Quark mass dependence of the ΛΛ interaction

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2016, Mar. 24th

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Result

- $\Lambda\Lambda$ **amplitude: SU(3) limit / physical point**

- **Extrapolation in quark mass plane**

Summary

H-dibaryon and ΛΛ interaction

H-dibaryon: uuddss **bound state in quark model**

R.L. Jaffe, Phys. Rev. Lett. 38, 195 (1977)

Experiments

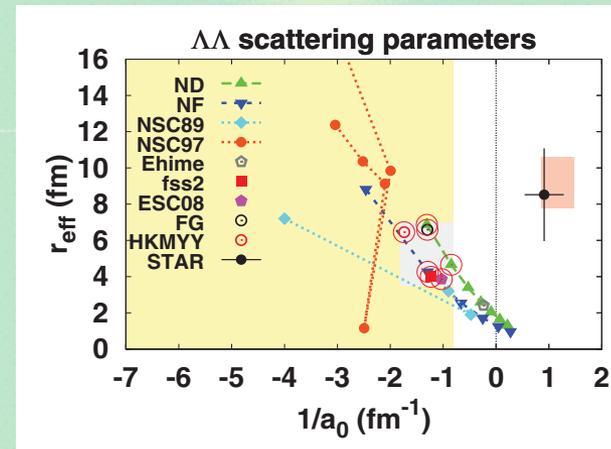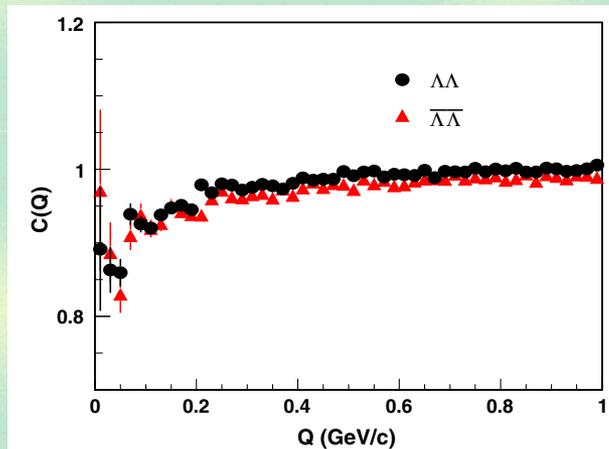## - Nagara event: no deeply bound H

H. Takahashi, *et al.*, Phys. Rev. Lett. 87, 212502 (2001)

- RHIC-STAR: ΛΛ correlation —> scattering length

L. Adamczyk, *et al.*, Phys. Rev. Lett. 114, 022301 (2015)

K. Morita, T. Furumoto, A. Ohnishi, Phys. Rev. C 91, 024916 (2015)

Lattice QCD and quark mass dependence

Bound H-dibaryon at unphysical quark masses

HAL QCD, T. Inoue *et al.*, Phys. Rev. Lett. 106, 162002 (2011);
NPLQCD, S. Beane *et al.*, Phys. Rev. Lett. 106, 162001 (2011);
HAL QCD, T. Inoue *et al.*, Nucl. Phys. A881, 28 (2012); …

- Physical point simulation is ongoing.

- Extrapolation: unbound at phys. point

S. Shanahan, A. Thomas, R. Young, Phys. Rev. Lett. 107, 092004 (2011);
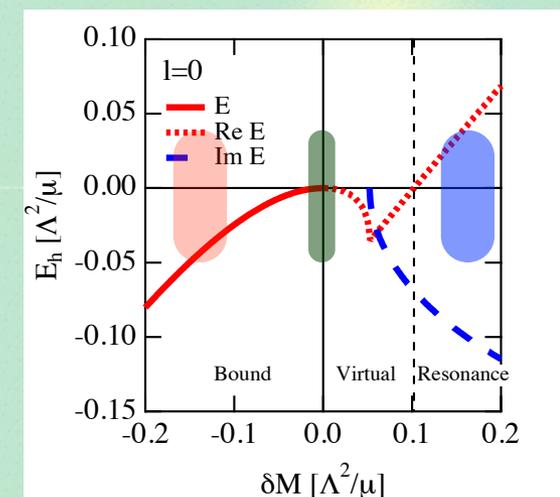J. Haidenbauer, U.G. Meissner, Phys. Lett. B 706, 100 (2011)
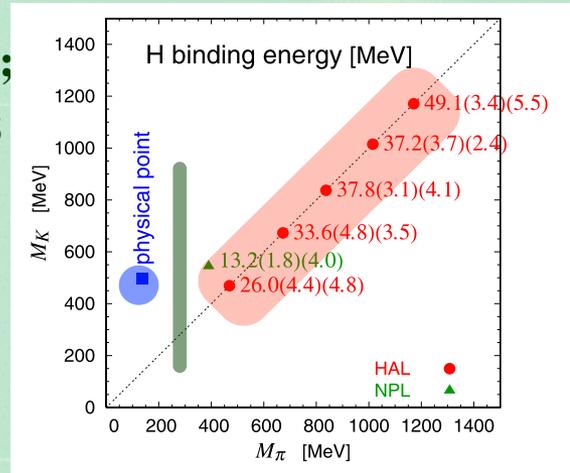
Near-threshold scaling in s-wave

