

Quark mass dependence of H-dibaryon in $\Lambda\Lambda$ scattering



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Contents



Introduction



Formulation

- Effective field theory
- Quark mass dependence



Result

- $\Lambda\Lambda$ amplitude: SU(3) limit / physical point
- Extrapolation in quark mass plane

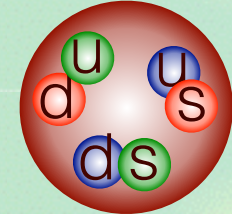


Summary

H-dibaryon in $\Lambda\Lambda$ scattering

H-dibaryon: $uuddss$ bound state predicted in a quark model

R.L. Jaffe, *Phys. Rev. Lett.* **38**, 195 (1977)



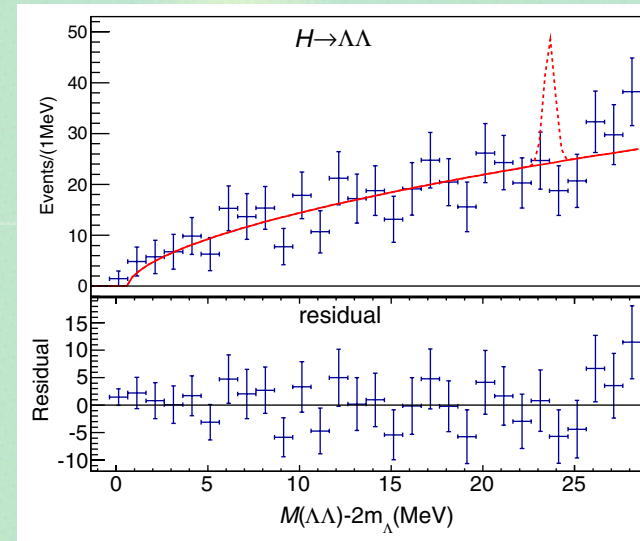
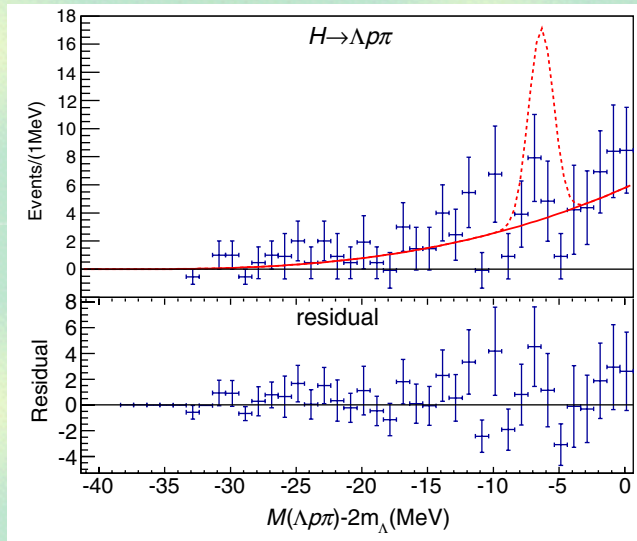
Experiments: Negative

- **Nagara event: double Λ hyper nuclei \rightarrow no deeply bound H**

H. Takahashi, *et al.*, *Phys. Rev. Lett.* **87**, 212502 (2001)

- **Belle: $Y(1S)$, $Y(2S)$ decay \rightarrow no signal (\ll deuteron)**

B.H. Kim, *et al.*, [Belle collaboration] *Phys. Rev. Lett.* **114**, 022301 (2015)

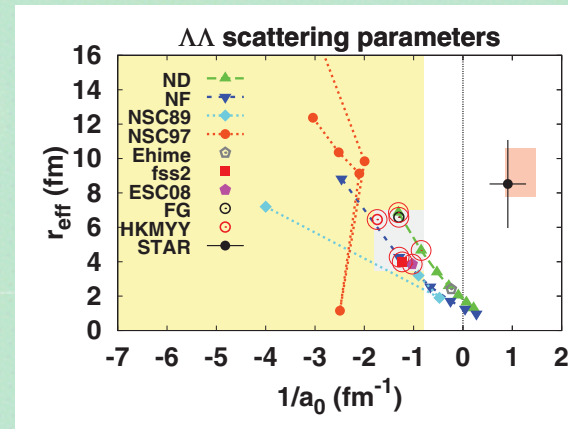
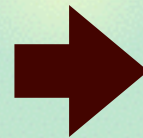
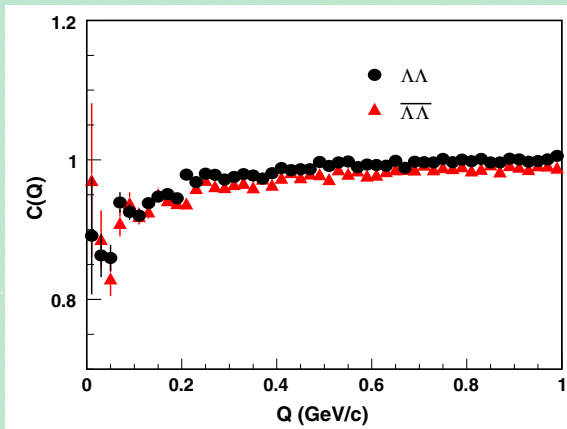


