Universal physics of three-bosons with isospin

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Universal physics

Universal: **different systems share the identical feature**

Critical phenomena around phase transition

- **large correlation length** ξ
- **scaling, critical exponent, ...**
- **liquid-gas transition ~ ferromagnet**

N. Goldenfeld, *"Lectures on phase transitions and the renormalization group"* (1992)

vdW

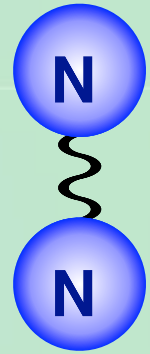
Universal physics in few-body system

- **large two-body scattering length** $|a|$

strong

- **scaling** $a \to \lambda a, \quad p \to \lambda^{-1} p \quad E \to \lambda^{-2} E$

E. Braaten, H.-W. Hammer, Phys. Rept. 428, 259 (2006)



$$B_2 = \frac{1}{ma^2}$$

	N [MeV]	^{4}He [mK]
B_2	**2.22**	**1.31**
$1/ma^2$	**1.41**	**1.12**

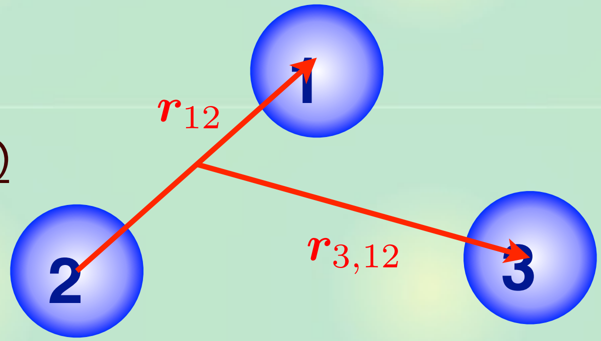
^{4}He

Three-body system: scaling and its violation

Three-body system in hyperspherical coordinates

$$(\boldsymbol{r}_{12}, \boldsymbol{r}_{3,12}) \leftrightarrow (R, \alpha_3, \hat{\boldsymbol{r}}_{12}, \hat{\boldsymbol{r}}_{3,12})$$

hyperradius **hyperangular variables Ω (dimensionless)**

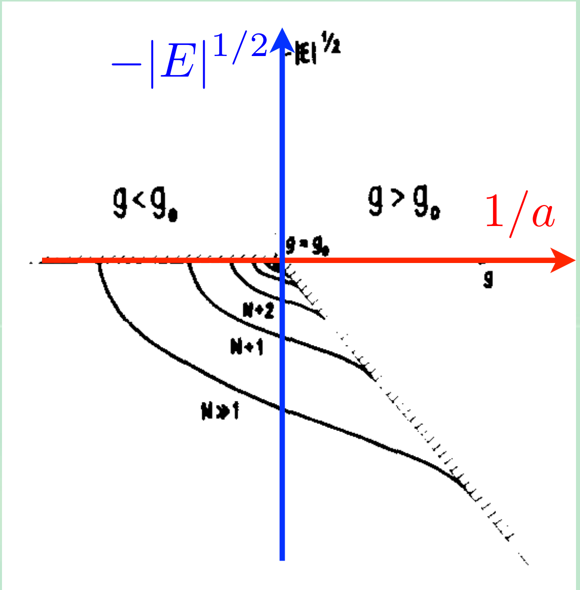


For |a| -> ∞, system is scale invariant.

$$V(R, \Omega) \propto \frac{1}{R^2}$$

Efimov effect : attractive $1/R^2$ for identical three bosons

V. Efimov, Phys. Lett. B 33, 563-564 (1970)



- **infinitely many bound states**
- **discrete scale invariance --> limit cycle**

P.F. Bedaque, H.-W. Hammer, U. van Kolck, Phys. Rev. Lett. 82, 463-437 (1999)

Pion interaction

$\pi\pi$ scattering length <- chiral low energy theorem

S. Weinberg, Phys. Rev. Lett. 17, 616-621 (1966)

$$a^{I=0} \propto -\frac{7}{4}\frac{m_\pi}{f_\pi^2}, \quad a^{I=2} \propto \frac{1}{2}\frac{m_\pi}{f_\pi^2}$$

- $1/f_\pi^2$ **~ spontaneous breaking of chiral symmetry**
- m_π **~ explicit breaking of chiral symmetry**

