Determination of the $\pi\Sigma$ scattering lengths from the weak decays of Λ_c

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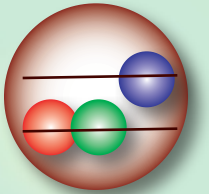
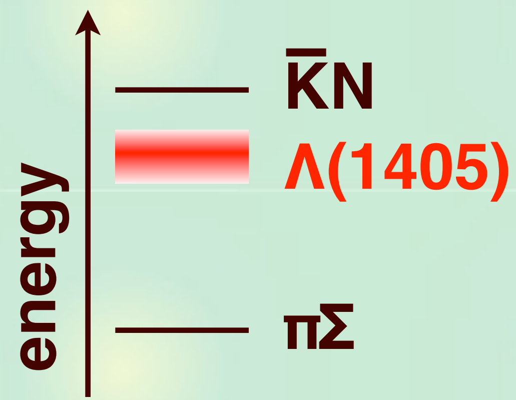
$\bar{K}$ meson and $\bar{K}N$ interaction

Two aspects of $\bar{K}$ meson

- NG boson of chiral SU(3) ⊗ SU(3) --> SU(3)
- relatively heavy mass: $M_K \sim 495$ MeV

--> peculiar role in hadron physics

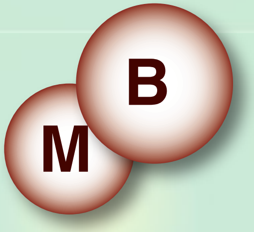




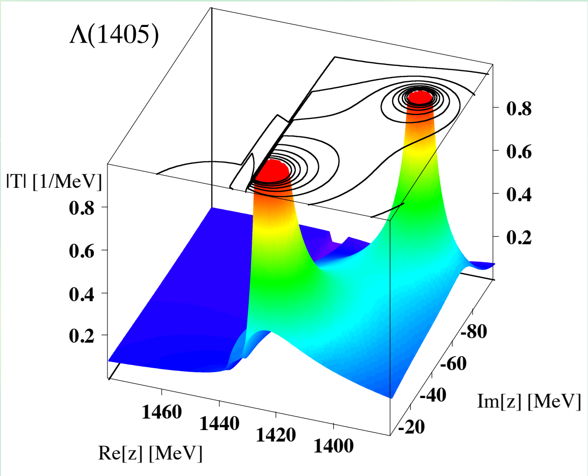
$\bar{K}N$ interaction is ...

- coupled with πΣ channel
- strongly attractive

--> quasi-bound state Λ(1405)
meson-baryon v.s. qqq state,
double pole, ...



- fundamental building block
for $\bar{K}$-nuclei, $\bar{K}$ in medium,...



T. Hyodo, D. Jido, Prog. Part. Nucl. Phys. 67, 55 (2012)

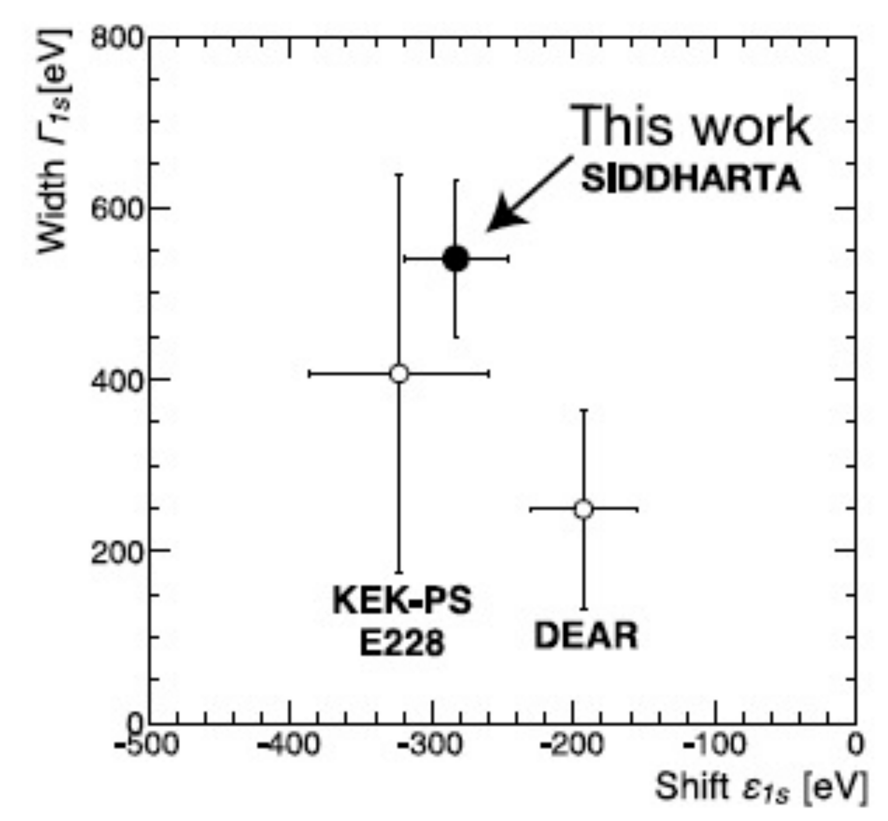
Recent progress

Accurate measurement of Kaonic hydrogen by SIDDHARTA

M. Bazzi, *et al.*, Phys. Lett. B704, 113 (2011)

- smallest uncertainties

$$\Delta E = -283 \pm 36 \pm 6 \text{ eV}, \quad \Gamma = 541 \pm 89 \pm 22 \text{ eV}$$



