Universal three-pion physics with a large scattering length

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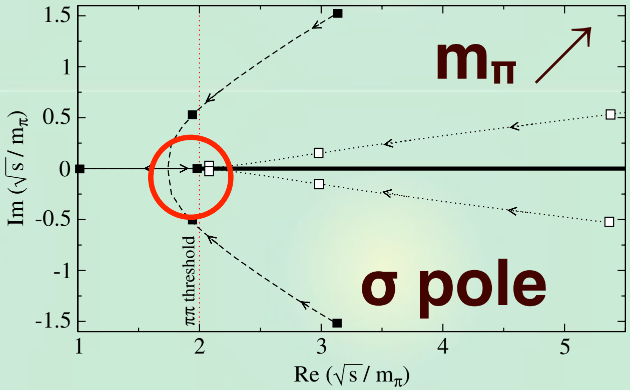
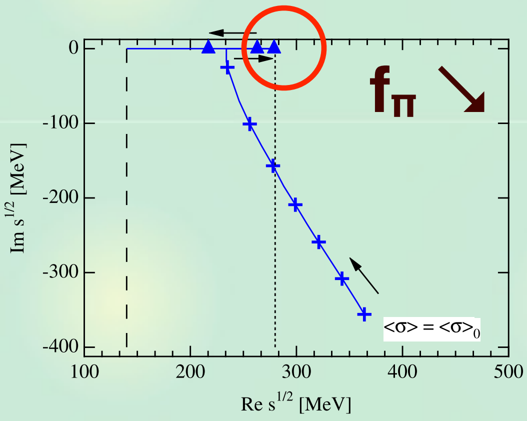
Universal phenomena in hadron physics

Universal few-body physics <-- large scattering length

S-wave $\pi\pi$ scattering length

- $a_{I=0}$ ~ -0.31 fm, $a_{I=2}$ ~ 0.06 fm / QCD scale ~ 1 fm
- I=0 component can be increased by m_π ↗ or f_π ↘





C. Hanhart, J.R. Pelaez, G. Rios, Phys. Rev. Lett. 100, 152001 (2008)
T. Hyodo, D. Jido, T. Kunihiro, Nucl. Phys. A848, 341-365 (2010)

- Realizable by lattice QCD / nuclear medium

==> Three-pion system with a large scattering length

Isospin symmetric three pions

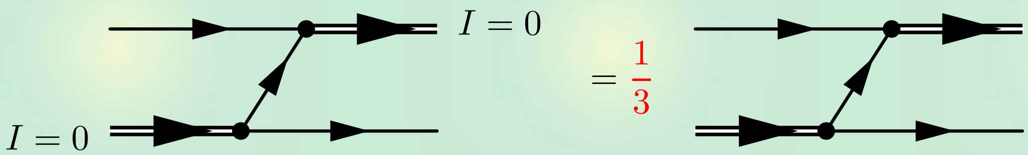
Pion has an internal degree of freedom : isospin I=1

- **s-wave two-body amplitude:** I=0 **and** I=2

$$it_0(p) = \frac{8\pi}{m}\frac{i}{\frac{1}{a} - \sqrt{\frac{\boldsymbol{p}^2}{4} - mp_0 - i0^+}}, \quad it_2(p) = 0$$

S-wave three-pion system in total I=1

$$\begin{pmatrix} |\,\pi \otimes [\pi \otimes \pi]_{I=0}\,\rangle_{I=1} \\ |\,\pi \otimes [\pi \otimes \pi]_{I=2}\,\rangle_{I=1} \end{pmatrix} = \begin{pmatrix} 1/3 & \sqrt{5}/3 \\ \sqrt{5}/3 & 1/6 \end{pmatrix} \begin{pmatrix} |\,[\pi \otimes \pi]_{I=0} \otimes \pi\,\rangle_{I=1} \\ |\,[\pi \otimes \pi]_{I=2} \otimes \pi\,\rangle_{I=1} \end{pmatrix}$$