In nature, the scattering lengths are small <-- m_π is small

- $a^{I=0}$ ~ -0.31 fm, $a^{I=2}$ ~ 0.06 fm **/ QCD scale ~** 1 fm

If we can adjust m_π or f_π, |a| increases by m_π ↗ or f_π ↘

- **sufficient attraction**
  - **--> bound state in** I=0
  - **--> diverging** |a|
- **sigma:** I=0 **resonance**

$f_0(500)$ or σ
was $f_0(600)$

$I^G(J^{PC}) = 0^+(0^{++})$

A REVIEW GOES HERE – Check our WWW List of Reviews

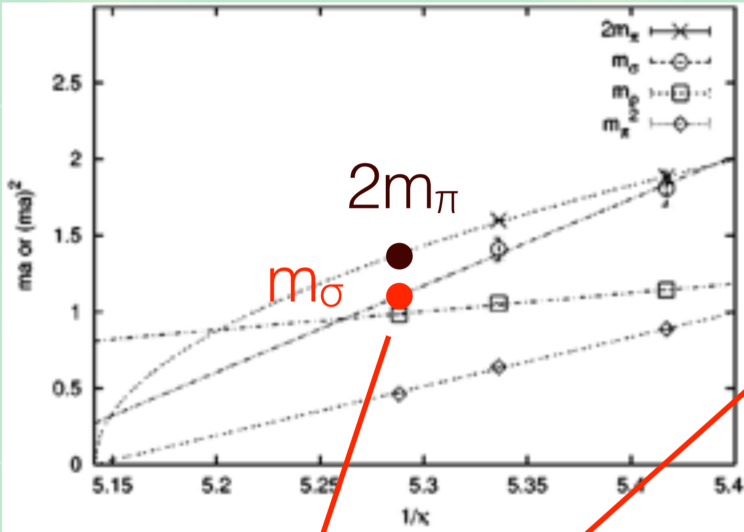
$f_0(500)$ T-MATRIX POLE $\sqrt{s}$

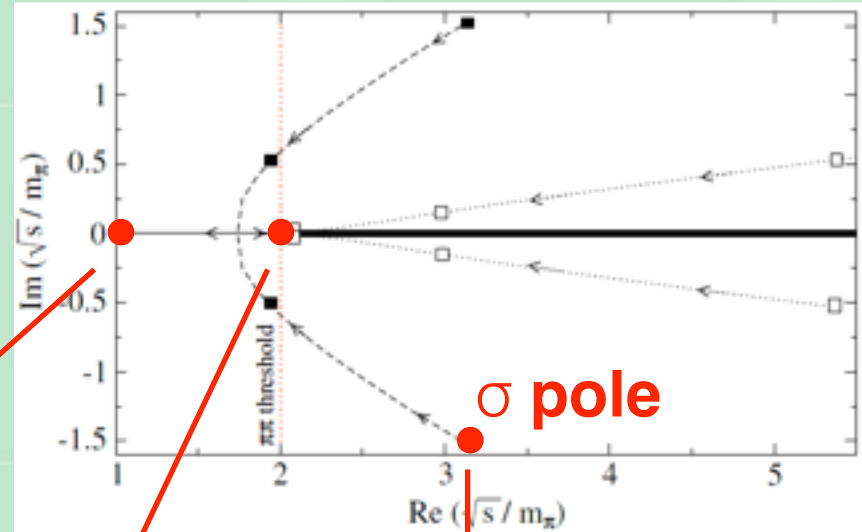
Note that $\Gamma \approx 2\,\mathrm{Im}(\sqrt{s_{pole}})$.

