Hadron mass scaling near an s-wave threshold

Tetsuo Hyodo

Yukawa Institute for Theoretical Physics, Kyoto Univ.

2014, Oct. 9th

Hadron mass scaling and threshold effect

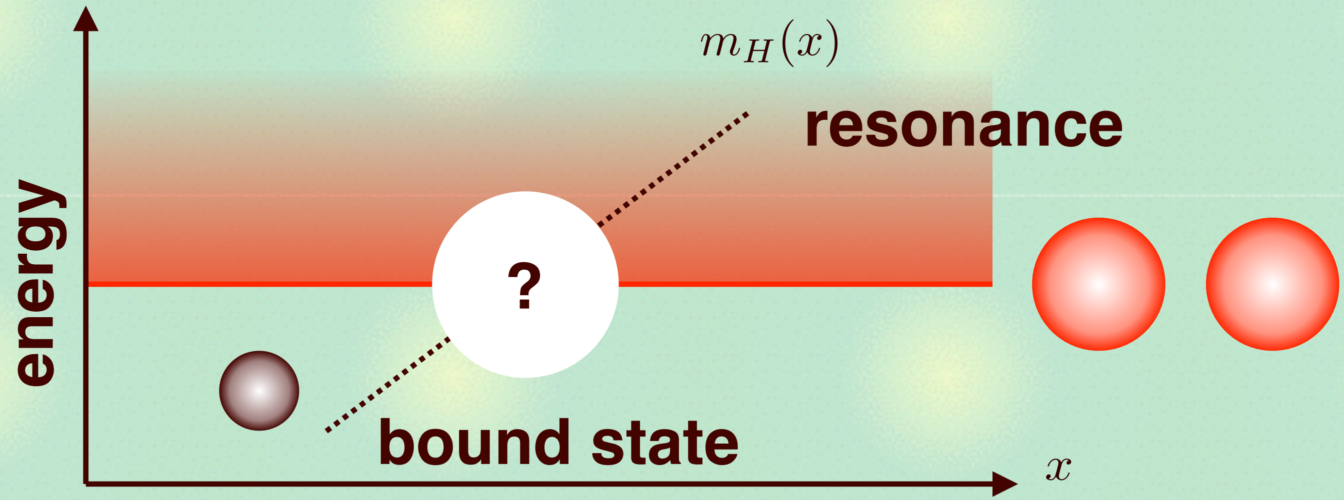
Systematic expansion of hadron masses

- ChPT: light quark mass m_q
- HQET: heavy quark mass m_Q
- large Nc: number of colors N_c

Hadron mass scaling

$$m_H(x); \quad x = \frac{m_q}{\Lambda}, \frac{\Lambda}{m_Q}, \frac{1}{N_c}$$

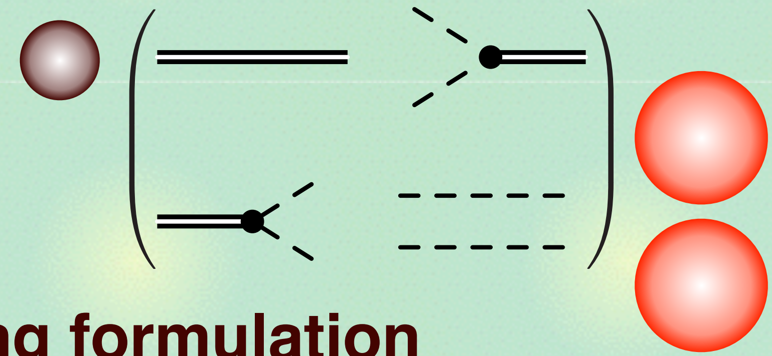
What happens at two-body threshold?