Recent activities

RHIC-STAR: $\Lambda\Lambda$ correlation \rightarrow scattering length

L. Adamczyk, *et al.*, [STAR collaboration] Phys. Rev. Lett. 114, 022301 (2015);

K. Morita, T. Furumoto, A. Ohnishi, Phys. Rev. C 91, 024916 (2015)



H-dibaryon in lattice QCD

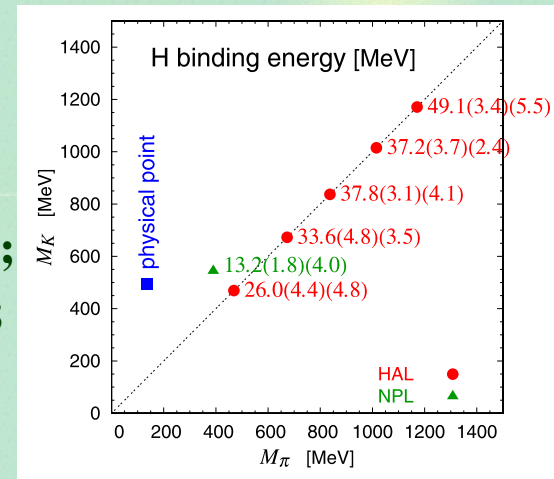
- **Bound** with unphysical quark masses

HAL QCD, T. Inoue *et al.*, Phys. Rev. Lett. 106, 162002 (2011);

NPLQCD, S. Beane *et al.*, Phys. Rev. Lett. 106, 162001 (2011);

HAL QCD, T. Inoue *et al.*, Nucl. Phys. A881, 28 (2012); ...

- Physical point simulation is ongoing.



Near-threshold scaling

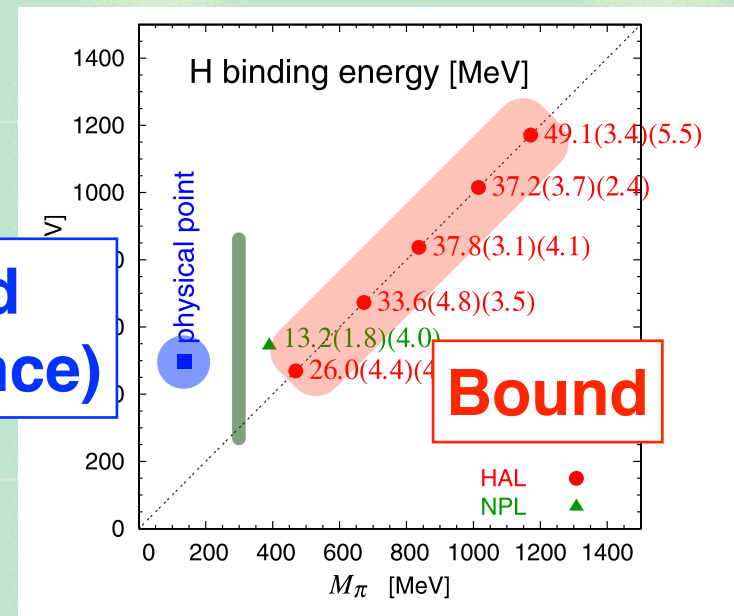
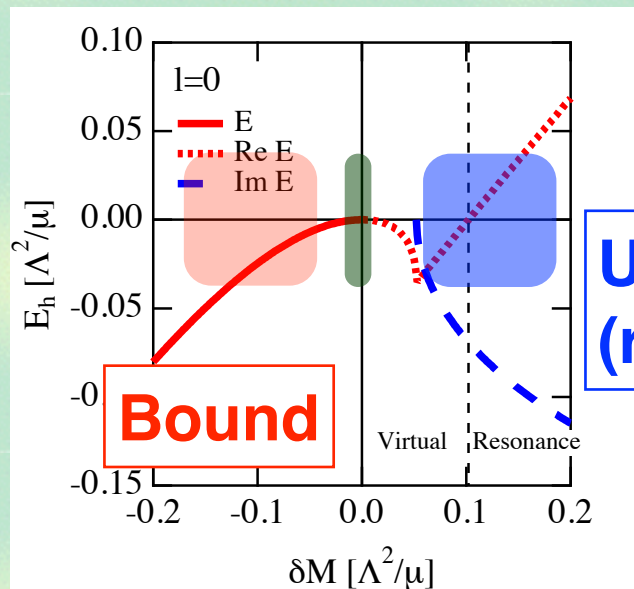
Extrapolation: **unbound** at physical point

S. Shanahan, A. Thomas, R. Young, *Phys. Rev. Lett.* **107**, 092004 (2011);

J. Haidenbauer, U.G. Meissner, *Phys. Lett. B* **706**, 100 (2011)

Near-threshold scaling in s-wave (bound \rightarrow unbound)




T. Hyodo, *Phys. Rev. C* **90**, 055208 (2014)



- unitary limit (infinitely large scattering length)

Unitary limit at unphysical quark masses?

Purpose of this talk

-  How does the H-dibaryon bound state in the $\Lambda\Lambda$ scattering change along with the **variation of the quark masses?**
-  Input: three lightest lattice data in **SU(3) limit.**
-  Effective framework which describes the $\Lambda\Lambda$ scattering in a relatively wide range of quark masses.

