

# The Origin of Dynamically Generated Resonances



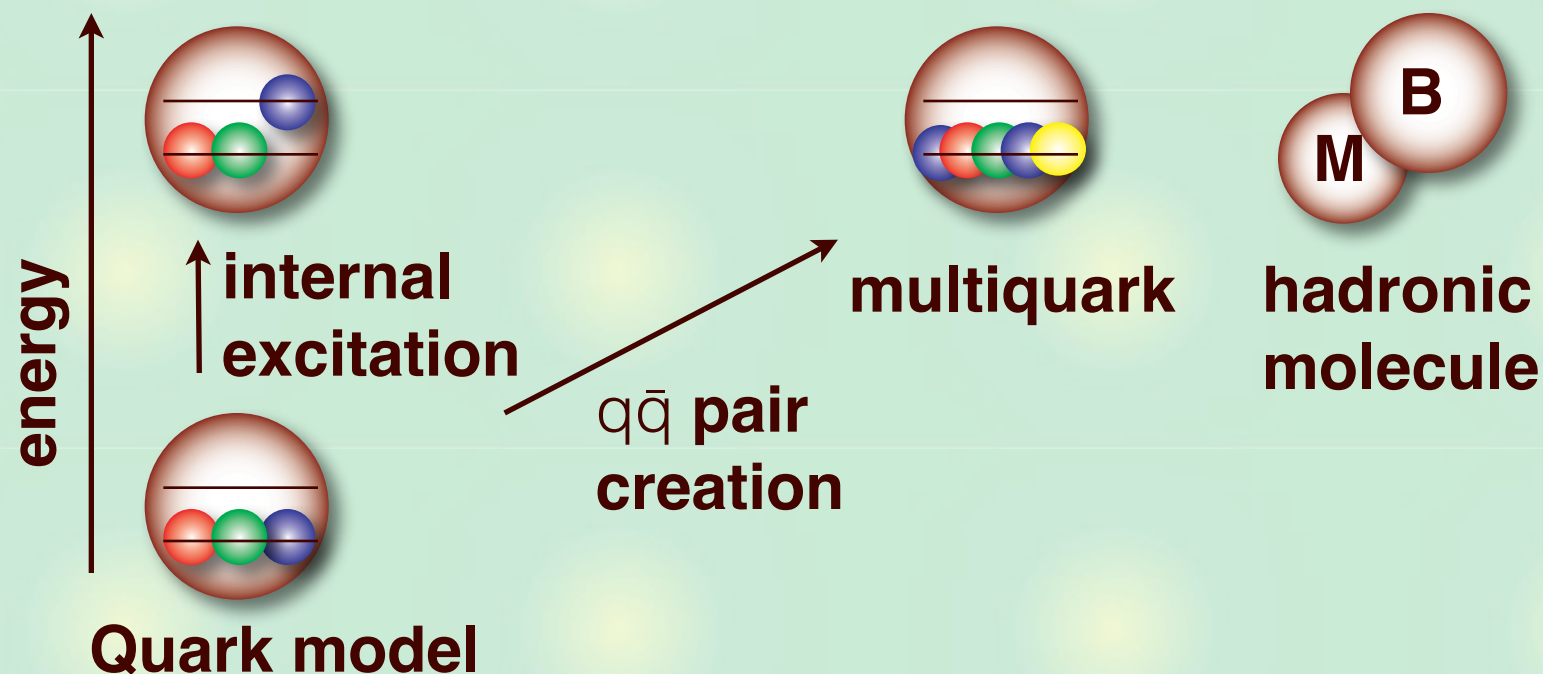
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2013, Jun. 25th 1

# Structure of hadron excited states

## Various excitations of baryons



What are 3q state, 5q state, MB state, ...?

- Comparison of data (spectrum, width,...) with quark **models**
- Analysis of scattering data by dynamical **models**

**Clear** (model-independent) **definition** of the structure?

# Difficulty 1 : definition and model space

Number of quarks + **antiquarks** ( $\neq$  quark number) ?

$$|\Lambda(1405)\rangle = \begin{array}{c} \text{red} \quad \text{blue} \\ \text{green} \end{array} + \begin{array}{c} \text{blue} \quad \text{yellow} \\ \text{red} \quad \text{green} \quad \text{blue} \end{array} + \dots$$

