Compositeness of hadrons and near-threshold dynamics

Tetsuo Hyodo

Yukawa Institute for Theoretical Physics, Kyoto Univ.

2015, Jun. 17th

Contents

- **Introduction: compositeness of hadrons**
- **Near-threshold bound state**

  S. Weinberg, Phys. Rev. 137, B672 (1965);
  T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)

- **Near-threshold resonance**

  Is $\Lambda_c(2595)$ a $\pi\Sigma_c$ molecule?

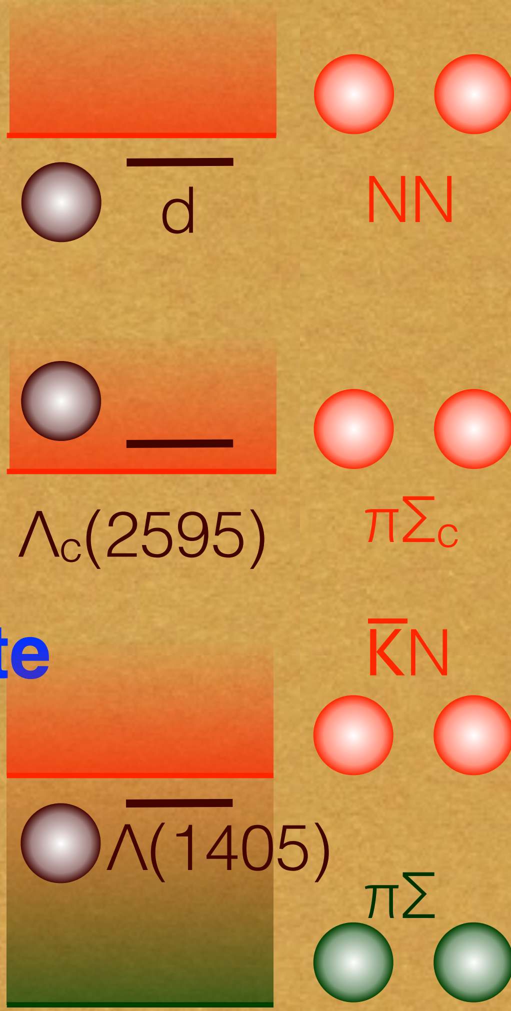
  T. Hyodo, Phys. Rev. Lett. 111, 132002 (2013)

- **Near-threshold quasi-bound state**

  Is $\Lambda(1405)$ a $\bar{K}N$ molecule?

