s波閾値近傍での ハドロン質量スケーリング

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Hadron mass scaling and threshold effect

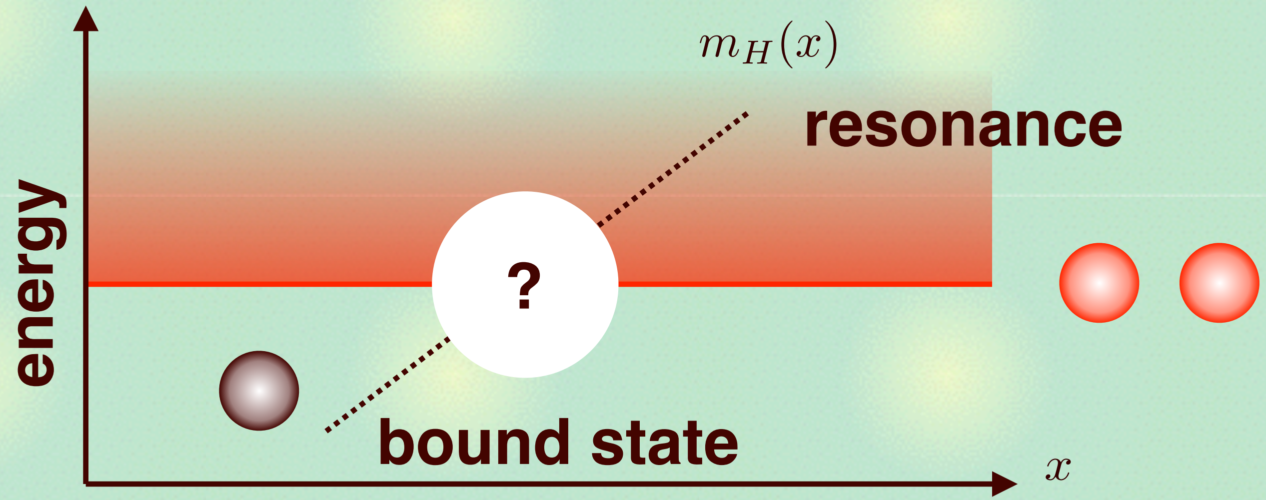
Systematic expansion of hadron masses

- **ChPT: light quark mass** m_q
- **HQET: heavy quark mass** m_Q
- **large Nc: number of colors** N_c

Hadron mass scaling

$$m_H(x); \quad x = \frac{m_q}{\Lambda}, \frac{\Lambda}{m_Q}, \frac{1}{N_c}$$

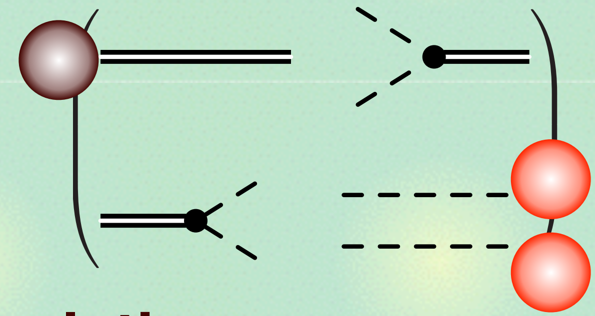
What happens at two-body threshold?



Formulation

Coupled-channel Hamiltonian (bare state + continuum)

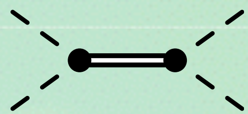
$$\begin{pmatrix} M_0 & \hat{V} \\ \hat{V} & \frac{p^2}{2\mu}(+\hat{V}_{\rm sc}) \end{pmatrix} |\,\Psi\,\rangle = E|\,\Psi\,\rangle, \quad |\,\Psi\,\rangle = \begin{pmatrix} c(E)|\,\psi_0\,\rangle \\ \chi_E(\boldsymbol{p})|\,\boldsymbol{p}\,\rangle \end{pmatrix}$$



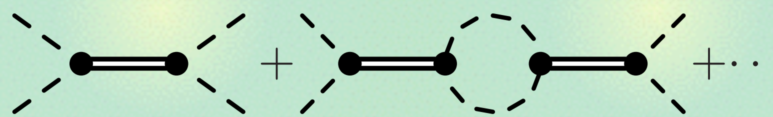
c.f. S. Weinberg, Phys. Rev. 131, 330 (1963)