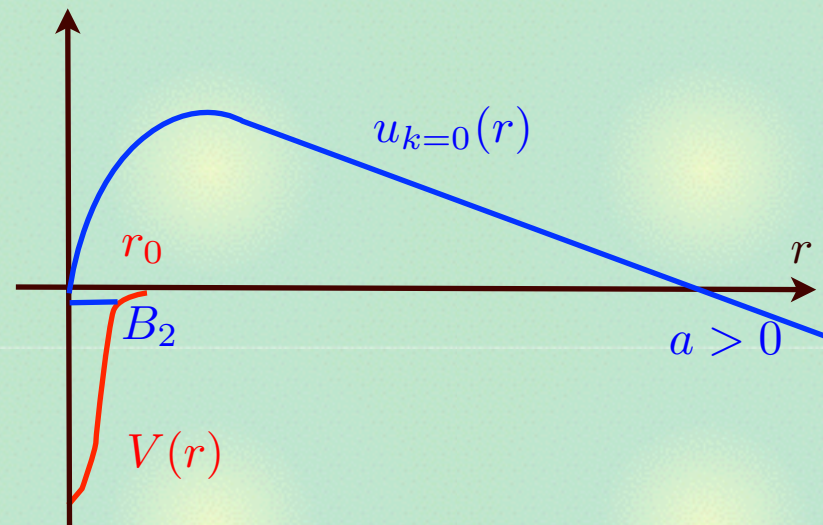
(Precise $\Lambda\Lambda$ interaction at physical point may be studied by lattice QCD / systematic ChPT.)

Low-energy baryon-baryon scattering

Length scales in the SU(3) limit

HAL QCD, T. Inoue *et al.*, Nucl. Phys. A881, 28 (2012)

- Interaction range by NG boson exchange: $r^0 \sim 0.24\text{-}0.42$ fm
- large scattering length: $a \sim 1.2\text{-}1.7$ fm
- large radius \leftarrow small binding energy: $0.77\text{-}1.14$ fm



At low energy, the interaction can be treated as **point like**.

Effective Lagrangian

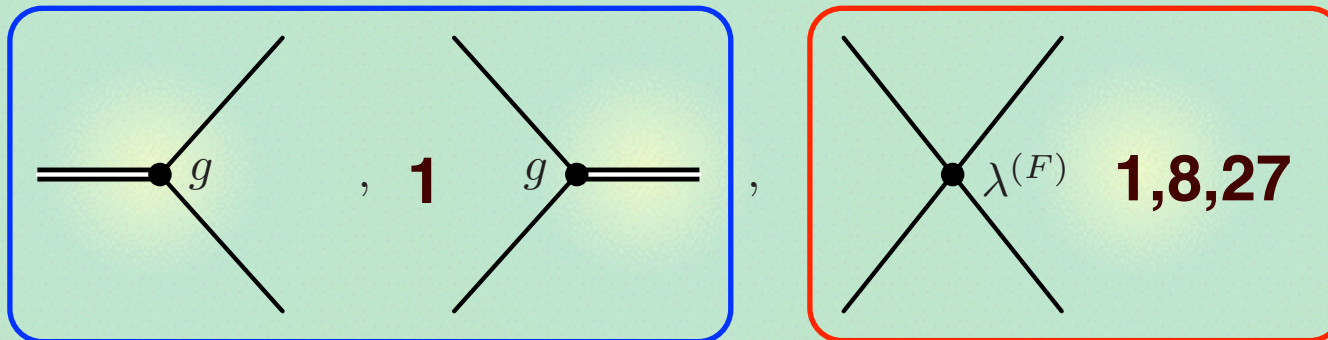
Low energy effective Lagrangian with contact interactions

c.f. D.B. Kaplan, Nucl. Phys. B494, 471 (1997)

$$\mathcal{L}_{\text{free}} = \sum_{a=1}^4 \sum_{\sigma=\uparrow,\downarrow} B_{a,\sigma}^\dagger \left(i \frac{\partial}{\partial t} + \frac{\nabla^2}{2M_a} + \delta_a \right) B_{a,\sigma} + H^\dagger \left(i \frac{\partial}{\partial t} + \frac{\nabla^2}{2M_H} + \nu \right) H$$

$$\mathcal{L}_{\text{int}} = \underline{-g[D^{(1)\dagger}H + H^\dagger D^{(1)}]} - \underline{\lambda^{(1)}D^{(1)\dagger}D^{(1)} - \lambda^{(8)}D^{(8)\dagger}D^{(8)} - \lambda^{(27)}D^{(27)\dagger}D^{(27)}}$$

$$D^{(F)} = [BB]_{J=0,S=-2,I=0}^{(F)}$$



Length scales at the physical point

- No π exchange in $\Lambda\Lambda$. π exchange in $N\Xi$ ($\Lambda + 25$ MeV)
- > safely applicable below $N\Xi$ threshold

Low energy scattering amplitude

Coupled-channel scattering amplitude ($i=\Lambda, N\Xi, \Sigma\Sigma$)

$$f_{ii}(E) = \frac{\mu_i}{2\pi} [(\mathcal{A}^{\text{tree}}(E))^{-1} + I(E)]_{ii}^{-1}$$

$$\mathcal{A}_{ij}^{\text{tree}}(E) = \begin{array}{c} \diagup \\ \bullet \\ \diagdown \end{array} \begin{array}{c} \diagdown \\ \bullet \\ \diagup \end{array} + \begin{array}{c} \diagup \\ \bullet \\ \diagdown \end{array} \begin{array}{c} \text{---} \\ \bullet \text{---} \\ \text{---} \end{array} \begin{array}{c} \diagdown \\ \bullet \\ \diagup \end{array}$$

$$= - \left(V_{ij} + \frac{g^2 d_i^\dagger d_j}{E - \nu + i0^+} \right), \quad V = U^{-1} \begin{pmatrix} \lambda^{(1)} & & \\ & \lambda^{(8)} & \\ & & \lambda^{(27)} \end{pmatrix} U, \quad d = \begin{pmatrix} -\sqrt{\frac{1}{8}} \\ -\sqrt{\frac{1}{2}} \\ \sqrt{\frac{3}{8}} \end{pmatrix}$$

$$I_i(E) = \begin{array}{c} \bullet \\ \circ \\ \bullet \end{array} \begin{array}{c} i \\ \bullet \\ \bullet \end{array}$$

$$= \frac{\mu_i}{\pi^2} \left(-\Lambda + k_i \operatorname{artanh} \frac{\Lambda}{k_i} \right), \quad k_i = \sqrt{2\mu_i(E - \Delta_i)}$$

EFT describes the low energy scattering **for a given** (m_l, m_s).