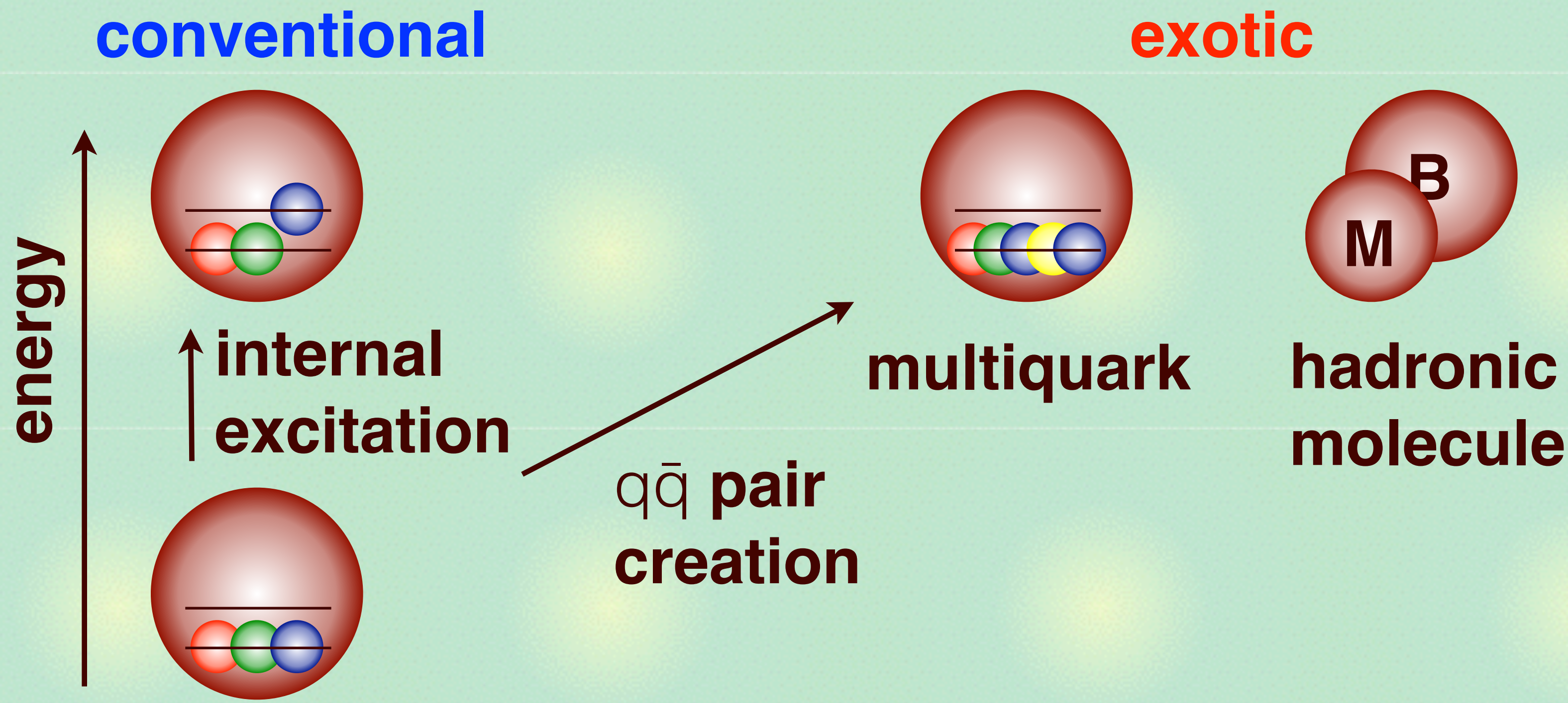
  Y. Kamiya, T. Hyodo, in preparation

- **Summary**



Exotic structure of hadrons

Various excitations of baryons



Physical state: superposition of 3q, 5q, MB, ...

$$|\,\Lambda(1405)\,\rangle = \underline{N_{3q}}|\,uds\,\rangle + \underline{N_{5q}}|\,uds\;q\bar{q}\,\rangle + \underline{N_{\bar{K}N}}|\,\bar{K}N\,\rangle + \cdots$$

Is this relevant strategy?

Ambiguity of definition of hadron structure

Decomposition of hadron “wave function”

$$|\,\Lambda(1405)\,\rangle = N_{3q}|\,uds\,\rangle + N_{5q}|\,uds\ q\bar{q}\,\rangle + N_{\bar{K}N}|\,\bar{K}N\,\rangle + \cdots$$

- N_X = **probability?**

- **5q v.s. MB: double counting (orthogonality)?**

$$\langle\,udsq\bar{q}\,|\,\bar{K}N\,\rangle \neq 0$$

- **3q v.s. 5q: not clearly separated in QCD**

$$\langle\,uds\,|\,udsq\bar{q}\,\rangle \neq 0$$

- **hadron resonances: unstable, finite decay width**

$|\,\Lambda(1405)\,\rangle = ?$ **complex** N_X

What is the suitable basis to classify the hadron structure?

Strategy

Elementary/composite nature of bound states near the lowest energy two-body threshold

elementary Z

- uududd
- ΔΔ
- uududduū
- NN(d-wave)
- …

composite X

- NN(s-wave)

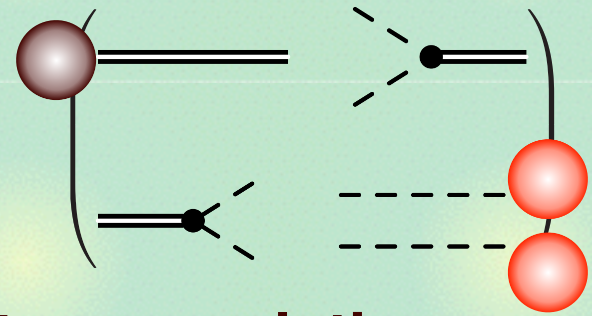
- **orthogonality <— eigenstates of bare Hamiltonian**
- **normalization <— eigenstate of full Hamiltonian**
- **model independence <— low-energy universality**

