Hadron composite systems in chiral dynamics

Tetsuo Hyodo

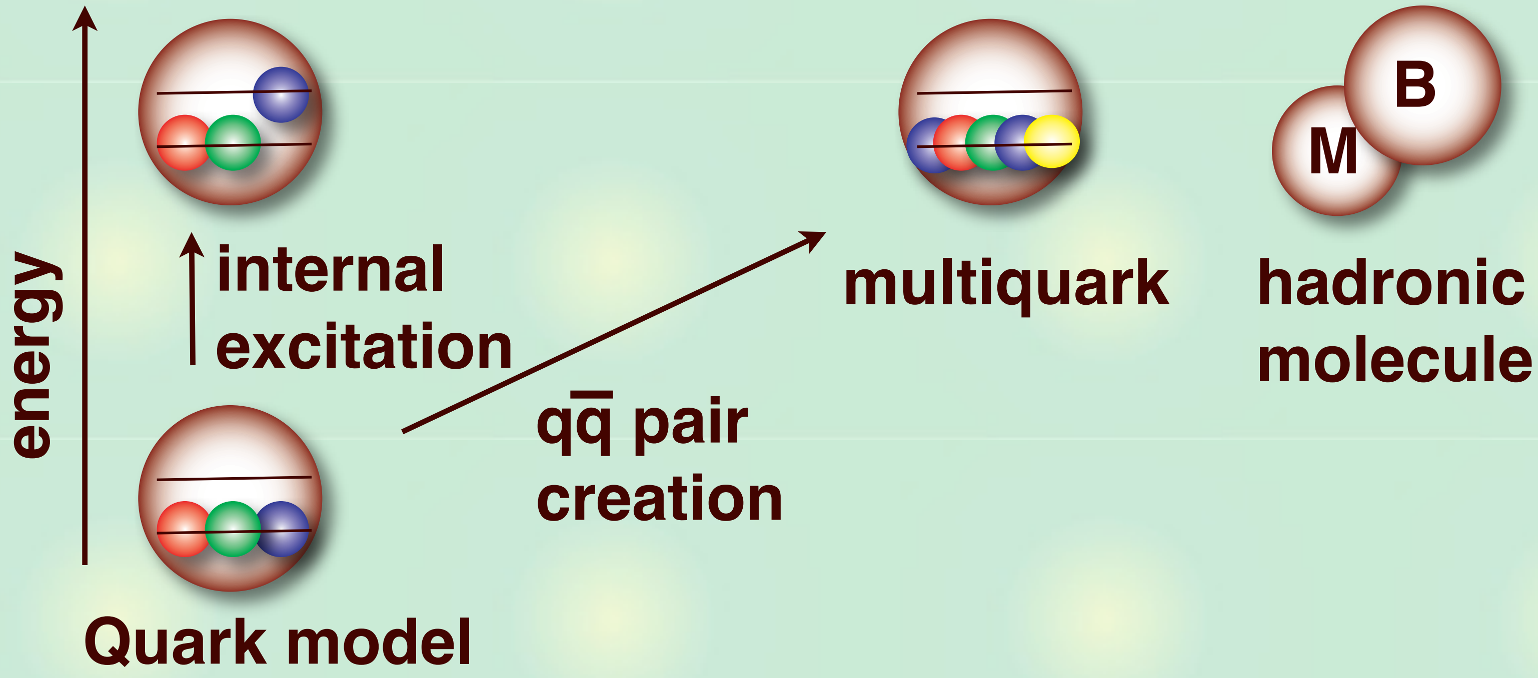
Tokyo Institute of Technology

supported by Global Center of Excellence Program
"Nanoscience and Quantum Physics"

2012, Jun. 28th

Structure of hadron resonances

Example) baryon excited state

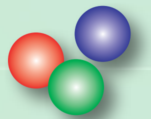
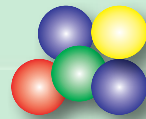


What are 3q state, 5q state, MB state, ...?

Clear (model-independent) definition of the structure?