- scattering length, bound state pole, ...

- Quark mass dep. \rightarrow baryon masses and couplings λ

Modeling quark mass dependence

“Quark masses” via GMOR relation

$$B_0 m_l = \frac{m_\pi^2}{2}, \quad B_0 m_s = m_K^2 - \frac{m_\pi^2}{2}$$

$$B_0 = -\frac{\langle \bar{q}q \rangle}{3F_0^2}$$

Baryon masses: linear in m_q

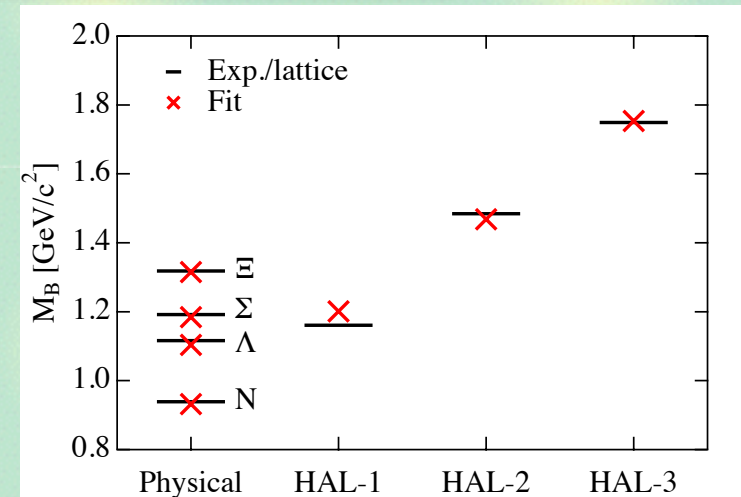
$$M_N(m_l, m_s) = M_0 - (2\alpha + 2\beta + 4\sigma)B_0 m_l - 2\sigma B_0 m_s,$$

$$M_\Lambda(m_l, m_s) = M_0 - (\alpha + 2\beta + 4\sigma)B_0 m_l - (\alpha + 2\sigma)B_0 m_s,$$

$$M_\Sigma(m_l, m_s) = M_0 - \left(\frac{5}{3}\alpha + \frac{2}{3}\beta + 4\sigma \right) B_0 m_l - \left(\frac{1}{3}\alpha + \frac{4}{3}\beta + 2\sigma \right) B_0 m_s,$$

$$M_\Xi(m_l, m_s) = M_0 - \left(\frac{1}{3}\alpha + \frac{4}{3}\beta + 4\sigma \right) B_0 m_l - \left(\frac{5}{3}\alpha + \frac{2}{3}\beta + 2\sigma \right) B_0 m_s$$

- three mass difference by (α, β) \rightarrow GMO relation
- fit to experiment/lattice \rightarrow reasonable



Modeling quark mass dependence

Coupling constants \leftarrow scattering length in SU(3) limit

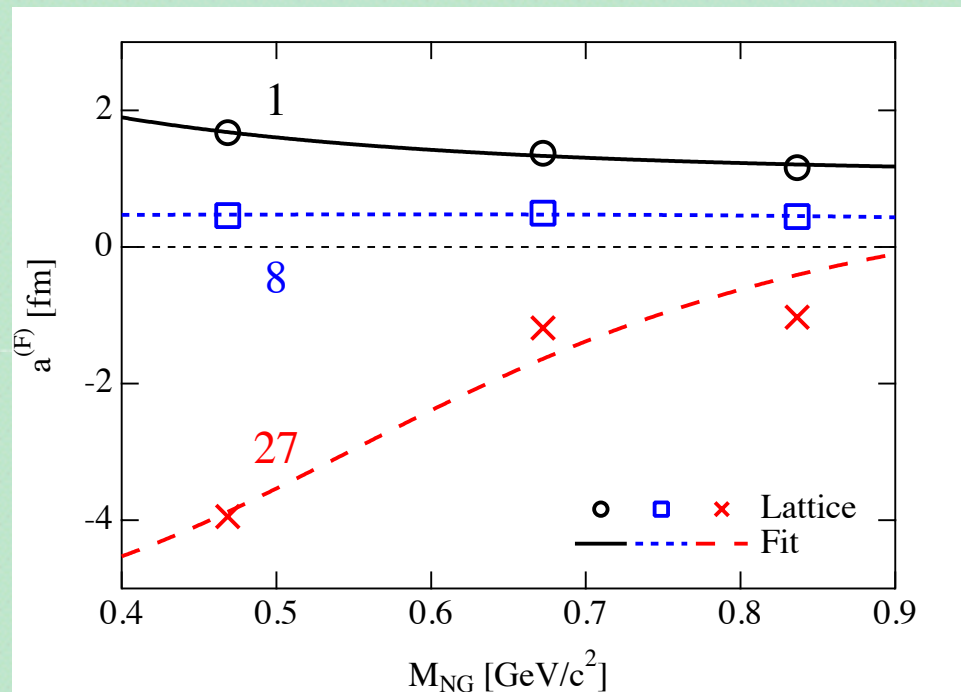
T. Inoue, private communication.