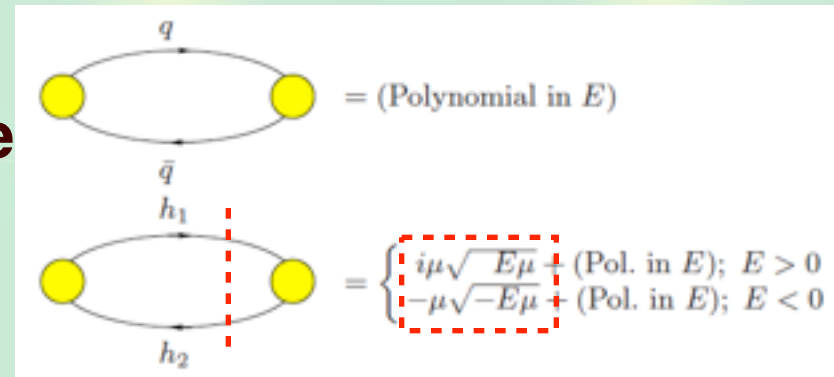
This may not be a good classification scheme.

Number of **hadrons**

$$|\Lambda(1405)\rangle = \boxed{\text{one large sphere} + \text{two smaller spheres}} + \dots$$

Hadrons are **asymptotic states**.  
--> different kinematical structure

C. Hanhart, Eur. Phys. J. A 35, 271 (2008)



--> **compositeness** in terms of **hadronic** degrees of freedom

# Difficulty 2 : resonances

Excited states : finite width  
(unstable against strong decay)

- stable (ground) states
- unstable states

Mostly resonances!

