Compositeness of hadrons and near-threshold dynamics

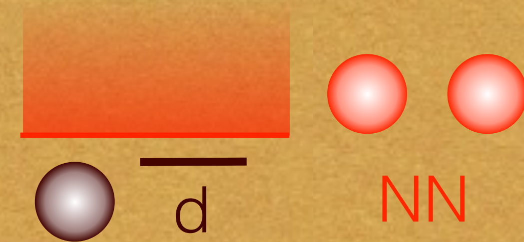
Tetsuo Hyodo

Yukawa Institute for Theoretical Physics, Kyoto Univ.

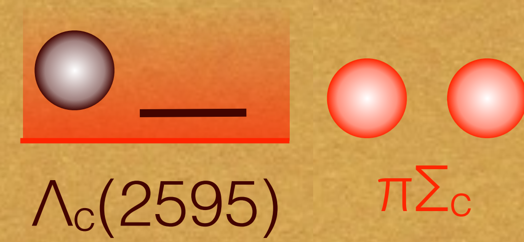
2015, Sep. 10th

Contents

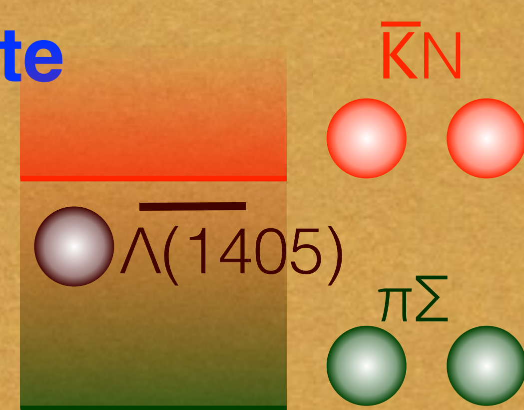
- **Introduction: compositeness of hadrons**
- **Near-threshold bound state**
  - d **is a** NN **molecule.**
  - S. Weinberg, Phys. Rev. 137, B672 (1965)



- **Near-threshold resonance**
  - **Is** $\Lambda_c(2595)$ **a** $\pi\Sigma_c$ **molecule?**
  - T. Hyodo, Phys. Rev. Lett. 111, 132002 (2013)



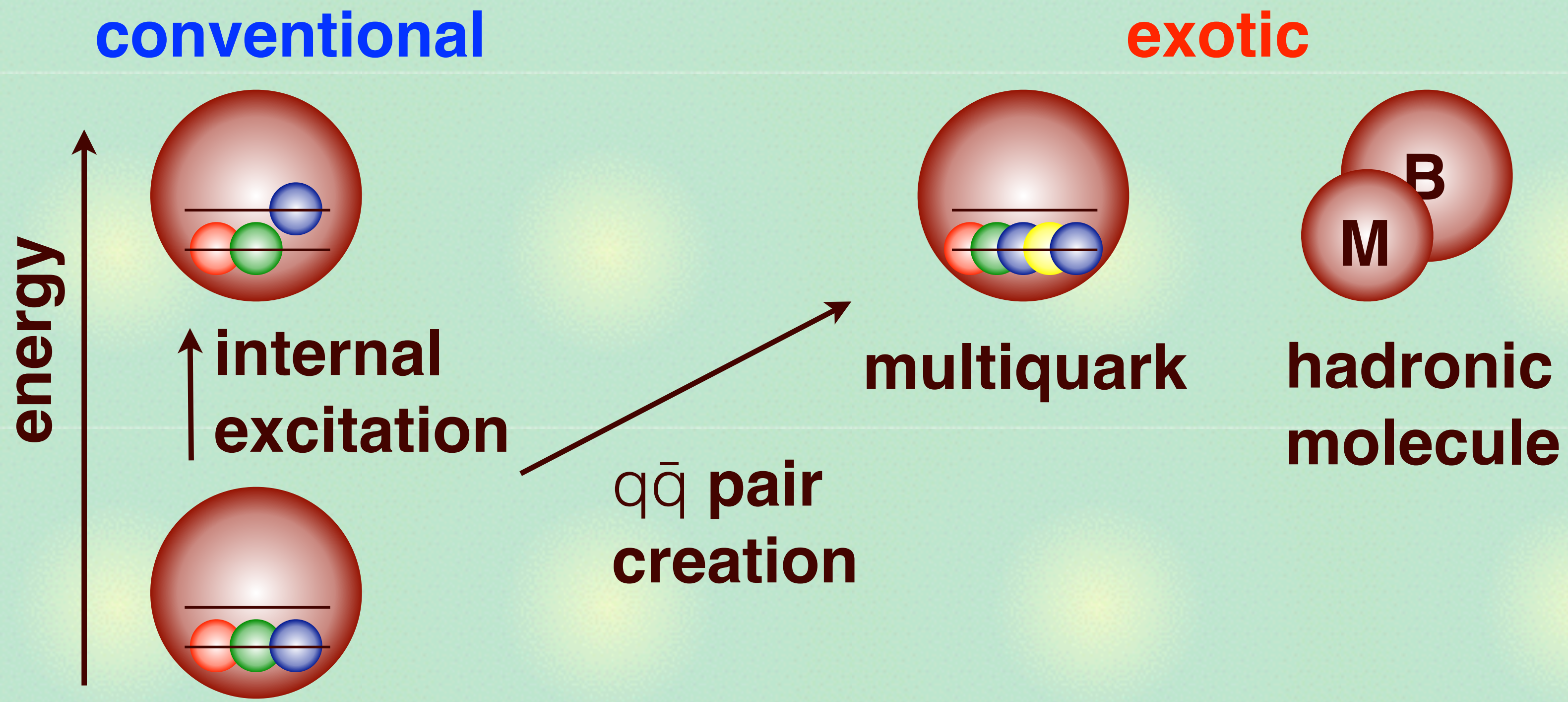
- **Near-threshold quasi-bound state**
  - **Is** $\Lambda(1405)$ **a** $\bar{K}N$ **molecule?**
  - Y. Kamiya, T. Hyodo, arXiv:1509.00146 [hep-ph]