- $a = -f(E=0)$ **1: bound, 8: repulsive, 27: attractive**

- **This talk: linear in m_q , no bare H**

$$\lambda^{(F)}(m_l, m_s) = \lambda_0^{(F)} + \lambda_1^{(F)} B_0 (2m_l + m_s)$$

$$g(m_l, m_s) = 0$$

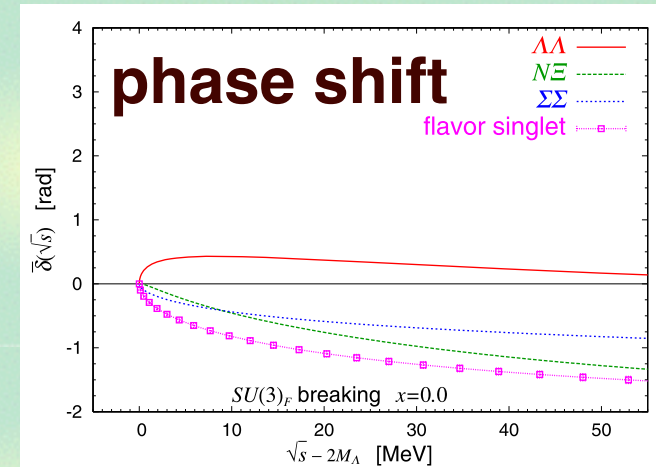
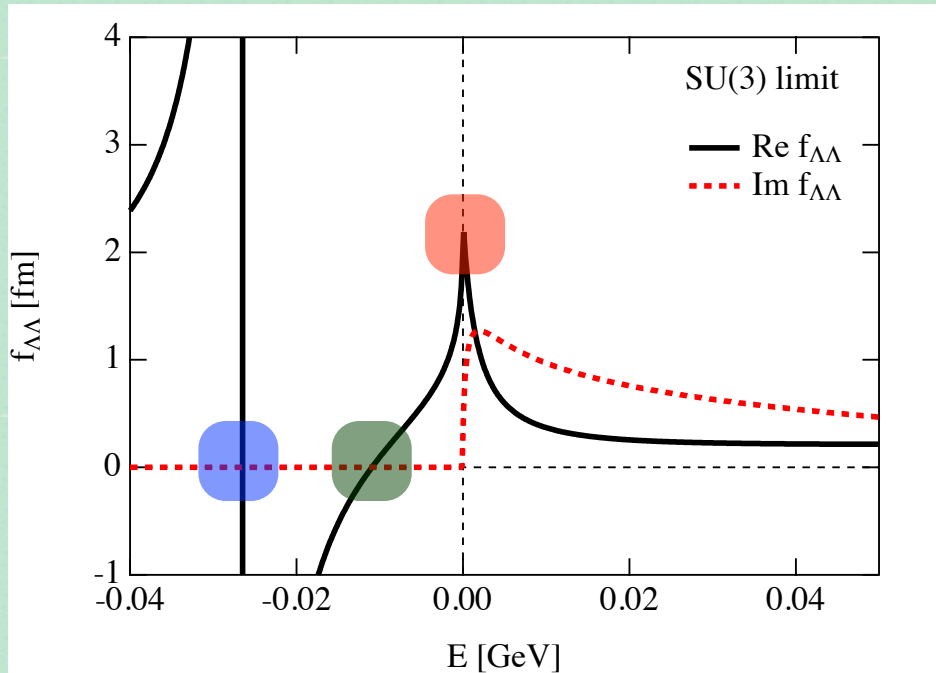


repulsive

attractive

$\Lambda\Lambda$ amplitude : SU(3) limit

$\Lambda\Lambda$ scattering amplitude in the SU(3) limit



HAL QCD, T. Inoue *et al.*,
Nucl. Phys. A881, 28 (2012)

- **bound H** ← bound state in 1
- **attractive** scattering length $a = -f(E=0)$ ← attraction in 27

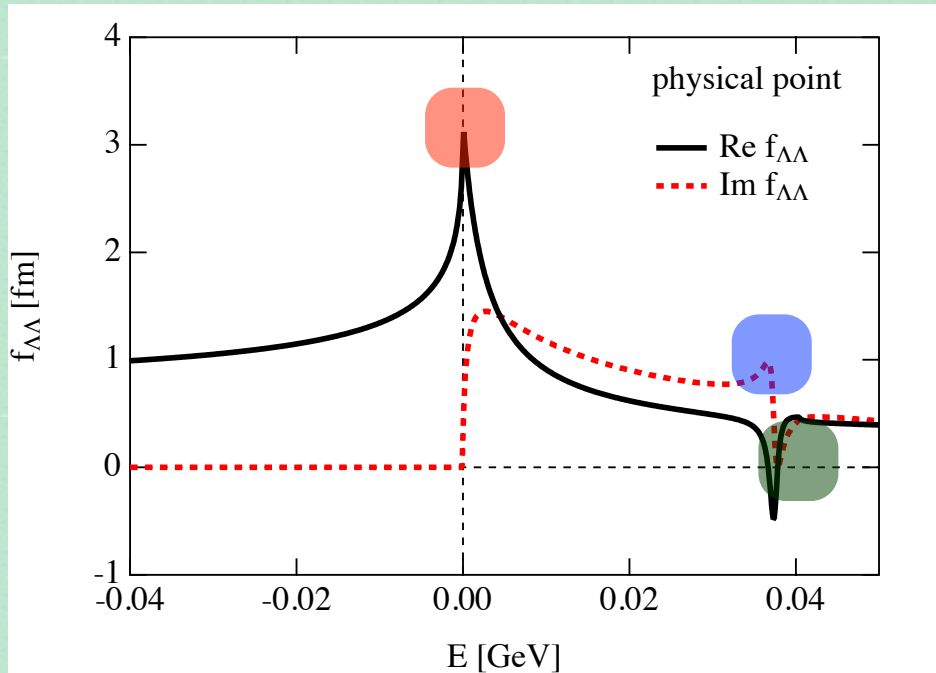
$$f_{\Lambda\Lambda}(E) = \frac{1}{8}f^{(1)}(E) + \frac{1}{5}f^{(8)}(E) + \frac{27}{40}f^{(27)}(E)$$

- **CDD pole below threshold:** $f(E)=0 \rightarrow$ ERE breaks down.