$\rho$	$1/2^+$ ****	$\Delta(1732)$	$3/2^+$ ****	$\Sigma^+$	$1/2^+$ ****	$\Sigma^+$	$1/2^+$ ****	$\Lambda^+$	$1/2^+$ ****
$\rho$	$1/2^+$ ****	$\Delta(1600)$	$3/2^+$ ***	$\Sigma^+$	$1/2^+$ ****	$\Sigma^+$	$1/2^+$ ****	$\Lambda(2595)^+$	$1/2^+$ ***
$\Lambda(1440)$	$1/2^+$ ****	$\Delta(1620)$	$1/2^+$ ****	$\Sigma^+$	$1/2^+$ ****	$\Sigma(1820)$	$3/2^+$ ****	$\Lambda(2625)^+$	$3/2^+$ ***
$\Lambda(1520)$	$3/2^+$ ****	$\Delta(1700)$	$3/2^+$ ****	$\Sigma(1895)$	$3/2^+$ ****	$\Sigma(1820)$	*	$\Lambda(2765)^+$	*
$\Lambda(1535)$	$1/2^+$ ****	$\Delta(1750)$	$1/2^+$ *	$\Sigma(1840)$	*	$\Sigma(1890)$	***	$\Lambda(2800)^+$	$5/2^+$ ***
$\Lambda(1650)$	$1/2^+$ ****	$\Delta(1900)$	$1/2^+$ **	$\Sigma(1960)$	**	$\Sigma(1820)$	$3/2^+$ ****	$\Lambda(2940)^+$	**
$\Lambda(1675)$	$5/2^+$ ****	$\Delta(1905)$	$5/2^+$ ****	$\Sigma(1950)$	$3/2^+$ **	$\Sigma(1950)$	***	$\Sigma_c(2455)$	$1/2^+$ ****
$\Lambda(1680)$	$5/2^+$ ****	$\Delta(1910)$	$1/2^+$ ****	$\Sigma(1920)$	$1/2^+$ **	$\Sigma(2030)$	$\geq \frac{1}{2}^+$ ****	$\Sigma_c(2520)$	$3/2^+$ ****
$\Lambda(1685)$	*	$\Delta(1920)$	$3/2^+$ ****	$\Sigma(1960)$	$1/2^+$ ****	$\Sigma(2120)$	**	$\Sigma_c(2600)$	***
$\Lambda(1700)$	$3/2^+$ ***	$\Delta(1930)$	$5/2^+$ ****	$\Sigma(1970)$	$3/2^+$ ****	$\Sigma(2250)$	**	$\Sigma_c^+$	$1/2^+$ ****
$\Lambda(1738)$	$1/2^+$ ****	$\Delta(1940)$	$3/2^+$ **	$\Sigma(1990)$	**	$\Sigma(2270)$	**	$\Sigma_c^+$	$1/2^+$ ****
$\Lambda(1720)$	$3/2^+$ ****	$\Delta(1950)$	$7/2^+$ ****	$\Sigma(1750)$	$1/2^+$ ****	$\Sigma(2500)$	*	$\Sigma_c^+$	$1/2^+$ ****
$\Lambda(1860)$	$5/2^+$ **	$\Delta(2000)$	$5/2^+$ **	$\Sigma(1770)$	$1/2^+$ *	$\Sigma_c^+$	$1/2^+$ ****	$\Sigma_c^+$	$1/2^+$ ****
$\Lambda(1875)$	$3/2^+$ ***	$\Delta(2150)$	$1/2^+$ *	$\Sigma(1775)$	$5/2^+$ ****	$\Sigma_c^+$	$3/2^+$ ****	$\Sigma_c(2645)$	$3/2^+$ ***
$\Lambda(1880)$	$1/2^+$ **	$\Delta(2200)$	$7/2^+$ *	$\Sigma(1840)$	$3/2^+$ *	$\Sigma(2250)$	***	$\Sigma_c(2790)$	$1/2^+$ ***
$\Lambda(1895)$	$1/2^+$ **	$\Delta(2300)$	$9/2^+$ **	$\Sigma(1880)$	$1/2^+$ **	$\Sigma(2300)^+$	**	$\Sigma_c(2835)$	$3/2^+$ ***
$\Lambda(1900)$	$3/2^+$ ***	$\Delta(2350)$	$5/2^+$ *	$\Sigma(1915)$	$5/2^+$ ****	$\Sigma(2900)$	*	$\Sigma_c(2900)$	*
$\Lambda(1990)$	$7/2^+$ **	$\Delta(2390)$	$7/2^+$ *	$\Sigma(1940)$	$3/2^+$ ****	$\Sigma_c(2960)$	***	$\Sigma_c(2960)$	***
$\Lambda(2000)$	$5/2^+$ **	$\Delta(2400)$	$9/2^+$ **	$\Sigma(2000)$	$1/2^+$ *	$\Sigma_c(3055)$	**	$\Sigma_c(3080)$	***
$\Lambda(2040)$	$3/2^+$ *	$\Delta(2420)$	$11/2^+$ ****	$\Sigma(2030)$	$7/2^+$ ****	$\Sigma_c(3123)$	*	$\Sigma_c^+$	$1/2^+$ ****
$\Lambda(2060)$	$5/2^+$ **	$\Delta(2750)$	$13/2^+$ **	$\Sigma(2070)$	$5/2^+$ *	$\Sigma_c^+$	$3/2^+$ ****	$\Sigma_c^+$	$1/2^+$ ****
$\Lambda(2100)$	$1/2^+$ *	$\Delta(2950)$	$15/2^+$ **	$\Sigma(2080)$	$3/2^+$ **	$\Sigma_c^+$	*	$\Sigma_c^+$	$1/2^+$ ****
$\Lambda(2120)$	$3/2^+$ **			$\Sigma(2100)$	$7/2^+$ **	$\Sigma_c^+$	*	$\Sigma_c^+$	$1/2^+$ ****
$\Lambda(2190)$	$7/2^+$ ****	$\Lambda$	$1/2^+$ ****	$\Sigma(2250)$	***	$\Sigma_c^+$	*	$\Sigma_c^+$	$1/2^+$ ****
$\Lambda(2220)$	$9/2^+$ ****	$\Lambda(1405)$	$1/2^+$ ****	$\Sigma(2455)$	**	$\Sigma_c^+$	*	$\Sigma_c^+$	$1/2^+$ ****
$\Lambda(2250)$	$9/2^+$ ****	$\Lambda(1520)$	$3/2^+$ ****	$\Sigma(2620)$	**	$\Sigma_c^+$	*	$\Sigma_c^+$	$1/2^+$ ****
$\Lambda(2600)$	$11/2^+$ ***	$\Lambda(1600)$	$1/2^+$ ****	$\Sigma(3000)$	*	$\Sigma_c^+$	$1/2^+$ ****	$\Sigma_c^+$	$1/2^+$ ****
$\Lambda(2700)$	$13/2^+$ **	$\Lambda(1670)$	$1/2^+$ ****	$\Sigma(3170)$	*	$\Sigma_c^+$	$1/2^+$ ****	$\Sigma_c^+$	$1/2^+$ ****
		$\Lambda(1690)$	$3/2^+$ ****			$\Sigma_c^+$	$1/2^+$ ****	$\Sigma_c^+$	$1/2^+$ ****
		$\Lambda(1800)$	$1/2^+$ ****			$\Sigma_c^+$	$1/2^+$ ****	$\Sigma_c^+$	$1/2^+$ ****
		$\Lambda(1810)$	$1/2^+$ ****			$\Sigma_c^+$	$1/2^+$ ****	$\Sigma_c^+$	$1/2^+$ ****
		$\Lambda(1820)$	$5/2^+$ ****			$\Sigma_c^+$	$1/2^+$ ****	$\Sigma_c^+$	$1/2^+$ ****
		$\Lambda(1830)$	$5/2^+$ ****			$\Sigma_c^+$	$1/2^+$ ****	$\Sigma_c^+$	$1/2^+$ ****
		$\Lambda(1890)$	$3/2^+$ ****			$\Sigma_c^+$	$1/2^+$ ****	$\Sigma_c^+$	$1/2^+$ ****
		$\Lambda(2000)$	*			$\Sigma_c^+$	$1/2^+$ ****	$\Sigma_c^+$	$1/2^+$ ****
		$\Lambda(2030)$	$7/2^+$ *			$\Sigma_c^+$	$1/2^+$ ****	$\Sigma_c^+$	$1/2^+$ ****
		$\Lambda(2200)$	$7/2^+$ ****			$\Sigma_c^+$	$1/2^+$ ****	$\Sigma_c^+$	$1/2^+$ ****
		$\Lambda(2310)$	$5/2^+$ ****			$\Sigma_c^+$	$1/2^+$ ****	$\Sigma_c^+$	$1/2^+$ ****
		$\Lambda(2325)$	$3/2^+$ *			$\Sigma_c^+$	$1/2^+$ ****	$\Sigma_c^+$	$1/2^+$ ****
		$\Lambda(2350)$	$9/2^+$ ****			$\Sigma_c^+$	$1/2^+$ ****	$\Sigma_c^+$	$1/2^+$ ****
		$\Lambda(2585)$	**			$\Sigma_c^+$	$1/2^+$ ****	$\Sigma_c^+$	$1/2^+$ ****