--> New constraint on the meson-baryon amplitude

Experimental constraints for S=-1 MB scattering

K-p total cross sections

- old bubble chamber data, large errors

$\bar{K}$N threshold observables

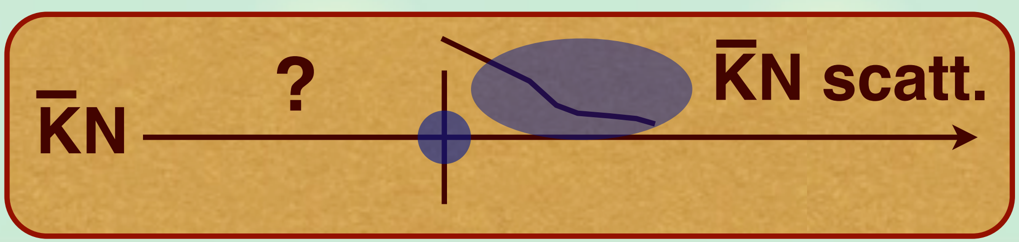
- threshold branching ratios (old but accurate)

$$\gamma = \frac{\Gamma(K^- p \to \pi^+ \Sigma^-)}{\Gamma(K^- p \to \pi^- \Sigma^+)} = 2.36 \pm 0.04$$

$$R_c = \frac{\Gamma(K^- p \to \pi^+ \Sigma^-, \pi^- \Sigma^+)}{\Gamma(K^- p \to \text{all inelastic channels})} = 0.664 \pm 0.011$$

$$R_n = \frac{\Gamma(K^- p \to \pi^0 \Lambda)}{\Gamma(K^- p \to \text{neutral states})} = 0.189 \pm 0.015$$

- K-p scattering length <-- SIDDHARTA



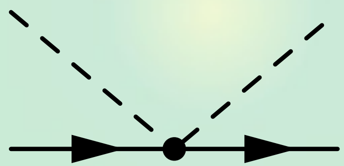
Construction of the realistic amplitude

Systematic analysis by chiral dynamics with SIDDHARTA

Y. Ikeda, T. Hyodo, W. Weise, Phys. Lett. B 706, 63 (2011); Nucl. Phys. A881, 98 (2012)

Interaction kernel: NLO ChPT

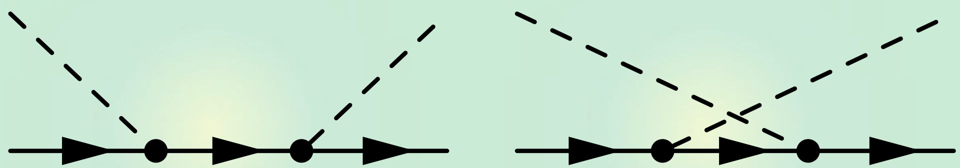
1) TW term



$\mathcal{O}(p)$

TW model

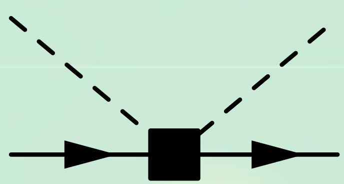
2) Born terms



$\mathcal{O}(p)$

TWB model

3) NLO terms



$\mathcal{O}(p^2)$

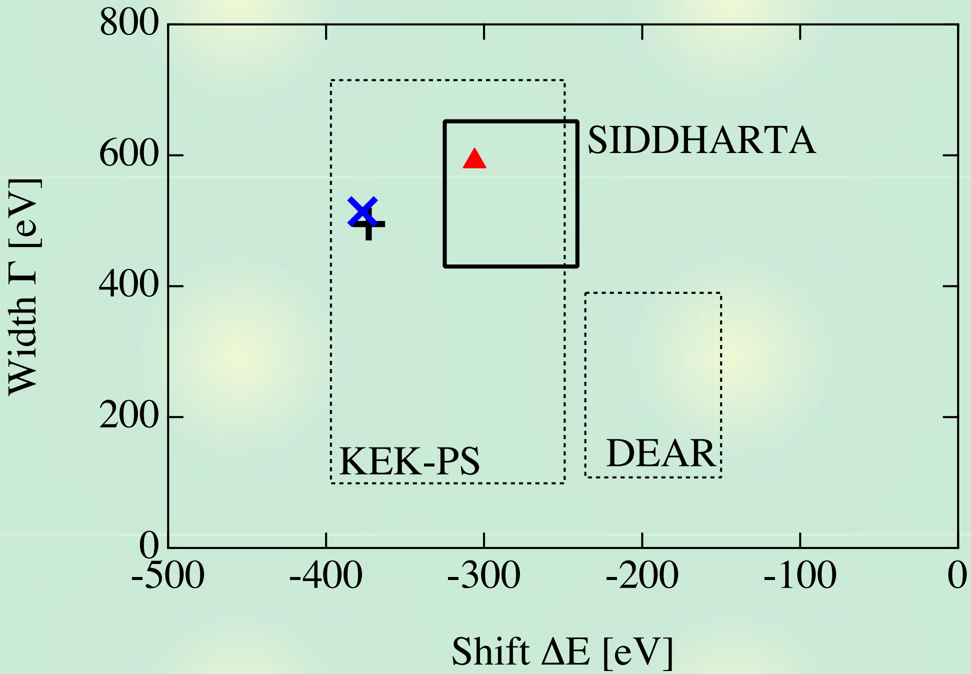
NLO model

Parameters: 6 cutoffs (+ 7 low energy constants in NLO)
--> fitted to cross section, branching ratio, and SIDDHARTA