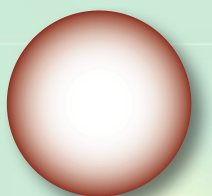
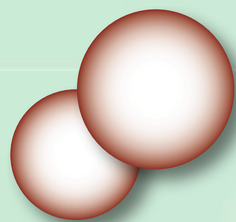
Definition of hadron structure

Number of quarks and antiquarks (≠ quark number) ?

$|\,\Lambda(1405)\,\rangle =$  $+$  $+\dots$

may not be a good classification scheme.

Number of hadrons

$|\,\Lambda(1405)\,\rangle =$  $+$  $+\dots$

Hadrons are asymptotic states
--> different kinematical structure

C. Hanhart, Eur. Phys. J. A 35, 271 (2008)

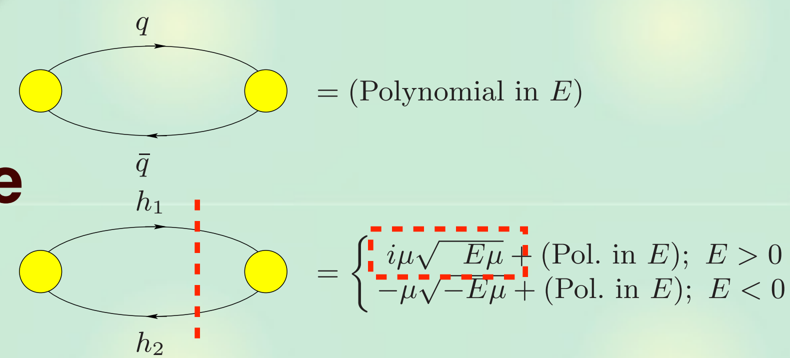


Fig. 1. Illustration of the essential difference between hadron loops (or loops of colour neutral objects) and quark loops (or loops of coloured objects): only the former have non-analyticities.

Contents

- Introduction
- Definition of compositeness
  - Nonrelativistic quantum mechanics

    S. Weinberg, Phys. Rev. 137, B672 (1965)
  - Yukawa field theory

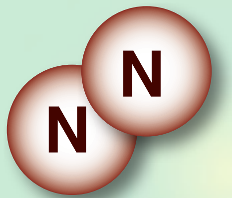
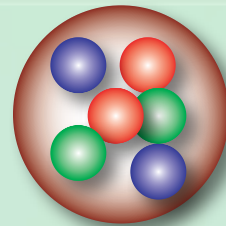
    D. Lurie and A. J. Macfarlane, Phys. Rev. 136, B816 (1963)
- Application to chiral dynamics
  - Compositeness of bound states

    T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C85, 015201 (2012)
- Summary

Weinberg's compositeness and deuteron

Z: probability of finding deuteron in a bare elementary state

S. Weinberg, Phys. Rev. 137, B672 (1965)

$|\mathrm{deuteron}\rangle =$  or 

$\notin$ NN model space

~ elementary particle

Z = 0 ⟷ Z = 1

model independent relation for weakly bound state

$$a_s = \left[\frac{2(1-Z)}{2-Z}\right] R + \mathcal{O}(m_\pi^{-1}), \quad r_e = \left[\frac{-Z}{1-Z}\right] R + \mathcal{O}(m_\pi^{-1})$$

a_s: scattering length

r_e: effective range <-- Experiments

R: deuteron radius (binding energy)

$$a_s = +5.41 \text{ [fm]}, \quad r_e = +1.75 \text{ [fm]}, \quad R \equiv (2\mu B)^{-1/2} = 4.31 \text{ [fm]}$$

