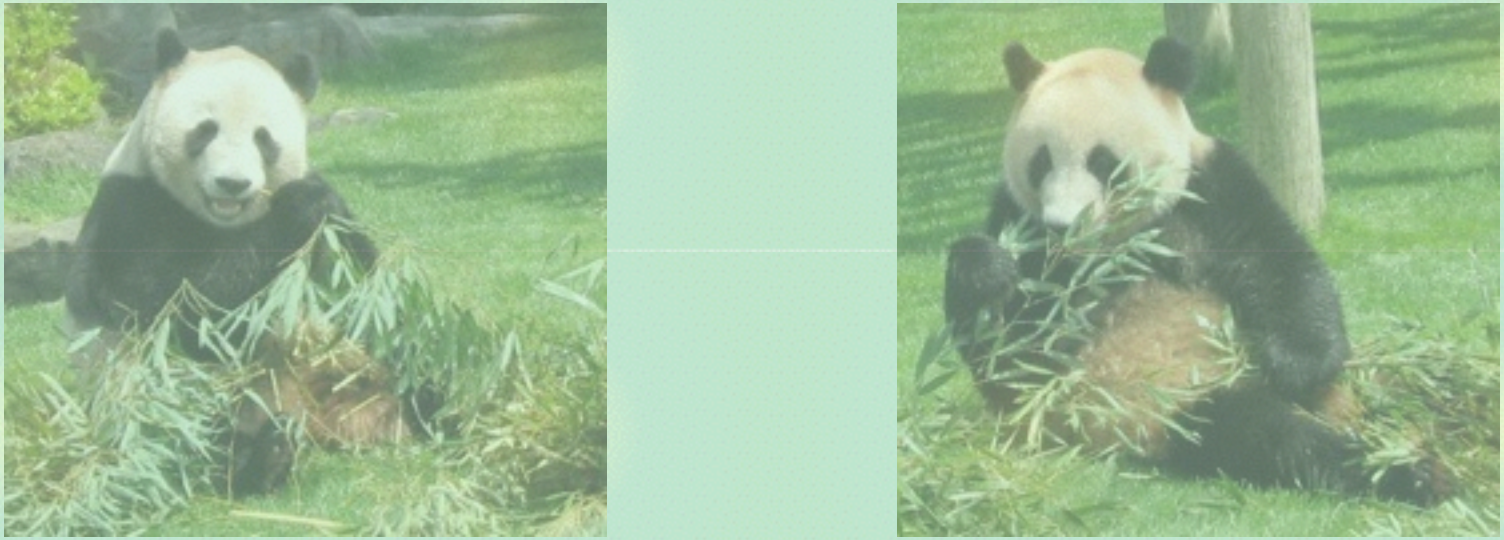
Compositeness of hadrons and near-threshold dynamics



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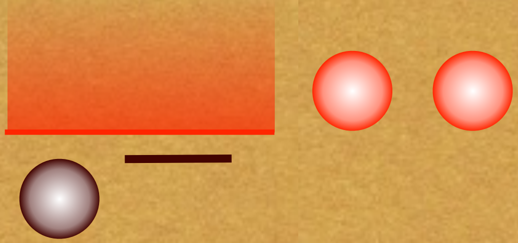

  S. Weinberg, Phys. Rev. 137, B672 (1965);
  T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)

- Near-threshold resonance

  T. Hyodo, Phys. Rev. Lett. 111, 132002 (2013)

- Mass scaling across threshold

  T. Hyodo, Phys. Rev. C90, 055208 (2014)

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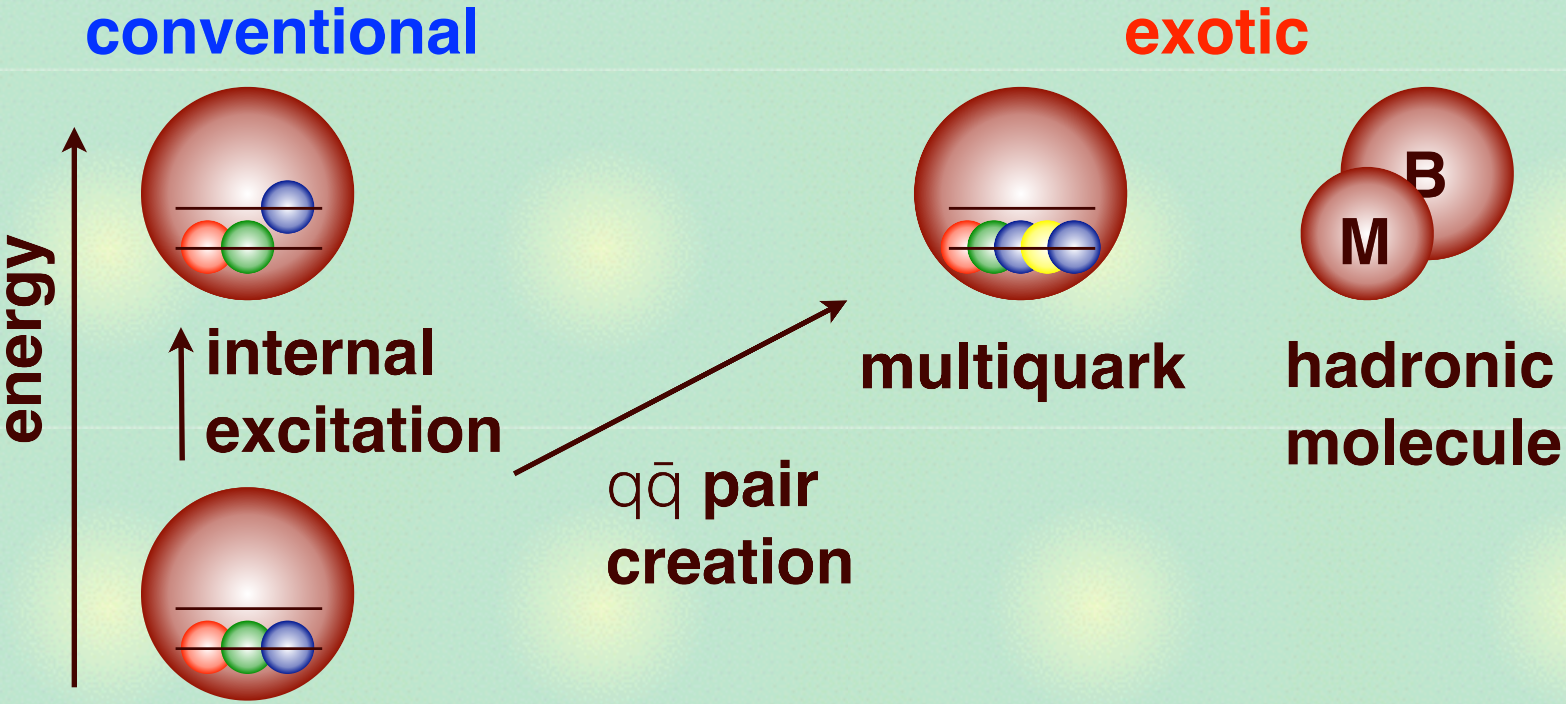
  Y. Kamiya, T. Hyodo, in preparation