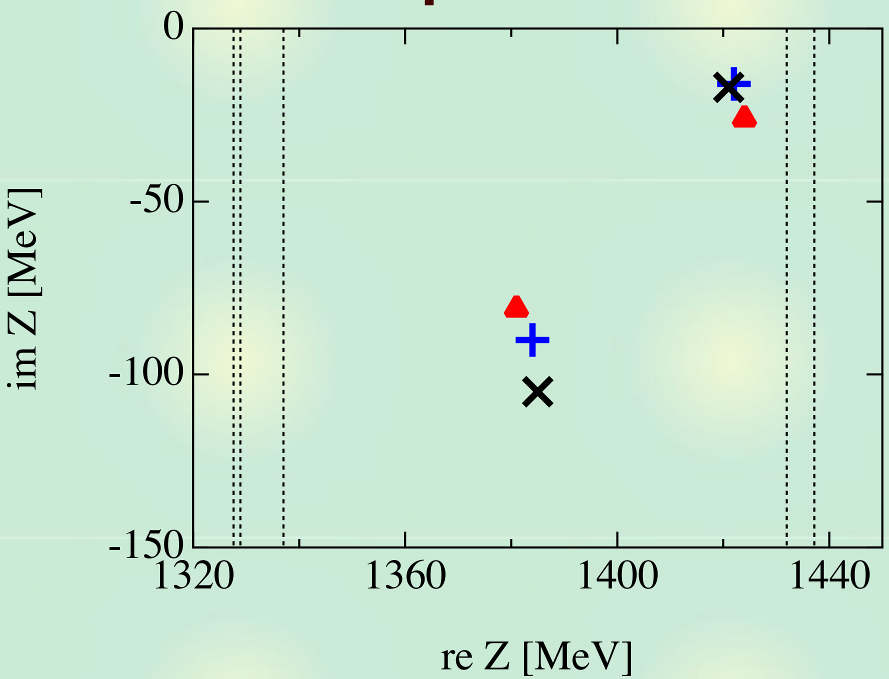
Shift, width, and pole positions

	TW	TWB	NLO
χ2/dof	1.12	1.15	0.957

Shift and width



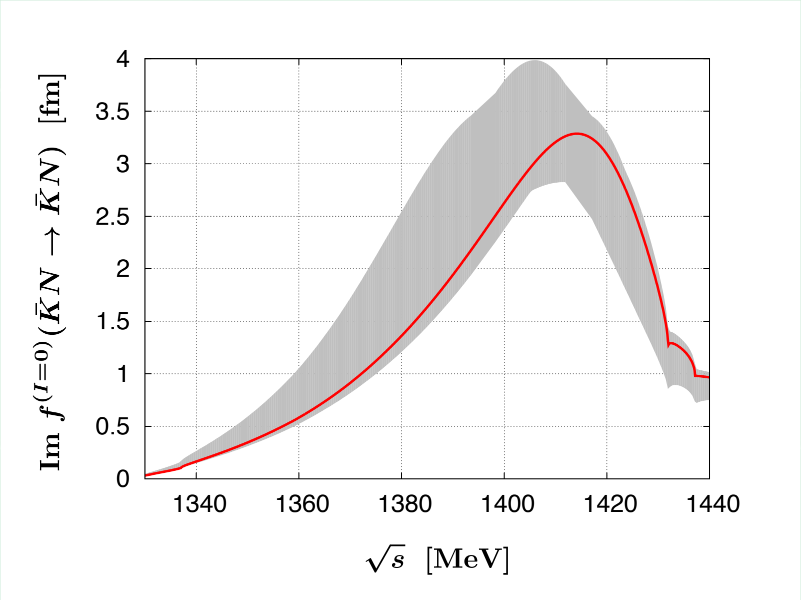
Pole positions

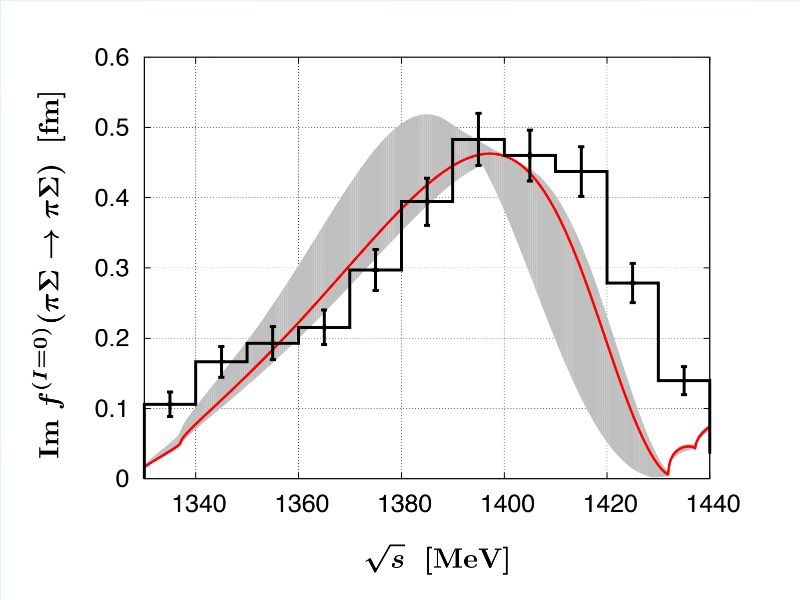


TW and TWB are reasonable, while best-fit requires NLO.
Pole positions are now converging.