T. Hyodo, Phys. Rev. C90, 055208 (2014)

- virtual state

- unitary limit (large scattering length)

Unitary limit at unphysical quark masses?

Purpose of this talk

- **How does the H-dibaryon bound state in the ΛΛ scattering change along with the variation of the quark masses?**

- **Input: three lightest lattice data in SU(3) limit.**

- **Effective framework which describes the ΛΛ scattering in a relatively wide range of quark masses.**

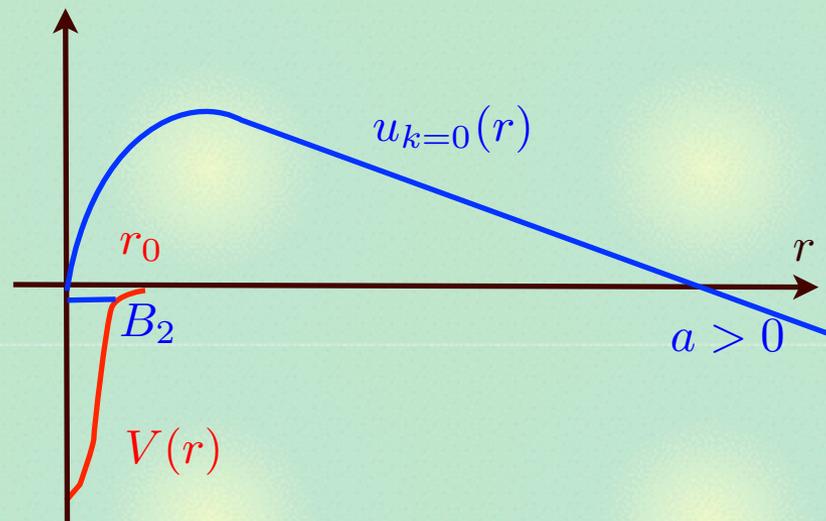- **(Precise ΛΛ interaction at physical point may be studied by lattice QCD / systematic ChPT.)**

Low-energy baryon-baryon scattering

Length scales in the SU(3) limit

HAL QCD, T. Inoue *et al*., Nucl. Phys. A881, 28 (2012)

- **Interaction range by NG boson exchange:** $r^0 \sim$ 0.24-0.42 **fm**

- **large scattering length:** $a \sim$ 1.2-1.7 **fm**

- **large radius <- small binding energy:** 0.77-1.14 **fm**

$u_{k=0}(r)$

r_0

r

B_2

$a > 0$

$V(r)$

At low energy, the interaction can be treated as point like.
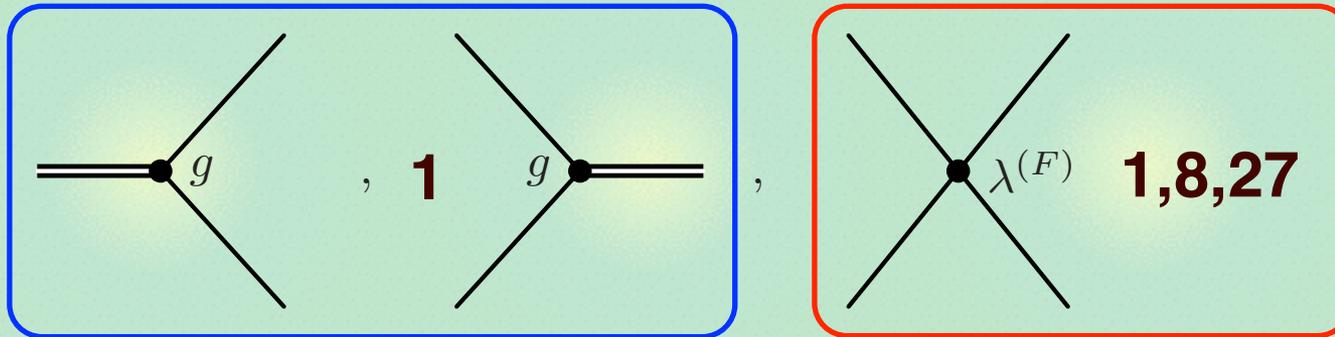
Effective Lagrangian