- Summary

Exotic structure of hadrons

Various excitations of baryons



conventional

exotic

energy

internal excitation

qq̄ pair creation

multiquark

hadronic molecule

Physical state: superposition of 3q, 5q, MB, ...

$$|\,\Lambda(1405)\,\rangle = \underline{N_{3q}}|\,uds\,\rangle + \underline{N_{5q}}|\,uds\ q\bar{q}\,\rangle + \underline{N_{\bar{K}N}}|\,\bar{K}N\,\rangle + \cdots$$

Is this relevant strategy?

Ambiguity of definition of hadron structure

Decomposition of hadron “wave function”

$$|\,\Lambda(1405)\,\rangle = N_{3q}|\,uds\,\rangle + N_{5q}|\,uds\;q\bar{q}\,\rangle + N_{\bar{K}N}|\,\bar{K}N\,\rangle + \cdots$$

- $N_X \neq$ **probability?**

- **5q v.s. MB: double counting (orthogonality)?**

$$\langle\, udsq\bar{q}\,|\,\bar{K}N\,\rangle \neq 0$$

- **3q v.s. 5q: not clearly separated in QCD**

$$\langle\, uds\,|\,udsq\bar{q}\,\rangle \neq 0$$

- **hadron resonances: unstable, finite decay width**

$$|\,\Lambda(1405)\,\rangle = \,?$$

What is the suitable basis to classify the hadron structure?

Strategy

Elementary/composite nature of bound states near the lowest energy two-body threshold

elementary Z

- 6q **for deuteron**
- $c\bar{c}$ **for** X(3872)

composite X

- NN **for deuteron**
- $\bar{D}D^*$ **for** X(3872)

- **orthogonality <— eigenstates of bare Hamiltonian**
- **normalization <— eigenstate of full Hamiltonian**
- **model dependence <— low energy universality**

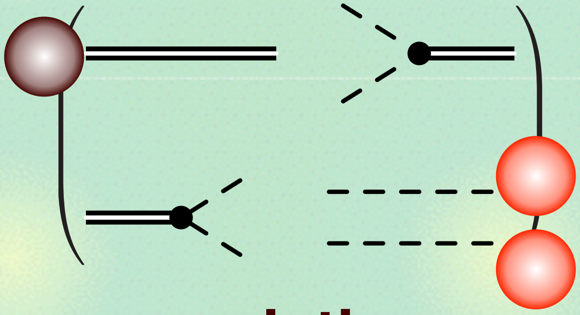
*** Basis must be asymptotic states (in QCD, hadrons).**

*** “Elementary” stands for any states other than two-body composite (missing channels, CDD pole, ...).**

Formulation

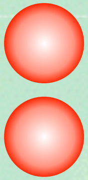
Coupled-channel Hamiltonian (bare state + continuum)

$$\begin{pmatrix} M_0 & \hat{V} \\ \hat{V} & \frac{p^2}{2\mu} \end{pmatrix} |\,\Psi\,\rangle = E|\,\Psi\,\rangle, \quad |\,\Psi\,\rangle = \begin{pmatrix} c(E)|\,\psi_0\,\rangle \\ \chi_E(\boldsymbol{p})|\,\boldsymbol{p}\,\rangle \end{pmatrix}$$



- Bound state normalization + completeness relation

$$\langle\,\Psi\,|\,\Psi\,\rangle = 1 \qquad 1 = |\,\psi_0\,\rangle\langle\,\psi_0\,| + \int d^3q |\,\boldsymbol{q}\,\rangle\langle\,\boldsymbol{q}\,|$$