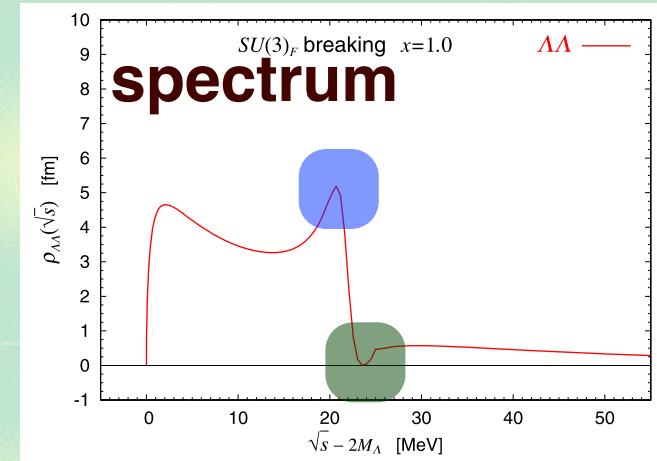
L. Castillejo, R.H. Dalitz, F.J. Dyson, Phys. Rev. 101, 453 (1956)

$\Lambda\Lambda$ amplitude : Physical point

$\Lambda\Lambda$ scattering amplitude at the physical point



SU(3) pot. + phys. mass

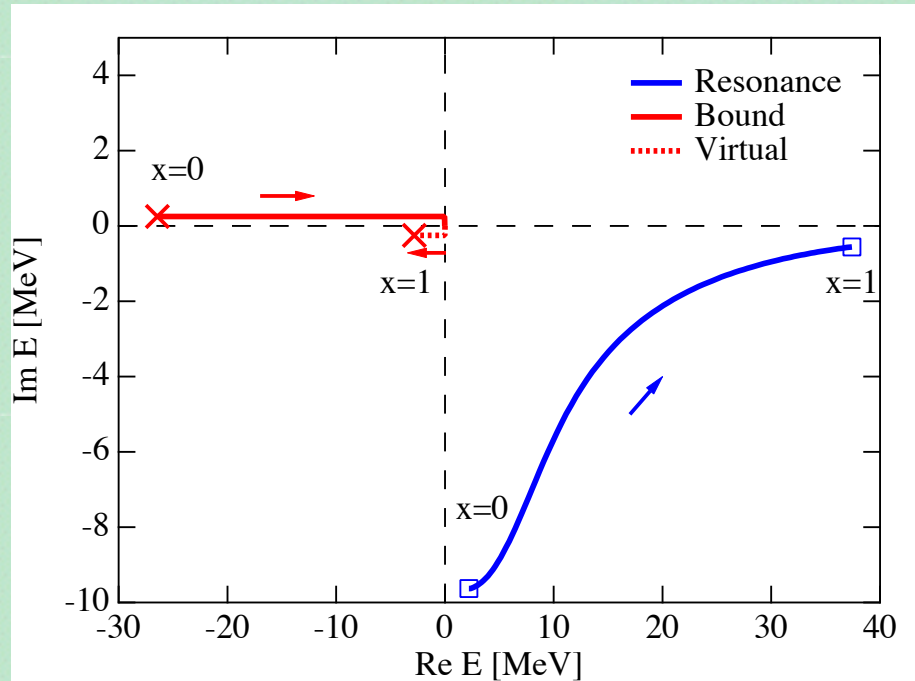


HAL QCD, T. Inoue *et al.*,
Nucl. Phys. A881, 28 (2012)

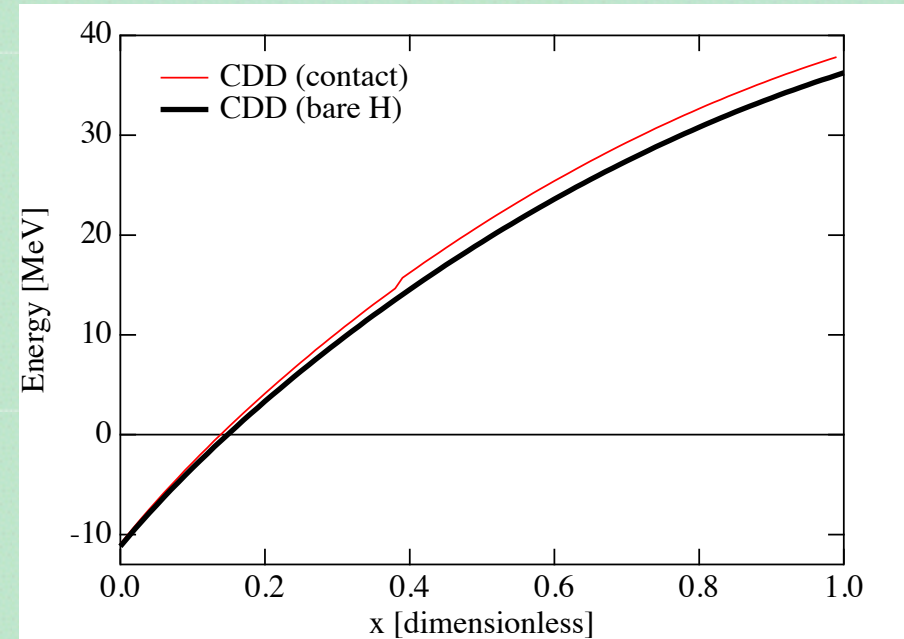
- no bound H, but a **resonance** below $N\Xi$ threshold
- **attractive** scattering length: $a_{\Lambda\Lambda} = -3.2$ fm
- **Ramsauer-Townsend effect near resonance** : $\delta = \pi \rightarrow f(E) = 0$
 ← remnant of the CDD pole