Low energy effective Lagrangian with contact interactions

c.f. D.B. Kaplan, Nucl. Phys. B494, 471 (1997)

$$\mathcal{L}_{\text{free}} = \sum_{a=1}^{4} \sum_{\sigma=\uparrow,\downarrow} B_{a,\sigma}^{\dagger} \left(i\frac{\partial}{\partial t} + \frac{\nabla^2}{2M_a} + \delta_a \right) B_{a,\sigma} + H^{\dagger} \left(i\frac{\partial}{\partial t} + \frac{\nabla^2}{2M_H} + \nu \right) H$$

$$\mathcal{L}_{\text{int}} = -g[D^{(1)\dagger}H + H^{\dagger}D^{(1)}] - \lambda^{(1)}D^{(1)\dagger}D^{(1)} - \lambda^{(8)}D^{(8)\dagger}D^{(8)} - \lambda^{(27)}D^{(27)\dagger}D^{(27)}$$

$$D^{(F)} = [BB]_{J=0,S=-2,I=0}^{(F)}$$

g , **1** g , $\lambda^{(F)}$ **1,8,27**

Length scales at the physical point

- **No π exchange in ΛΛ. π exchange in NΞ (ΛΛ + 25 MeV)**

—> **safely applicable below NΞ threshold**

Low energy scattering amplitude

Coupled-channel scattering amplitude (i=ΛΛ, NΞ, ΣΣ**)**

$$f_{ii}(E) = \frac{\mu_i}{2\pi}[(\mathcal{A}^{\text{tree}}(E))^{-1} + I(E)]_{ii}^{-1}$$

$$\mathcal{A}_{ij}^{\text{tree}}(E) = \quad i \quad \diagup\!\!\!\!\diagdown \quad j \quad + \quad i \quad \diagup\!\!\!\!\diagdown\!=\!\diagdown\!\!\!\!\diagup \quad j$$

$$= -\left(V_{ij} + \frac{g^2 d_i^\dagger d_j}{E - \nu + i0^+}\right), \quad V = U^{-1}\begin{pmatrix}\lambda^{(1)} & & \\ & \lambda^{(8)} & \\ & & \lambda^{(27)}\end{pmatrix}U, \quad d = \begin{pmatrix}-\sqrt{\frac{1}{8}} \\ -\sqrt{\frac{1}{2}} \\ \sqrt{\frac{3}{8}}\end{pmatrix}$$

$$I_i(E) = \quad \bullet\!\!\bigcirc\!\!\bullet \; i$$

$$= \frac{\mu_i}{\pi^2}\left(-\Lambda + k_i \operatorname{artanh}\frac{\Lambda}{k_i}\right), \quad k_i = \sqrt{2\mu_i(E - \Delta_i)}$$

EFT describes the low energy scattering for a given (m_l**, **m_s**).**

- scattering length, bound state pole, …

- Quark mass dep. —> baryon masses and couplings λ

Modeling quark mass dependence

"Quark masses" via GMOR relation

$$B_0 m_l = \frac{m_\pi^2}{2}, \quad B_0 m_s = m_K^2 - \frac{m_\pi^2}{2}$$
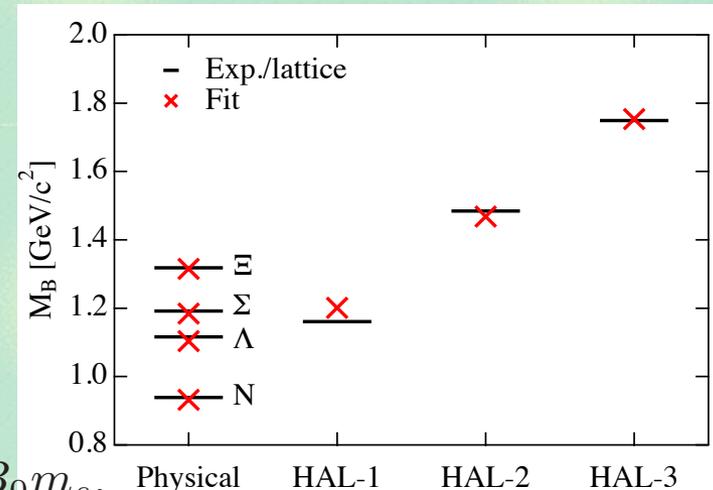
$$B_0 = -\frac{\langle \bar{q}q \rangle}{3F_0^2}$$