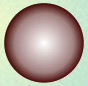
$$1 = \underline{\left|\langle\,\Psi\,|\begin{pmatrix} |\,\psi_0\,\rangle \\ 0 \end{pmatrix}\right|^2} + \underline{\int d^3q \left|\langle\,\Psi\,|\begin{pmatrix} 0 \\ |\,\boldsymbol{q}\,\rangle \end{pmatrix}\right|^2} \equiv Z + X \longleftarrow$$

compositeness

bare state contribution

continuum contribution

elementariness (field renormalization constant)



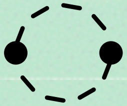
Z, X**: real and nonnegative —> probabilistic interpretation**

Z in model calculations

In general, Z is determined by the potential V.

$$Z(B) = \frac{1}{1 - \frac{d}{dE}\int \frac{|\langle \psi_0 | \hat{V} | \boldsymbol{q} \rangle|^2}{E - q^2/(2\mu) + i0^+} d^3q \Big|_{E=-B}} \equiv \frac{1}{1 - \Sigma'(-B)}$$

$\Sigma(E) \sim$



- Z is model dependent (c.f. potential, wave function)

Applications:

Baryons	Z	$\lvert Z\rvert$	Mesons	Z	$\lvert Z\rvert$
$\Lambda(1405)$ higher pole (Ref. 58)	$0.00 + 0.09i$	0.09	$f_0(500)$ or σ (Ref. 58)	$1.17 - 0.34i$	1.22
$\Lambda(1405)$ lower pole (Ref. 58)	$0.86 - 0.40i$	0.95	$f_0(980)$ (Ref. 58)	$0.25 + 0.10i$	0.27
$\Delta(1232)$ (Ref. 60)	$0.43 + 0.29i$	0.52	$a_0(980)$ (Ref. 58)	$0.68 + 0.18i$	0.70
$\Sigma(1385)$ (Ref. 60)	$0.74 + 0.19i$	0.77	$\rho(770)$ (Ref. 55)	$0.87 + 0.21i$	0.89
$\Xi(1535)$ (Ref. 60)	$0.89 + 0.99i$	1.33	$K^*(892)$ (Ref. 59)	$0.88 + 0.13i$	0.89
Ω (Ref. 60)	0.74	0.74			
$\Lambda_c(2595)$ (Ref. 56)	$1.00 - 0.61i$	1.17			

for details, see T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)

Z can be evaluated in specific models.

Weak binding limit

Z of **weakly-bound** ($R \gg R_{\text{typ}}$) **s-wave state <— observables.**

S. Weinberg, Phys. Rev. 137, B672 (1965);
T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)

$$a = \frac{2(1-Z)}{2-Z}R + \mathcal{O}(R_{\text{typ}}), \quad r_e = \frac{-Z}{1-Z}R + \mathcal{O}(R_{\text{typ}}),$$

a **: scattering length,** r_e **: effective range**
$R = (2\mu B)^{-1/2}$ **: radius <— binding energy**
R_{typ} **: typical length scale of the interaction**

- Deuteron is found to be composite ($Z \sim 0$**),**
without referring to the nuclear force/wave function.

- Another derivation (expansion of the amplitude):
T. Sekihara, T. Hyodo, D. Jido, arXiv: 1411.2308 [hep-ph], to appear in PTEP

Scaling limit

Scaling (zero-range) limit: scattering length a ≠ 0, R_{typ} -> 0

E. Braaten, H.-W. Hammer, Phys. Rept. 428, 259 (2006)

- All (2-body) quantities are expressed by a**: universality**

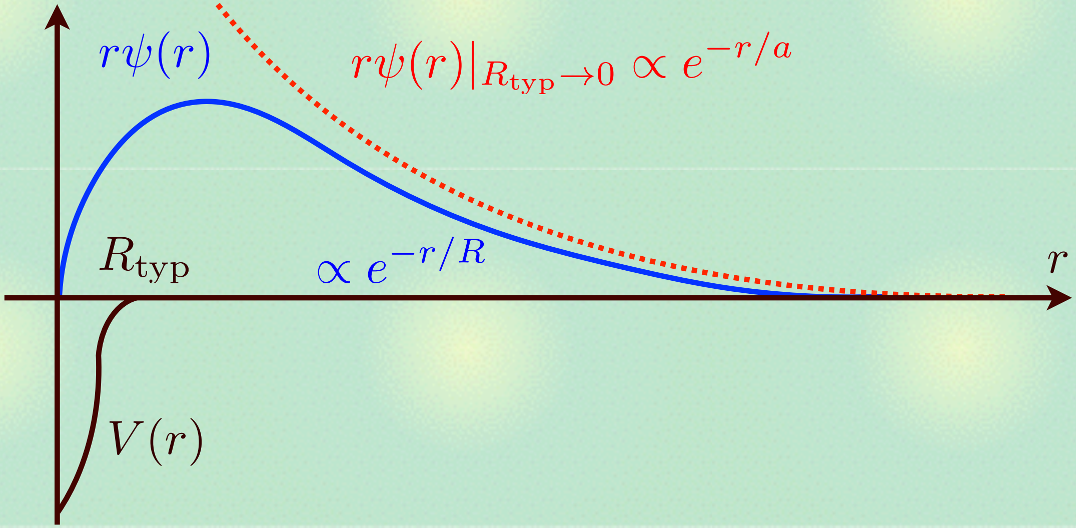
$$\psi(r) = \frac{e^{-r/a}}{\sqrt{2\pi a r}}$$

$$B = 1/(2\mu a^2) \quad \Rightarrow \quad R = a \quad \Rightarrow \quad Z = 0$$

- Bound state is always composite in the scaling limit.