VALUE (MeV)	DOCUMENT ID	TECN	COMMENT
(400–550)−i(200–350) OUR ESTIMATE			

Increase pion mass

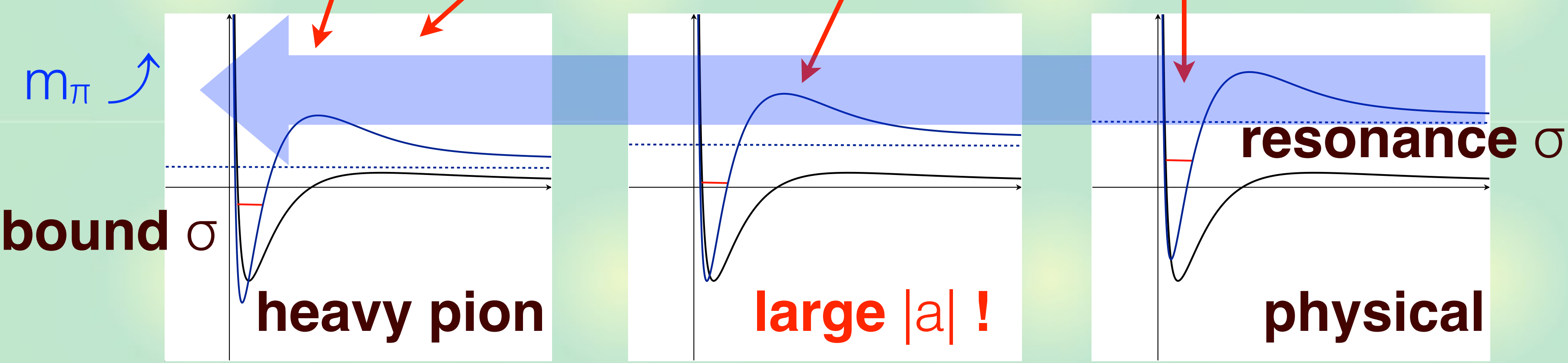
Lattice QCD/chiral EFT can tune the pion mass





T. Kunihiro *et al.* (SCALAR Collaboration), Rev. Rev. D70, 034504 (2004)

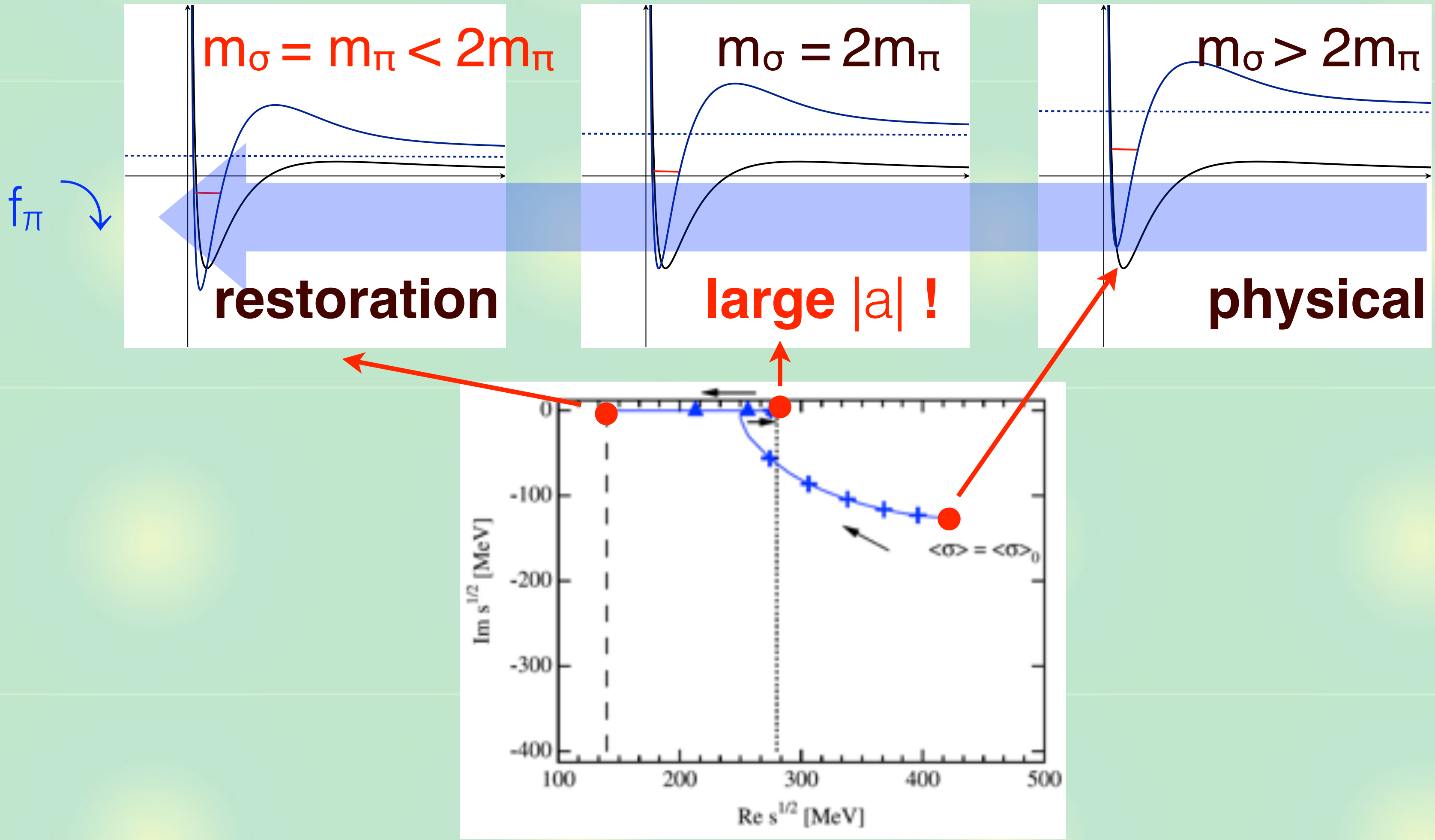
C. Hanhart, J.R. Pelaez, G. Rios, Phys. Rev. Lett. 100, 152001 (2008)