Equivalent single-channel scattering formulation

$$\hat{V}_{\rm eff}(E) = \frac{\hat{V}|\,\psi_0\,\rangle\langle\,\psi_0\,|\hat{V}}{E - M_0} \sim$$



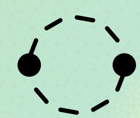
$$f(\boldsymbol{p}, \boldsymbol{p}', E) = -\frac{4\pi^2\mu\langle\,\boldsymbol{p}\,|\hat{V}|\,\psi_0\,\rangle\langle\,\psi_0\,|\hat{V}|\,\boldsymbol{p}'\,\rangle}{E - M_0 - \Sigma(E)} \sim$$



Pole condition:

$$E_h - M_0 = \Sigma(E_h)$$

$$\Sigma(E) = \int \frac{\langle\,\psi_0\,|\hat{V}|\,\boldsymbol{q}\,\rangle\langle\,\boldsymbol{q}\,|\hat{V}|\,\psi_0\,\rangle}{E - q^2/(2\mu) + i0^+} d^3q \sim$$

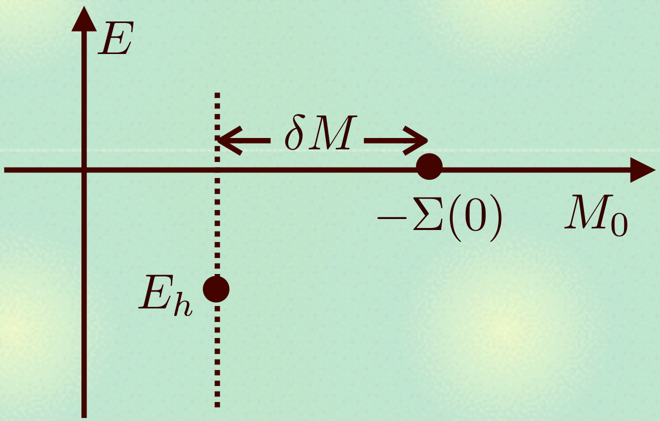


Question: How E_h behaves against M_0 around $E_h=0$?

Near-threshold bound state

Bound state condition around $E_h=0$

$$E_h + \Sigma(0) - \delta M = \Sigma(E_h)$$



Leading contribution of the expansion:

$$E_h = \frac{1}{1-\Sigma'(0)}\delta M = Z(0)\delta M, \quad \Sigma'(E) \equiv \frac{d\Sigma(E)}{dE}$$

Field renormalization constant

$$\left|\langle \Psi | \begin{pmatrix} |\psi_0\rangle \\ 0 \end{pmatrix}\right|^2 + \int \left|\langle \Psi | \begin{pmatrix} 0 \\ |\boldsymbol{q}\rangle \end{pmatrix}\right|^2 d^3q = 1$$

T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)

$Z(0)$ vanishes for $l=0$: compositeness theorem

$$E_h \propto \begin{cases} \mathcal{O}(\delta M^2) & l=0 \\ \delta M & l \neq 0 \end{cases}$$

Near-threshold bound state (general)

General argument by Jost function (Fredholm determinant)

J.R. Taylor, *Scattering Theory* (Wiley, New York, 1972)

$$f_l(p) = \frac{\not{f}_l(-p) - \not{f}_l(p)}{2ip\not{f}_l(p)} \qquad p = \sqrt{2\mu E}$$

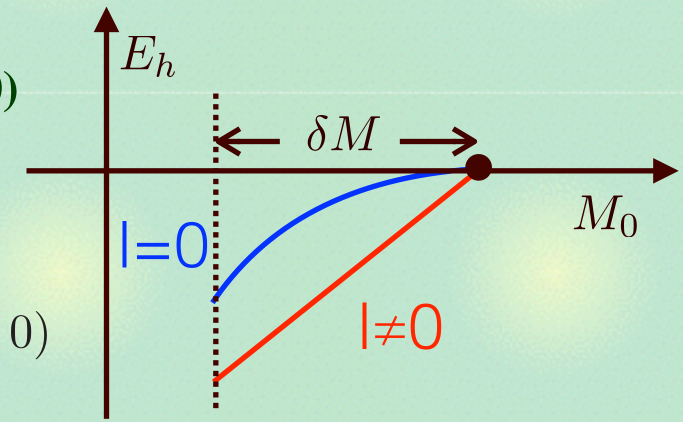
pole (eigenstate)
= Jost function zero

Expansion of the Jost function:

