Compositeness of hadrons from effective field theory

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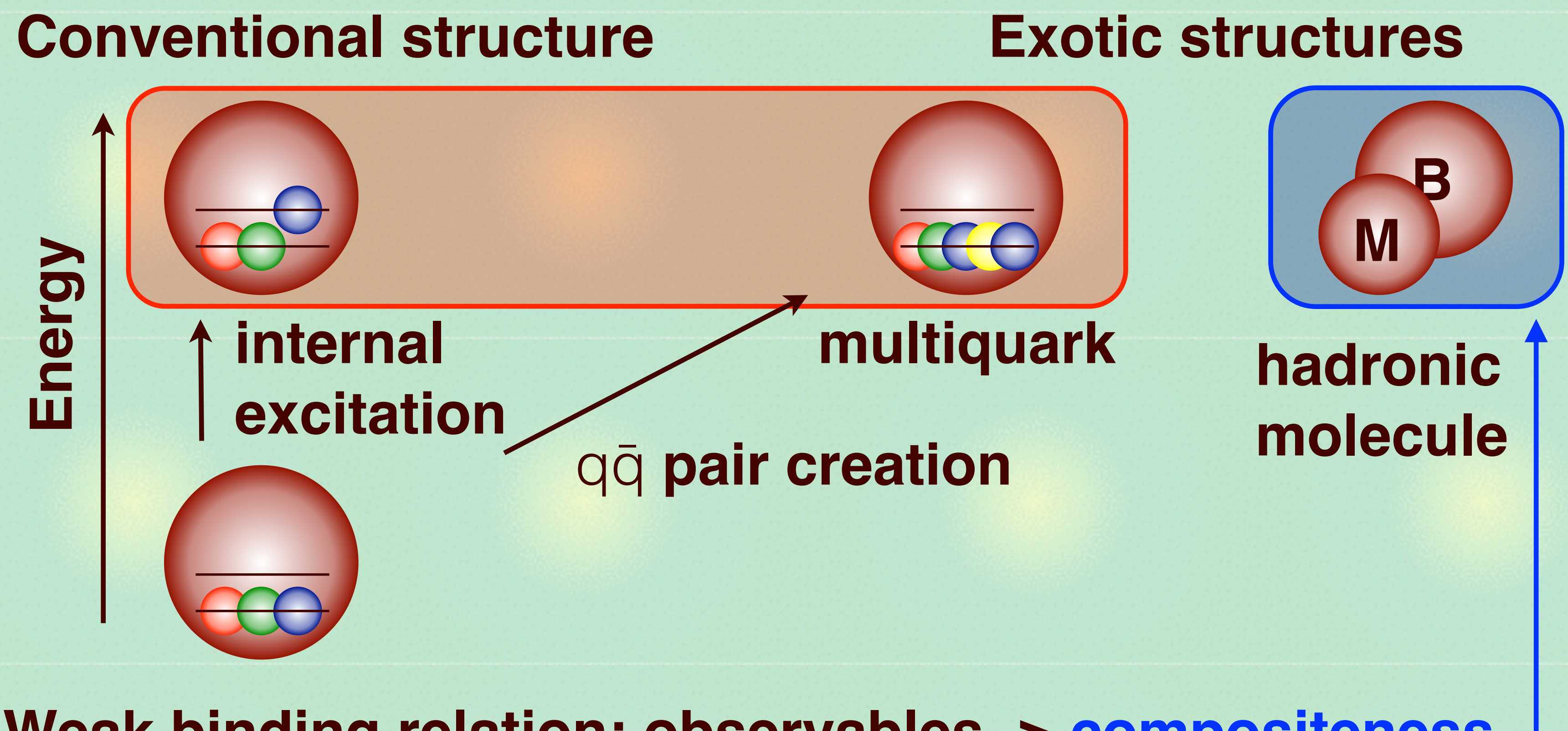
Method to study the internal structure

Internal structure of excited hadrons?