*** "Elementary" stands for anything other than the composite channel of interest (missing channels, CDD pole, …).**

Formulation

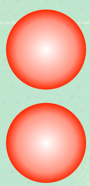
Coupled-channel Hamiltonian (bare state + continuum)

$$\begin{pmatrix} M_0 & \hat{V} \\ \hat{V} & \frac{p^2}{2\mu} \end{pmatrix} |\,\Psi\,\rangle = E|\,\Psi\,\rangle, \quad |\,\Psi\,\rangle = \begin{pmatrix} c(E)|\,\psi_0\,\rangle \\ \chi_E(\boldsymbol{p})|\,\boldsymbol{p}\,\rangle \end{pmatrix}$$



- Bound state normalization + completeness relation

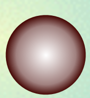
$$\langle\,\Psi\,|\,\Psi\,\rangle = 1 \qquad 1 = |\,\psi_0\,\rangle\langle\,\psi_0\,| + \int d^3q|\,\boldsymbol{q}\,\rangle\langle\,\boldsymbol{q}\,|$$



$$1 = \underbrace{\left|\langle\,\Psi\,|\begin{pmatrix}|\,\psi_0\,\rangle \\ 0\end{pmatrix}\right|^2}_{\text{bare state contribution}} + \underbrace{\int d^3q \left|\langle\,\Psi\,|\begin{pmatrix}0 \\ |\,\boldsymbol{q}\,\rangle\end{pmatrix}\right|^2}_{\text{continuum contribution}} \equiv Z + X$$

X ← **compositeness**

Z ↑ **elementariness (field renormalization constant)**

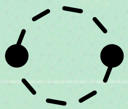


Z, X**: real and nonnegative —> probabilistic interpretation**

Z in model calculations

In general, Z is model dependent (~ potential, wave function)

$$Z(B) = \frac{1}{1 - \frac{d}{dE}\int \frac{|\langle \psi_0 | \hat{V} | \boldsymbol{q} \rangle|^2}{E - q^2/(2\mu) + i0^+} d^3 q \Big|_{E=-B}} \equiv \frac{1}{1 - \Sigma'(-B)}$$

$\Sigma(E) \sim$ 

- Z can be calculated by employing models.

Baryons	Z	$\lvert Z\rvert$	Mesons	Z	$\lvert Z\rvert$
$\Lambda(1405)$ higher pole (Ref. 58)	$0.00 + 0.09i$	0.09	$f_0(500)$ or σ (Ref. 58)	$1.17 - 0.34i$	1.22
$\Lambda(1405)$ lower pole (Ref. 58)	$0.86 - 0.40i$	0.95	$f_0(980)$ (Ref. 58)	$0.25 + 0.10i$	0.27
$\Delta(1232)$ (Ref. 60)	$0.43 + 0.29i$	0.52	$a_0(980)$ (Ref. 58)	$0.68 + 0.18i$	0.70
$\Sigma(1385)$ (Ref. 60)	$0.74 + 0.19i$	0.77	$\rho(770)$ (Ref. 55)	$0.87 + 0.21i$	0.89
$\Xi(1535)$ (Ref. 60)	$0.89 + 0.99i$	1.33	$K^*(892)$ (Ref. 59)	$0.88 + 0.13i$	0.89
Ω (Ref. 60)	0.74	0.74			
$\Lambda_c(2595)$ (Ref. 56)	$1.00 - 0.61i$	1.17			

[55] F. Aceti, E. Oset, Phys. Rev. D86, 014012 (2012), [56] T. Hyodo, Phys. Rev. Lett. 111, 132002 (2013), [58] T. Sekihara, T. Hyodo, Phys. Rev. C 87, 045202 (2012),
[59] C.W. Xiao, F. Aceti, M. Bayar, Eur. Phys. J. A49, 22 (2013), [60], F. Aceti, *et al.*, Eur. Phys. J. A50, 57 (2014).

Model-independent determination?

Weak binding limit

Z of weakly-bound ($R \gg R_{typ}$) s-wave state <— observables.