PDG12


“Wave function” of resonance?

$$|\Lambda(1405)\rangle = \text{[Diagram: a large sphere] + \text{[Diagram: two smaller spheres overlapping]} + \dots$$

↑ ?

--> First consider stable states, then extend it to resonances.

# Contents




## Introduction



## Compositeness 1 - $Z$ (bound states)

[S. Weinberg, Phys. Rev. 137, B672 \(1965\)](#)

[D. Lurie and A. J. Macfarlane, Phys. Rev. 136, B816 \(1963\)](#)



## Baryon resonances in a chiral model

- **Compositeness of bound states**

[T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C85, 015201 \(2012\)](#)

- **Structure of resonances**

[T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C78, 025203 \(2008\)](#)

[T. Hyodo, arXiv:1305.1999 \[hep-ph\]](#)

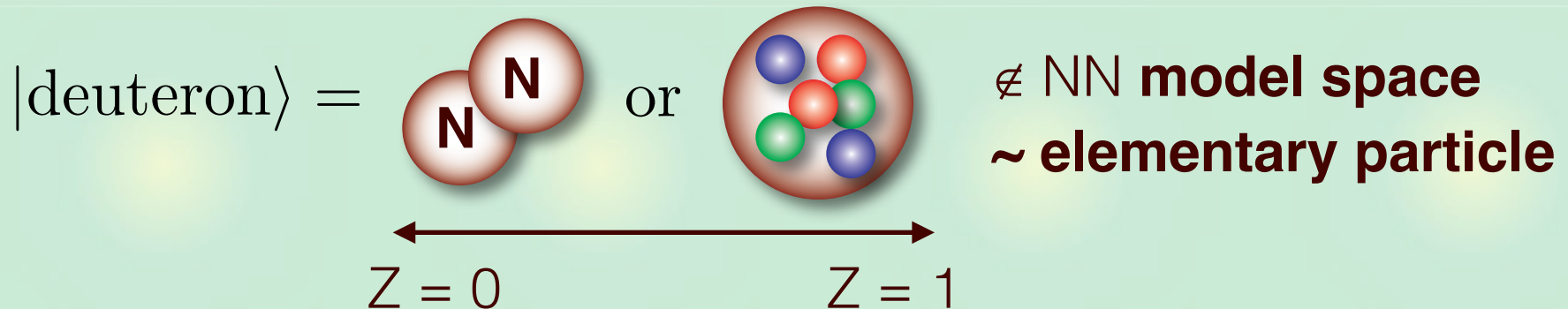


## Summary

# Weinberg's compositeness and deuteron

Z: probability of finding deuteron in a bare elementary state

S. Weinberg, Phys. Rev. 137, B672 (1965)



**model-independent** relation for weakly bound state

$$\boxed{a_s} = \left[ \frac{2(1-Z)}{2-Z} \right] \boxed{R} + \mathcal{O}(m_\pi^{-1}), \quad \boxed{r_e} = \left[ \frac{-Z}{1-Z} \right] \boxed{R} + \mathcal{O}(m_\pi^{-1})$$