$\Rightarrow Z \lesssim 0.2$ --> **deuteron is almost composite!**

Definition of the compositeness 1-Z

Hamiltonian of two-body system: free + interaction V

$$\mathcal{H} = \mathcal{H}_0 + V$$

Complete set for free Hamiltonian: bare |B_0> + continuum

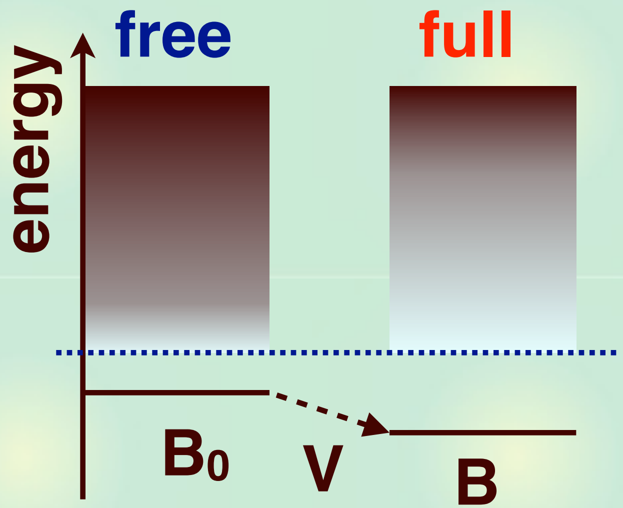
$$1 = |\, B_0 \,\rangle\langle\, B_0 \,| + \int d\boldsymbol{k} |\, \boldsymbol{k} \,\rangle\langle\, \boldsymbol{k} \,|$$

$$\mathcal{H}_0 |\, B_0 \,\rangle = E_0 |\, B_0 \,\rangle, \quad \mathcal{H}_0 |\, \boldsymbol{k} \,\rangle = E(\boldsymbol{k}) |\, \boldsymbol{k} \,\rangle$$

Physical bound state |B> : eigenstate of full Hamiltonian

$$(\mathcal{H}_0 + V) |\, B \,\rangle = -B |\, B \,\rangle$$

B: binding energy



Define Z as the overlap of B and B_0
: probability of finding the physical bound state in the bare state |B>

$$Z \equiv |\langle\, B_0 \,|\, B \,\rangle|^2$$

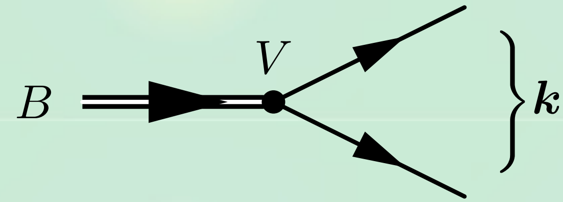
1 - Z : Compositeness of the bound state

Model-independent but approximated method

With the Schrödinger equation, we obtain

$$1 - Z = \int d\boldsymbol{k} \frac{|\langle \boldsymbol{k} | V | B \rangle|^2}{[E(\boldsymbol{k}) + B]^2}$$

$\langle \boldsymbol{k} | V | B \rangle :$



$$= 4\pi\sqrt{2\mu^3} \int_0^\infty dE \frac{\sqrt{E}|G_W(E)|^2}{(E+B)^2}$$

$\langle \boldsymbol{k} | V | B \rangle \equiv G_W[E(\boldsymbol{k})]$ **for s-wave**

Approximation: For small binding energy B<<1, the vertex G_W(E) can be regarded as a constant: $G_W(E) \sim g_W$

Then the integration can be done analytically, leading to

$$1 - Z = 2\pi^2\sqrt{2\mu^3}\frac{g_W^2}{\sqrt{B}}$$

Compositeness <-- coupling g_w and binding energy B

S. Weinberg, Phys. Rev. 137 B672-B678 (1965)

- **Model-independent: no information of V**
- **Approximated: valid only for small B**

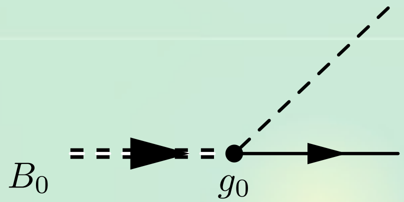
Z in Yukawa model

Field theory with Yukawa coupling (ψ,ϕ,B_0)

see D. Lurie and A. J. Macfarlane, Phys. Rev. 136, B816 (1963)

$$\mathcal{L}_0 = \bar{\psi}(i\partial\!\!\!/ - M)\psi + \frac{1}{2}(\partial_\mu\phi\partial^\mu\phi - m^2\phi^2) + \bar{B}_0(i\partial\!\!\!/ - M_{B_0})B_0$$