Eigenvalue equation (eigenvalue B_3 for eigenfunction $z(|\mathbf{p}|)$)

$$z(|\boldsymbol{p}|) = \frac{2}{3\pi}\int_0^\infty d|\boldsymbol{q}|\frac{|\boldsymbol{q}|}{|\boldsymbol{p}|}\ln\left(\frac{\boldsymbol{q}^2 + \boldsymbol{p}^2 + |\boldsymbol{q}||\boldsymbol{p}| + mB_3}{\boldsymbol{q}^2 + \boldsymbol{p}^2 - |\boldsymbol{q}||\boldsymbol{p}| + mB_3}\right)\frac{z(|\boldsymbol{q}|)}{\sqrt{\frac{3}{4}\boldsymbol{q}^2 + mB_3} - \frac{1}{a}}$$

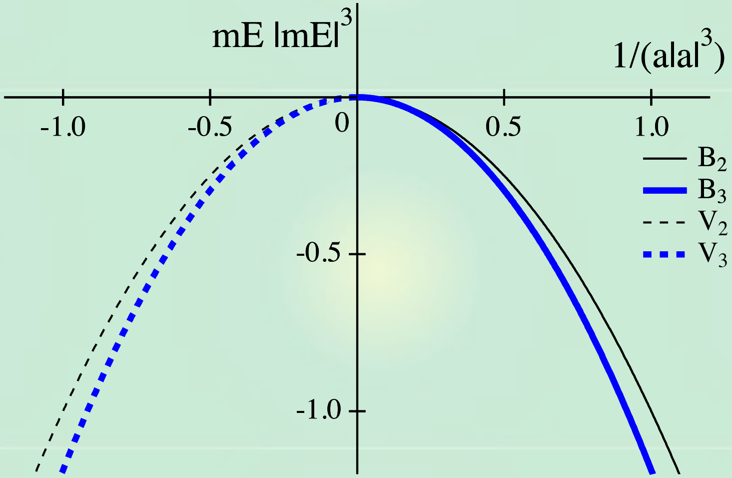
Factor 1/3 difference from the identical boson case

Spectrum in the isospin symmetric limit

Result: one universal three-pion bound state

$$B_3 = \frac{1.04391}{ma^2} \quad \text{for } 1/a > 0$$

$$\text{c.f.} \quad B_2 = \frac{1}{ma^2}$$



Resonances?

- phase rotation of binding energy = phase rotation of a

$$B_3 \to B_3 e^{i\theta} \quad \Leftrightarrow \quad \frac{1}{a} \to \frac{1}{a} e^{-i\theta/2}$$

Negative a: virtual state

<-- rotation of B_3 **by** 2π **= sign flip of** a

No resonance for all a

<-- interchange of Riemann sheet = sign flip of a

With isospin breaking

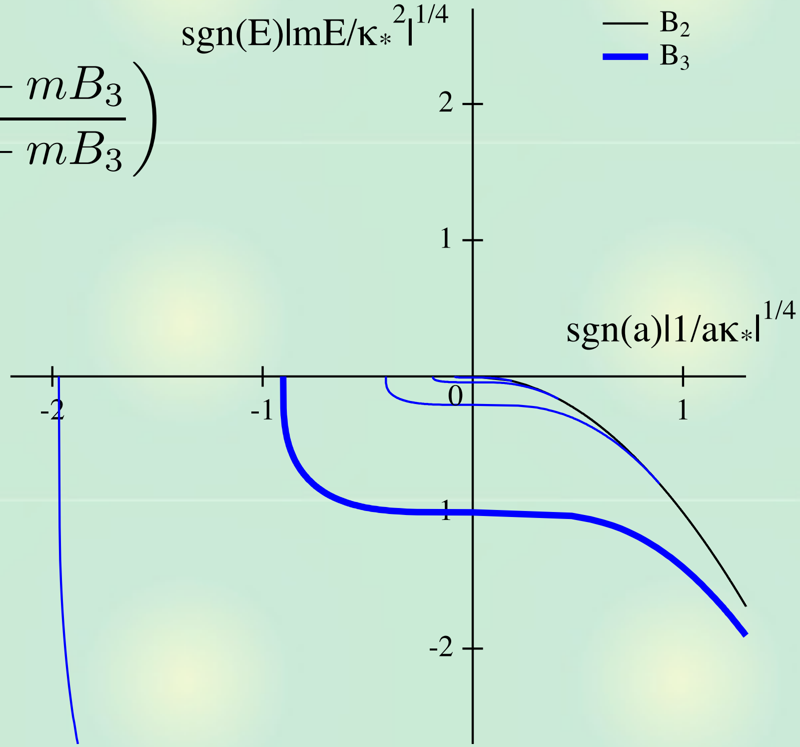
In nature, $m_{\pi^\pm} = m_{\pi^0} + \Delta$ **with** $\Delta > 0$