S. Weinberg, Phys. Rev. 137, B672 (1965);
T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)

$$a = \frac{2(1-Z)}{2-Z}R + \mathcal{O}(R_{\text{typ}}), \quad r_e = \frac{-Z}{1-Z}R + \mathcal{O}(R_{\text{typ}}),$$

a : scattering length, r_e : effective range
$R = (2\mu B)^{-1/2}$: radius <— binding energy
R_{typ} : typical length scale of the interaction

- **Deuteron is NN composite ($Z \sim 0$), only from observables, without referring to the nuclear force/wave function.**

- **A derivation by the expansion of the amplitude:**

T. Sekihara, T. Hyodo, D. Jido, arXiv: 1411.2308 [hep-ph], to appear in PTEP

Scaling limit

Scaling (zero-range) limit: scattering length a ≠ 0, R_{typ} -> 0

E. Braaten, H.-W. Hammer, Phys. Rept. 428, 259 (2006)

- All (2-body) quantities are expressed by a: **universality**

$$\psi(r) = \frac{e^{-r/a}}{\sqrt{2\pi a r}} \qquad B = 1/(2\mu a^2) \quad \Rightarrow \quad R = a \quad \Rightarrow \quad Z = 0$$

- Bound state is always composite in the scaling limit.

T. Hyodo, Phys. Rev. C90, 055208 (2014)

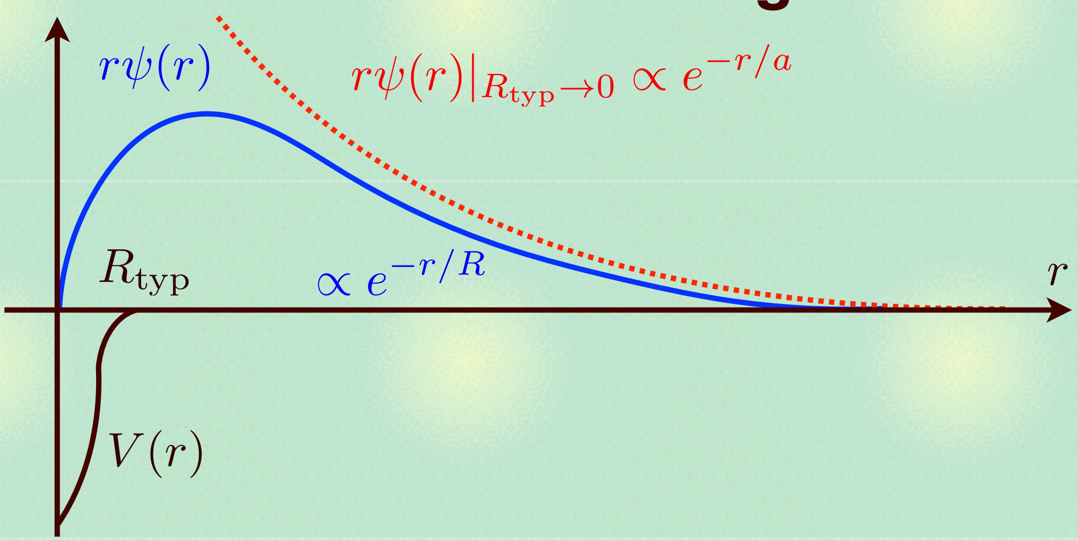
C. Hanhart, J.R. Pelaez, G. Rios, Phys. Lett. B 738, 375 (2014)

Finite R_{typ}: Z **expresses the violation of the scaling**

$$a = \frac{2(1-Z)}{2-Z} R + \mathcal{O}(R_{\rm typ})$$

model independent

model dependent



Interpretation of negative effective range

For $Z > 0$ and $R \gg R_{\mathrm{typ}}$, effective range is always negative.

$$a = \frac{2(1-Z)}{2-Z}R + \mathcal{O}(R_{\mathrm{typ}}), \quad r_e = \frac{-Z}{1-Z}R + \mathcal{O}(R_{\mathrm{typ}}),$$

$$\begin{cases} a \sim R_{\mathrm{typ}} \ll -r_e & \text{(elementary dominance)}, \\ a \sim R \gg r_e \sim R_{\mathrm{typ}} & \text{(composite dominance)}. \end{cases}$$

Simple (e.g. square-well) attractive potential: $r_e > 0$

- Only "composite dominance" is possible.