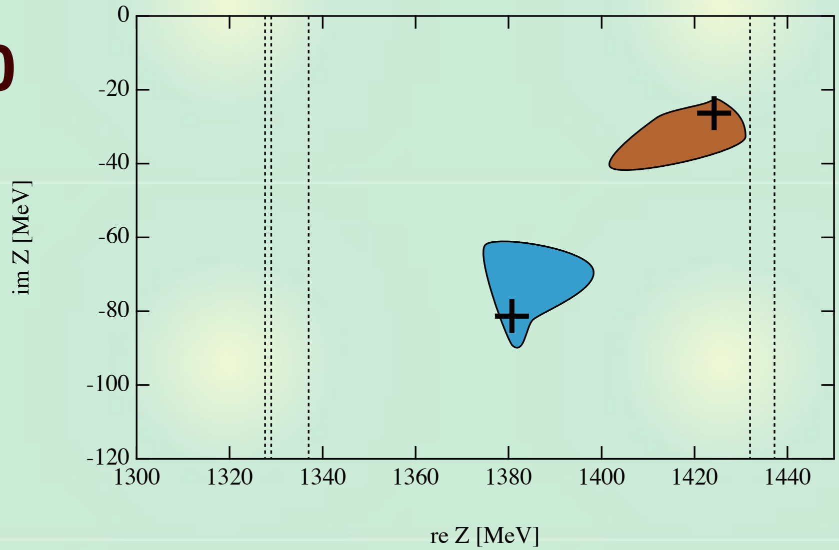
Subthreshold extrapolation

Predicted πΣ spectrum in comparison with K̄N





Note: Hemingway data is not I=0



Shift of the peak position
<-- two poles

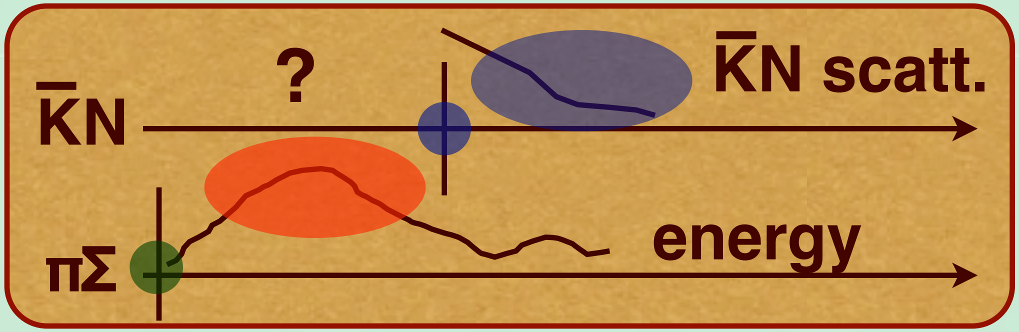
Uncertainty is reduced.

Experimental constraints for S=-1 MB scattering

K-p total cross sections

$\bar{K}$N threshold observables

- threshold branching ratios
- K-p scattering length <-- SIDDHARTA



πΣ mass spectra

- new data is becoming available (LEPS, CLAS, HADES,...)

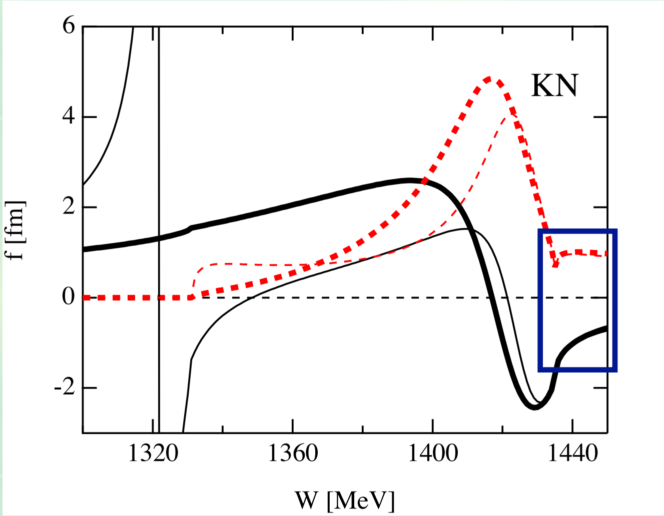
πΣ threshold observables (so far no data) ?

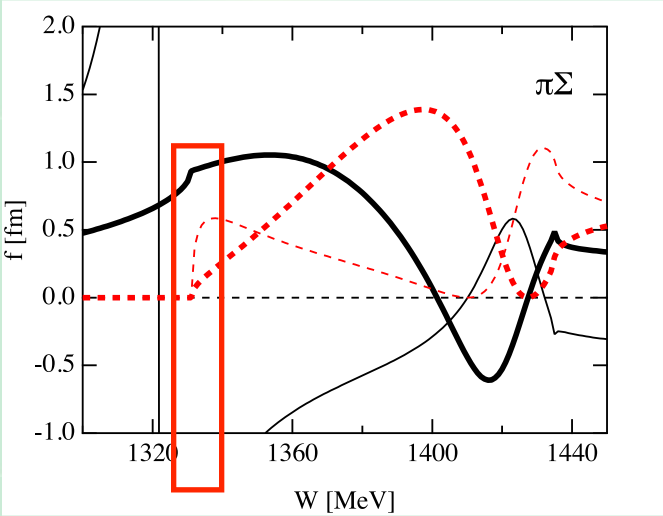
Importance of πΣ scattering length

Why threshold behavior of πΣ channel?

Y. Ikeda, T. Hyodo, D. Jido, H. Kamano, T. Sato, K. Yazaki, PTP 125, 1205 (2011)

Simple model extrapolation with $\bar{K}$N(I=0) being fixed
--> large uncertainty at πΣ threshold





Determination of πΣ threshold observables
--> understanding of Λ(1405), K nuclei, DISTO result,...