- In the energy region $E \ll \Delta$**, heavy** $\pi^\pm$ **can be neglected.**

Identical three-boson system with a large scattering length --> Efimov effect

$$z(|\boldsymbol{p}|) = \frac{2}{\pi}\int_0^\infty d|\boldsymbol{q}|\frac{|\boldsymbol{q}|}{|\boldsymbol{p}|}\ln\left(\frac{\boldsymbol{q}^2+\boldsymbol{p}^2+|\boldsymbol{q}||\boldsymbol{p}|+mB_3}{\boldsymbol{q}^2+\boldsymbol{p}^2-|\boldsymbol{q}||\boldsymbol{p}|+mB_3}\right) \times \frac{z(|\boldsymbol{q}|)}{\sqrt{\frac{3}{4}\boldsymbol{q}^2+mB_3}-\frac{1}{a}} f_\Lambda(|\boldsymbol{q}|)$$

cutoff



Universal physics at $E \ll (2m\Lambda)^{1/2}$

<-- Efimov parameter κ_*

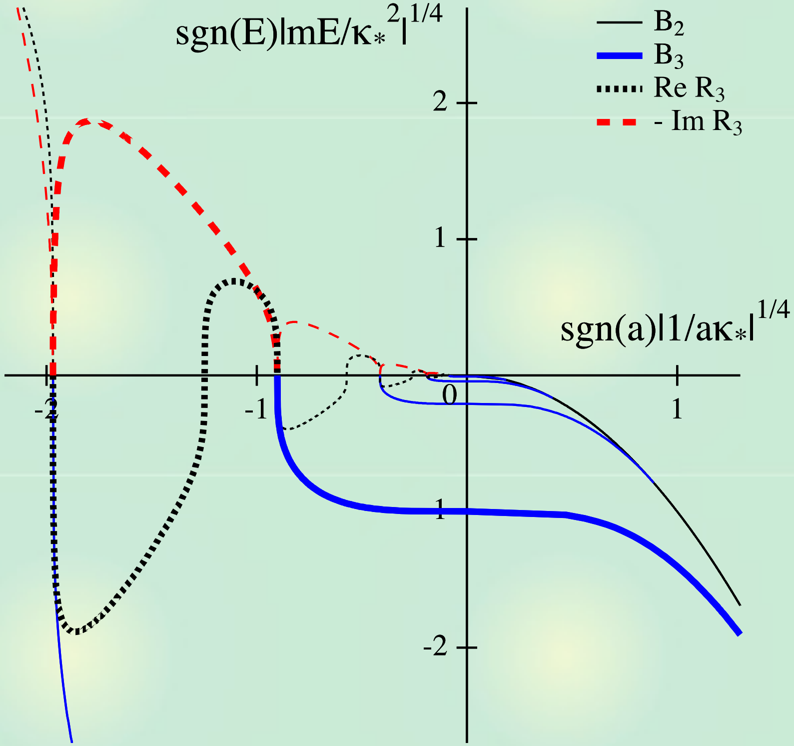
E. Braaten and H.-W. Hammer, Phys. Rept. 428, 259 (2006); Annals Phys. 322, 120 (2007)