- Weak binding relation: observables -> compositeness

S. Weinberg, Phys. Rev. 137, B672 (1965)

c.f. Talk by
T. Sekihara (Fri)

Weak binding relation for stable states

Compositeness of s-wave weakly bound state ($R \gg R_{typ}$)

S. Weinberg, Phys. Rev. 137, B672 (1965);
T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right)\right\}, \quad r_e = R\left\{\frac{X-1}{X} + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right)\right\}$$

a_0: **scattering length,** r_e: **effective range**
$R = (2\mu B)^{-1/2}$: **radius of wave function**
R_{typ}: **length scale of interaction**
X: **probability of finding composite component**

- **deuteron is** NN **composite (**$a_0 \sim R \gg r_e$**) —>** $X \sim 1$
- **internal structure from observable**
- **no nuclear force potential / wavefunction of deuteron**

Note: applicable only for stable states

Effective field theory

Low-energy scattering with near-threshold bound state

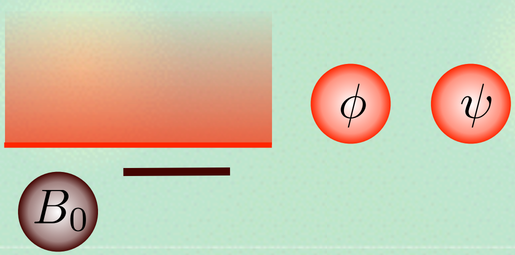
- **Nonrelativistic EFT with contact interaction**

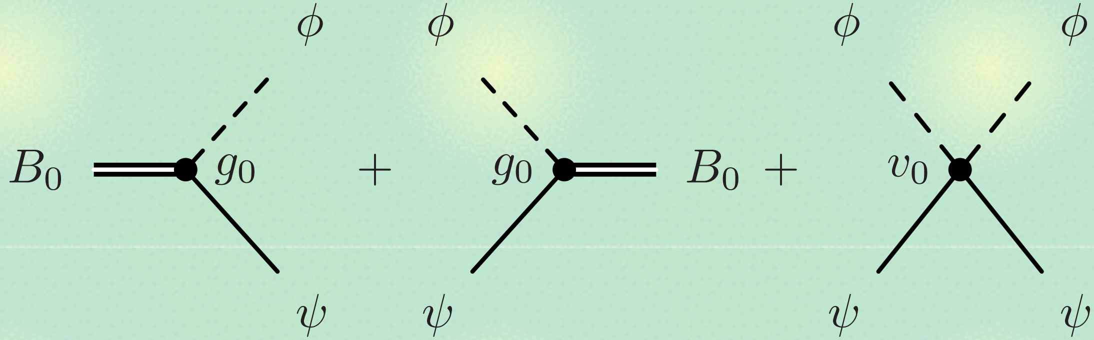
D.B. Kaplan, Nucl. Phys. B494, 471 (1997)

Y. Kamiya, T. Hyodo, Phys. Rev. C93, 035203 (2016)

$$H_{\text{free}} = \int d\boldsymbol{r} \left[\frac{1}{2M} \nabla\psi^\dagger \cdot \nabla\psi + \frac{1}{2m} \nabla\phi^\dagger \cdot \nabla\phi + \frac{1}{2M_0} \nabla B_0^\dagger \cdot \nabla B_0 + \nu_0 B_0^\dagger B_0 \right],$$

$$H_{\text{int}} = \int d\boldsymbol{r} \left[g_0 \left(B_0^\dagger \phi\psi + \psi^\dagger\phi^\dagger B_0 \right) + v_0 \psi^\dagger\phi^\dagger\phi\psi \right]$$





- **cutoff** : Λ ~ 1/R_{typ} **(interaction range of microscopic theory)**
- **At low momentum** p ≪ Λ, **interaction ~ contact**

Compositeness and "elementariness"

Eigenstates

$$H_{\rm free}|\, B_0\, \rangle = \nu_0 |\, B_0\, \rangle, \quad H_{\rm free}|\, \boldsymbol{p}\, \rangle = \frac{p^2}{2\mu}|\, \boldsymbol{p}\, \rangle$$

free (discrete + continuum)

$$(H_{\rm free} + H_{\rm int})|\, B\, \rangle = -B|\, B\, \rangle$$

full (bound state)