$a_s$ : scattering length

$r_e$ : effective range

**<-- Experiments**

$R$ : deuteron radius (binding energy)

$$a_s = +5.41 \text{ [fm]}, \quad r_e = +1.75 \text{ [fm]}, \quad R \equiv (2\mu B)^{-1/2} = 4.31 \text{ [fm]}$$

$\Rightarrow Z \lesssim 0.2$  **--> deuteron is almost composite!**

# Compositeness in quantum mechanics

## Hamiltonian of a single channel scattering system

$$\mathcal{H} = \boxed{\mathcal{H}_0} + V$$

## Complete set for **free** Hamiltonian: bare $|B_0\rangle$ + continuum

$$1 = |B_0\rangle\langle B_0| + \int dk |k\rangle\langle k|$$

## Physical bound state $|B\rangle$ with binding energy $B$

$$(\mathcal{H}_0 + V)|B\rangle = -B|B\rangle$$

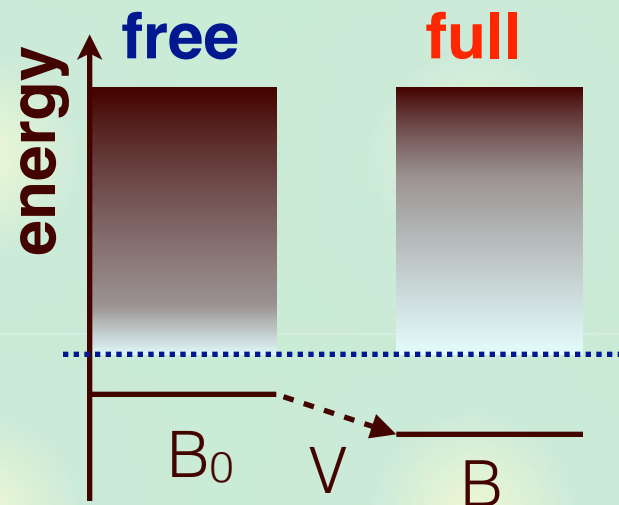
## Z : overlap of B and $B_0$

$$Z \equiv |\langle B_0 | B \rangle|^2$$

## For **small B**, compositeness 1 - Z is

$$1 - Z = 2\pi^2 \sqrt{2\mu^3} \frac{g_w^2}{\sqrt{B}}$$

$g_w$  : coupling constant



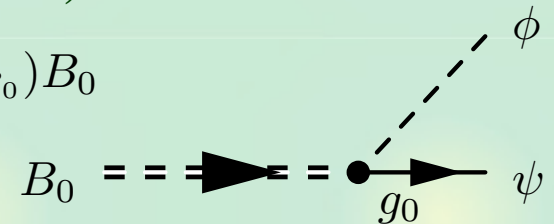
# Compositeness in Yukawa theory

## Field theory with Yukawa coupling ( $\psi, \phi, B_0$ )

c.f. D. Lurie and A. J. Macfarlane, Phys. Rev. 136, B816 (1963)

$$\mathcal{L}_0 = \bar{\psi}(i\partial - M)\psi + \frac{1}{2}(\partial_\mu\phi\partial^\mu\phi - m^2\phi^2) + \bar{B}_0(i\partial - M_{B_0})B_0$$

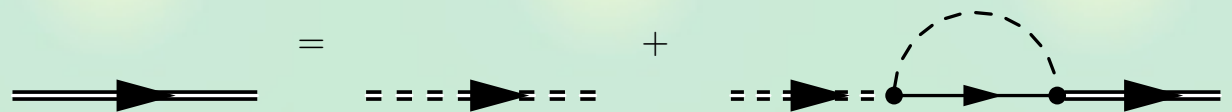
$$\mathcal{L}_{\text{int}} = g_0\bar{\psi}\phi B_0 + (\text{h.c.})$$



## Physical bound state B at total energy $W=M_B$

### Free (full) propagator of $B_0$ (B) field (positive energy part)

$$\Delta_0(W) = \frac{1}{W - M_{B_0}}, \quad \Delta(W) = \Delta_0(W) + \Delta_0(W)g_0G(W)g_0\Delta(W) = \frac{Z}{W - M_B}$$