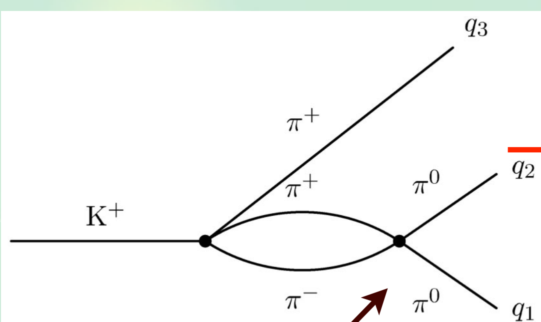
Determination of hadron scattering length

Extraction of hadron scattering length

- shift and width of atomic state (c.f. Kaonic hydrogen)
- extrapolation of low energy phase shift
- final state interaction from heavy particle's decay

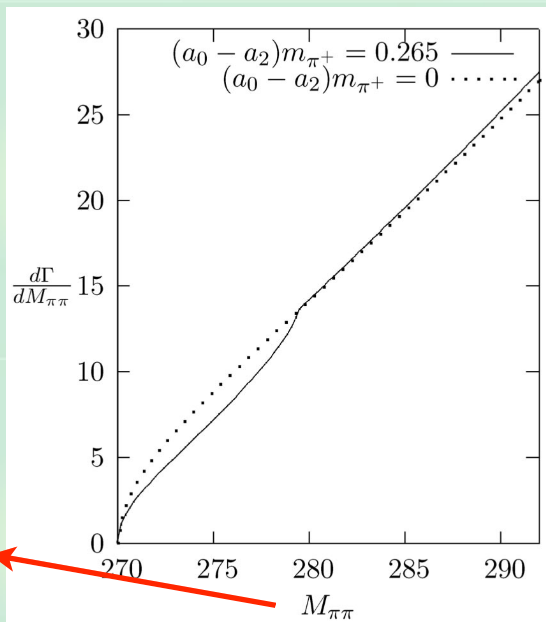
Isospin violation + threshold cusp + amplitude interference

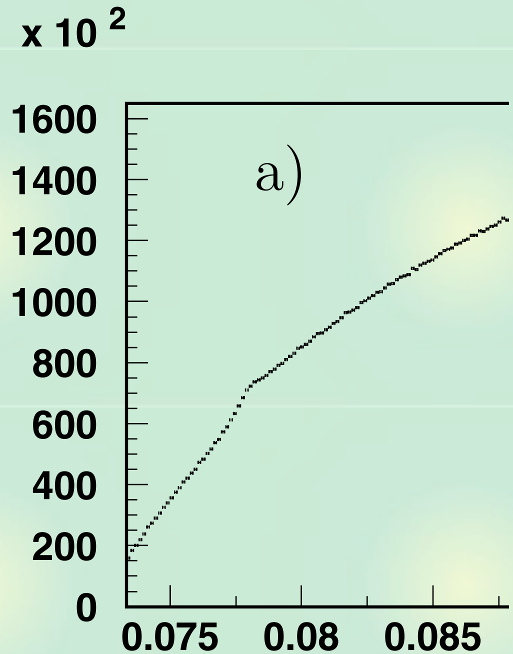
N. Cabibbo, Phys. Rev. Lett. 93, 121801 (2004)