$$\mathcal{L}_{\rm int} = g_0\bar{\psi}\phi B_0 + ({\rm h.c.})$$



Physical bound state B at total energy $W=M_B$

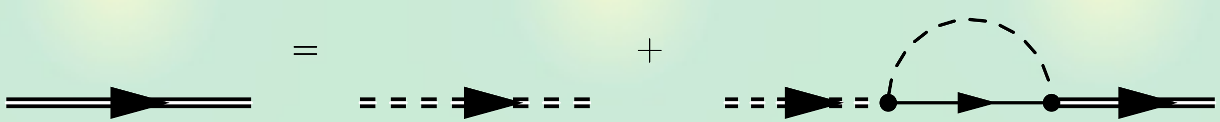
Free (full) propagator of B_0 (B) field (positive energy part)

$$\Delta_0(W) = \frac{1}{W - M_{B_0}}, \quad \Delta(W) = \frac{Z}{W - M_B}$$

Z: field renormalization constant

Dyson equation: relation between full and free propagators

$$\Delta(W) = \Delta_0(W) + \Delta_0(W)g_0G(W)g_0\Delta(W)$$



Master formula of compositeness

Solution of Dyson equation and renormalization

$$\Delta(W) = \frac{1}{W - M_{B_0} - g_0^2 G(W)} \to \frac{1}{W - g_0^2 G(W;a)}$$

Renormalization condition, pole at W=M$_B$: $M_B = g_0^2 G(M_B;a)$

The field renormalization constant: residue of the propagator

$$Z = \lim_{W\to M_B} \frac{W - M_B}{W - g_0^2 G(W;a)} = \frac{1}{1 - g_0^2 G'(M_B)}$$

Physical coupling constant: residue of T-matrix

$$g^2 = g_0^2 Z$$

Compositeness in Yukawa theory

$$1 - Z = -g^2 G'(M_B)$$

Compositeness: summary

Compositeness of the bound state <-- g and M_B

Method 1: nonrelativistic quantum mechanics

$$1 - Z_{NR} = g^2 \frac{M|\lambda^{1/2}(M_B^2, M^2, m^2)|}{16\pi M_B^2 (M + m - M_B)} \quad \text{for } M_B \to M + m$$

model independent, but valid only for weak binding

Method 2: field theory with Yukawa coupling

$$1 - Z = -g^2 G'(M_B)$$

exact (any M_B), but Lagrangian dependent

Application?

For a bound state, compositeness is determined by physical mass ``$M_B$” and coupling constant ``g”.

Model calculation, Lattice QCD, Experiments, ...

Chiral dynamics: overview

Description of S = -1, $\bar{K}N$ s-wave scattering: Λ(1405) in I=0

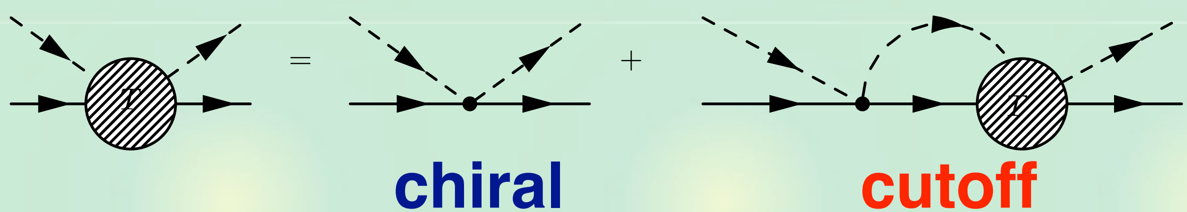
- Interaction <-- chiral symmetry

Y. Tomozawa, Nuovo Cim. 46A, 707 (1966); S. Weinberg, Phys. Rev. Lett. 17, 616 (1966)

- Amplitude <-- unitarity in coupled channels

R.H. Dalitz, T.C. Wong, G. Rajasekaran, Phys. Rev. 153, 1617 (1967)

$$T = \frac{1}{1 - VG} V$$



N. Kaiser, P. B. Siegel, W. Weise, Nucl. Phys. A594, 325 (1995),
E. Oset, A. Ramos, Nucl. Phys. A635, 99 (1998),
J. A. Oller, U. G. Meissner, Phys. Lett. B500, 263 (2001),
M.F.M. Lutz, E. E. Kolomeitsev, Nucl. Phys. A700, 193 (2002), ... many others

It works successfully in various hadron scatterings.