**Z: field renormalization constant**

## Compositeness in Yukawa theory

$$1 - Z = -g^2G'(M_B)$$

$G(W)$  : loop function,  $g$  : coupling



# Compositeness: summary

## Compositeness of bound states

- Quantum mechanics (model independent for small  $B$ )

$$1 - Z = 2\pi^2 \sqrt{2\mu^3} \frac{g_W^2}{\sqrt{B}}$$

- Yukawa theory (any  $M_B$ , but Lagrangian dependent)

$$1 - Z = -g^2 G'(M_B)$$

## Mass $M_B$ and coupling constant $g^2$

- are the pole position and the residue of amplitude

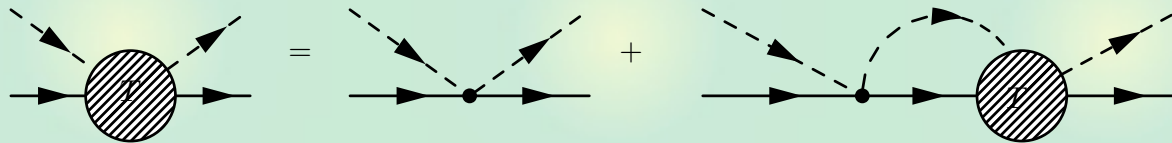
$$T(W) = \frac{g^2}{W - M_B} + T_{BG}(W)$$

- can be calculated in models, lattice QCD, experiments, ...?
- can be calculated for resonances?

# Chiral coupled-channel model

## Baryon resonances in s-wave meson-baryon scattering

- Interaction  $\leftarrow$  chiral perturbation theory
- Amplitude  $\leftarrow$  unitarity in coupled channels



A review: [T. Hyodo, D. Jido, Prog. Part. Nucl. Phys. 67, 55 \(2012\)](#)

## Unitarized amplitude

$$T(W) = \frac{1}{1 - V(W)G(W; a)} V(W) \quad \leftarrow \text{cutoff parameter}$$

- generate physical resonances,  $\Lambda(1405)$ ,  $N(1535)$ , ...
- pole position and residue by analytic continuation

## Natural renormalization scheme

Model contains no bare pole states.

--> generated states are all two-body **composite**?

This is **not always** the case: CDD pole contribution

L. Castillejo, R.H. Dalitz, F.J. Dyson, *Phys. Rev.* **101**, 453 (1956)

G.F. Chew, S.C. Frautschi, *Phys. Rev.* **124**, 264 (1961)

ambiguity of subtractions of dispersion relation

--> contribution from “independent” particles

Natural renormalization condition

<-- to exclude CDD pole contributions from the loop function

T. Hyodo, D. Jido, A. Hosaka, *Phys. Rev. C* **78**, 025203 (2008)

$$G(W = M; a_{\text{natural}}) = 0$$

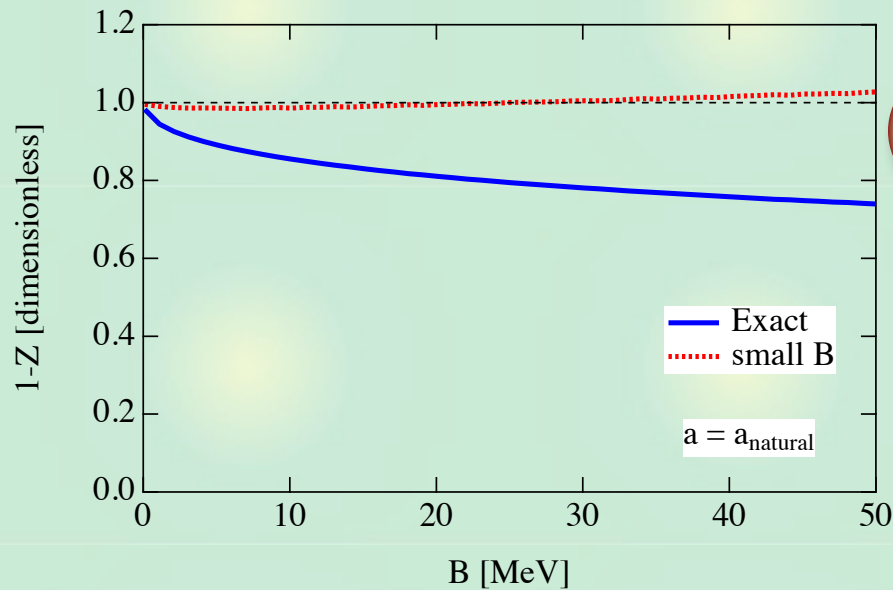
**Compositeness** in natural renormalization scheme?