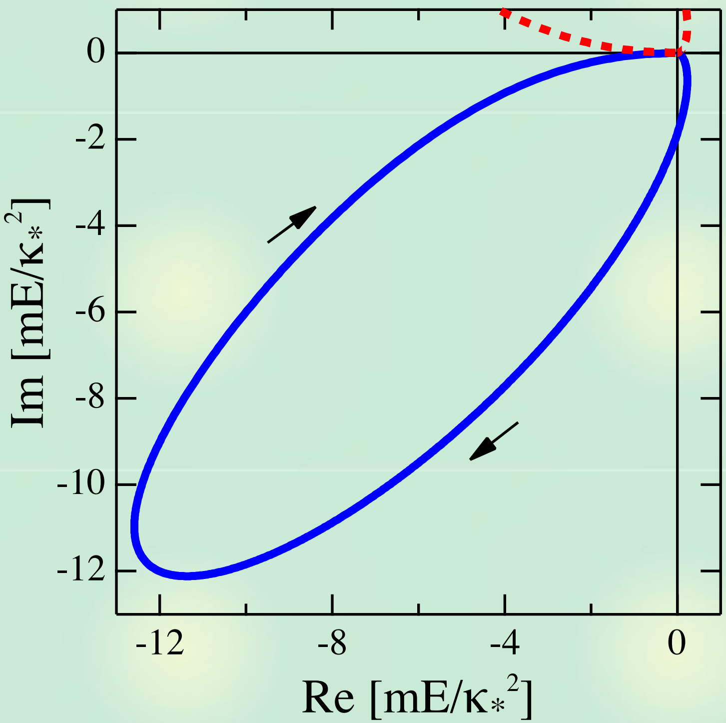
Efimov resonances

Resonance solution is now possible.

- phase rotation of binding energy = phase rotation of a and Λ + proper treatment of singularity in $f_\Lambda(|\mathbf{q}|)$

$$B_3 \to B_3 e^{i\theta} \quad \Leftrightarrow \quad \frac{1}{a} \to \frac{1}{a} e^{-i\theta/2} \quad \text{and} \quad \Lambda \to \Lambda e^{-i\theta/2}$$