Baryon masses: linear in m_q

$$M_N(m_l, m_s) = M_0 - (2\alpha + 2\beta + 4\sigma)B_0 m_l - 2\sigma B_0 m_s,$$

$$M_\Lambda(m_l, m_s) = M_0 - (\alpha + 2\beta + 4\sigma)B_0 m_l - (\alpha + 2\sigma)B_0 m_s,$$

$$M_\Sigma(m_l, m_s) = M_0 - \left(\frac{5}{3}\alpha + \frac{2}{3}\beta + 4\sigma\right)B_0 m_l - \left(\frac{1}{3}\alpha + \frac{4}{3}\beta + 2\sigma\right)B_0 m_s,$$

$$M_\Xi(m_l, m_s) = M_0 - \left(\frac{1}{3}\alpha + \frac{4}{3}\beta + 4\sigma\right)B_0 m_l - \left(\frac{5}{3}\alpha + \frac{2}{3}\beta + 2\sigma\right)B_0 m_s$$

- three mass difference by (α, β) —> GMO relation

- fit to experiment/lattice —> reasonable

HAL QCD, T. Inoue *et al.*, Nucl. Phys. A881, 28 (2012)

Modeling quark mass dependence

Coupling constants <— scattering length in SU(3) limit
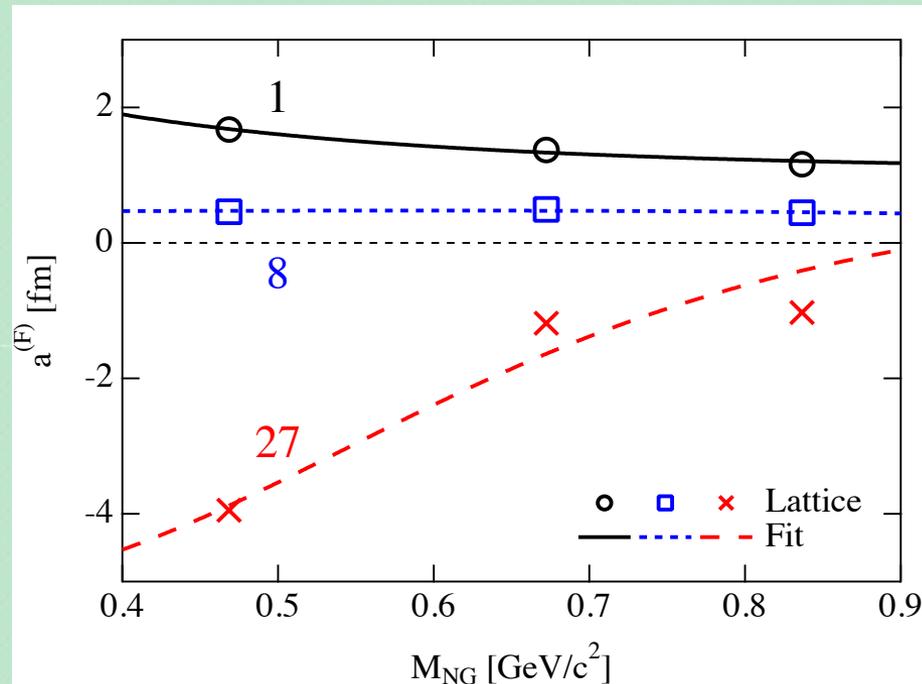
T. Inoue, private communication.