$r_e < 0$ **: energy- (momentum-)dependence of the potential**

D. Phillips, S. Beane, T.D. Cohen, Annals Phys. 264, 255 (1998);
E. Braaten, M. Kusunoki, D. Zhang, Annals Phys. 323, 1770 (2008)

- pole term/Feshbach projection of coupled-channel effect

Large negative r_e <—> elementary dominance

Short summary of bound states

Classification:

elementary Z

- uududd
- ΔΔ - πNN - …

composite X

- NN(s-wave)

Conditions for model-independent formula:

- stable s-wave bound state near threshold

Applicability:

- Deuteron only!

—> Generalization to unstable particles

(remark: large $-r_e$ **<—> elementary dominance)**

Generalization to resonances

Compositeness of bound states

$$Z(B) = \frac{1}{1 - \Sigma'(-B)}$$

Naive generalization to resonances:

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C85, 015201 (2012)

$$\underbrace{Z(E_R)}_{\text{complex}} = \frac{1}{1 - \Sigma'(E_R)}$$

complex

complex

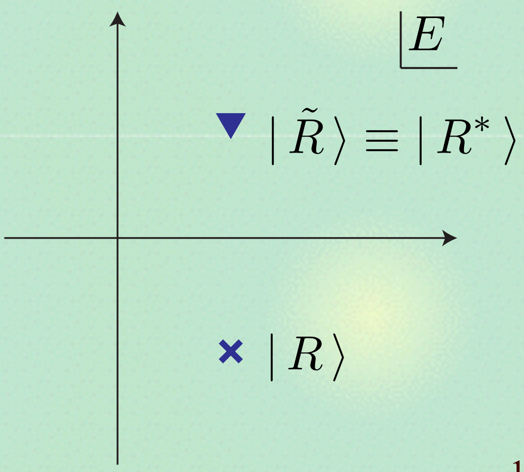
- Problem of interpretation (not probability!)

<— Normalization of resonances

$$\langle R | R \rangle \to \infty, \quad \langle \tilde{R} | R \rangle = 1$$

$$1 = \underbrace{\langle \tilde{R} | \psi_0 \rangle \langle \psi_0 | R \rangle}_{\text{complex}} + \int d\boldsymbol{p} \langle \tilde{R} | \boldsymbol{p} \rangle \langle \boldsymbol{p} | R \rangle$$

$$\langle \tilde{R} | \psi_0 \rangle = \langle \psi_0 | R \rangle \neq \langle \psi_0 | R \rangle^*$$



T. Berggren, Nucl. Phys. A 109, 265 (1968)

Interpretation of complex numbers

Suppose that we obtain…

case	Z	X	|Z|+|X|-1
1	0.94 + i 0.01	0.06 - i 0.01	0.00088
2	0.94 + i 5.3	0.06 - i 5.3	9.7
3	4.45 + i 0.01	-3.45 - i 0.01	6.9

Ideal case 1: Z **dominance, elementary.**

<— wave function is similar to the bound state with Z = 0.94

Problematic cases: large imaginary part (case 2) and/or large cancellation of the real part (case 3)

A measure: |Z|+|X|-1 **~ uncertainty?**

c.f. T. Berggren, Phys. Lett. 33B, 547 (1970)

- If $|Z| + |X| - 1 \gtrsim 0.5$ **, then** Z **and** X **should not be interpreted.**

Near-threshold resonances

Weak binding limit for bound states

- Model-independent (no potential, wavefunction, ...)
- Related to experimental observables

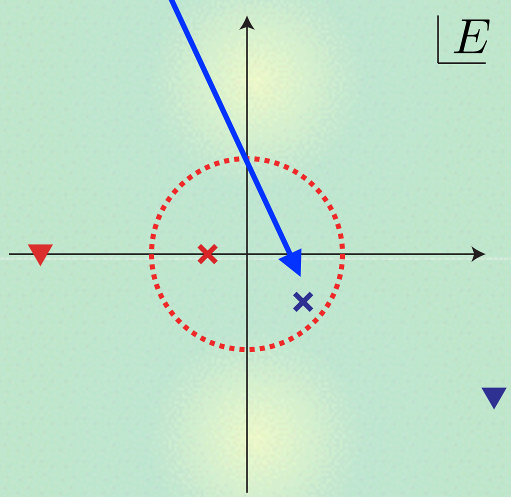
What about near-threshold resonances (~ small binding)?