# Check of natural renormalization scheme

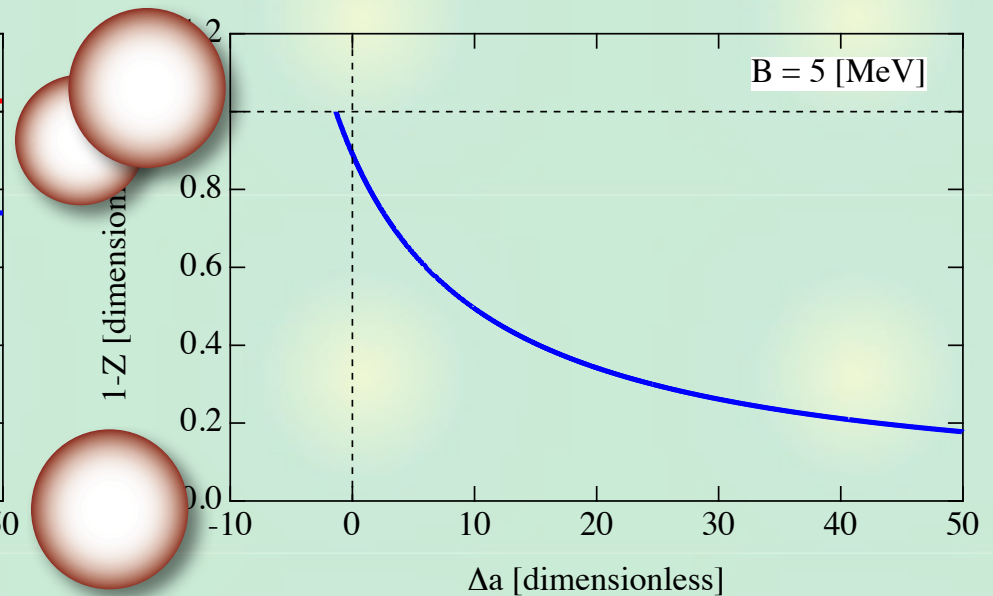
Test: single-channel scattering with a **bound state**

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C85, 015201 (2012)

1)  $a = a_{\text{natural}}$ , vary  $B$



2)  $B = 5 \text{ MeV}$ , vary  $a$



natural scheme  $\rightarrow Z \sim 0$

large deviation  $\rightarrow Z \sim 1$

dynamical state in natural scheme  $\Leftrightarrow$  compositeness

## Compositeness of resonances

### Application to resonances

- integration of spectral density (for narrow resonances)

*V. Baru et al., Phys. Lett. B586, 53 (2004)*

- analytic continuation of the bound state formula

$$1 - Z = -g^2 G'(M_B)$$

residue  $\uparrow$        $\uparrow$  pole position

Interpretation: absolute value ( $|Z| > 1$ )? real part ( $\text{Re}Z < 0$ )?

- no completeness:  $Z$  is **not normalized**
- $g, M_B$  are complex:  $Z$  is **complex**

Example:  $\Lambda(1405)$

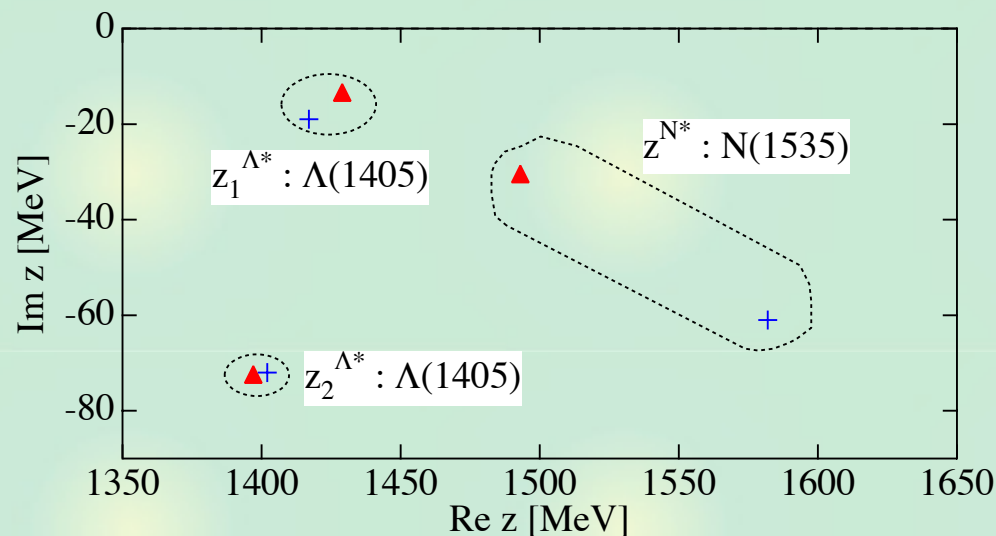
- higher energy pole:  $Z = 0.00 + 0.09i$
- lower energy pole:  $Z = 0.86 - 0.40i$

*T. Sekihara, T. Hyodo, Phys. Rev. C87, 045202 (2013)*

# Analysis by natural scheme

## Pole positions in natural / phenomenological scheme

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C78, 025203 (2008)