Finite R_{typ}**:** Z **expresses the violation of the scaling**

$$a = \frac{2(1-Z)}{2-Z} R + \mathcal{O}(R_{\text{typ}})$$



Interpretation of negative effective range

For $Z > 0$ **and** $R \gg R_{\text{typ}}$**, effective range is always negative.**

$$a = \frac{2(1-Z)}{2-Z}R + \mathcal{O}(R_{\text{typ}}), \quad r_e = \frac{-Z}{1-Z}R + \mathcal{O}(R_{\text{typ}}),$$

$$\begin{cases} a \sim R_{\text{typ}} \ll -r_e & \text{(elementary dominance)}, \\ a \sim R \gg r_e \sim R_{\text{typ}} & \text{(composite dominance)}. \end{cases}$$

Simple (e.g. square-well) attractive potential: $r_e > 0$

- Only "composite dominance" is possible.

$r_e < 0$ **: energy- (momentum-)dependence of the potential**

D. Phillips, S. Beane, T.D. Cohen, Annals Phys. 264, 255 (1998);
E. Braaten, M. Kusunoki, D. Zhang, Annals Phys. 323, 1770 (2008)

- pole term/Feshbach projection of coupled-channel effect

Negative r_e **—> something other than |p>: CDD pole**

Generalization to resonances

Compositeness of bound states

$$Z(B) = \frac{1}{1-\Sigma'(-B)}$$

Naive generalization to resonances:

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C85, 015201 (2012)

$$\underset{\text{complex}}{\underline{Z(E_R)}} = \frac{1}{1-\underset{\substack{\uparrow\\ \text{complex}}}{\Sigma'(E_R)}}$$

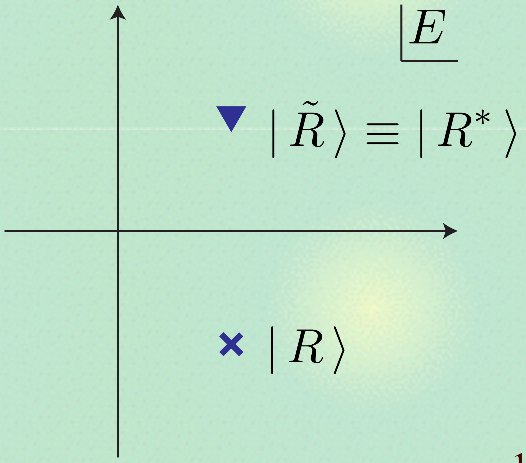
- Problem of interpretation (probability?)

<— Normalization of resonances

$$\langle R|R\rangle \to \infty, \quad \langle \tilde{R}|R\rangle = 1$$

$$1 = \underset{\text{complex}}{\underline{\langle \tilde{R}|\psi_0\rangle\langle\psi_0|R\rangle}} + \int d\boldsymbol{p}\langle \tilde{R}|\boldsymbol{p}\rangle\langle\boldsymbol{p}|R\rangle$$

$$\langle \tilde{R}|\psi_0\rangle = \langle\psi_0|R\rangle \neq \langle\psi_0|R\rangle^*$$



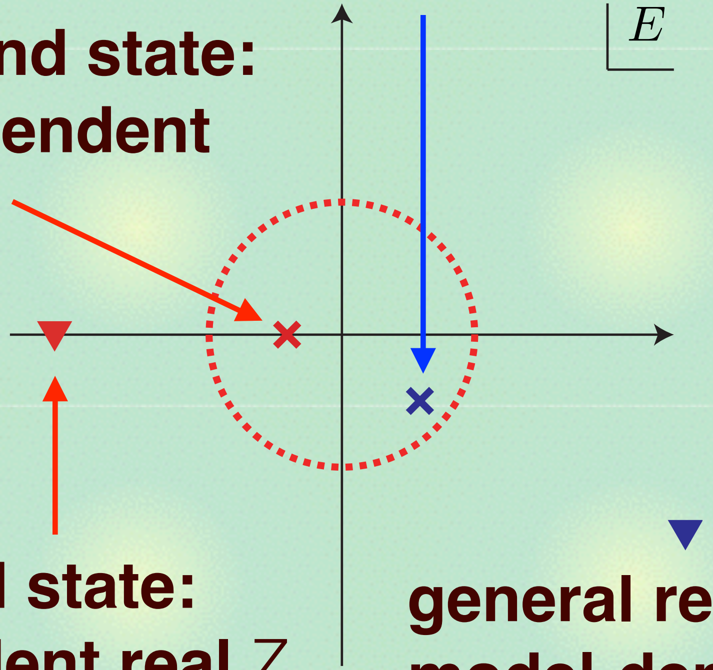
T. Berggren, Nucl. Phys. A 109, 265 (1968)

Near-threshold resonances

Weak binding limit for bound states

- Model-independent (no potential, wavefunction, ...)
- Related to experimental observables

What about **near-threshold resonances** (~ small binding)?



