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EXOTIC HADRONS AND SU(3) CHIRAL DYNAMICS

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We explore the possibility to generate exotic hadrons dynamically in the scattering of hadrons. The s-wave scattering amplitude of an arbitrary hadron with the Nambu-Goldstone boson is constructed so as to satisfy the unitarity condition and the chiral low energy theorem. We write down the general expression of the coupling strength of the low energy interaction for flavor SU(3) case, and introduce the exoticness quantum number in order to classify the flavor representations. We find that the interaction for the exotic channels is in most cases repulsive, and that the strength of the possible attractive interaction is uniquely determined. We show that the attractive interaction in exotic channels is not strong enough to generate a bound state, while the interaction in nonexotic channel generate bound states which are considered to be the origin of some resonances observed in nature.

1 Introduction

Strong interaction of QCD exhibits a rich spectrum of hadrons, in which about 300 hadronic states have been identified[1]. It is important to investigate the properties of hadrons to understand the low energy dynamics of QCD. Chiral symmetry provides us the way to study hadron properties in connection with the fundamental theory of QCD.

A dynamical model based on chiral symmetry, called chiral unitary approach, successfully describes the two-body scattering of a hadron with the

Nambu-Goldstone (NG) boson, dynamically generating some s-wave resonances in the scattering[2,3,4,5]. These studies are along the same line with the coupled-channel dynamical models for the meson-baryon scattering[6,7], where the vector meson exchange interaction was adopted. This phenomenological interaction is now identified as the Weinberg-Tomozawa (WT) term[8,9], which is the leading order term in chiral perturbation theory. In this respect, one can introduce higher order corrections into the interaction systematically[5,10,11,12,13]. The WT interaction was originally derived in current algebra[8,9]. Since current algebra tells us about the interaction for arbitrary target hadrons, it is possible to apply the chiral unitary approach to the system with $J^P = 3/2^+$ baryon target[14,15] and to the heavy quark sectors[16,17]. In the series of studies, the properties of the generated resonances are in fair agreement with experimental data.

On the other hand, the hadrons observed so far can be classified by their flavor quantum numbers. *Empirically*, there is a regularity in the quantum numbers of the observed hadrons: the states with the valence quark contents of $\bar{q}q$ or qqq were observed, while no state was established with larger number of valence quarks (4, 5, 6,... quarks). The latter states, called exotic hadrons, were intensively studied recently after the result by LEPS collaboration[18]. In spite of the large amount of theoretical work, it is not clear why the exotic hadrons are difficult (or impossible) to observe.

In order to clarify this issue, we have recently performed an analysis of exotic hadrons in s-wave chiral dynamics[19,20,21,22]. We utilize the framework of the chiral unitary approach, since it is naively expected that the resonances produced in the dynamical model should have large component of the multiquark configuration, which is the flavor partner of the exotic hadrons. We simplify the framework as possible, in order to obtain a general and model-independent result.

We construct the scattering amplitude of an arbitrary hadron with the Nambu-Goldstone boson $t(\sqrt{s})$ as

$$t(\sqrt{s}) \to V^{\text{chiral}}(\sqrt{s}) \quad \text{at low energy,} \tag{1}$$

$$\text{Im}t^{-1}(\sqrt{s}) = \rho(\sqrt{s}), \tag{2}$$

where $V^{\text{chiral}}(\sqrt{s})$ is the low energy interaction based on chiral symmetry and $\rho(\sqrt{s})$ is the phase space of the two-body scattering. Eq. (1) is the constraint from the chiral low energy theorem, whereas Eq. (2) guarantees the unitarity of the S-matrix. Utilizing this approach, we would like to study what chiral dynamics tells us about the existence of the exotic hadrons.

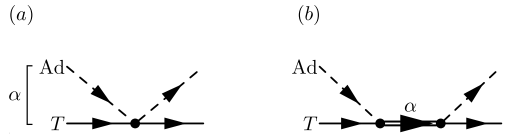

Figure 1. Diagrammatic representation of the scattering. (a) : Notation of the representations α, Ad and T for the WT term. (b) : The bound state pole diagram after unitarization of the amplitude.