- **1: bound, 8: repulsive, 27: attractive**

- **This talk: linear in m_q, no bare H**

$$\lambda^{(F)}(m_l, m_s) = \lambda_0^{(F)} + \lambda_1^{(F)} B_0 (2m_l + m_s)$$
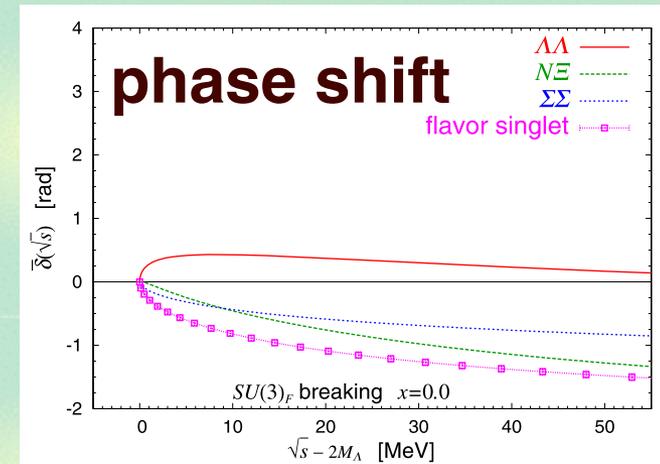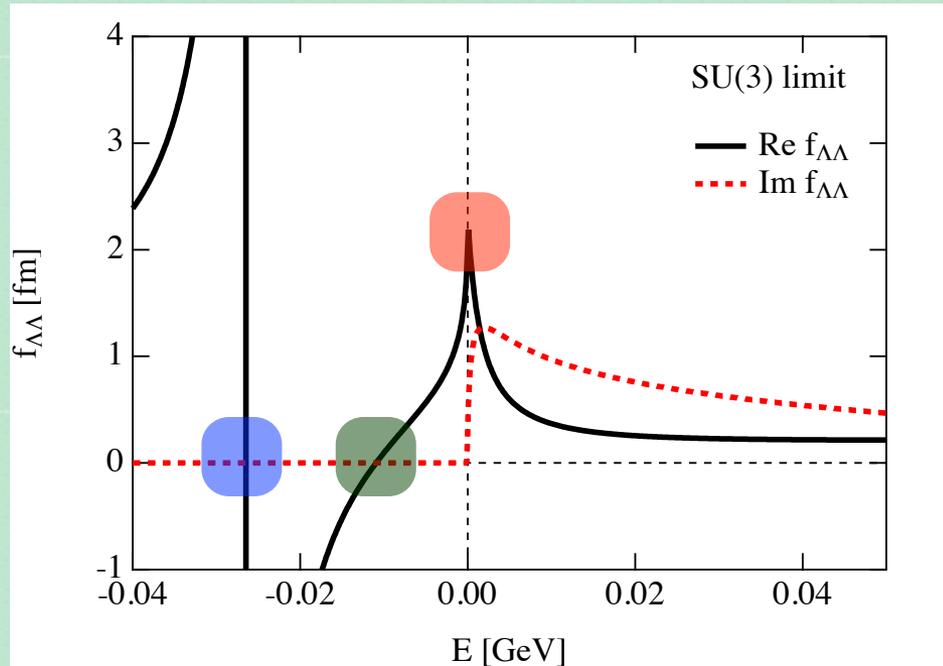
$$g(m_l, m_s) = 0$$