==> Numerical experiment (lattice QCD)!

Decrease pion decay constant

Chiral symmetry restoration ~ reduction of f_π



T. Hyodo, D. Jido, T. Kunihiro, Nucl. Phys. A848, 341-365 (2010)

==> Real experiment (in-medium symmetry restoration) !

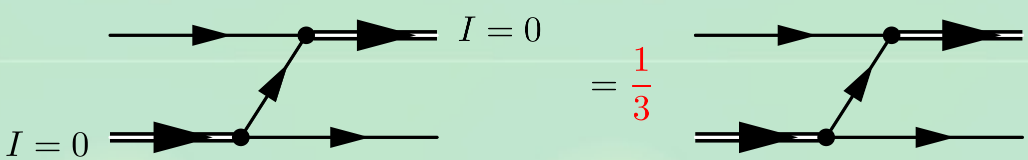
Three pions with isospin symmetry

Large I=0 scattering length

$$f_{I=0} = \frac{1}{-1/a - ip}, \quad f_{I=2} = 0$$

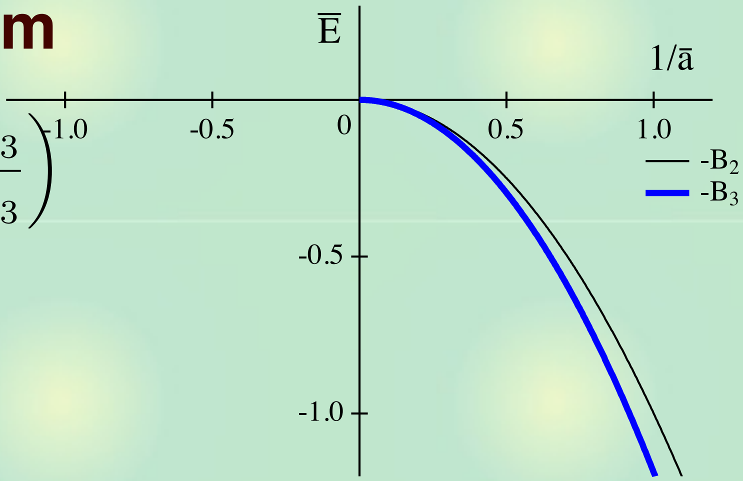
S-wave three-pion system in total I=1

$$\begin{pmatrix} |\,\pi \otimes [\pi \otimes \pi]_{I=0}\,\rangle_{I=1} \\ |\,\pi \otimes [\pi \otimes \pi]_{I=2}\,\rangle_{I=1} \end{pmatrix} = \begin{pmatrix} 1/3 & \sqrt{5}/3 \\ \sqrt{5}/3 & 1/6 \end{pmatrix} \begin{pmatrix} |\,[\pi \otimes \pi]_{I=0} \otimes \pi\,\rangle_{I=1} \\ |\,[\pi \otimes \pi]_{I=2} \otimes \pi\,\rangle_{I=1} \end{pmatrix}$$



