Structure of near-threshold s-wave resonances

Tetsuo Hyodo

Yukawa Institute for Theoretical Physics, Kyoto University

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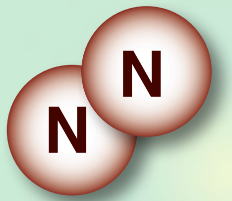
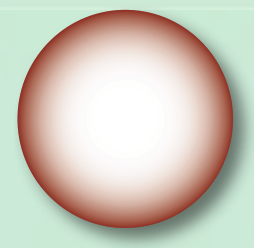
Weinberg's compositeness and deuteron

Z**: probability of finding deuteron in a bare elementary state**

S. Weinberg, Phys. Rev. 137, B672 (1965)

$|\text{deuteron}\rangle =$  or 

$\notin$ NN **model space**

~ elementary particle

$\longleftrightarrow$

Z = 0 Z = 1

Model-independent relation for a shallow bound state

$$a_s = \left[\frac{2(1-Z)}{2-Z}\right] R + \mathcal{O}(m_\pi^{-1}), \quad r_e = \left[\frac{-Z}{1-Z}\right] R + \mathcal{O}(m_\pi^{-1})$$

a_s ~ 5.41 [fm] **: scattering length**

r_e ~ 1.75 [fm] **: effective range**

R ~ $(2\mu B)^{-1/2}$ ~ 4.31 [fm] **: deuteron radius (binding energy)**

--> Z ≲ 0.2 **: Deuteron is almost composite!**

Application to resonances

Features of the Weinberg's argument:

- Model-independent approach (no potential, wave-fn, ...)
- Relation with experimental observables
- Only for bound states with small binding

Interesting (exotic) hadrons: resonances
--> application to resonances by analytic continuation

$$1 - Z = \int d\boldsymbol{q} \frac{|\langle \boldsymbol{q}|V|B\rangle|^2}{[E(q)+B]^2} \sim -g^2 \left.\frac{dG(W)}{dW}\right|_{W=M_B}$$

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C85, 015201 (2012)
F. Aceti, E. Oset, Phys. Rev. D86, 014012 (2012)

- Z can be complex and larger than unity. Interpretation?

What about near-threshold resonances (~ small binding) ?

Effective range expansion

S-wave scattering amplitude at low momentum

$$f(k) = \frac{1}{k\cot\delta - ki} \to \left(\frac{1}{a} - ki + \frac{r_e}{2}k^2\right)^{-1}$$

Truncation is valid only at small k.

Scattering length a

- **strength of the interaction**
- **cross section at zero momentum : $4\pi a^2$**

Effective range r_e

- **typical length scale of the interaction**
- **can be negative (energy-dep., Feshbach resonance, ...)**

D. Phillips, S. Beane, T.D. Cohen, Annals Phys. 264, 255 (1998)
E. Braaten, M. Kusunoki, D. Zhang, Annals Phys. 323, 1770 (2008)