Λ amplitude : Physical point

pole trajectory



CDD pole trajectory

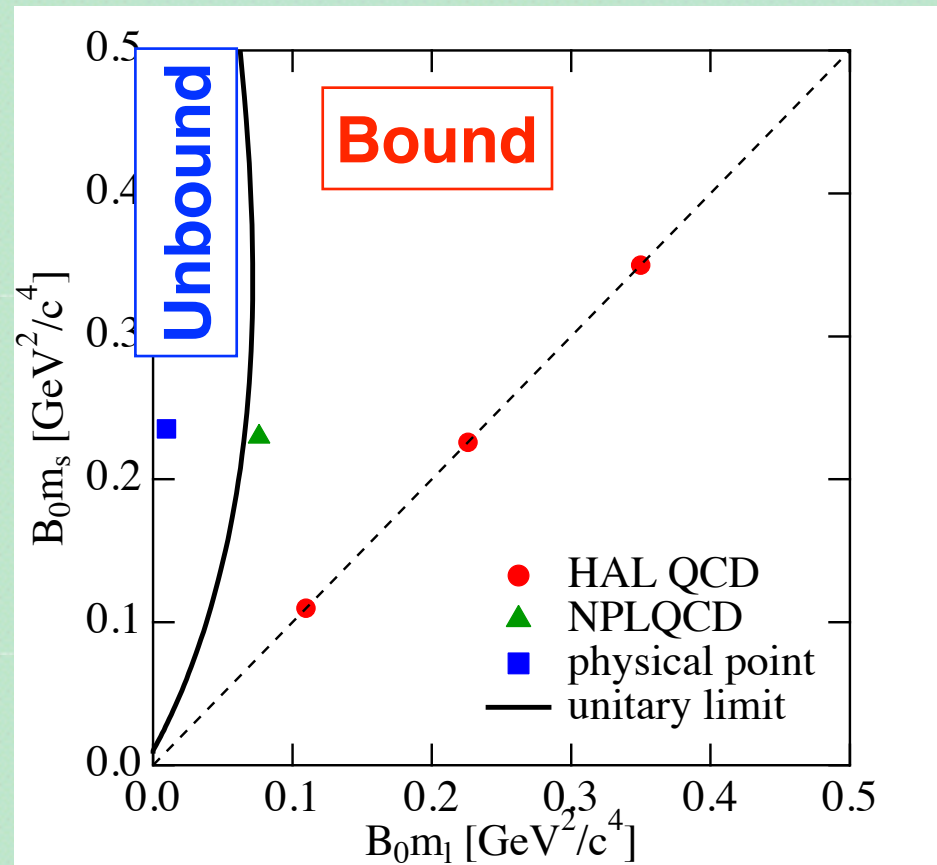
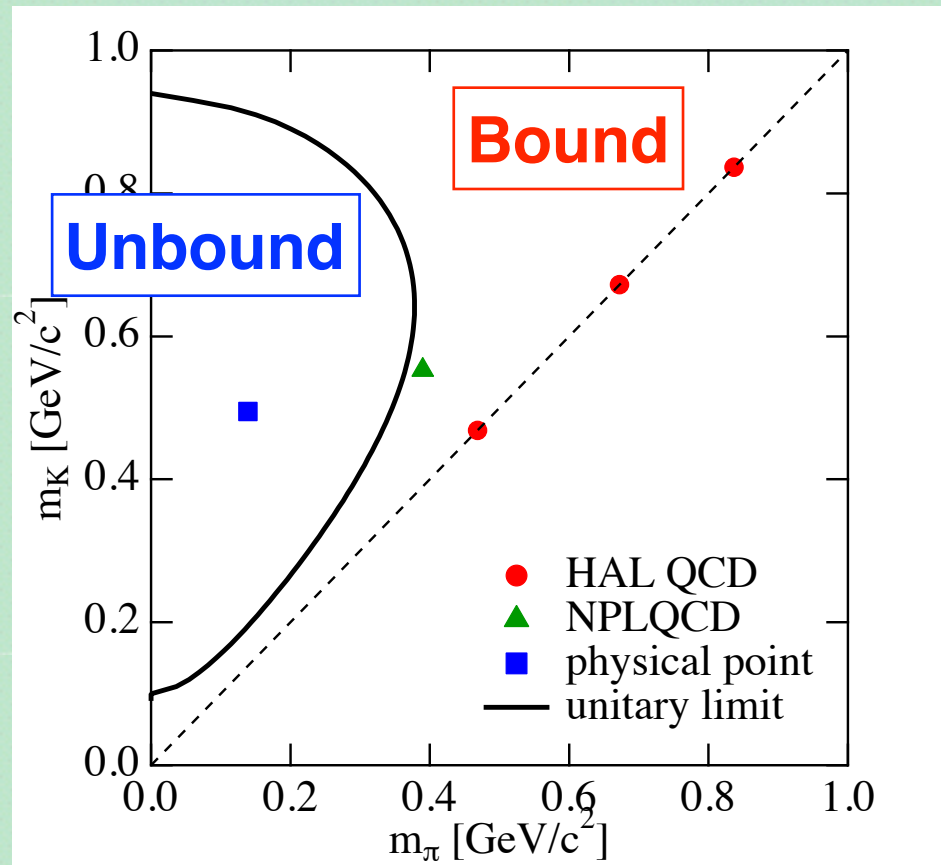