repulsive

attractive

∧∧ amplitude : SU(3) limit

∧∧ scattering amplitude in the SU(3) limit

c.f. HAL QCD, T. Inoue *et al.*,
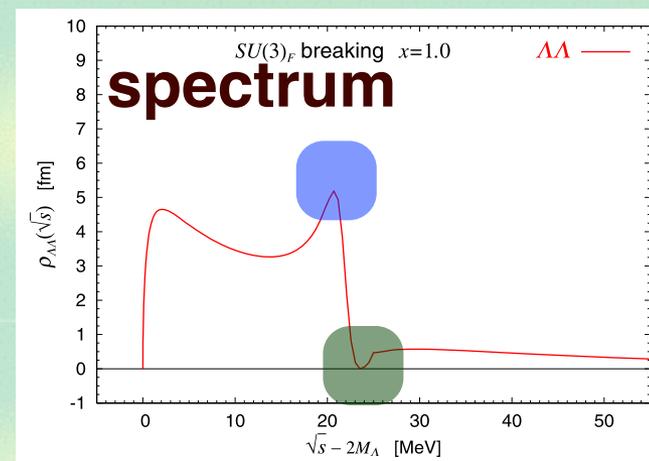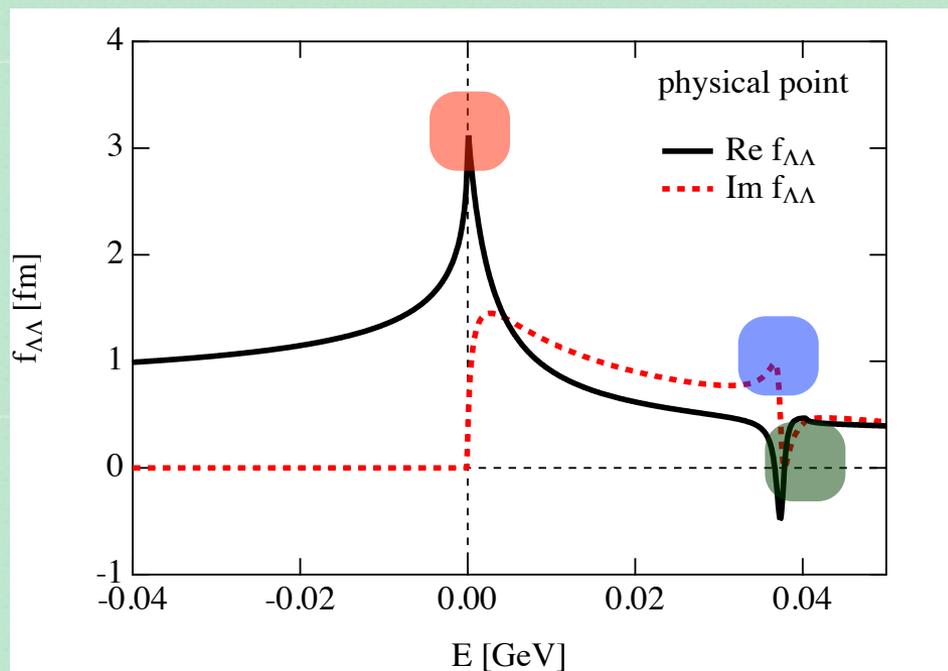Nucl. Phys. A881, 28 (2012)

- **bound H <— bound state in 1**

- **attractive scattering length** $a = -f(E=0)$ **<— attraction in 27**

$$f_{\Lambda\Lambda}(E) = \frac{1}{8}f^{(1)}(E) + \frac{1}{5}f^{(8)}(E) + \frac{27}{40}f^{(27)}(E)$$

