\pi f_\pi^2} \sim -0.22 \text{ fm},$$

$$a_{I=2} = \frac{1}{2}\frac{m_\pi}{8\pi f_\pi^2} \sim 0.06 \text{ fm}$$



- proportional to m_π : zero in the chiral limit

- no other constant than $f_\pi \sim \langle\bar{q}q\rangle$ (order parameter)

- recent determination

R. Garcia-Martin, *et al.*, Phys. Rev. D 83, 074007 (2011)

$$a_{I=0} \sim -0.31 \text{ fm}, \quad a_{I=2} \sim 0.06 \text{ fm}$$

Scattering length reflects the “NG mode” nature.

NG modes in Nonrelativistic systems

Classification of NG modes without Lorentz invariance

Y. Hidaka, Phys. Rev. Lett. 110, 091601 (2013),
H. Watanabe, H. Murayama, Phys. Rev. Lett. 108, 251602 (2012)

- number of broken charges > number of NG modes
- Type-I ($\omega \sim k$), Type-II ($\omega \sim k^2$)

$$N_{\rm BS} = N_I + 2N_{II}$$

$$N_{II} = \frac{1}{2}\mathrm{rank}\langle\, 0\,|[iQ^\alpha, Q^\beta]|\,0\,\rangle, \quad \alpha,\beta = 1,\ldots,N_{\rm BS}$$

Type-II mode <— linear dependence of the NG fields

H. Nielsen, S. Chadha, Nucl. Phys. B 105, 445 (1976)

$$\langle\, 0\,|j_0^\alpha|\,\pi^\alpha\,\rangle \neq 0, \quad \alpha,\beta = 1,\ldots,N_{\rm BS}$$

$$\sum_\alpha C_\alpha|\pi^\alpha\rangle = 0, \qquad \{\omega\sim k,\ \omega\sim k\}\to\omega\sim k^2$$

- existence of canonical conjugate pair

Y. Nambu, J. Statist. Phys. 115, 7 (2004)

Gapped NG modes

Gapped NG modes

S. Gongyo, S. Karasawa, Phys. Rev. D 90, 085014 (2014),
T. Hayata, Y. Hidaka, Phys. Rev. D 91, 056006 (2015),
M. Kobayashi, M. Nitta, Phys. Rev. D 92, 045028 (2015)

- pairwise mode with type II with $\partial_0\partial_0$ term in EFT

$$\{\omega \sim k,\ \omega \sim k\} \to \{\omega \sim k^2,\ \omega \sim m^2 + k^2\}$$

- number of the gapped NG modes (ϕ_i : an operator $\neq j_0$)

$$N_{\text{gapped}} = \frac{1}{2}(\text{rank}\langle\, 0\, |[iQ^\alpha, \phi_i]|\, 0\,\rangle - N_I)$$

- gap is SSB origin; no explicit breaking is needed
- different from “massive NG mode” (external field origin)

A. Nicolis, F. Piazza, Phys. Rev. Lett. 110, 011602 (2013),
H. Watanabe, T. Brauner, H. Murayama, Phys. Rev. Lett 111, 021601 (2013)

In what system the gapped NG modes appear?

How can we identify the gapped NG modes?

Effective Lagrangian

SO(3) —> SO(2): spin system (e.g. Heisenberg model)

$$\mathcal{L} = \frac{i\Sigma}{2}\mathrm{Tr}\left[T^3 U^{-1}\partial_0 U\right] - \frac{F_t^2}{8}\mathrm{Tr}\left[T^\alpha U^{-1}\partial_0 U\right]\mathrm{Tr}\left[T^\alpha U^{-1}\partial_0 U\right] + \frac{F^2}{8}\mathrm{Tr}\left[T^\alpha U^{-1}\partial_i U\right]\mathrm{Tr}\left[T^\alpha U^{-1}\partial_i U\right] + O(\partial_0^3, \partial_i^4),$$

- **2 broken generators** T^α**, 2 NG fields** π^α **(**α = 1,2**)**
- **representative of SO(3)/SO(2):** $U = e^{i\pi^\alpha T^\alpha/F} \to gU(\pi)h(\pi,g)^{-1}$

Quadratic terms of π field: dispersion relation

$$\mathcal{L} = -\frac{\Sigma}{2F^2}\epsilon^{\alpha\beta}\pi^\alpha\partial_0\pi^\beta + \frac{1}{2v^2}\partial_0\pi^\alpha\partial_0\pi^\alpha - \frac{1}{2}\partial_i\pi^\alpha\partial_i\pi^\alpha + \mathcal{O}(\pi^4),$$

- $\Sigma\neq0$, $1/v=0$ (∂_0, $\partial_i\partial_i$): **type II mode**
- $\Sigma=0$, $1/v\neq0$ ($\partial_0\partial_0$, $\partial_i\partial_i$): **type I mode + type I mode**
- $\Sigma\neq0$, $1/v\neq0$ (∂_0, $\partial_0\partial_0$, $\partial_i\partial_i$): **type II mode + gapped mode**