shallow bound state:
model-independent
structure

general bound state:
model-dependent real Z

general resonance:
model-dependent complex Z

Poles in the effective range expansion

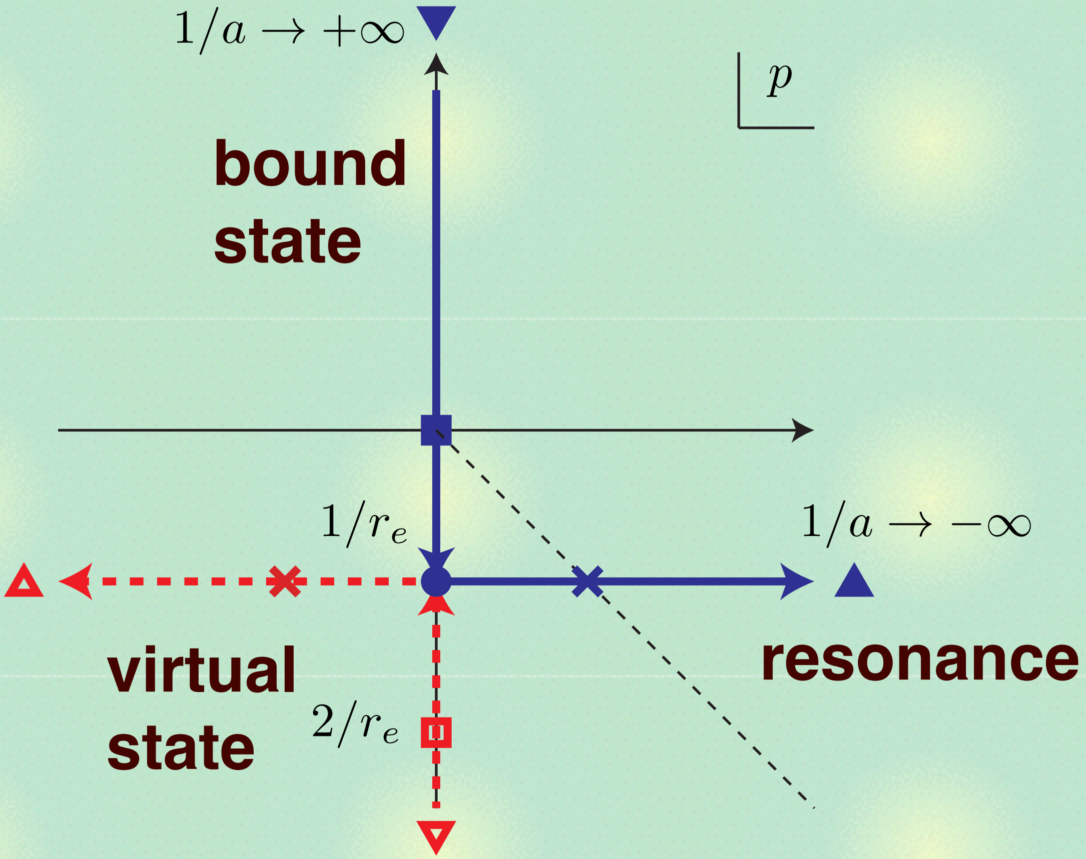
Near-threshold pole: effective range expansion

T. Hyodo, Phys. Rev. Lett. 111, 132002 (2013) with opposite sign of scattering length

$$f(p)=\left(-\frac{1}{a}+\frac{r_e}{2}p^2-ip\right)^{-1}$$

$$p^{\pm}=\frac{i}{r_e}\pm\frac{1}{r_e}\sqrt{\frac{2r_e}{a}-1}$$

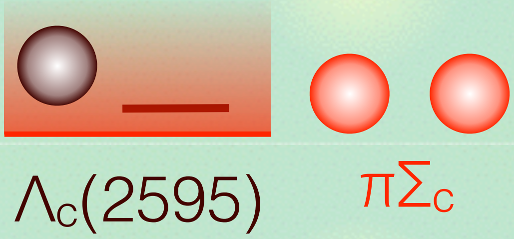
- pole trajectories with a fixed $r_e < 0$



Resonance pole position —> (a, r_e) —> elementariness

Application: $\Lambda_c(2595)$

Pole position of $\Lambda_c(2595)$ in $\pi\Sigma_c$ scattering



- central values in PDG

$$E = 0.67\ \text{MeV}, \quad \Gamma = 2.59\ \text{MeV} \qquad p^{\pm} = \sqrt{2\mu(E \mp i\Gamma/2)}$$

- deduced threshold parameters of $\pi\Sigma_c$ scattering

$$a = -\frac{p^+ + p^-}{ip^+p^-} = -10.5\ \text{fm}, \quad r_e = \frac{2i}{p^+ + p^-} = \boxed{-19.5\ \text{fm}}$$

- field renormalization constant: complex

$$Z = 1 - 0.608i$$

Large negative effective range

<— substantial elementary contribution other than $\pi\Sigma_c$
(three-quark, other meson-baryon channel, or ...)