▲ natural  
+ phenomenological

Magnitude of deviation : amount of CDD pole contribution

-->  $\Lambda(1405)$  : mostly composite,  $N(1535)$  : mixture

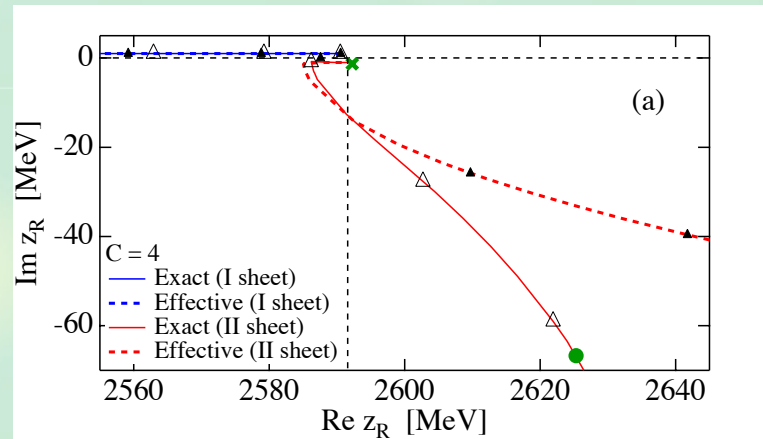
- higher energy pole:  $Z = 0.00 + 0.09i$

- lower energy pole:  $Z = 0.86 - 0.40i$  ??

# Near-threshold states

## Near-threshold resonances: effective range expansion

$$f(k) = \left( \frac{1}{a} - ki + \frac{r_e}{2} k^2 \right)^{-1}$$



Amplitude is given by 2 parameters

--> residue is determined by the pole position

T. Hyodo, arXiv:1305.1999 [hep-ph]

$$Z = 1 - \sqrt{1 - \frac{1}{1 + a/(2r_e)}}$$

$1 - Z$  is **pure imaginary** for resonances and  $0 < |1 - Z| < 1$ .

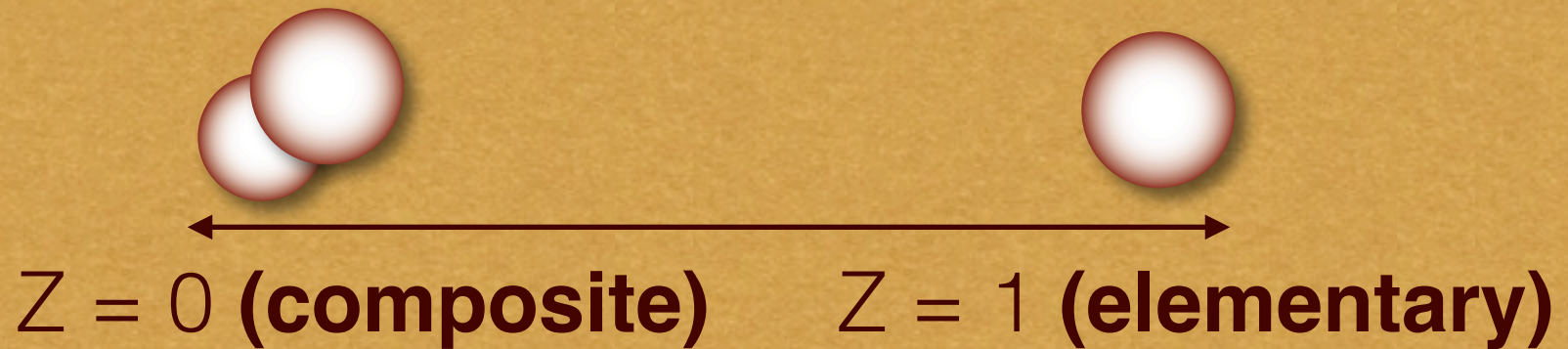
- **normalized quantity**

- measures deviation from  $1 - Z = i$  ?

# Summary 1

## Classification of hadron structure


 Field renormalization constant  $Z$



  $Z \leftarrow$  mass + coupling (pole + residue)

S. Weinberg, *Phys. Rev.* 137, B672 (1965)


D. Lurie and A. J. Macfarlane, *Phys. Rev.* 136, B816 (1963)


 Models, lattice, experiments,...



## Summary 2


### Application to resonances

  $Z$  for resonances is in general **complex** and **not normalized**. Interpretation?

 Combination with other analysis (natural scheme,  $N_c$  scaling, ...) may help it.

[T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C78, 025203 \(2008\)](#)

[T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C85, 015201 \(2012\)](#)

 Near-threshold resonances: effective range expansion may help it.

[T. Hyodo, arXiv:1305.1999 \[hep-ph\]](#)