Formulation

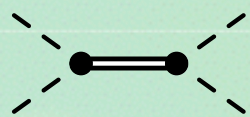
Coupled-channel Hamiltonian (bare state + continuum)

$$\begin{pmatrix} M_0 & \hat{V} \\ \hat{V} & \frac{p^2}{2\mu} \end{pmatrix} \begin{pmatrix} c(E)|\,\psi_0\,\rangle \\ \chi_E(\boldsymbol{p})|\,\boldsymbol{p}\,\rangle \end{pmatrix} = E \begin{pmatrix} c(E)|\,\psi_0\,\rangle \\ \chi_E(\boldsymbol{p})|\,\boldsymbol{p}\,\rangle \end{pmatrix}$$

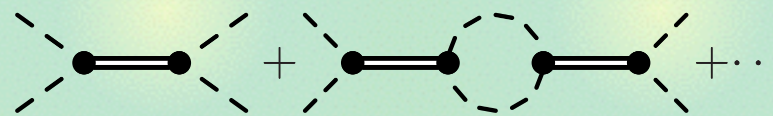


Equivalent single-channel scattering formulation

$$\hat{V}_{\mathrm{eff}}(E) = \frac{\hat{V}|\,\psi_0\,\rangle\langle\,\psi_0\,|\hat{V}}{E - M_0} \sim$$



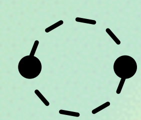
$$f(\boldsymbol{p}, \boldsymbol{p}', E) = -\frac{4\pi^2\mu\langle\,\boldsymbol{p}\,|\hat{V}|\,\psi_0\,\rangle\langle\,\psi_0\,|\hat{V}|\,\boldsymbol{p}'\,\rangle}{E - M_0 - \Sigma(E)} \sim$$



Pole condition:

$$E_h - M_0 = \Sigma(E_h)$$

$$\Sigma(E) = \int \frac{\langle\,\psi_0\,|\hat{V}|\,\boldsymbol{q}\,\rangle\langle\,\boldsymbol{q}\,|\hat{V}|\,\psi_0\,\rangle}{E - q^2/(2\mu) + i0^+} d^3q \sim$$

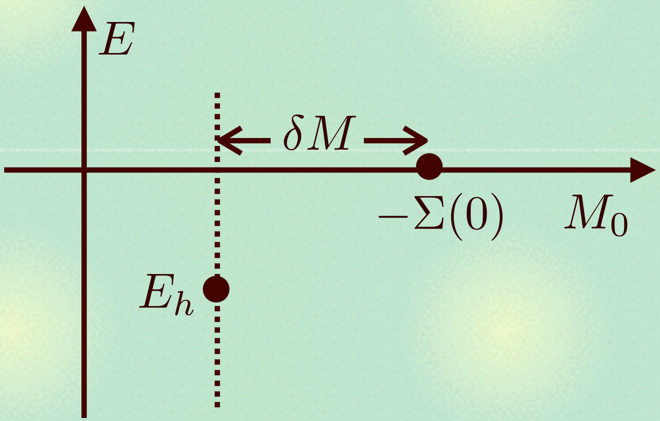


Question: How E_h behaves against M_0 around $E_h=0$?

Near-threshold bound state

Bound state condition around $E_h=0$

$$E_h + \Sigma(0) - \delta M = \Sigma(E_h)$$

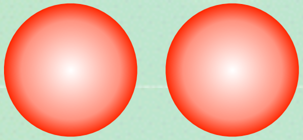


Leading contribution of the expansion:

$$E_h = \frac{1}{1-\Sigma'(0)}\delta M = Z(0)\delta M, \quad \Sigma'(E) \equiv \frac{d\Sigma(E)}{dE}$$

Field renormalization constant

$$\left|\langle \Psi | \begin{pmatrix} |\psi_0\rangle \\ 0 \end{pmatrix}\right|^2 + \int \left|\langle \Psi | \begin{pmatrix} 0 \\ |\boldsymbol{q}\rangle \end{pmatrix}\right|^2 d^3q = 1$$



$Z(0)$ vanishes for $l=0$: compositeness theorem (shown later)

$$E_h \propto \begin{cases} \mathcal{O}(\delta M^2) & l=0 \\ \delta M & l \neq 0 \end{cases}$$