- **Summary**

Exotic structure of hadrons

Various excitations of baryons



Physical state: superposition of 3q, 5q, MB, ...

$$|\,\Lambda(1405)\,\rangle \stackrel{?}{=} \underline{N_{3q}}|\,uds\,\rangle + \underline{N_{5q}}|\,uds\;q\bar{q}\,\rangle + \underline{N_{\bar{K}N}}|\,\bar{K}N\,\rangle + \cdots$$

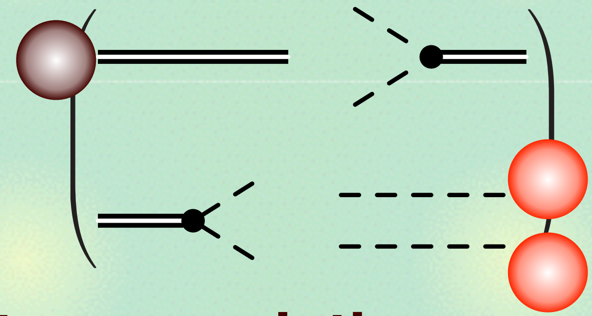
probability?

- What is the **appropriate basis**?
- How can we **interpret** the complex weights?

Compositeness and elementariness

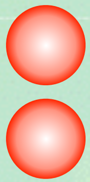
Example: Coupled-channel Hamiltonian model

$$\begin{pmatrix} M_0 & \hat{V} \\ \hat{V} & \frac{p^2}{2\mu} \end{pmatrix} |\,\Psi\,\rangle = E|\,\Psi\,\rangle, \quad |\,\Psi\,\rangle = \begin{pmatrix} c(E)|\,\psi_0\,\rangle \\ \chi_E(\boldsymbol{p})|\,\boldsymbol{p}\,\rangle \end{pmatrix}$$



- Bound state normalization + completeness relation

$$\langle\,\Psi\,|\,\Psi\,\rangle = 1 \qquad 1 = |\,\psi_0\,\rangle\langle\,\psi_0\,| + \int d^3q|\,\boldsymbol{q}\,\rangle\langle\,\boldsymbol{q}\,|$$

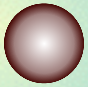


$$1 = \left|\langle\,\Psi\,|\begin{pmatrix} |\,\psi_0\,\rangle \\ 0 \end{pmatrix}\right|^2 + \int d^3q\left|\langle\,\Psi\,|\begin{pmatrix} 0 \\ |\,\boldsymbol{q}\,\rangle \end{pmatrix}\right|^2 \equiv Z + X$$

← **compositeness**

bare state contribution — **continuum contribution**

↑ **elementariness (field renormalization constant)**

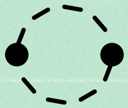


Z, X**: real and nonnegative —> probabilistic interpretation**

Z in model calculations

In general, Z is model dependent (~ potential, wave function)

$$Z(B) = \frac{1}{1 - \frac{d}{dE}\int \frac{|\langle \psi_0 | \hat{V} | \boldsymbol{q} \rangle|^2}{E - q^2/(2\mu) + i0^+} d^3q \Big|_{E=-B}} \equiv \frac{1}{1 - \Sigma'(-B)}$$

$\Sigma(E) \sim$ 

- Z can be calculated by employing models.