$a_{I=0} - a_{I=2}$

$M_{\pi\pi}$

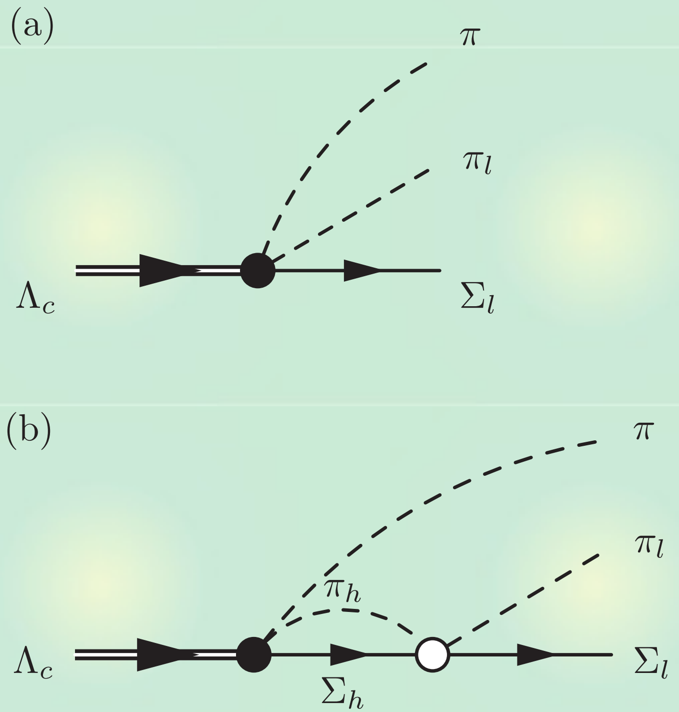


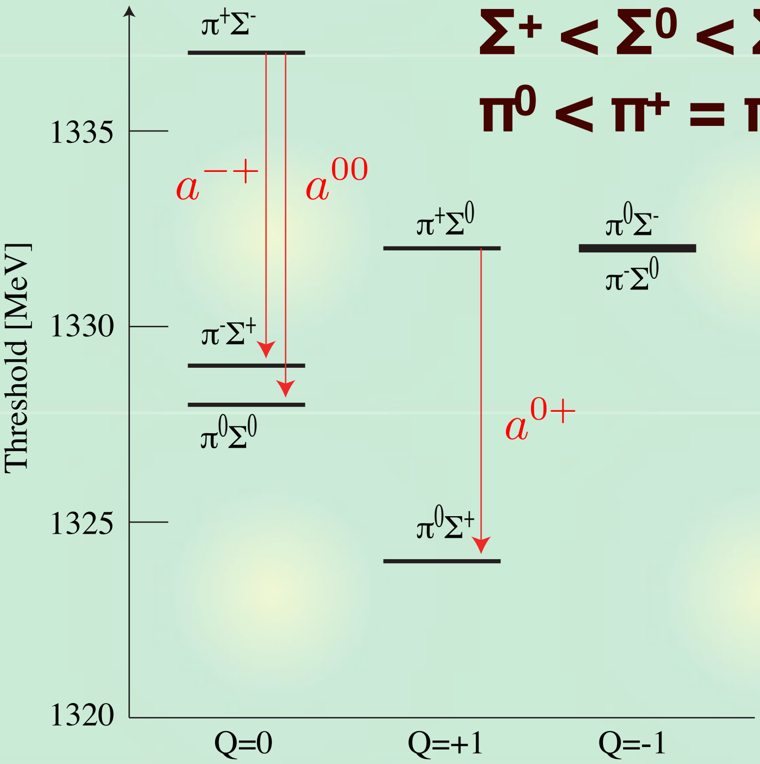


NA48/2, J.R. Batley et al, Phys. Lett. B686, 101 (2010) possible decay modes

Threshold difference of πΣ channels

Isospin violation in πΣ channels





$\Sigma^+ < \Sigma^0 < \Sigma^-$

$\pi^0 < \pi^+ = \pi^-$

To utilize threshold cusp, appreciable mass difference between $(\pi\Sigma)_h$ and $(\pi\Sigma)_l$ is necessary.

$$a^{-+} : \pi^+\Sigma^- \to \pi^-\Sigma^+, \quad a^{00} : \pi^+\Sigma^- \to \pi^0\Sigma^0, \quad a^{0+} : \pi^+\Sigma^0 \to \pi^0\Sigma^+$$

Determination of πΣ scattering length

Structure around the cusp in $(\pi\Sigma)_l$ + spectrum in $(\pi\Sigma)_h$
--> extraction of the scattering length

mode	$\Lambda_c \to \pi(\pi\Sigma)_h$	$\Lambda_c \to \pi(\pi\Sigma)_l$
a^{-+}	1.7 ± 0.5 %	3.6 ± 1.0 %
a^{00}	1.7 ± 0.5 %	1.8 ± 0.8 %
a^{0+}	1.8 ± 0.8 %	not known

A lot of Λ_c (Belle, Babar, LHC, ...) --> feasible?

Isospin decomposition of three channels

$$
\begin{aligned}
a^{-+} &= \tfrac{1}{3}a^0 - \tfrac{1}{2}a^1 + \tfrac{1}{6}a^2 + \cdots , \\
a^{00} &= \tfrac{1}{3}a^0 - \tfrac{1}{3}a^2 + \cdots , \\
a^{0+} &= -\tfrac{1}{2}a^1 + \tfrac{1}{2}a^2 + \cdots ,
\end{aligned}
$$

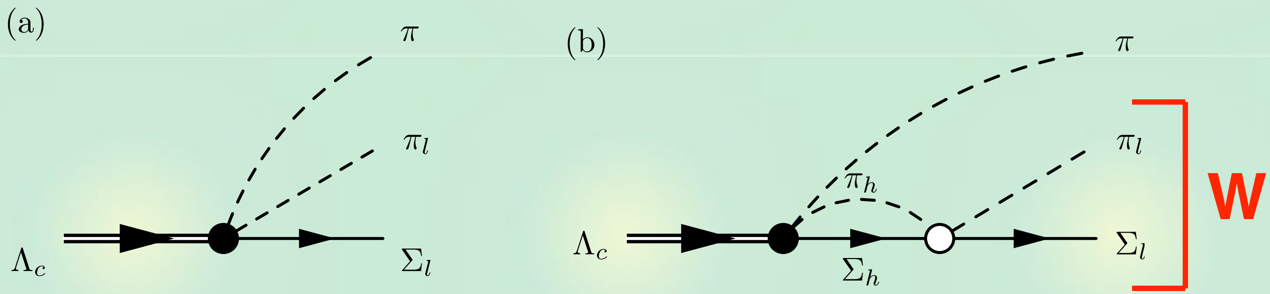
Three unknown scattering lengths, two constraints

$$a^{-+} - a^{00} = a^{0+} + \cdots$$

I=2 scattering length: lattice QCD (HAL QCD, NPLQCD,...)

Decay process and intermediate loop

Decay diagrams for Λ_c --> $\pi\ \pi_l\ \Sigma_l$ process



Spectral representation of the loop function

$$G(W) = \frac{1}{2\pi}\int_{W_{th}}^{\infty} dW' \frac{\rho(W')}{W - W' + i\epsilon} + \text{(subtractions)} \qquad \rho(W) = 2M_h \frac{q(W)}{4\pi W}$$

ρ: phase space, q: three-momentum

Imaginary part of the loop function (on-shell part):

$$\text{Im } G(W) = -\frac{\rho(W)}{2}\Theta(W - W_{th})$$

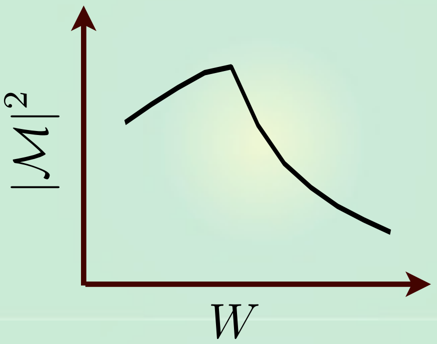
amplitude (a) : real
amplitude (b) : real ($W < W_{th}$), complex ($W > W_{th}$)

Threshold cusp in the spectrum

Decomposition of the amplitude

$$\mathcal{M}(W) = \mathcal{M}_0(W) + \tilde{\mathcal{M}}_1(W) m_h \delta$$

δ ~ real ($W < W_{th}$), imaginary ($W > W_{th}$)