$$\not{f}_l(p) = \begin{cases} 1 + \alpha_0 + i\gamma_0 p + \mathcal{O}(p^2) & l = 0 \\ 1 + \alpha_l + \beta_l p^2 + \mathcal{O}(p^3) & l \neq 0 \end{cases}$$

- γ_0 **and** β_l **are nonzero for a general potential**
- **zero at** $p=0$ ($1+\alpha_l=0$) **must be simple (double) for** $l=0$ ($l\neq0$)

R.G. Newton, J. Math. Phys. 1, 319 (1960)
H.-W. Hammer, D. Lee, Annals Phys. 325, 2212 (2010)



Near-threshold scaling:

$$1 + \alpha_l \sim \delta M \quad \Rightarrow \quad E_h \propto \begin{cases} -\delta M^2 & l = 0 \\ \delta M & l \neq 0 \end{cases} \quad (\delta M < 0)$$

General threshold behavior

Near threshold scaling:

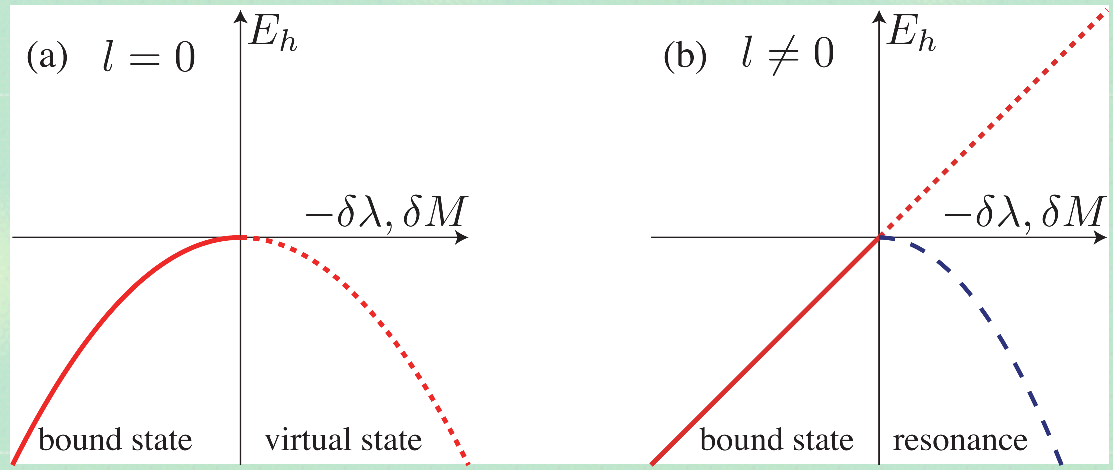
- $\delta M < 0$

$$E_h \propto \begin{cases} -\delta M^2 & l = 0 \\ \delta M & l \neq 0 \end{cases}$$

- $\delta M > 0$

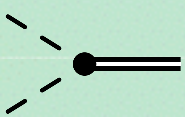
$$E_h \propto -\delta M^2 \quad l = 0$$

$$\begin{cases} \mathrm{Re}\, E_h \propto \delta M \\ \mathrm{Im}\, E_h \propto -(\delta M)^{l+1/2} \end{cases} \quad l \neq 0$$