Baryons	Z	$\|Z\|$	Mesons	Z	$\|Z\|$
$\Lambda(1405)$ higher pole (Ref. 58)	$0.00 + 0.09i$	0.09	$f_0(500)$ or σ (Ref. 58)	$1.17 - 0.34i$	1.22
$\Lambda(1405)$ lower pole (Ref. 58)	$0.86 - 0.40i$	0.95	$f_0(980)$ (Ref. 58)	$0.25 + 0.10i$	0.27
$\Delta(1232)$ (Ref. 60)	$0.43 + 0.29i$	0.52	$a_0(980)$ (Ref. 58)	$0.68 + 0.18i$	0.70
$\Sigma(1385)$ (Ref. 60)	$0.74 + 0.19i$	0.77	$\rho(770)$ (Ref. 55)	$0.87 + 0.21i$	0.89
$\Xi(1535)$ (Ref. 60)	$0.89 + 0.99i$	1.33	$K^*(892)$ (Ref. 59)	$0.88 + 0.13i$	0.89
Ω (Ref. 60)	0.74	0.74			
$\Lambda_c(2595)$ (Ref. 56)	$1.00 - 0.61i$	1.17			

[55] F. Aceti, E. Oset, Phys. Rev. D86, 014012 (2012), [56] T. Hyodo, Phys. Rev. Lett. 111, 132002 (2013), [58] T. Sekihara, T. Hyodo, Phys. Rev. C 87, 045202 (2012),
[59] C.W. Xiao, F. Aceti, M. Bayar, Eur. Phys. J. A49, 22 (2013), [60], F. Aceti, *et al.*, Eur. Phys. J. A50, 57 (2014).

Model-independent determination?

Weak binding limit

Z **of weakly-bound ($R \gg R_{typ}$) s-wave state <— observables.**

S. Weinberg, Phys. Rev. 137, B672 (1965);
T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)

$$a = \frac{2(1-Z)}{2-Z}R + \mathcal{O}(R_{\text{typ}}), \quad r_e = \frac{-Z}{1-Z}R + \mathcal{O}(R_{\text{typ}}),$$

$$\begin{cases} a \sim R_{\text{typ}} \ll -r_e & \text{(elementary dominance)}, \\ a \sim R \gg r_e \sim R_{\text{typ}} & \text{(composite dominance)}. \end{cases}$$

a **: scattering length,** r_e **: effective range**
$R = (2\mu B)^{-1/2}$ **: radius <— binding energy**
R_{typ} **: typical length scale of the interaction**

- **Deuteron is NN composite ($a \sim R \gg r_e$), only from observables, without referring to the nuclear force/wave function.**

- **Derivation by $1/R$ expansion of the scattering length**

T. Sekihara, T. Hyodo, D. Jido, PTEP2015, 063D04 (2015)
Y. Kamiya, T. Hyodo, arXiv:1509.00146 [hep-ph]

Short summary for bound states

Appropriate basis for bound states

elementary Z
- uududd
- ΔΔ - πNN - ...

composite X
- NN(s-wave)

Conditions for model-independent formula:

- **stable s-wave bound state near threshold**

Applicability:

- **Deuteron only!**

Application to exotic hadrons

—> Generalization to unstable particles

Generalization to unstable states

Compositeness of bound states

$$Z(B) = \frac{1}{1 - \Sigma'(-B)}$$

Naive generalization to resonances:

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C85, 015201 (2012)

$$\underline{Z(E_R)} = \frac{1}{1 - \Sigma'(E_R)}$$

complex ↑ complex

- Problem of interpretation (not probability!)