T. Hyodo, Phys. Rev. Lett. 111, 132002 (2013)

Effective range expansion

$$f(p) = \left(-\frac{1}{a} + \frac{r_e}{2}p^2 - ip\right)^{-1}$$

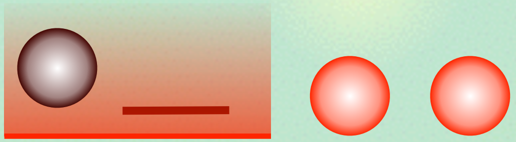
$$p^{\pm} = \frac{i}{r_e} \pm \frac{1}{r_e}\sqrt{\frac{2r_e}{a} - 1}$$



Resonance pole position —> (a, r_e) —> elementariness

Application: $\Lambda_c(2595)$

Pole position of $\Lambda_c(2595)$ in $\pi\Sigma_c$ scattering



- central values in PDG

$$E = 0.67\ \mathrm{MeV}, \quad \Gamma = 2.59\ \mathrm{MeV} \qquad p^{\pm} = \sqrt{2\mu(E \mp i\Gamma/2)}$$

- deduced threshold parameters of $\pi\Sigma_c$ scattering

$$a = -\frac{p^{+} + p^{-}}{ip^{+}p^{-}} = -10.5\ \mathrm{fm}, \quad r_e = \frac{2i}{p^{+} + p^{-}} = -19.5\ \mathrm{fm}$$

- Field renormalization constant cannot be interpreted.

$$Z = 1 - 0.608i, \quad |Z| + |X| - 1 = 0.78 \gtrsim 0.5$$

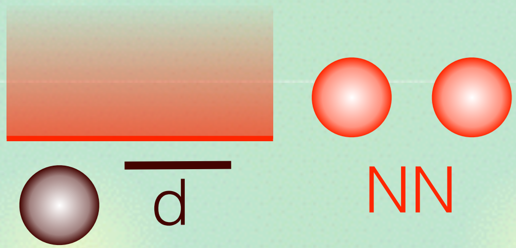
Large negative effective range

<— substantial elementary contribution other than $\pi\Sigma_c$
(three-quark, other meson-baryon channel, or ...)

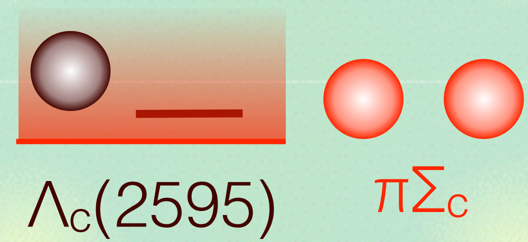
$\Lambda_c(2595)$ **is not likely a $\pi\Sigma_c$ composite**

Generalization to quasi-bound state

So far, we consider the lowest energy threshold.