$\Lambda_c(2595)$ **is not likely a $\pi\Sigma_c$ composite**

Hadron mass scaling and threshold effect

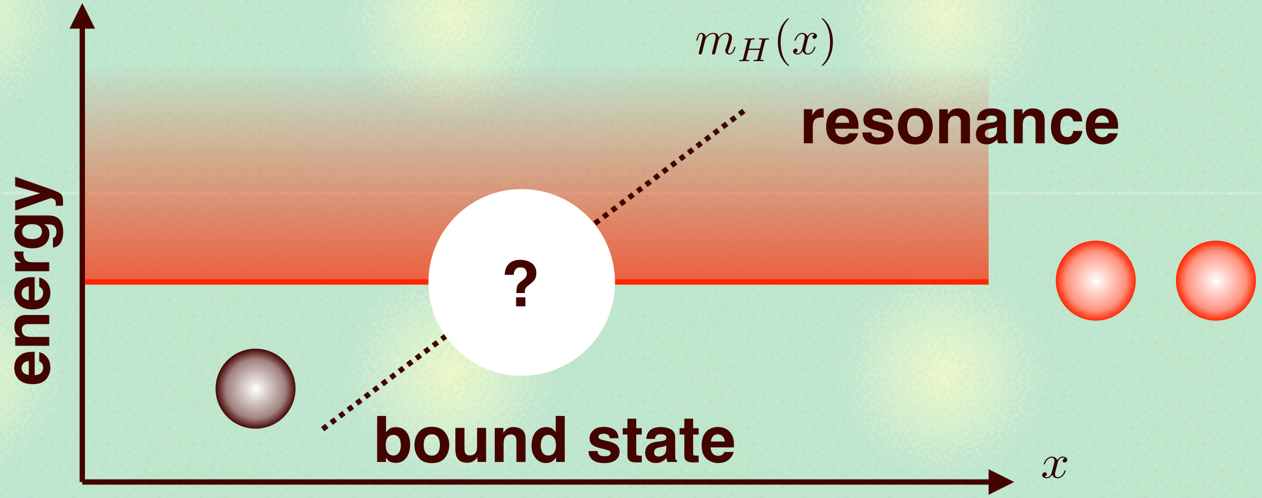
Systematic expansion of hadron masses

- ChPT: light quark mass m_q
- HQET: heavy quark mass m_Q
- large Nc: number of colors N_c

Hadron mass scaling

$$m_H(x); \quad x = \frac{m_q}{\Lambda}, \frac{\Lambda}{m_Q}, \frac{1}{N_c}$$

What happens at two-body threshold?



General threshold behavior

Expansion of the Jost function (δM: small perturbation)

T. Hyodo, Phys. Rev. C90, 055208 (2014)

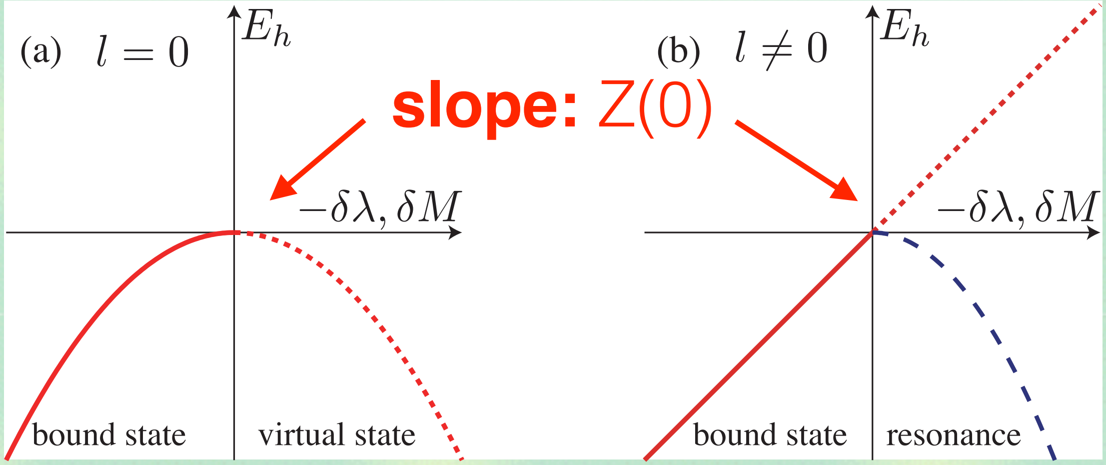
- δM < 0

$$E_h \propto \begin{cases} -\delta M^2 & l=0 \\ \delta M & l \neq 0 \end{cases}$$

- δM > 0

$$E_h \propto -\delta M^2 \quad l=0$$

$$\begin{cases} \mathrm{Re}\ E_h \propto \delta M \\ \mathrm{Im}\ E_h \propto -(\delta M)^{l+1/2} \end{cases} \quad l \neq 0$$



slope: Z(0)