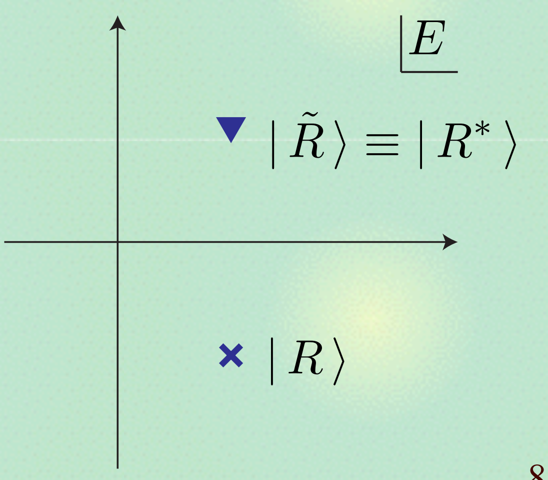
<— Normalization of resonances

$$\langle\, R \,|\, R \,\rangle \to \infty, \quad \langle\, \tilde{R} \,|\, R \,\rangle = 1$$

$$1 = \underline{\langle\, \tilde{R} \,|\, \psi_0 \,\rangle \langle\, \psi_0 \,|\, R \,\rangle} + \int d\boldsymbol{p} \langle\, \tilde{R} \,|\, \boldsymbol{p} \,\rangle \langle\, \boldsymbol{p} \,|\, R \,\rangle$$

complex

$$\langle\, \tilde{R} \,|\, \psi_0 \,\rangle = \langle\, \psi_0 \,|\, R \,\rangle \neq \langle\, \psi_0 \,|\, R \,\rangle^*$$



T. Berggren, Nucl. Phys. A 109, 265 (1968)

Near-threshold resonances

Weak binding limit for bound states

- Model-independent (no potential, wavefunction, ...)
- Related to experimental observables

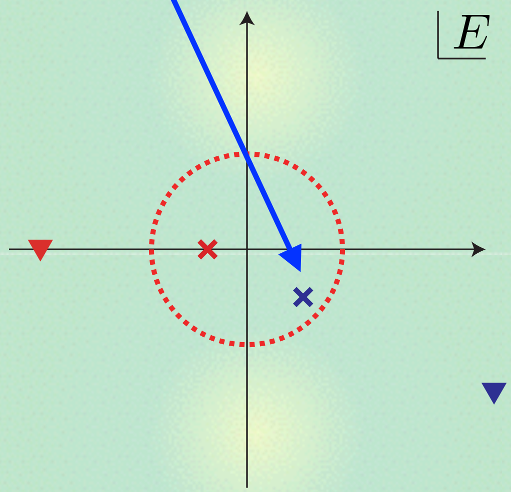
What about near-threshold resonances (~ small binding)?

T. Hyodo, Phys. Rev. Lett. 111, 132002 (2013)

Effective range expansion

$$f(p) = \left(-\frac{1}{a} + \frac{r_e}{2}p^2 - ip\right)^{-1}$$

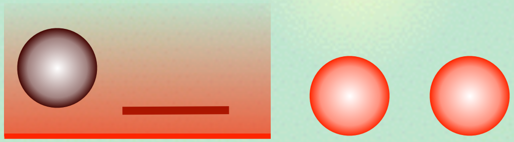
$$p^{\pm} = \frac{i}{r_e} \pm \frac{1}{r_e}\sqrt{\frac{2r_e}{a} - 1}$$



Resonance pole position —> (a, r_e) —> elementariness

Application: $\Lambda_c(2595)$

Pole position of $\Lambda_c(2595)$ in $\pi\Sigma_c$ scattering



$\Lambda_c(2595)$ $\pi\Sigma_c$

- central values in PDG

$$E = 0.67\ \text{MeV}, \quad \Gamma = 2.59\ \text{MeV} \qquad p^{\pm} = \sqrt{2\mu(E \mp i\Gamma/2)}$$

- deduced threshold parameters of $\pi\Sigma_c$ scattering

$$a = -\frac{p^+ + p^-}{ip^+p^-} = -10.5\ \text{fm}, \quad r_e = \frac{2i}{p^+ + p^-} = -19.5\ \text{fm}$$

- Elementariness Z=1-0.6i cannot be interpreted.

Large negative effective range

$$a \sim R_{\text{typ}} \ll -r_e \quad \text{(elementary dominance)}$$

<— substantial elementary contribution other than $\pi\Sigma_c$
(three-quark, other meson-baryon channel, or ...)