Near-threshold bound state (general)

General argument by Jost function (Fredholm determinant)

J.R. Taylor, *Scattering Theory* (Wiley, New York, 1972)

$$f_l(p) = \frac{\mathscr{f}_l(-p) - \mathscr{f}_l(p)}{2ip\,\mathscr{f}_l(p)}$$

- amplitude pole (eigenstate): Jost function zero

If there is a zero at p=0, then the expansion is

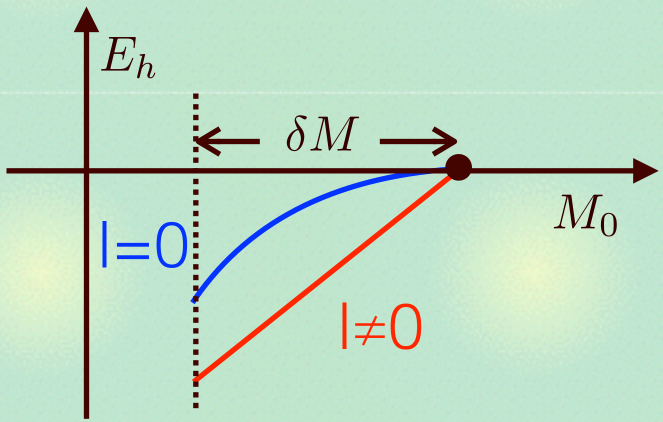
$$\mathscr{f}_l(p) = \begin{cases} i\gamma_0 p + \mathcal{O}(p^2) & l = 0 \\ \beta_l p^2 + \mathcal{O}(p^3) & l \neq 0 \end{cases}$$

- γ_0 and β_l are nonzero for a general local potential
  : simple (double) zero for l=0 (l≠0)

R.G. Newton, J. Math. Phys. 1, 319 (1960)

Near-threshold scaling:

$$E_h \propto \begin{cases} -\delta M^2 & l = 0 \\ \delta M & l \neq 0 \end{cases}, \quad \delta M < 0$$



General threshold behavior

Near threshold scaling:

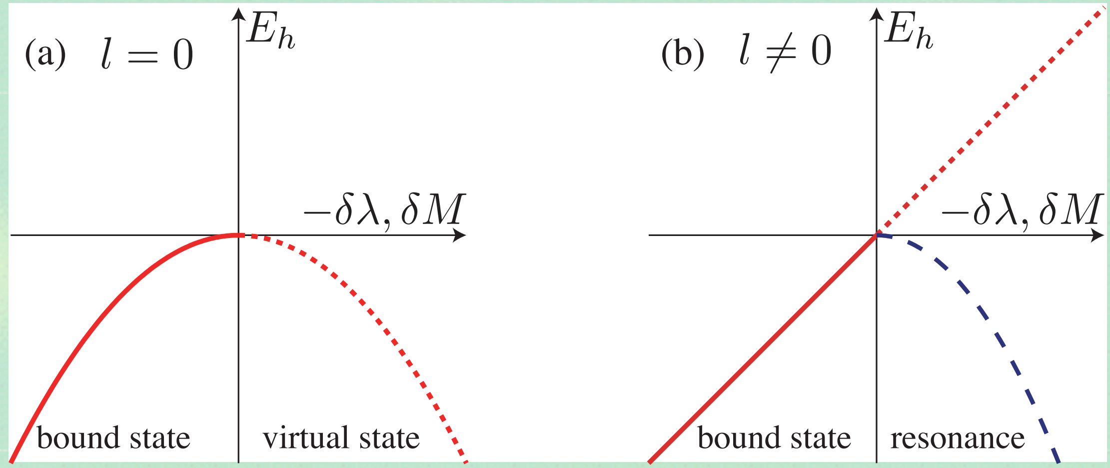
- $\delta M < 0$

$$E_h \propto \begin{cases} -\delta M^2 & l = 0 \\ \delta M & l \neq 0 \end{cases}$$