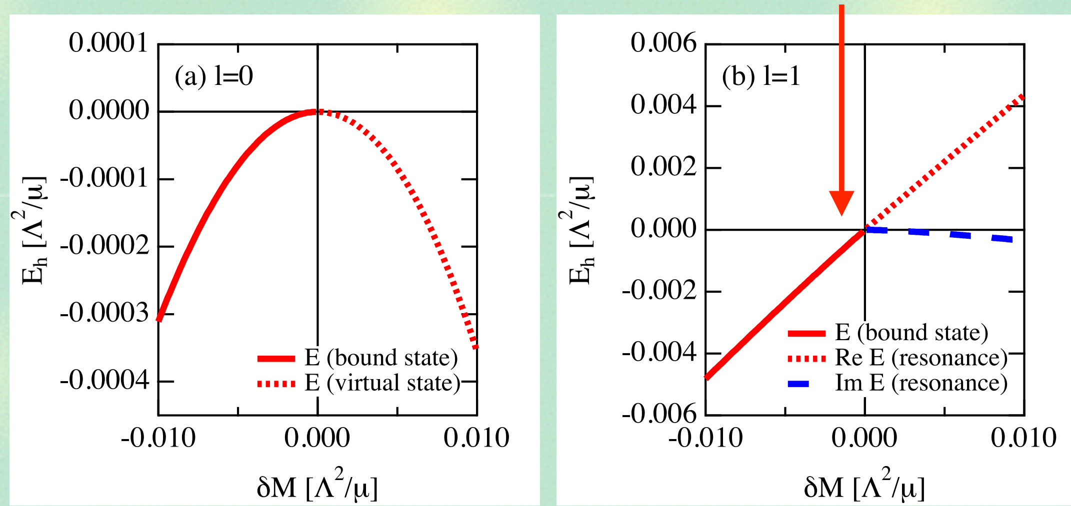
c.f. NN $^1S_0$

Numerical calculation



$$\langle \boldsymbol{q} | \hat{V} | \psi_0 \rangle = g_l |\boldsymbol{q}|^l \Theta(\Lambda - |\boldsymbol{q}|)$$

slope: Z(0)



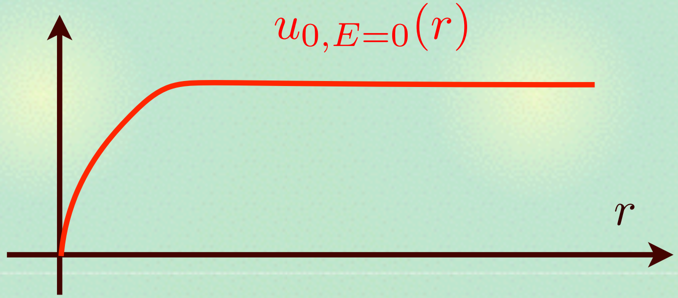
Compositeness theorem

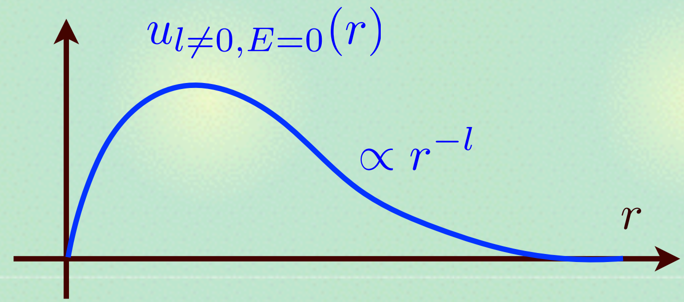
Theorem: Z(0)=0 **for s wave**

If the s-wave scattering amplitude has a pole exactly at the threshold with a finite range interaction, then the field renormalization constant vanishes.

T. Hyodo, Phys. Rev. C90, 055208 (2014)

Two-body wave function at E=0**:** $u_{l,E=0}(r) \xrightarrow{r\to\infty} r^{-l}$