A review: T. Hyodo, D. Jido, Prog. Part. Nucl. Phys. 67, 55 (2012)

Natural renormalization condition

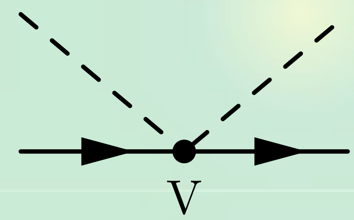
Single-channel scattering of meson m and baryon M.

$$T(W) = \frac{1}{1 - V(W)G(W;a)} V(W)$$

← **cutoff parameter**

V: 4-point interaction, attractive

$$V(W) = \begin{cases} V^{(\text{const})} = Cm & \text{constant interaction} \\ V^{(\text{WT})}(W) = C(W - M) & \text{WT interaction} \end{cases}$$



Bound state condition: pole at $W = M_B$

$$1 - V(M_B)G(M_B;a) = 0$$

Coupling constant: residue of the pole

$$g^2 = \lim_{W \to M_B} (W - M_B) T(W) = \begin{cases} -[G'(M_B)]^{-1} & \text{constant interaction} \\ -\left[G'(M_B) + \dfrac{G(M_B;a)}{M_B - M} \right]^{-1} & \text{WT interaction} \end{cases}$$

We determine mass and coupling of the bound state

Compositeness of bound states

Compositeness in Yukawa theory

$$1 - Z = -g^2 G'(M_B) = \begin{cases} 1 & \text{constant interaction} \\ \left[1 + \dfrac{G(M_B; a)}{(M_B - M)G'(M_B)}\right]^{-1} & \text{WT interaction} \end{cases}$$

- constant interaction --> purely composite bound state

- WT interaction --> mixture of composite and elementary

- Purely composite bound state for WT interaction:

$$G'(M_B) = -\infty \quad \text{or} \quad G(M_B; a) = 0$$

$$M_B = M + m \quad \text{or} \quad C \to -\infty$$

1) zero energy bound state
2) infinitely strong two-body attraction

Relation with natural renormalization scheme?

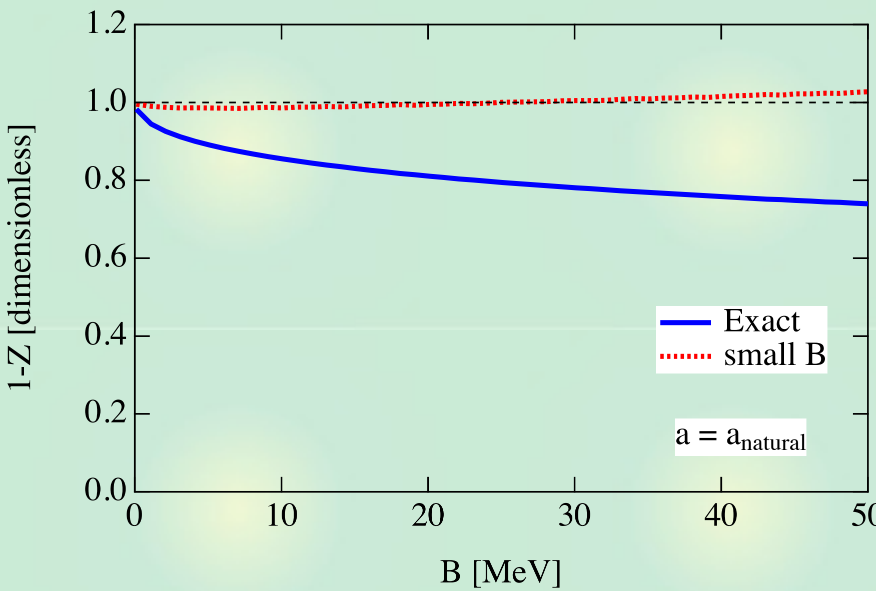
Consistency check of the natural renormalization scheme