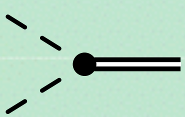
- $\delta M > 0$

$$E_h \propto -\delta M^2 \quad l = 0$$

$$\begin{cases} \mathrm{Re}\ E_h \propto \delta M \\ \mathrm{Im}\ E_h \propto -(\delta M)^{l+1/2} \end{cases} \quad l \neq 0$$

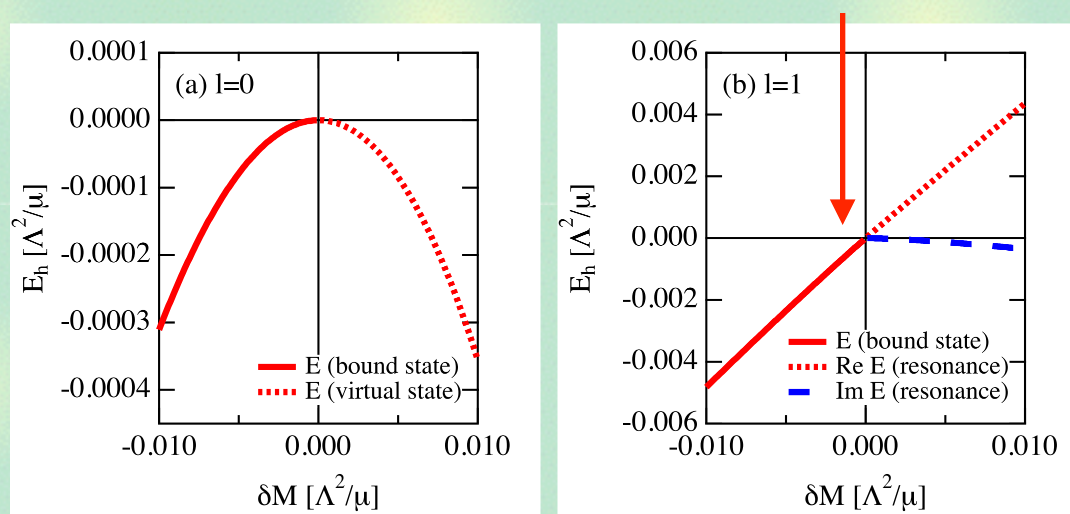


Numerical calculation



$$\langle\, \boldsymbol{q}\, |\hat{V}|\, \psi_0 \,\rangle = g_l |\boldsymbol{q}|^l \Theta(\Lambda - |\boldsymbol{q}|)$$

slope: Z(0)



Compositeness theorem

Theorem: $Z(0)=0$ **for s wave**

If the s-wave scattering amplitude has a pole exactly at the threshold with a finite range interaction, then the field renormalization constant vanishes.

T. Hyodo, arXiv:1407.2372 [hep-ph]

For bare state-continuum model (c**: nonzero constant)**

$$Z(B) = \frac{1}{1-\Sigma'(-B)} \approx \frac{1}{1-c\frac{g_0^2}{\sqrt{B}}}$$

$$\mathrm{Im}\,\Sigma(p^2/2\mu) \propto p^{2l+1}$$

$Z(0)$ **vanishes for** $g_0 \neq 0$**. If** $g_0=0$**, no pole in the amplitude.**

For a local potential: poles in the effective range expansion

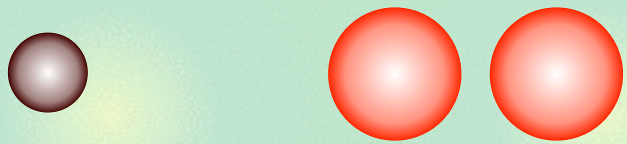
$$p_1 = i\sqrt{2\mu B}, \quad p_2 = -i\sqrt{2\mu B}\frac{2-Z(B)}{Z(B)}$$

If $Z(0) \neq 0$**, then both** p_1 **and** p_2 **go to zero for** $B \to 0$
: contradiction with the simple pole at $p=0$