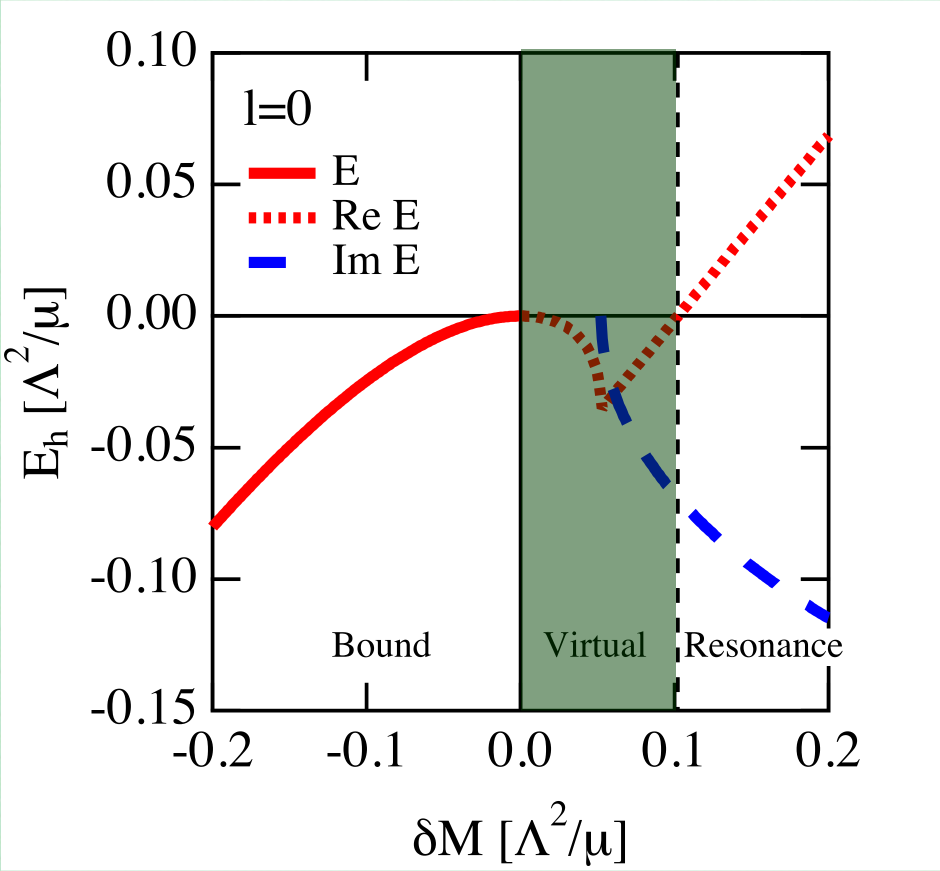




Z(0)=0**: ~~Bound state is completely composite.~~**
Composite component is infinitely large so that the fraction of any finite admixture of bare state vanishes.

Chiral extrapolation across s-wave threshold

s-wave: bound state —> virtual state —> resonance



Near-threshold scaling: nonperturbative phenomenon

—> Naive ChPT does not work; resummation required.