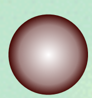
- normalization of |B> + completeness relation

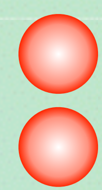
$$\langle\, B\, |\, B\, \rangle = 1, \quad 1 = |\, B_0\, \rangle\langle\, B_0\, | + \int \frac{d\boldsymbol{p}}{(2\pi)^3} |\, \boldsymbol{p}\rangle\langle\, \boldsymbol{p}\, |$$

- projections onto bare states

$$1 = Z + X, \quad Z \equiv |\langle\, B_0\, |\, B\, \rangle|^2, \quad X \equiv \int \frac{d\boldsymbol{p}}{(2\pi)^3} |\langle\, \boldsymbol{p}\, |\, B\, \rangle|^2$$

"elementariness" **compositeness**



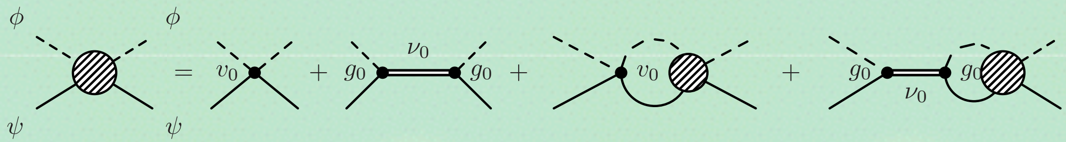


Z, X**: real and nonnegative —> interpreted as probability**

Weak binding relation

ΨΦ scattering amplitude (exact result)

$$f(E) = -\frac{\mu}{2\pi}\frac{1}{[v(E)]^{-1} - G(E)}$$



$$v(E) = v_0 + \frac{g_0^2}{E-\nu_0}, \quad G(E) = \frac{1}{2\pi^2}\int_0^{\Lambda} dp \frac{p^2}{E - p^2/(2\mu) + i0^+}$$

Compositeness X <— v(E), G(E)

T. Sekihara, T. Hyodo, D. Jido, PTEP2015, 063D04 (2015)

$$X = \{1 + G^2(-B)v'(-B)\,[G'(-B)]^{-1}\}^{-1}$$

$1/R=(2\mu B)^{1/2}$ expansion: leading term <— X

$$a_0 = -f(E=0) = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right)\right\}$$

renormalization dependent

renormalization independent

If $R \gg R_{\rm typ}$, correction terms neglected: X <— (B, a_0)

Introduction of decay channel

Introduce decay channel

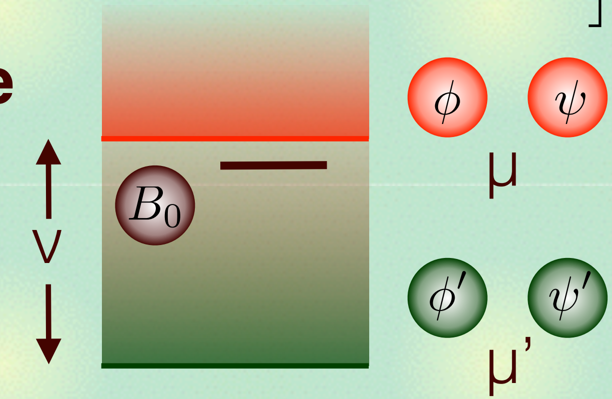
$$H'_{\mathrm{free}} = \int d\boldsymbol{r}\left[\frac{1}{2M'}\nabla\psi'^{\dagger}\cdot\nabla\psi' - \nu_{\psi}\psi'^{\dagger}\psi' + \frac{1}{2m'}\nabla\phi'^{\dagger}\cdot\nabla\phi' - \nu_{\phi}\phi'^{\dagger}\phi'\right],$$

$$H'_{\mathrm{int}} = \int d\boldsymbol{r}\left[g'_0\left(B_0^{\dagger}\phi'\psi' + \psi'^{\dagger}\phi'^{\dagger}B_0\right) + v'_0\psi'^{\dagger}\phi'^{\dagger}\phi'\psi' + v_0^t(\psi^{\dagger}\phi^{\dagger}\phi'\psi' + \psi'^{\dagger}\phi'^{\dagger}\phi\psi)\right],$$