Interpretation of the compositeness theorem

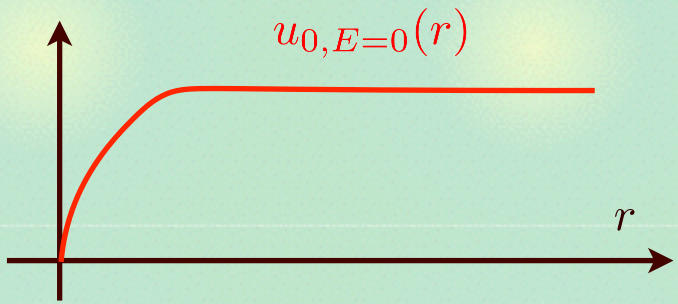
Z(B)**: overlap of the bound state with bare state**

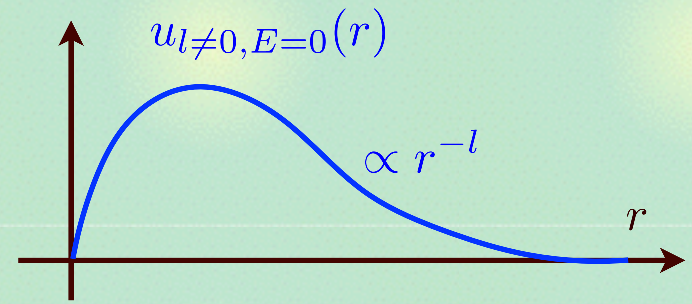
$$\left|\langle \Psi | \begin{pmatrix} |\psi_0\rangle \\ 0 \end{pmatrix}\right|^2 + \int \left|\langle \Psi | \begin{pmatrix} 0 \\ |\boldsymbol{q}\rangle \end{pmatrix}\right|^2 d^3q = 1$$



- Z(B≠0)=0 **—> Bound state is completely composite.**

Two-body wave function at E=0**:** $u_{l,E=0}(r) \xrightarrow{r\to\infty} r^{-l}$



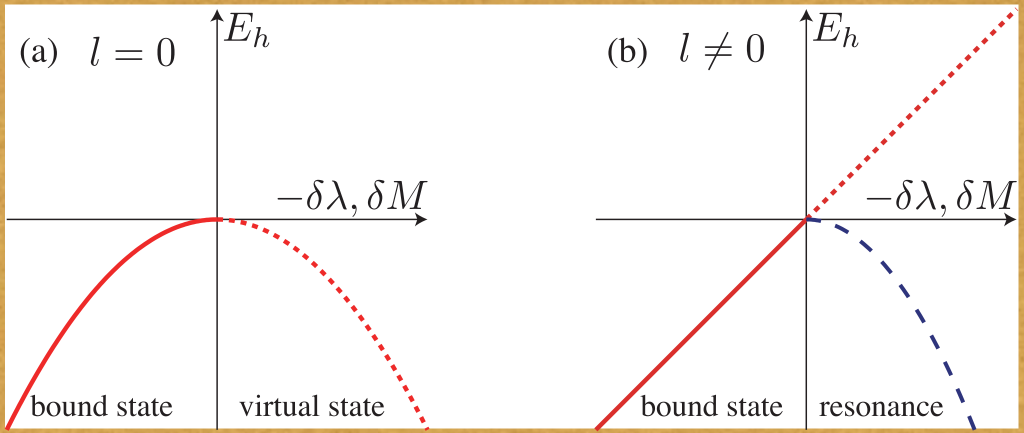


Z(0)=0**: ~~Bound state is completely composite.~~**
Composite component is infinitely large so that the fraction of any finite admixture of bare state is zero.

Summary

We study the hadron mass scaling near threshold.

- **General scaling laws:**



- **Compositeness theorem:**

$$Z(B = 0) = 0 \quad \text{for } l = 0$$

- **Chiral extrapolation across the s-wave threshold should be carefully performed.**

T. Hyodo, arXiv:1407.2372 [hep-ph]