$\Lambda_c(2595)$ **is not likely a $\pi\Sigma_c$ composite**

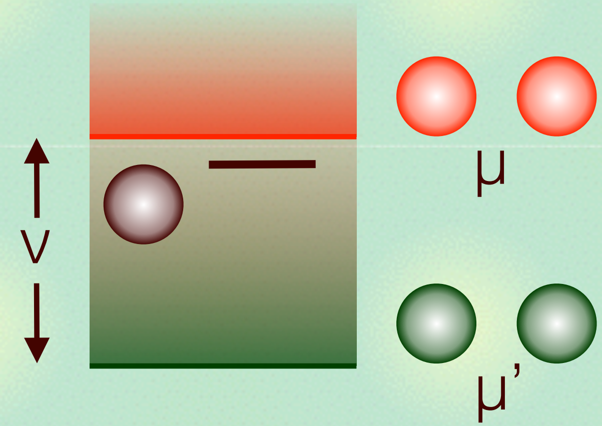
Generalized formula for quasi-bound state

Generalization by Effective Field Theory

Y. Kamiya, T. Hyodo, arXiv:1509.00146 [hep-ph]

$$a = R\left\{ \frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\rm typ}}{R}\right|\right) + \sqrt{\frac{\mu'^3}{\mu^3}}\mathcal{O}\left(\left|\frac{l}{R}\right|^3\right) \right\}$$

$$R = 1/\sqrt{-2\mu E_{QB}}, \quad l = 1/\sqrt{2\mu\nu}$$



- Formula is valid for complex a, R **and** X

Interpretation of $Z + X = 1, \quad Z, X \in \mathbb{C}$

$$\tilde{Z} \equiv \frac{1 - |X| + |Z|}{2}, \quad \tilde{X} \equiv \frac{1 - |Z| + |X|}{2}, \quad U \equiv |Z| + |X| - 1$$

probabilities / **uncertainty**

$$\tilde{Z} + \tilde{X} = 1, \quad \tilde{Z}, \tilde{X} \in [0, 1]$$

c.f. spectral density/unitary transformation

V. Baru, *et al.*, Phys. Lett. B 586, 53 (2004)

Z.H. Guo, J.A. Oller, arXiv:1508.06400 [hep-ph]

Application: Λ(1405)

Recent analyses of Λ(1405) with SIDDHARTA (χ^2/dof ~ 1)

Ref.	E_{QB} (MeV)	a_0 (fm)	$X_{\bar{K}N}$	$\tilde{X}_{\bar{K}N}$	U	$\lvert r_e/a_0\rvert$
[43]	$-10-i26$	$1.39-i0.85$	$1.2+i0.1$	1.0	0.5	0.2
[44]	$-4-i8$	$1.81-i0.92$	$0.6+i0.1$	0.6	0.0	0.7
[45]	$-13-i20$	$1.30-i0.85$	$0.9-i0.2$	0.9	0.1	0.2
[46]	$2-i10$	$1.21-i1.47$	$0.6+i0.0$	0.6	0.0	0.7
[46]	$-3-i12$	$1.52-i1.85$	$1.0+i0.5$	0.8	0.6	0.4

[43] Y. Ikeda, T. Hyodo, W. Weise, Phys. Lett. B706, 63 (2011); Nucl. Phys. A881 98 (2012)
[44] M. Mai, U.-G. Meissner, Nucl. Phys. A900, 51 (2013), [45] Z.H. Guo, J.A. Oller, Phys. Rev. C87, 035202 (2013), [46] M. Mai, U.-G. Meissner, Eur. Phys. J. A 51, 30 (2015)

$$a = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\rm typ}}{R}\right|\right) + \sqrt{\frac{\mu'^3}{\mu^3}}\mathcal{O}\left(\left|\frac{l}{R}\right|^3\right)\right\}$$

- Correction terms and U are small $|R_{\rm typ}/R| \lesssim 0.17, \quad |l/R|^3 \lesssim 0.04$
- $\bar{\rm K}$N compositeness: close to unity

Λ(1405) is dominated by the $\bar{\rm K}$N composite component.

Summary

Model-independent aspect of compositeness

- **Structure of near-threshold bound state:**

  S. Weinberg, Phys. Rev. 137, B672 (1965)

  - Observables (B, a**) —> structure**

- **Near-threshold resonance:**

  T. Hyodo, Phys. Rev. Lett. 111, 132002 (2013)

  - Pole position —> (a, r_e**) —> structure**

  - $\Lambda_c(2595)$ **is not a** $\pi\Sigma_c$ **molecule.**

- **Near-threshold quasi-bound state:**

  Y. Kamiya, T. Hyodo, arXiv:1509.00146 [hep-ph]

  - (Pole position, a**) —> structure**

  - $\Lambda(1405)$ **is a** $\bar{K}N$ **molecule.**