Eigenvalue equation for 3-body system

$$z(|\boldsymbol{p}|) = \frac{2}{3\pi} \int_0^\infty d|\boldsymbol{q}| \frac{|\boldsymbol{q}|}{|\boldsymbol{p}|} \ln\left(\frac{\boldsymbol{q}^2 + \boldsymbol{p}^2 + |\boldsymbol{q}||\boldsymbol{p}| + mB_3}{\boldsymbol{q}^2 + \boldsymbol{p}^2 - |\boldsymbol{q}||\boldsymbol{p}| + mB_3} \right) \times \frac{z(|\boldsymbol{q}|)}{\sqrt{\frac{3}{4}\boldsymbol{q}^2 + mB_3} - \frac{1}{a}}$$



$$B_3 = \frac{1.04391}{ma^2} \quad \text{for } 1/a > 0 \qquad \text{c.f.} \quad B_2 = \frac{1}{ma^2}$$

Three pions with isospin breaking

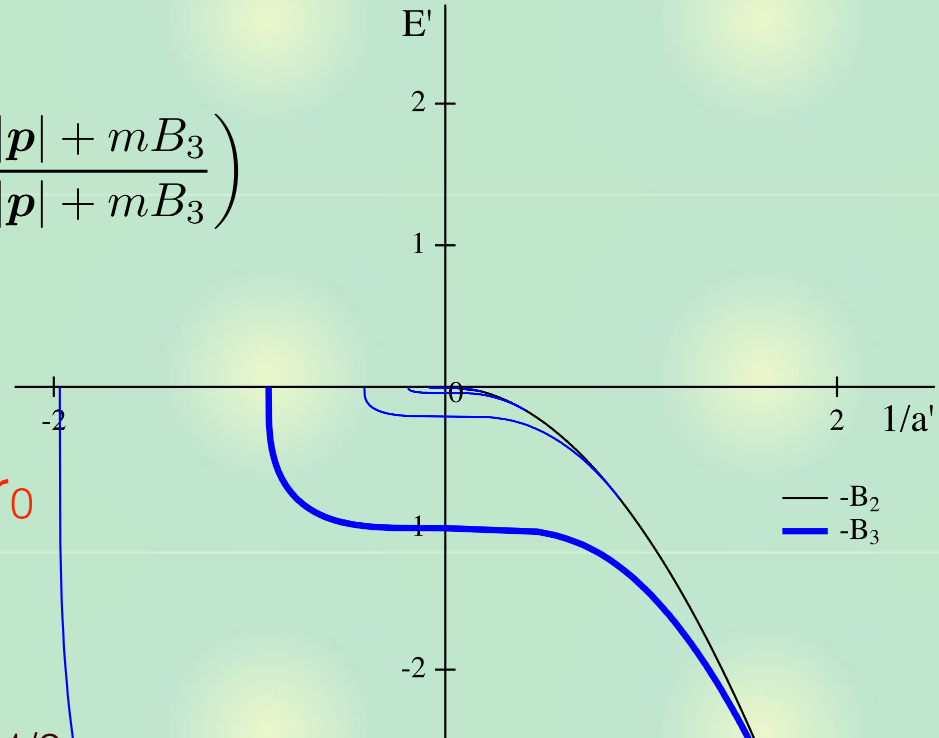
Isospin breaking: $m_{\pi^\pm} = m_{\pi^0} + \Delta$ **with** $\Delta > 0$

- In the energy region $E \ll \Delta$**, heavy** $\pi^\pm$ **can be neglected.**

Identical three-boson system with a large scattering length
--> Efimov effect

$$z(|\boldsymbol{p}|) = \frac{2}{\pi}\int_0^\infty d|\boldsymbol{q}|\frac{|\boldsymbol{q}|}{|\boldsymbol{p}|}\ln\left(\frac{\boldsymbol{q}^2+\boldsymbol{p}^2+|\boldsymbol{q}||\boldsymbol{p}|+mB_3}{\boldsymbol{q}^2+\boldsymbol{p}^2-|\boldsymbol{q}||\boldsymbol{p}|+mB_3}\right)$$
$$\times\frac{z(|\boldsymbol{q}|)}{\sqrt{\frac{3}{4}\boldsymbol{q}^2+mB_3}-\frac{1}{a}} f_\Lambda(|\boldsymbol{q}|)$$