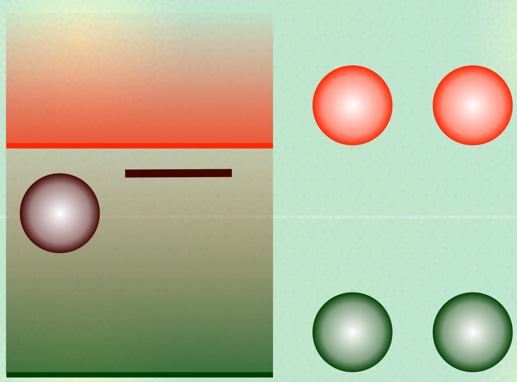
bound state



resonance

- **Scattering length is real.**

Quasi-bound state



channel 1: $\bar{K}N$

quasi-bound state: $\Lambda(1405)$

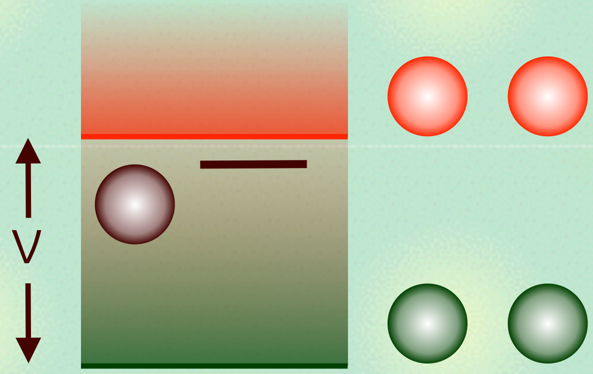
channel 2 (decay): $\pi\Sigma$

- **Scattering length of channel 1 is complex.**
- **decomposition:** $1 = X_1 + X_2 + Z \quad \Rightarrow \quad 1 = X_{\bar{K}N} + Z_{\text{others}}$

Generalized formula for quasi-bound state

Generalization of the formula

Y. Kamiya, T. Hyodo, in preparation



$$a_{\bar{K}N} = R_{\bar{K}N}\left\{\frac{2X_{\bar{K}N}}{1+X_{\bar{K}N}} + \mathcal{O}(|\frac{R_{\text{typ}}}{R_{\bar{K}N}}|) + \mathcal{O}(|\frac{l}{R_{\bar{K}N}}|^3)\right\}$$

$$R_{\bar{K}N} = (-2\mu_{\bar{K}N}E_{\Lambda^*})^{-1/2}, \quad l = (2\mu_{\bar{K}N}\nu)^{-1/2}$$

- **Formula is valid for complex $a_{\bar{K}N}$, $R_{\bar{K}N}$.**
- $X_{\bar{K}N}$ **is obtained as a complex number.**
- **Uncertainty is estimated.**

c.f. integration of the spectral density

V. Baru, *et al.*, Phys. Lett. B 586, 53 (2004)

$$\int_0^\infty w(E)dE = 1 - Z$$ **<— real, because of** $$\int d\boldsymbol{p}|\langle\,\boldsymbol{p}\,|\,R\,\rangle|^2$$

Application: Λ(1405)

(Higher) pole of Λ(1405) and the scattering length <— NLO chiral approach with SIDDHARTA (χ^2/dof = 0.96)

E_{Λ^*} [MeV]	$a_{\bar{K}N}$ [fm]	$\lvert R_{\bar{K}N}\rvert$ [fm]
-9.436 -i26	1.19 -i0.929	1.47

Y. Ikeda, T. Hyodo, W. Weise, Phys. Lett. B706, 63 (2011); Nucl. Phys. A881, 98 (2012)

Estimation of uncertainty:

- Typical scale of the interaction <— vector meson exchange

$$R_{\mathrm{typ}} = 1/m_\rho \sim 0.25 \text{ fm} \quad \Rightarrow \quad \left| \frac{R_{\mathrm{typ}}}{R_{\bar{K}N}} \right| = 0.17$$

- Mass difference from πΣ

$$\nu = 103 \text{ MeV}, \quad l = 0.76 \text{ fm} \quad \Rightarrow \quad \left| \frac{l}{R_{\bar{K}N}} \right|^3 = 0.14$$

Compositeness of Λ(1405)

$$X_{\bar{K}N} = 1.04 - i0.11, \quad |X_{\bar{K}N}| + |Z_{\mathrm{others}}| - 1 = 0.16 \ll 0.5$$

Λ(1405) is dominated by the $\bar{K}N$ composite component.

Summary

Model-independent aspect of compositeness

- **Complex X and Z for unstable particles:**
  - **can be interpreted only when** $|Z| + |X| - 1 \ll 0.5$
- **Near-threshold resonance:**

  T. Hyodo, Phys. Rev. Lett. 111, 132002 (2013)

  - **Pole position determines (a, r_e).**
  - **Elementariness <— r_e.**
  - $\Lambda_c(2595)$ **is not a** $\pi\Sigma_c$ **molecule.**
- **Near-threshold quasi-bound state:**

  Y. Kamiya, T. Hyodo, in preparation

  - **Generalized formula for complex numbers.**
  - $\Lambda(1405)$ **is a** $\bar{K}N$ **molecule.**