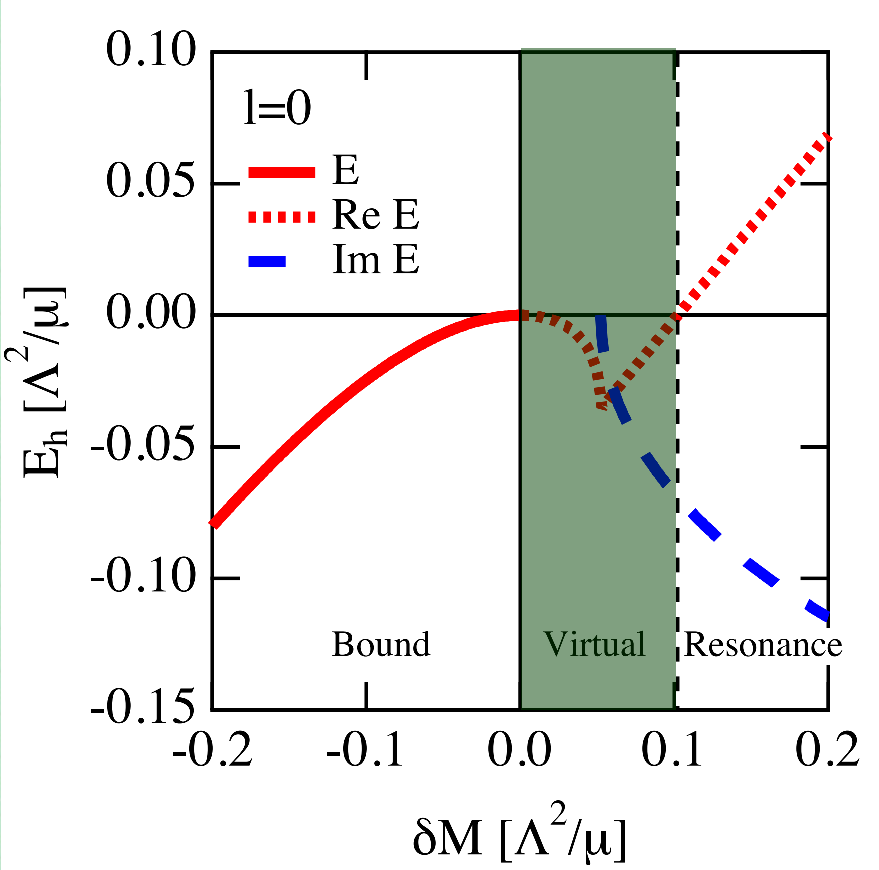
Slope at E_h=0: field renormalization constant

$$E_h = \frac{1}{1-\Sigma'(0)}\delta M = Z(0)\delta M, \quad \Sigma'(E) \equiv \frac{d\Sigma(E)}{dE}$$

Z(0)=0 for s-wave —> quadratic scaling

Chiral extrapolation across s-wave threshold

s-wave: bound state —> virtual state —> resonance



Near-threshold scaling: nonperturbative phenomenon

—> Naive ChPT does not work; resummation required.

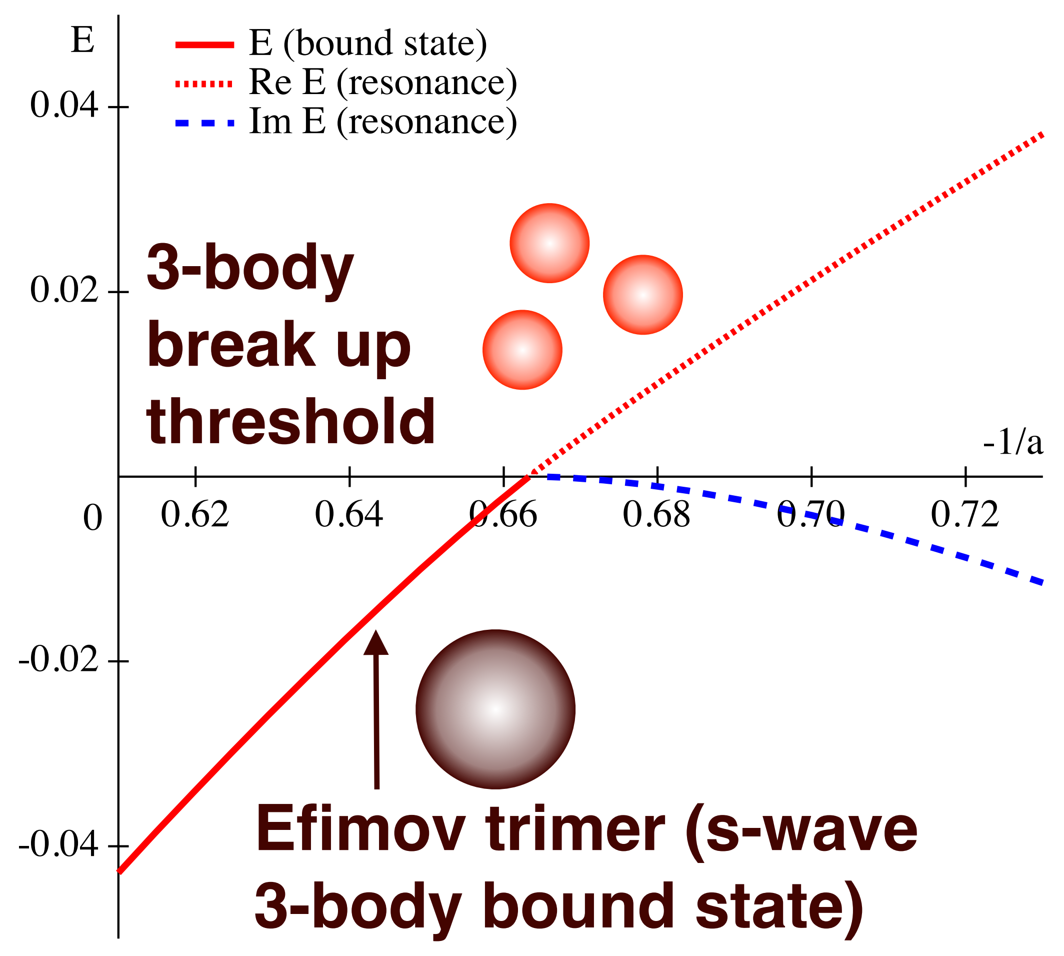
c.f.) NN **sector,** $\bar{\mathrm{K}}$N **sector, …**

Scaling of three-body bound state

Near-threshold scaling is universal for 2-body system.