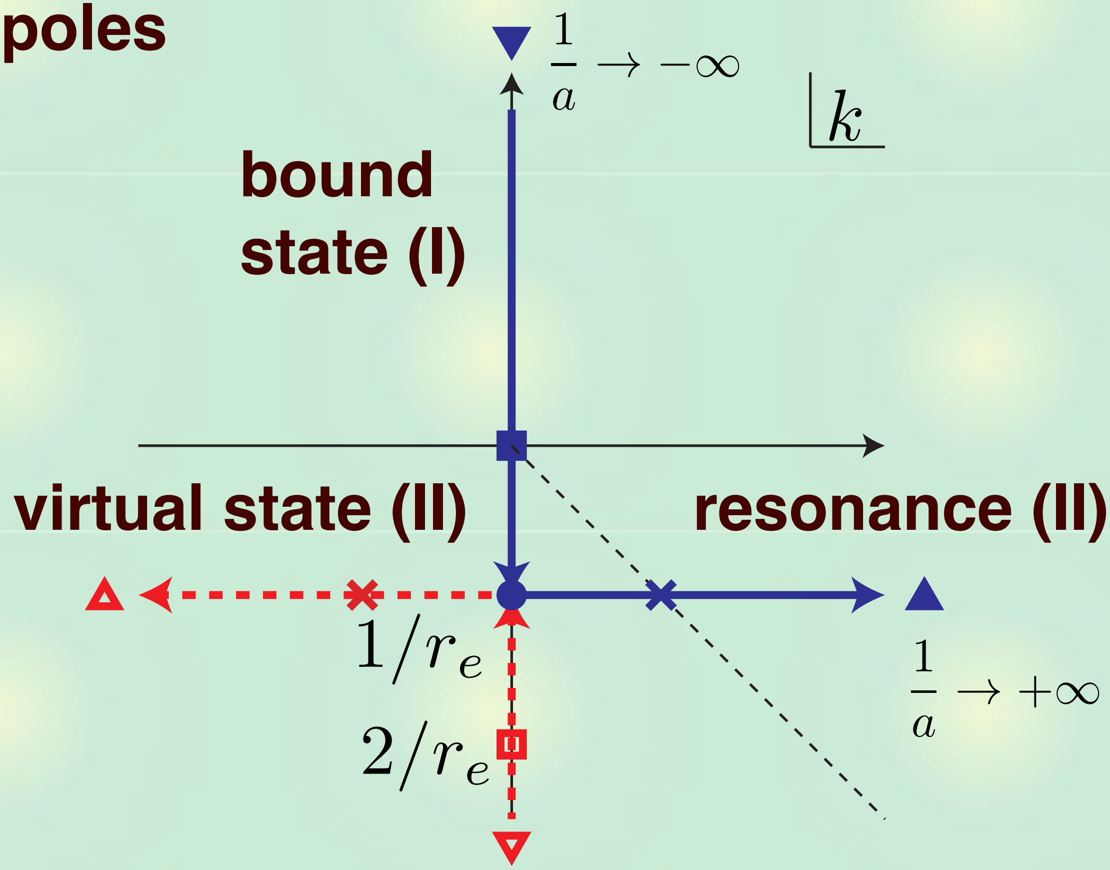
Poles of the amplitude

The amplitude has two poles

$$f(k) = \left(\frac{1}{a} - ki + \frac{r_e}{2} k^2 \right)^{-1}$$

$$k^{\pm} = \frac{i}{r_e} \pm \frac{1}{r_e} \sqrt{-\frac{2r_e}{a} - 1}$$

Pole trajectories with a fixed $r_e < 0$



Positions of poles <--> scattering length + effective range

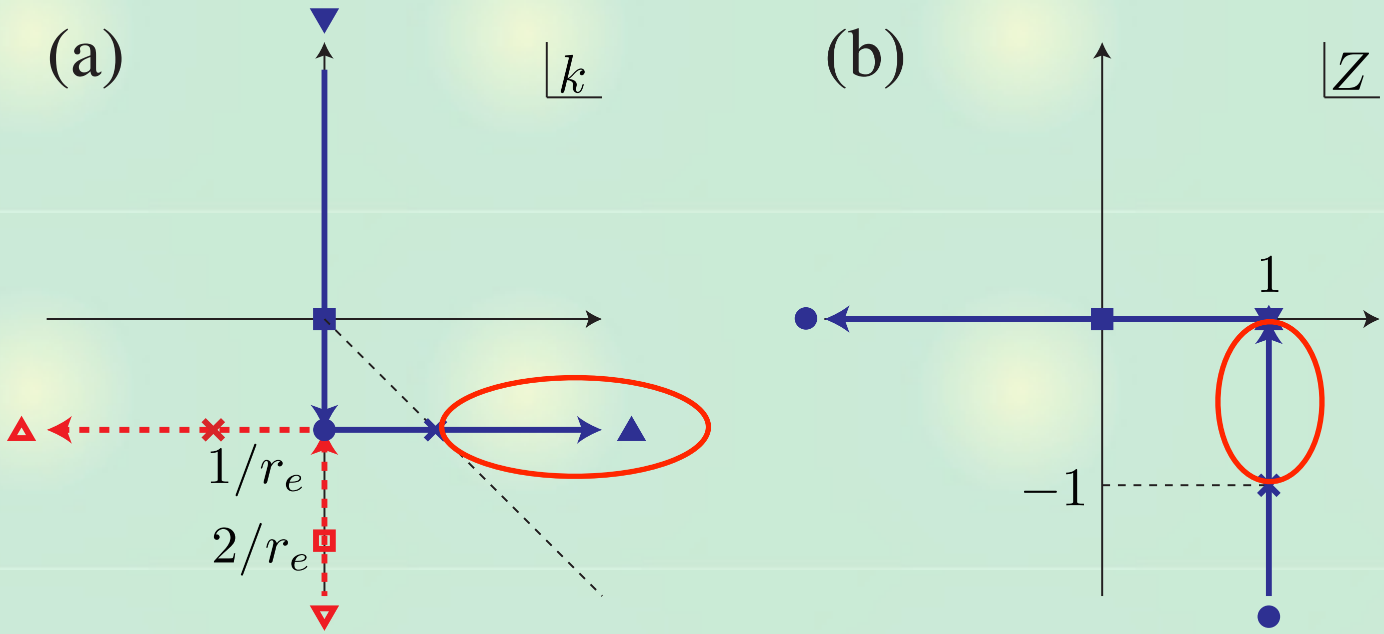
$$a = \frac{k^+ + k^-}{ik^+k^-}, \quad r_e = \frac{2i}{k^+ + k^-}$$