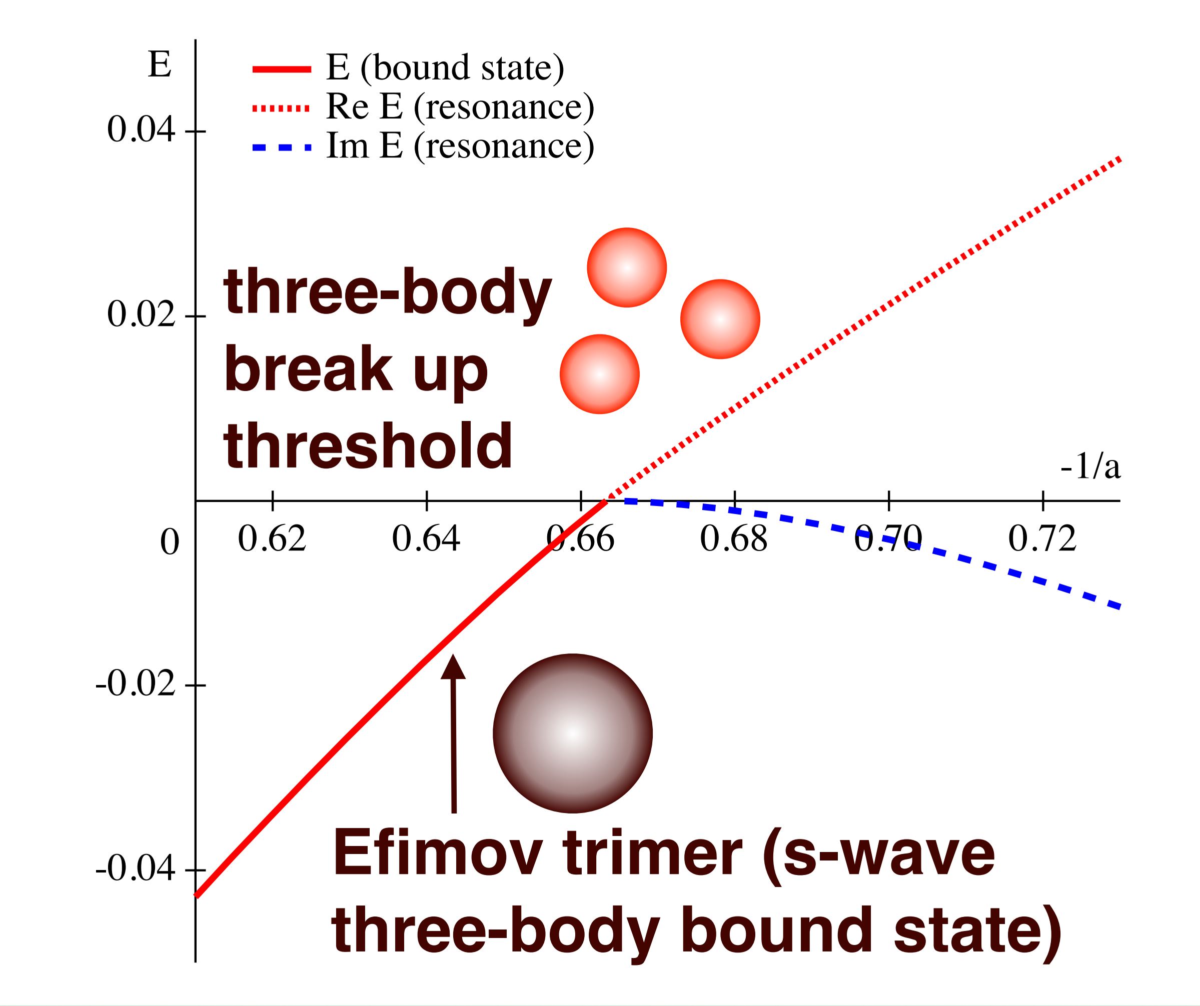
c.f.) NN sector, $\bar{K}$N sector, ...

Scaling of three-body bound state

Near-threshold scaling is universal for two-body system.

- Three-body case? Not always…

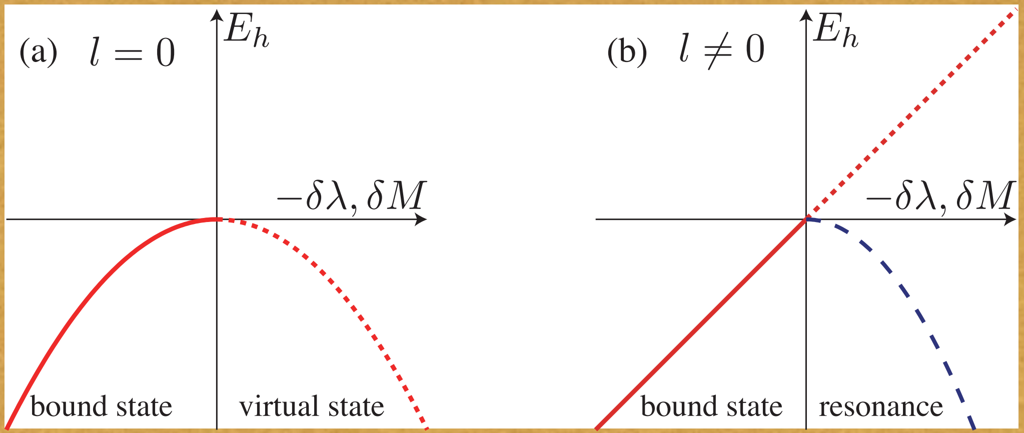
T. Hyodo, T. Hatsuda, Y. Nishida, Phys. Rev. C89, 032201 (2014)



Summary

We study the hadron mass scaling near threshold.

- **General scaling laws:**



- **Compositeness theorem:**

$$Z(B=0) = 0 \quad \text{for } l = 0$$

- **Chiral extrapolation across the s-wave threshold should be carefully performed.**

T. Hyodo, Phys. Rev. C90, 055208 (2014)