- **CDD pole below threshold:** $f(E)=0$ **—> ERE breaks down.**

ΛΛ amplitude : Physical point

ΛΛ scattering amplitude at the physical point
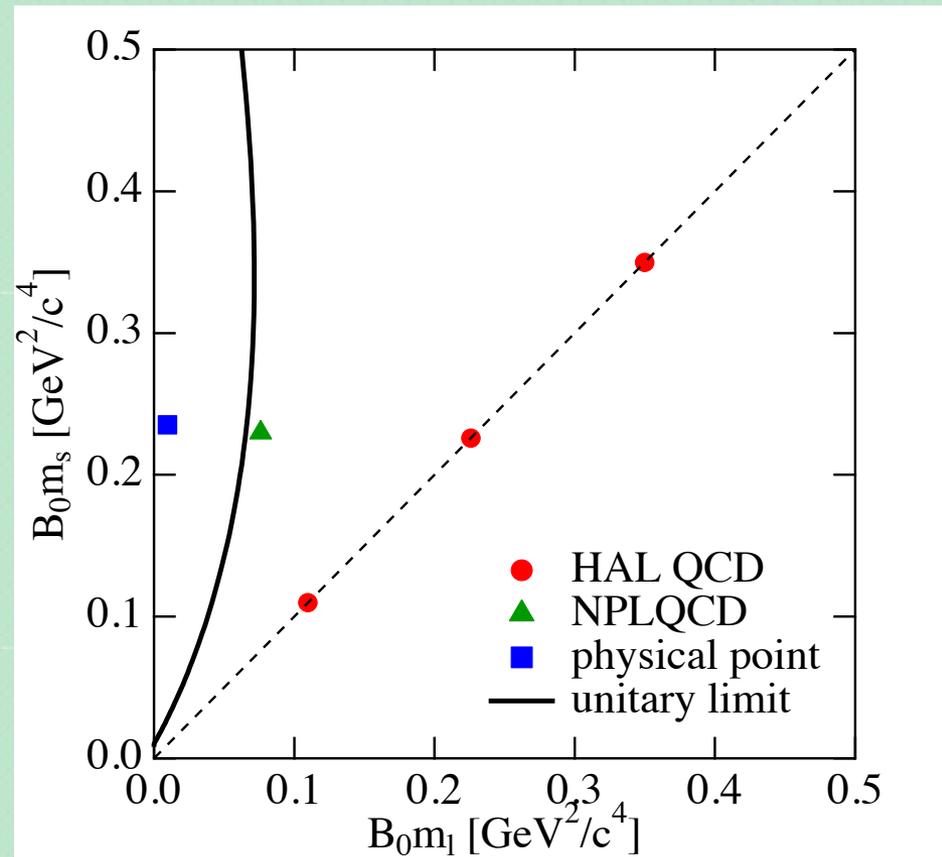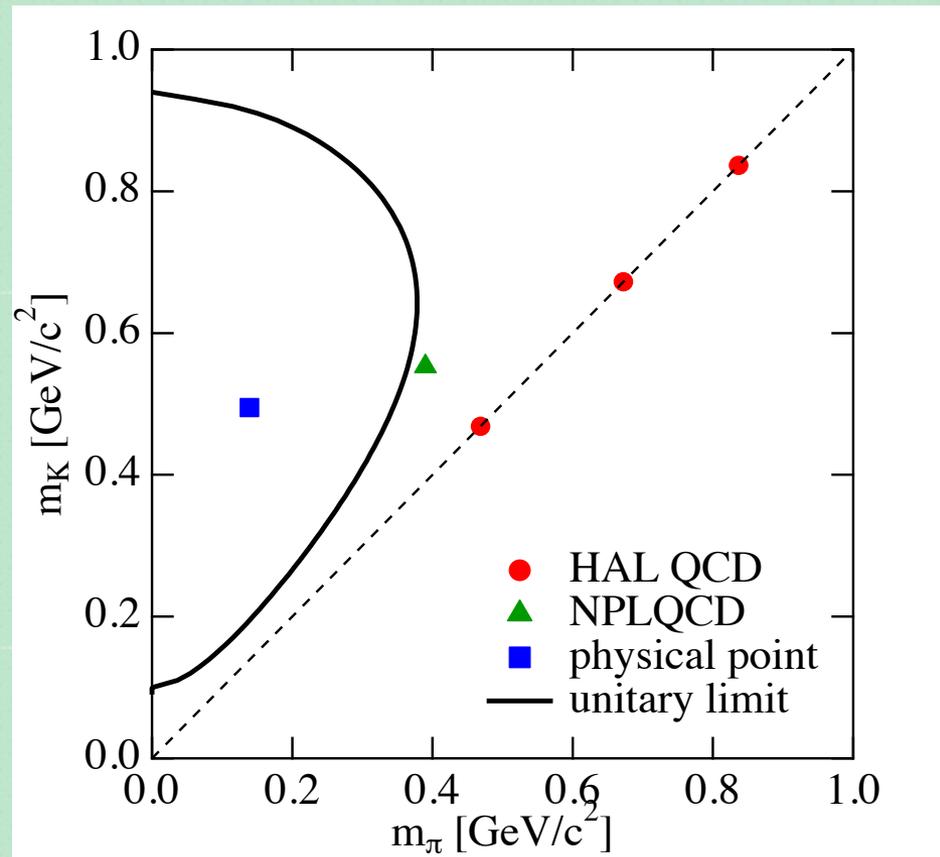

c.f. HAL QCD, T. Inoue *et al.*,
Nucl. Phys. A881, 28 (2012)

- no bound H, but a **resonance**

- **attractive** scattering length: $a_{ΛΛ}$ = -3.2 fm

- **Ramsauer-Townsend effect** near resonance : $\delta=\pi$ -> $f(E)=0$
  <— remnant of the CDD pole

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Extrapolation and unitary limit

Extrapolation in the NGboson/quark mass plane

$$B_0 m_l = \frac{m_\pi^2}{2}, \quad B_0 m_s = m_K^2 - \frac{m_\pi^2}{2}$$

- unitary limit between SU(3) limit and physical point

Summary

We study the quark mass dependence of the H-dibaryon and the ΛΛ interaction using EFT.

SU(3) limit: **bound H** with attractive scattering length <— **CDD pole below the threshold.**

Physical point: **Ramsauer-Townsend effect** near **resonance H** <— remnant of the CDD pole.

The ΛΛ scattering undergoes the **unitary limit** between SU(3) limit and physical point.

Y. Yamaguchi, T. Hyodo, in preparation.