2 Low energy theorem and chiral interaction

The low energy s-wave interaction of a target hadron (T) with the NG boson (Ad) in channel α is given by (see Fig. 1)

$$V_\alpha = -\frac{\omega}{2f^2} C_{\alpha,T}, \tag{3}$$

where ω and f are the energy and the decay constant of the NG boson, and the group theoretical factor $C_{\alpha,T}$ is given for flavor SU(3) by

$$C_{\alpha,T} = -\left\langle 2\boldsymbol{F}_T \cdot \boldsymbol{F}_{\mathrm{Ad}} \right\rangle_\alpha = C_2(T) - C_2(\alpha) + 3, \tag{4}$$

with $C_2(R)$ being quadratic Casimir of the representation R. The representation of the combined channel α corresponds to that of the bound state, if it is generated after ressumation (Fig. 1, (b)). Since the low energy theorem is the consequence of the chiral symmetry, Eq. (3) can be derived either in current algebra[19] or in chiral perturbation theory[20]. Note that the information of the target hadron is reflected in the interaction only through its group theoretical representation T.

We assign an arbitrary representation $[p,q]$ to the target T. The possible representations for α are read from the irreducible decomposition

$$\begin{aligned}[p,q]\otimes[1,1] =&[p+1,q+1]\oplus[p+2,q-1]\oplus[p-1,q+2]\\ &\oplus[p,q]\oplus[p,q]\oplus[p+1,q-2]\oplus[p-2,q+1]\oplus[p-1,q-1].\end{aligned}$$

The coupling strength of each representation is summarized in Table 1. Notice that the sign of the interaction is in most cases determined from the nonnegativeness of the Dynkin indices, $p, q \geq 0$. It is also interesting to recall that the baryons in flavor SU(3) changes its representation when we vary N_c[23,24]. With Eq. (4), we can calculate the N_c dependence of the coupling strength. The results are also shown in the fourth column of the Table 1.

At this moment, we do not know which state is exotic. Here we introduce the exoticness quantum number E[25,26,27,28] as the minimal number of valence quark-antiquark pairs to construct the given flavor multiplet $[p, q]$ with the baryon number B carried by the u, d, and s quarks. For $B > 0$, the exoticness takes on the form[19,20]

$$E = \epsilon\theta(\epsilon) + \nu\theta(\nu),$$

with

$$\epsilon \equiv \frac{p+2q}{3} - B, \quad \nu \equiv \frac{p-q}{3} - B.$$

With this index, we can specify the exotic channels as the channel in which α has the larger exoticness than the target T does. Since α is obtained by multiplying $[1, 1]$ to the target representation T, the increase of the exoticness is at most unity. We denote ΔE as the difference between the exoticness of α and that of T which are shown in Table 1. Taking into account that $p, q \geq 0$, we find that the interaction for $\Delta E = +1$ channels is in most cases repulsive, and the strength of the possible attractive interaction is uniquely determined as

$$C_{\text{exotic}} = 1. \tag{5}$$

In this way, we construct the low energy interaction V_{chiral} in Eq. (1).

Table 1. Properties of the WT interaction in the channel α of the NG boson scattering on the target hadron with the $T = [p, q]$ representation. The coupling strengths of the WT term is denoted as $C_{\alpha,T}$, ΔE is the differences of the exoticness E between the channel α and the target hadron T, and $C_{\alpha,T}(N_c)$ denotes the coupling strengths for arbitrary N_c.

α	$C_{\alpha,T}$	ΔE	$C_{\alpha,T}(N_c)$
$[p+1, q+1]$	$-p-q$	1 or 0	$\frac{3-N_c}{2} - p - q$
$[p+2, q-1]$	$1-p$	1 or 0	$1-p$
$[p-1, q+2]$	$1-q$	1 or 0	$\frac{5-N_c}{2} - q$
$[p, q]$	3	0	3
$[p+1, q-2]$	$3+q$	0 or -1	$\frac{3+N_c}{2} + q$
$[p-2, q+1]$	$3+p$	0 or -1	$3+p$
$[p-1, q-1]$	$4+p+q$	0 or -1	$\frac{5+N_c}{2} + p + q$

3 Unitarity condition

Next we construct the scattering amplitude which satisfies Eq. (2). We utilize the N/D method[4]. In this method, the scattering amplitude t_α is given by

$$t_\alpha(\sqrt{s}) = \frac{1}{1 - V_\alpha(\sqrt{s})G(\sqrt{s})} V_\alpha(\sqrt{s}), \tag{6}$$

where V_α determines the dynamics of the system and we choose the chiral interaction V_α in Eq. (3). G is given by

$$G(\sqrt{s}) = -\tilde{a}(s_0) - \frac{1}{2\pi}\int_{s^+}^{\infty} ds' \left(\frac{\rho(s')}{s' - s} - \frac{\rho(s')}{s' - s_0} \right),$$

with $\rho(s) = 2M_T\sqrt{(s - s^+)(s - s^-)}/(8\pi s)$ and $s^\pm = (m \pm M_T)^2$. This function can be identified as the meson-hadron loop function, renormalized by the dimensional regularization.