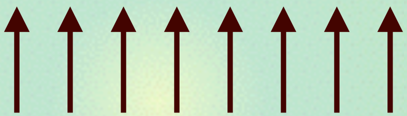
S. Gongyo, S. Karasawa, Phys. Rev. D 90, 085014 (2014)

Low energy constants and oder parameters

Magnetization : Σ (∂_0 term)

H. Leutwyler, Phys. Rev. D 49, 3033 (1994),
C.P. Hofman, Phys. Rev. B 60, 388 (1999)

$$\Sigma = \lim_{V\to\infty} \frac{1}{V} \sum_{m}^{N} \langle 0| S^3_m |0\rangle$$

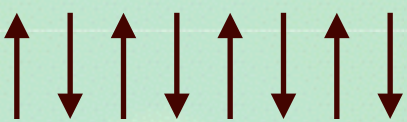


- **ferromagnet ($\Sigma \neq 0$, $1/v = 0$): type II mode**
- **n.b.** $S^3 \propto [S^1, S^2]$, $\quad \Sigma \sim \langle\, 0\,|[iQ, j_0]|\, 0\,\rangle$

Staggered Magnetization : $1/v$ ($\partial_0\partial_0$ term)

S. Gongyo, Y. Kikuchi, T. Hyodo, T. Kunihiro, PTEP 2016, 083B01 (2016)

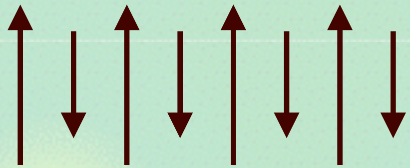
$$\Sigma_h = \lim_{V\to\infty} \frac{1}{V} \sum_{m}^{N} \langle 0| (-1)^m S^3_m |0\rangle$$



- **antiferromagnet ($\Sigma = 0$, $1/v \neq 0$): type I mode + type I mode**
- **n.b.** $1/v \sim \langle\, 0\,|[iQ, \pi]|\, 0\,\rangle$

Realization of the gapped mode

Ferrimagnet



- **magnetization + staggered magnetization**
- $\Sigma \neq 0$, $1/v \neq 0$: **type II mode + gapped mode**

$$\mathcal{L} = -\frac{\Sigma}{2F^2}\epsilon^{\alpha\beta}\pi^\alpha\partial_0\pi^\beta + \frac{1}{2v^2}\partial_0\pi^\alpha\partial_0\pi^\alpha - \frac{1}{2}\partial_i\pi^\alpha\partial_i\pi^\alpha + \mathcal{O}(\pi^4),$$

- **gap is determined by the order parameters**

$$\nu_M = \frac{\Sigma v^2}{F^2}$$

- **consistent with Holstein-Primakoff transformation**

S. Brehmer, H.J. Mikeska, S. Yamamoto, J. Phys.: Cond. Matt. 9, 3921 (1997),
S.K. Pati, S. Ramasesha, D. Sen, J. Phys.: Cond. Matt. 9, 8707 (1997).

Scattering lengths

Scattering lengths: π^4 terms in effective Lagrangian

$$\mathcal{L}^4 = \frac{\Sigma}{24F^4}\epsilon^{\alpha\beta}\pi^\alpha\partial_0\pi^\beta\pi^\gamma\pi^\gamma - \frac{1}{6v^2F^2}\left[\partial_0\pi^\alpha\partial_0\pi^\alpha\pi^\beta\pi^\beta - \pi^\alpha\partial_0\pi^\alpha\pi^\beta\partial_0\pi^\beta\right]$$
$$+ \frac{1}{6F^2}\left[\partial_i\pi^\alpha\partial_i\pi^\alpha\pi^\beta\pi^\beta - \pi^\alpha\partial_i\pi^\alpha\pi^\beta\partial_i\pi^\beta\right] + \cdots$$

- vanish among Type I / Type II modes (c.f. chiral limit)

Scattering lengths including gapped modes

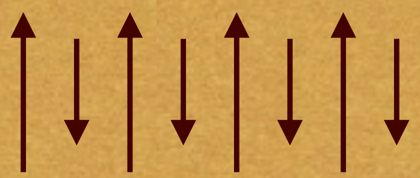
$$a^{II+M\to II+M} = \frac{\nu_M}{12F^2}, \quad a^{M+M\to M+M} = \frac{\nu_M}{6F^2}$$

- finite and proportional to the gap

- no other constant than the order parameters (c.f. Weinberg's result)

Scattering lengths <— NG boson nature of the gapped mode

Summary

- We construct EFT for SO(3) —> SO(2)
- Ferrimagnet 
  - magnetization + staggered magnetization
  - type II mode + gapped NG mode $\nu_M = \frac{\Sigma v^2}{F^2}$
- Scattering length of the gapped NG modes
  - finite and proportional to the gap

$$a^{II+M \to II+M} = \frac{\nu_M}{12F^2}, \quad a^{M+M \to M+M} = \frac{\nu_M}{6F^2}$$

S. Gongyo, Y. Kikuchi, T. Hyodo, T. Kunihiro, PTEP 2016, 083B01 (2016)