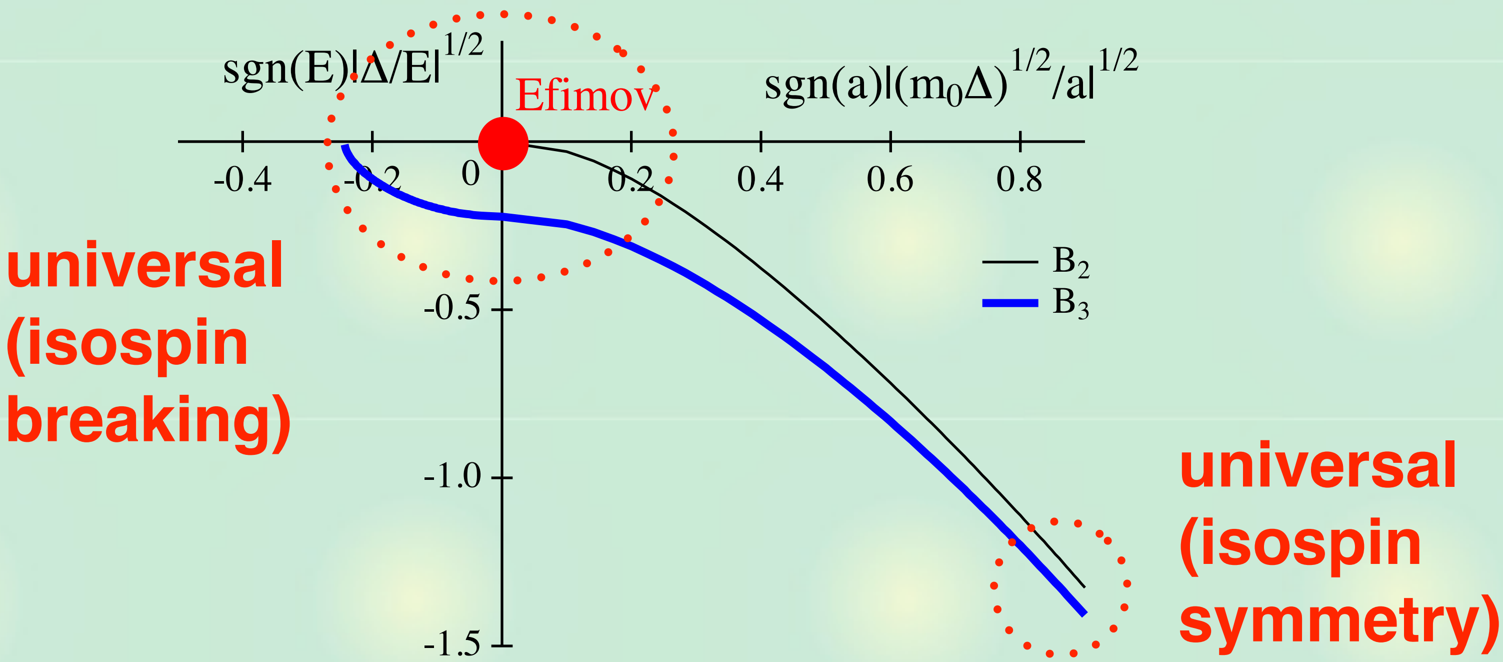
Efimov bound state --> resonance

Interpolation by model

A model with finite mass difference Δ = $m_\pm$ - m_0

$$\mathcal{L} = \sum_{i=0,\pm} \pi_i^\dagger \left(i\partial_t + \frac{\nabla^2}{2m_i} - m_i \right) \pi_i + \frac{g}{4} \frac{\pi_0^\dagger \pi_0^\dagger - 2\pi_+^\dagger \pi_-^\dagger}{\sqrt{3}} \frac{\pi_0 \pi_0 - 2\pi_- \pi_+}{\sqrt{3}}$$

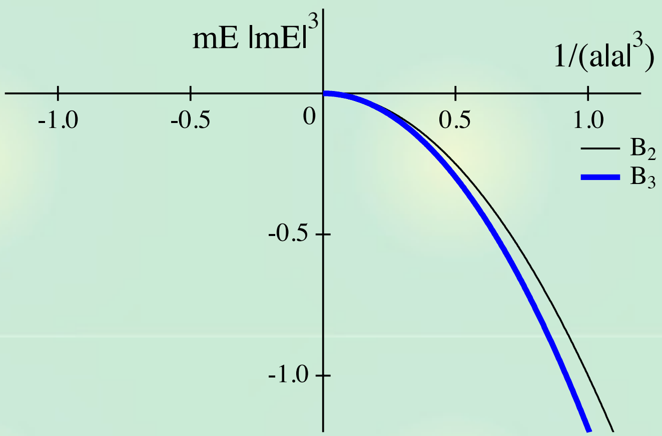
- E ≪ Δ : **Efimov states,** (Λ ≫) E ≫ Δ : **single bound state**
- **cutoff for the Efimov effect is introduced by Δ.**

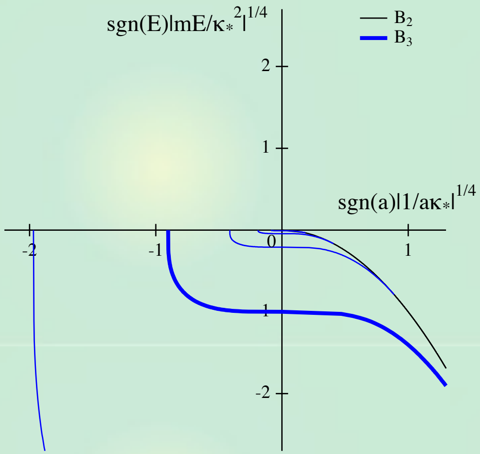


Lowest Efimov level --> universal bound state

Implication in hadron physics

Two-body ππ bound state (σ) --> at least one bound state in three-body channel with I=1 **and** J=0 **channel:** π*





Remnant of universal bound state : π*(1300)

M = 1300 ± 100 MeV, Γ = 200-600 MeV,

$\Gamma(\pi(\pi\pi)_{\text{s-wave}})/\Gamma(\pi\rho) \sim 2.2$

When the σ softens, π* **also softens simultaneously.**

- caveats for the σ softening in practice: final state interaction, mixing with quark number fluctuation, ...

Summary

Universal physics of three pions

- **Large $\pi\pi$ scattering length ($I=0$) can be realized by $m_\pi \nearrow$ or $f_\pi \searrow$.**
- **Universal phenomena with large a:**
  - **single bound state (isospin symmetry)**
  - **Efimov states (isospin breaking)**
- **Consequence in hadron physics:**
  - **simultaneous softening of σ and π^***

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