We determine the renormalization constants from the requirement (1). We first note that if $G = 0$ in Eq. (6), then the full amplitude t_α coincides with the kernel interaction V_α which is chosen to be the chiral interaction in the present case. Therefore, in order to satisfy Eq. (1), we need to make $G(\sqrt{s}) = 0$, which is only possible within $M_T - m \leq \sqrt{s} \leq M_T + m$ (otherwise G is complex). Here we choose

$$G(M_T) = 0, \tag{7}$$

where M_T is the mass of the target hadron. This condition is consistent for the requirement of the low energy, since the energy of the NG boson is zero $\omega = 0$ at $\sqrt{s} = \sqrt{M_T^2 - m^2} \sim M_T$, (with on-shell kinematics). In fact, this condition is the most advantageous to generate a bound state within the region where a natural matching of the full amplitude to the low energy interaction can be performed[21,22].

Thus, we obtain the scattering amplitude $t_\alpha(\sqrt{s})$ which satisfies both Eqs. (1) and (2). Now we search for the pole of the bound state in the amplitude, which corresponds to zero of the denominator $1 - V_\alpha(\sqrt{s})G(\sqrt{s})$. From the energy dependence of the interaction and the loop function, we find the critical value for the attractive interaction strength

$$C_{\text{crit}} = \frac{2f^2}{m\big[-G(M_T + m)\big]}. \tag{8}$$

If the interaction strength $C_{\alpha,T}$ is larger than this critical value, a bound state is generated in the amplitude (6). In Fig. 2, we plot C_{crit} as a function of M_T, with $m = 368$ MeV and $f = 93$ MeV. We also plot the attractive interaction

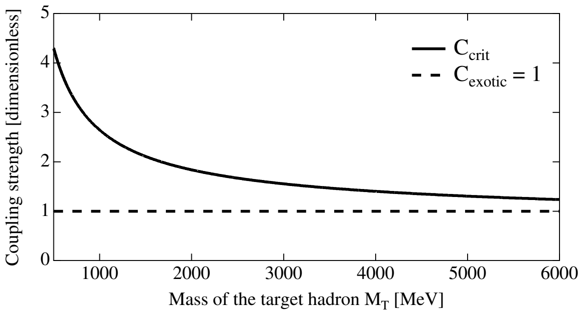

Figure 2. Critical coupling strength $C_{\rm crit}$ for $f = 93$ MeV and $m = 368$ MeV (Solid line). The dashed line denotes the universal attractive coupling strength in exotic channels $C_{\rm exotic} = 1$.

in the exotic channel (5). As we see in the figure, the interaction is not strong enough to generate a bound state for the mass of the target hadron ≤ 6 GeV, where possible target hadrons exist.

4 Summary and conclusion

We have studied the exotic states in the NG boson-hadron scattering. We construct the scattering amplitude which satisfies the chiral low energy theorem and unitarity condition. Considering the general target hadrons, we find that the interaction in the exotic channels are in most cases repulsive, and possible attractive interaction is uniquely given as $C_{\rm exotic} = 1$. We show that the strength of the attractive interaction is not sufficient to generate a bound state for the physically known masses of the target hadrons.

In order to draw a general and model-independent conclusion, we have simplified the framework. Our basic assumptions are 1) flavor SU(3) symmetric limit and 2) convergence of the chiral expansion. Once we accept these conditions, the subsequent arguments are straightforward. In practice, however, the SU(3) symmetry is broken and the higher order terms of the chiral expansion would play a substantial role, especially for the larger mass of the NG boson. These effects could be included in the kernel interaction based on chiral perturbation theory, but we need experimental data to determine the low energy constants.

In this study, we stress that the WT term is the leading order term of the chiral expansion and the strength is only determined by the group theoretical factor. We can therefore argue that the leading order term does not provides a bound state in exotic channel, without performing experiments. Given the success of the chiral unitary approach in the nonexotic sectors, our result may partly explain the difficulty to observe exotic hadrons in nature.

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