Quasi-bound state: complex eigenvalue

$$H = H_{\mathrm{free}} + H'_{\mathrm{free}} + H_{\mathrm{int}} + H'_{\mathrm{int}}$$

$$H|\,QB\,\rangle = E_{QB}|\,QB\,\rangle, \quad E_{QB}\in\mathbb{C}$$



Generalized relation: correction term <— threshold difference

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\mathrm{typ}}}{R}\right|\right) + \underline{\mathcal{O}\left(\left|\frac{l}{R}\right|^3\right)}\right\}, \quad R = \frac{1}{\sqrt{-2\mu E_{QB}}}\in\mathbb{C}, \quad l\equiv\frac{1}{\sqrt{2\mu\nu}}$$

Y. Kamiya, T. Hyodo, Phys. Rev. C93, 035203 (2016)

c.f. V. Baru, *et al.*, Phys. Lett. B586, 53 (2004),…

If $|R| \gg (R_{\mathrm{typ}}, l)$ correction terms neglected: X <— (E_{QB}, a_0)

Application

Generalized weak binding relation X <— (E_{QB}, a_0)

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\rm typ}}{R}\right|\right) + \mathcal{O}\left(\left|\frac{l}{R}\right|^3\right)\right\}, \quad R = \frac{1}{\sqrt{-2\mu E_{QB}}}, \quad l \equiv \frac{1}{\sqrt{2\mu\nu}}$$

- Λ(1405) **(higher) pole position and** $\bar{K}N$ **scattering length**

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881 98 (2012), …

- E_{QB} = -10 -26i **MeV —>** |R| ~ 2 fm **—> small correction term**

$$\left|\frac{R_{\rm typ}}{R}\right| \lesssim 0.12, \quad \left|\frac{l}{R}\right|^3 \lesssim 0.16$$ **(rho exchange, πΣ threshold)**

Ref.	E_{QB} (MeV)	a_0 (fm)	$X_{\bar{K}N}$	$\tilde{X}_{\bar{K}N}$	U
[45]	$-10 - i26$	$1.39 - i0.85$	$1.2 + i0.1$	1.0	0.5
[46]	$-4 - i8$	$1.81 - i0.92$	$0.6 + i0.1$	0.6	0.0
[47]	$-13 - i20$	$1.30 - i0.85$	$0.9 - i0.2$	0.9	0.1
[48]	$2 - i10$	$1.21 - i1.47$	$0.6 + i0.0$	0.6	0.0
[48]	$-3 - i12$	$1.52 - i1.85$	$1.0 + i0.5$	0.8	0.6

systematic error (↑ ↓)

$$\tilde{X} = \frac{1 - |Z| + |X|}{2}$$

$$U = |Z| + |X| - 1$$

Λ(1405) is $\bar{K}N$ composite <— observables

Summary

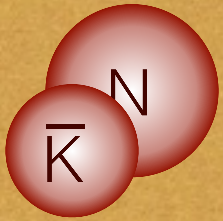
- **Compositeness of near-threshold bound state can be determined only by observables.**

  S. Weinberg, Phys. Rev. 137, B672 (1965)

- **Weak binding relation can be generalized to unstable states with effective field theory.**

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\rm typ}}{R}\right|\right) + \mathcal{O}\left(\left|\frac{l}{R}\right|^3\right)\right\}, \quad R = \frac{1}{\sqrt{-2\mu E_{QB}}}, \quad l \equiv \frac{1}{\sqrt{2\mu\nu}}$$

- **Precise determination of the pole position and scattering length shows that Λ(1405) is dominated by $\bar{K}N$ composite component.**



Y. Kamiya, T. Hyodo, Phys. Rev. C93, 035203 (2016), arXiv:1607.01899[hep-ph]