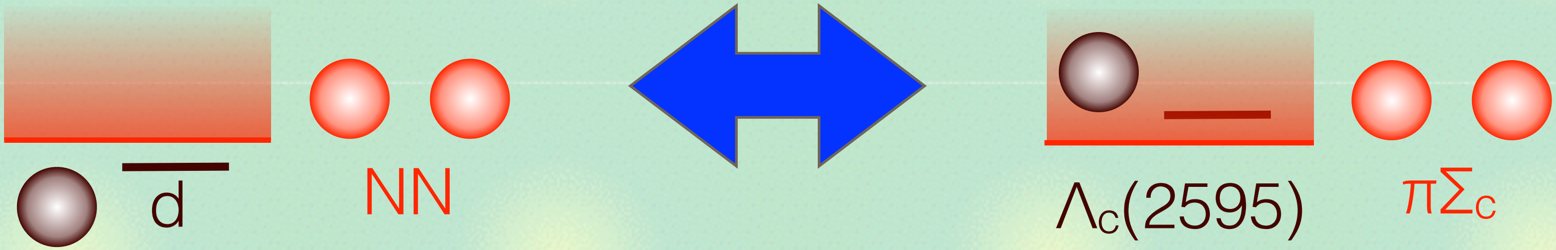
- 3-body case?

T. Hyodo, T. Hatsuda, Y. Nishida, Phys. Rev. C89, 032201 (2014)



Generalization to quasi-bound state

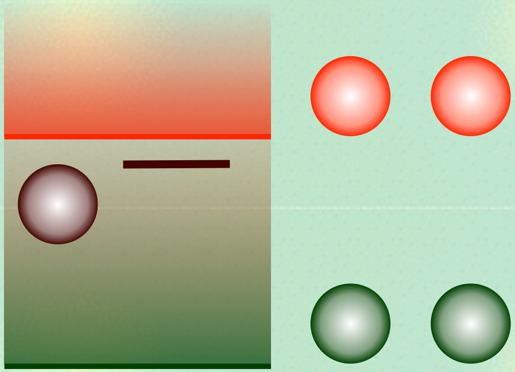
So far, we consider the lowest energy threshold.



bound state **scaling** **resonance**

- **Scattering length is real.**

Physically relevant situation: quasi-bound state



channel 1:	$\bar{K}N$	$\bar{D}D^*$
quasi-bound state:	Λ(1405)	X(3872)
channel 2 (decay):	$\pi\Sigma$	$\pi\pi\Psi$

- **Scattering length of channel 1 is complex.**
- **decomposition:** $1 = Z + X_1 + X_2$

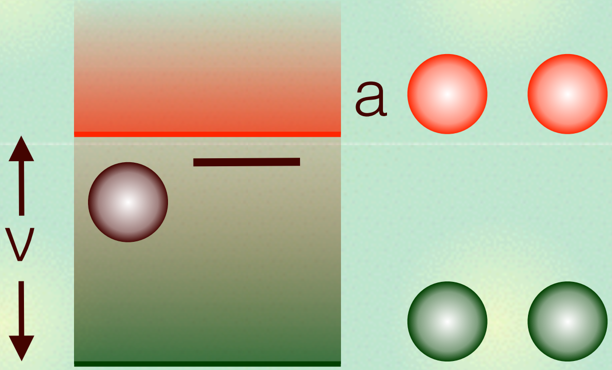
Application: Λ(1405)

Generalization of the formula

Y. Kamiya, T. Hyodo, in preparation

$$a = R_1 \left\{ \frac{2X_1}{1+X_1} + \mathcal{O}(|\frac{R_{\rm typ}}{R_1}|) + \mathcal{O}(|\frac{l_1}{R_1}|^3) \right\}$$

$$R_1 = (-2\mu_1 E)^{-1/2}, \quad l_1 = (2\mu_1 \nu)^{-1/2}$$



- **Formula is valid for complex** a, R₁, X₁.

Example: Λ(1405)

E **[MeV] ***	a **[fm] ***	X₁
-9.436 -i26	1.19 -i0.929	1.04 -i0.11

* Y. Ikeda, T. Hyodo, W. Weise, Phys. Lett. B706, 63 (2011); Nucl. Phys. A881, 98 (2012)

- **consistent with the residue calculation:** 1.14 +i0.01

T. Sekihara, T. Hyodo, D. Jido, arXiv: 1411.2308 [hep-ph], to appear in PTEP

Λ(1405) **is dominated by the** $\bar{\rm K}$N **composite** **component.**

Summary

Near-threshold states: structure <—> observables

Near-threshold **resonance**:

T. Hyodo, Phys. Rev. Lett. 111, 132002 (2013)

- Pole position determines (a, r_e).
- Instead of complex Z, effective range serves as the measure of the elementariness.

Mass **scaling** across threshold:

T. Hyodo, Phys. Rev. C90, 055208 (2014)

- Quadratic scaling in s wave <— $Z(0)=0$

Near-threshold **quasi-bound** state:

Y. Kamiya, T. Hyodo, in preparation

- Generalized formula for complex numbers.