$\pi_I \Sigma_I$ invariant mass spectrum (M_0, M_1: real)

$$|\mathcal{M}|^2 = \begin{cases} (\mathcal{M}_0)^2 + (\tilde{\mathcal{M}}_1 m_h)^2 |\delta|^2 & \text{for} \quad W > W_{\rm th} \\ (\mathcal{M}_0)^2 + \underline{2\mathcal{M}_0 \tilde{\mathcal{M}}_1 m_h \delta} + (\tilde{\mathcal{M}}_1 m_h)^2 \delta^2 & \text{for} \quad W < W_{\rm th} \end{cases}$$

- Spectrum is continuous (δ vanishes at threshold)
- Derivative of the spectrum is discontinuous

$$\left.\frac{d|\mathcal{M}|^2}{dW}\right|_{W\to W_{\rm th}-0} - \left.\frac{d|\mathcal{M}|^2}{dW}\right|_{W\to W_{\rm th}+0} \propto -\frac{2\mathcal{M}_0 \tilde{\mathcal{M}}_1 m_h M_h}{M_h + m_h} \frac{1}{\delta} + \mathcal{O}(\delta)$$

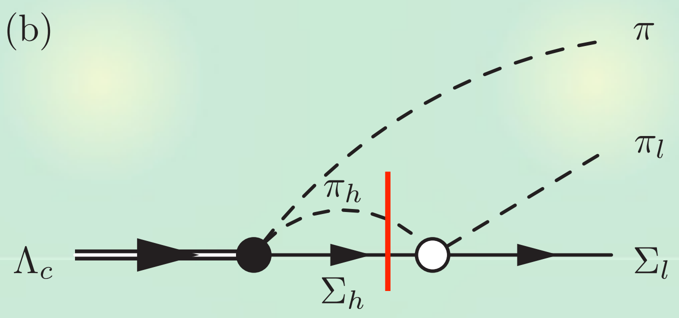
--> threshold cusp

It is purely kinematical effect. General phenomena.

Relation to scattering length

The term which produces the cusp

- Energy is fixed at $W = W_{th}$
- On-shell kinematics for $\pi_h \Sigma_h$ channel



--> amplitude of $\pi_h \Sigma_h$ --> $\pi_l \Sigma_l$ at threshold: scattering length

General decomposition of the amplitude

$$\mathcal{M}(W) = \mathcal{M}_0(W) + \tilde{\mathcal{M}}_1(W) e^{i\theta} m_h \delta$$

- Cusp appears, but relative phase affect to the structure.
- Relative phase can be calculated by the dynamical model of final state interactions.

Determination of πΣ scattering length

Expansion of the decay spectrum (M_0, M_1: real)

weak process

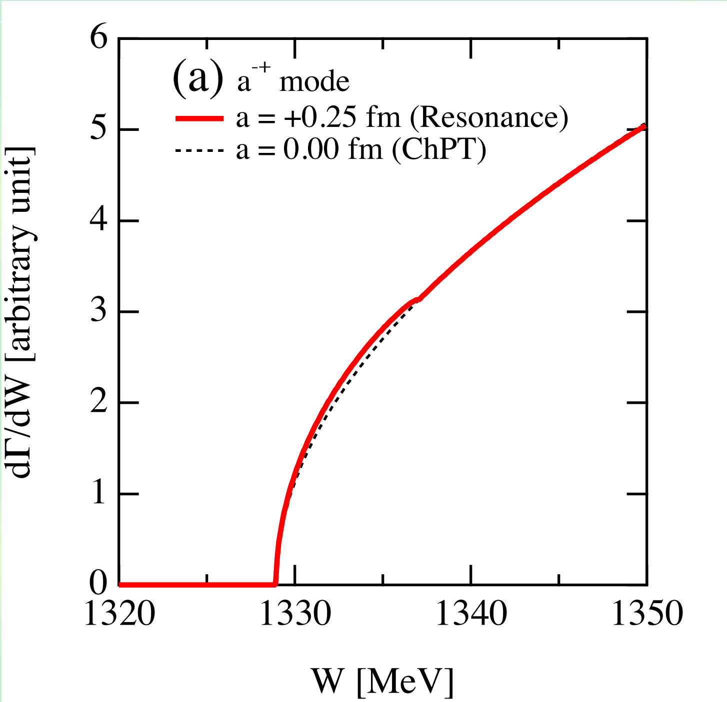
$$M_0^{h(0)}, M_0^{(0)}$$

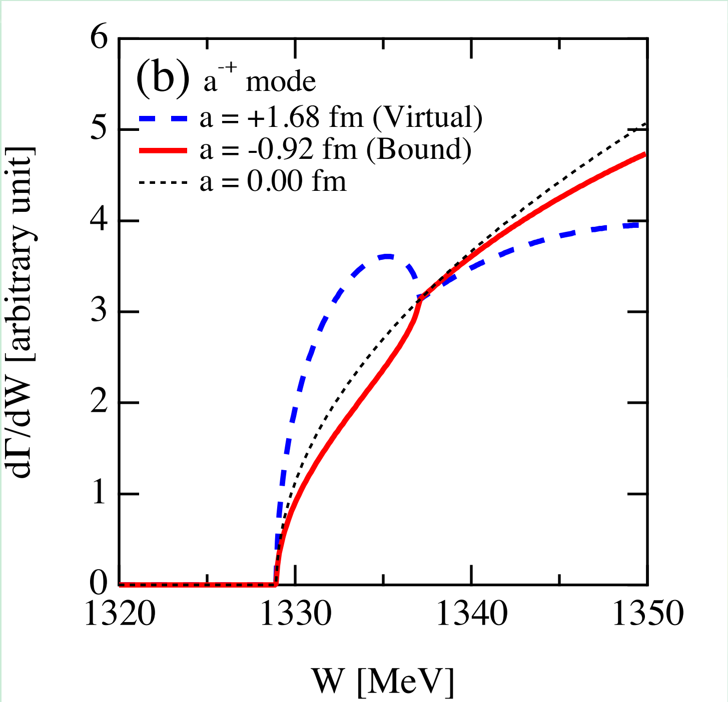
$$|\mathcal{M}|^2 = \begin{cases} A + C'|\delta|^2 + \mathcal{O}(|\delta|^4) & \text{for} \quad W > W_{\rm th} \\ A + B\delta + C\delta^2 + \mathcal{O}(\delta^3) & \text{for} \quad W < W_{\rm th} \end{cases}$$