Natural renormalization condition
<-- to exclude elementary contribution from the loop function

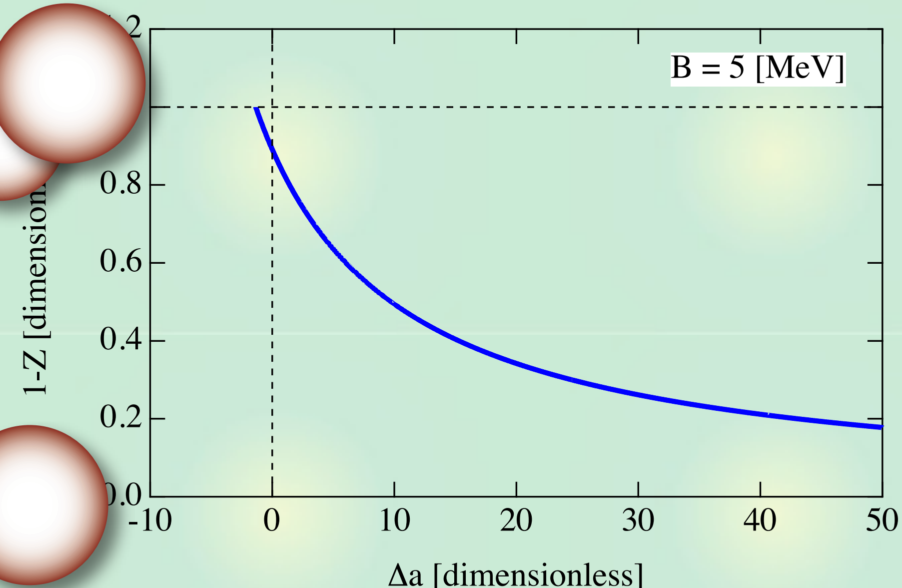
T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C78, 025203 (2008)

$$G(W = M; a_{\text{natural}}) = 0$$

1) $a = a_{\text{natural}}$, vary B



2) B = 5 MeV, vary a



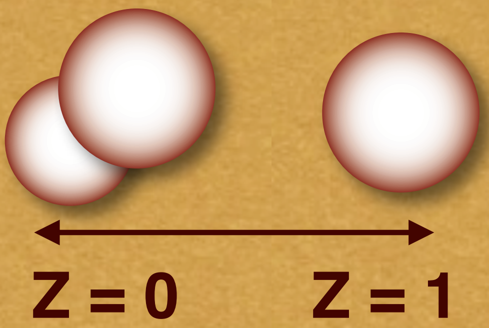
natural scheme --> Z ~ 0

large deviation --> Z ~ 1

Summary 1

Compositeness of the bound state

- **Field renormalization constant Z: compositeness**



- **Model independent formula**

$$1 - Z_{NR} = g^2 \frac{M|\lambda^{1/2}(M_B^2, M^2, m^2)|}{16\pi M_B^2 (M + m - M_B)} \quad \text{for } M_B \to M + m$$

S. Weinberg, Phys. Rev. 137 B672 (1965)

- **Exact formula**

$$1 - Z = -g^2 G'(M_B)$$

D. Lurie and A. J. Macfarlane, Phys. Rev. 136, B816 (1963)

- **Expressed in terms of physical quantities**

Summary 2

Application to chiral unitary approach

- Bound state in chiral dynamics

  Energy independent interaction
  --> purely composite bound state

  Energy-dependent chiral interaction
  --> mixture of composite and elementary

- Natural scheme corresponds to Z ~ 0

  --> composite particle is generated

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C85, 015201 (2012)

Summary 3

Future perspective

- Coupled-channel problem
  - maybe possible?
- Hadron resonances
  - Z becomes complex, not normalized?
- Application to other fields
  - Higgs boson
  - polaron-molecule transition in ultra-cold atoms

R. Schmidt, T. Enss, Phys. Rev. A 83, 063620 (2011)