- pole is not continuously connected (shadow pole)
- CDD pole is continuously connected

Extrapolation and unitary limit

Extrapolation in the NGboson/quark mass plane

$$B_0 m_l = \frac{m_\pi^2}{2}, \quad B_0 m_s = m_K^2 - \frac{m_\pi^2}{2}$$

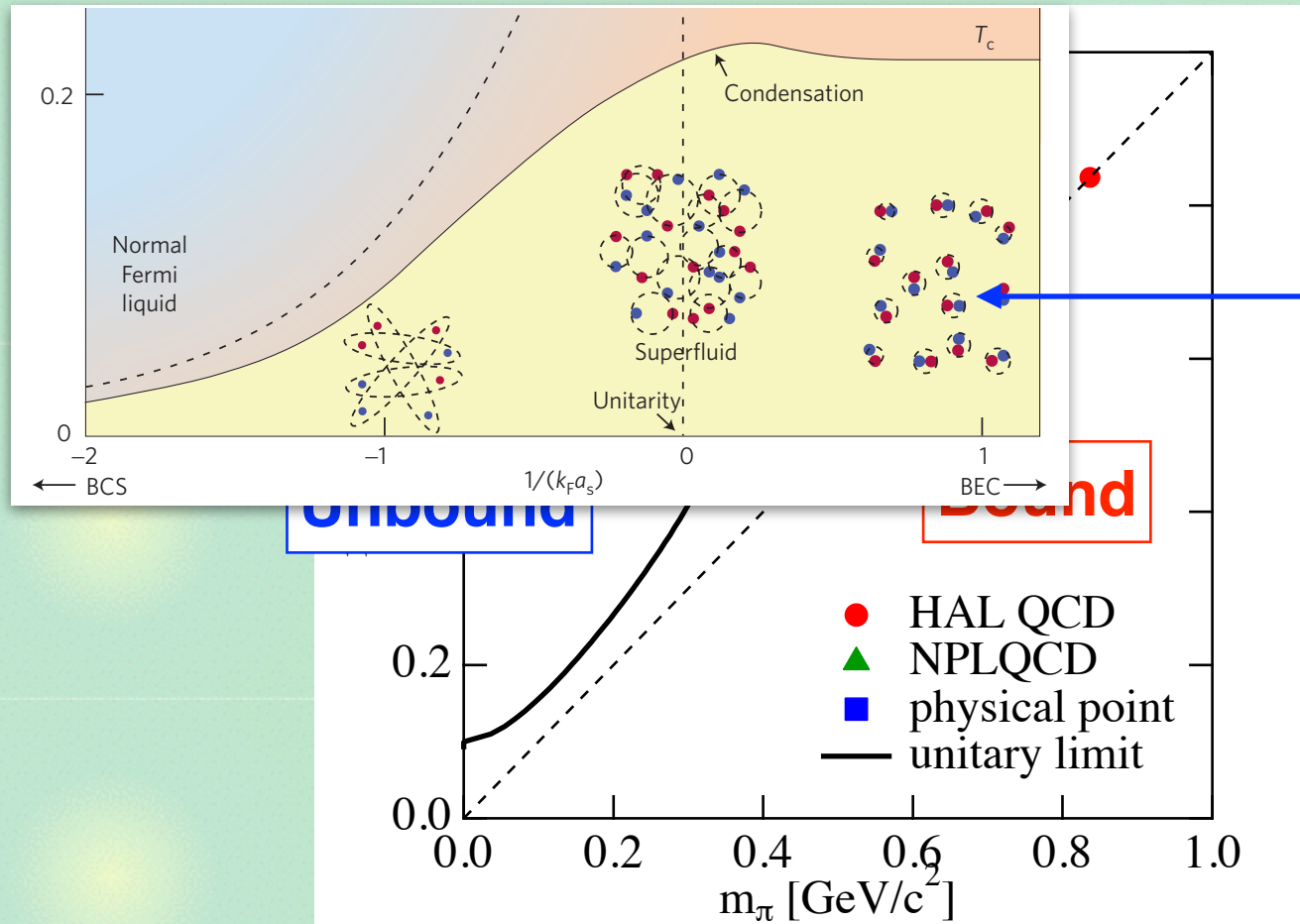


- unitary limit between SU(3) limit and physical point

Implication to many-body system

Many-body system of Λ baryons: BEC-BCS crossover

W. Zwerger, *Lect. Notes Phys.* 836, 1 (2012); M. Randeria, *Nature Phys.* 6, 561 (2010)







**Superfluid of
H-dibaryons
= "H-matter"**

R. Tamagaki,
Prog. Theor. Phys.
85, 321 (1991)

- "H-matter" may be realized with unphysical quark masses.

Summary

-  We study the quark mass dependence of the H-dibaryon and the $\Lambda\Lambda$ interaction using EFT.
-  SU(3) limit: **bound H** with attractive scattering length \leftarrow **CDD pole below the threshold**.
-  Physical point: **Ramsauer-Townsend effect** near **resonance H** \leftarrow remnant of the CDD pole.
-  The $\Lambda\Lambda$ scattering undergoes the **unitary limit** between SU(3) limit and physical point.