(a,r_e) are real for resonances

Field renormalization constant

Eliminate R **from the Weinberg's relations**

$$Z = 1 - \sqrt{1 - \frac{1}{1 + a/(2r_e)}} = \frac{2k^-}{k^- - k^+}$$



Z **(residue) is determined by the pole position**

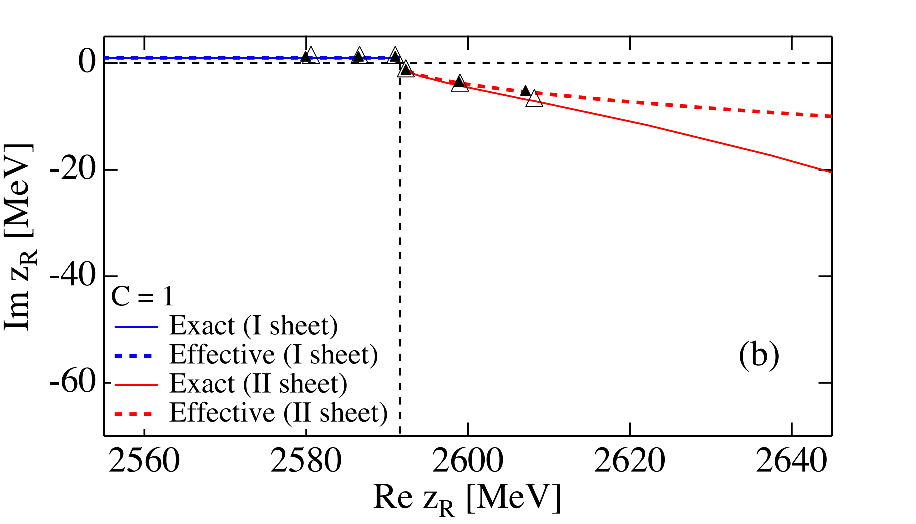
<-- Amplitude is given by two parameters.

1-Z **is pure imaginary and** $0 \leq |1-Z| \leq 1$

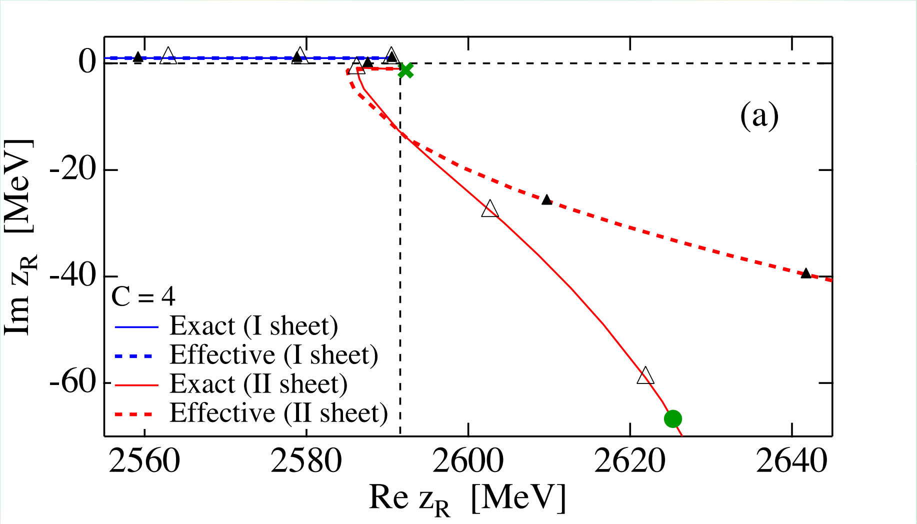
Validity of the effective range expansion

A model calculation

- solid lines: pole position in a scattering model
- dashed lines: position deduced from (a, r_e)



large r_e



small r_e

If the effective range is large, the expansion works well.

Example: $\Lambda_c(2595)$

Pole position of $\Lambda_c(2595)$ with $\pi\Sigma_c$ threshold in PDG

E [MeV]	Γ [MeV]	a [fm]	r_e [fm]
0.67	2.59	10.5	-19.5

- Isospin symmetry is assumed.
- $\pi\pi\Lambda$ channel is not taken into account.
- Decay of Σ_c is not taken into account.

$|1-Z| \sim 0.6$ **Interpretation ?**

Larger effective range than typical hadronic scale
Chiral interaction gives $r_e \sim -4.6$ fm

--> $\Lambda_c(2595)$ is not likely a $\pi\Sigma_c$ molecule

Summary

Near-threshold s-wave resonances

- **Effective range expansion :**

  Resonance structure <--> (a, r_e)

- **Compositeness $1-Z$:**

  pure imaginary and normalized

- **Application to $\Lambda_c(2595)$**

  Large r_e --> not likely a molecule

T. Hyodo, arXiv:1305.1999 [hep-ph], to appear in Phys. Rev. Lett.