cutoff ~ $1/r_0$



Universal physics at $E \ll (2m\Lambda)^{1/2}$

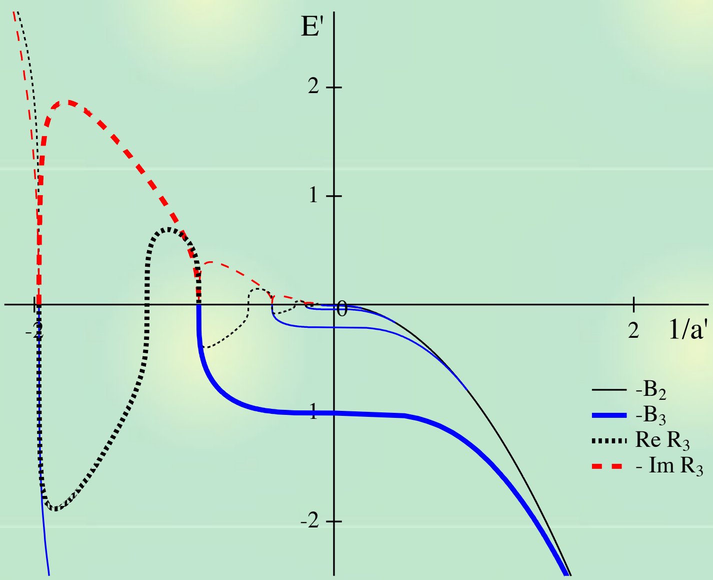
<-- Efimov parameter κ_*

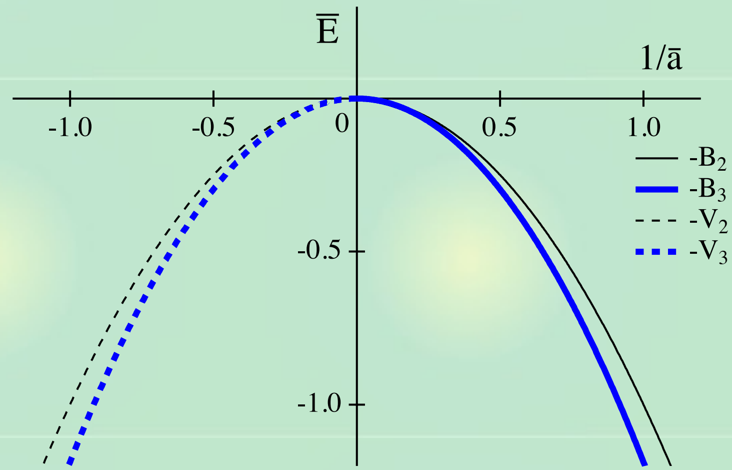
Coupled-channel effect

Two universal phenomena : existence of the coupled channel

$$z(|\boldsymbol{p}|) = \frac{2}{\lambda\pi}\int_0^\infty d|\boldsymbol{q}|\frac{|\boldsymbol{q}|}{|\boldsymbol{p}|}\ln\left(\frac{\boldsymbol{q}^2+\boldsymbol{p}^2+|\boldsymbol{q}||\boldsymbol{p}|+mB_3}{\boldsymbol{q}^2+\boldsymbol{p}^2-|\boldsymbol{q}||\boldsymbol{p}|+mB_3}\right)\frac{z(|\boldsymbol{q}|)}{\sqrt{\frac{3}{4}\boldsymbol{q}^2+mB_3}-\frac{1}{a}}$$

$\lambda < 2.41480$ $\quad$ $2.41480 < \lambda < 3.66811$ $\quad$ $3.66811 < \lambda$





no universal bound state

discrete scale invariance

scale invariance

Both cases can be realized in three-pion systems.

Summary

Universal physics of three pions

- **Large $\pi\pi$ scattering length ($I=0$) can be obtained by $m_\pi \nearrow$ or $f_\pi \searrow$.**
- **Universal phenomena with large a:**
  - **single bound state (isospin symmetry)**
  - **Efimov states (isospin breaking)**
- **Consequence in hadron physics:**
  - **realization in lattice QCD**
  - **simultaneous softening of σ and π^***

T. Hyodo, T. Hatsuda, Y. Nishida, Phys. Rev. C89, 032201(R) (2014)