spectrum

$$A, B, C$$

$$|a_{h\to l}| = \frac{|B|}{2m_h\sqrt{A}|\mathcal{M}_0^{h(0)}|}, \qquad \frac{a_{h\to l}}{|a_{h\to l}|} = \frac{\mathcal{M}_0^{(0)}}{|\mathcal{M}_0^{(0)}|}\cdot\frac{\mathcal{M}_0^{h(0)}}{|\mathcal{M}_0^{h(0)}|}\cdot\frac{B}{|B|}$$



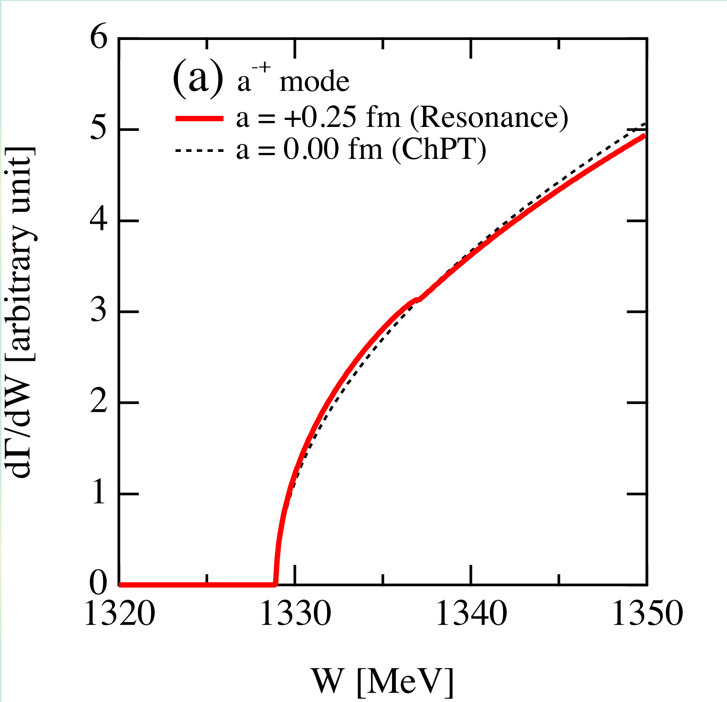


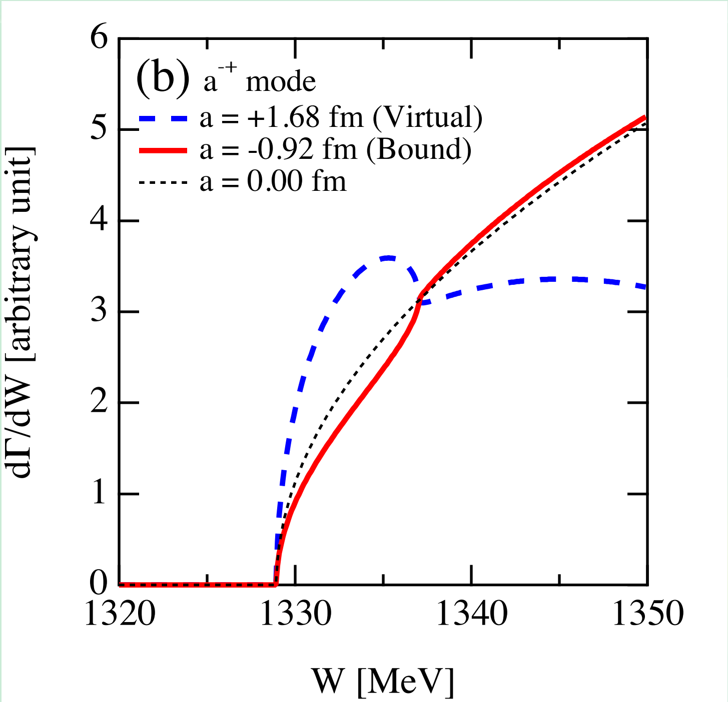
Determination of πΣ scattering length

Expansion of the decay spectrum (relative phase θ= -12 deg)

$$|\mathcal{M}|^2 = \begin{cases} A + B'|\delta| + C'|\delta|^2 + \mathcal{O}(|\delta|^4) & \text{for} \quad W > W_{\rm th} \\ A + B\delta + C\delta^2 + \mathcal{O}(\delta^3) & \text{for} \quad W < W_{\rm th} \end{cases},$$

$$|a_{h\to l}| = \frac{\sqrt{B^2 + (B')^2}}{2m_h\sqrt{A}|\mathcal{M}_0^{h(0)}|}, \qquad \frac{a_{h\to l}}{|a_{h\to l}|} = \frac{\mathcal{M}_0^{(0)}}{|\mathcal{M}_0^{(0)}|}\cdot\frac{\mathcal{M}_0^{h(0)}}{|\mathcal{M}_0^{h(0)}|}\cdot\frac{B/\cos\theta}{|B/\cos\theta|}$$





Summary

$\pi\Sigma$ scattering length from Λ_c decay

- **$\pi\Sigma$ scattering length** is important for low energy $\bar{K}N$-$\pi\Sigma$ amplitude.
  --> $\bar{K}$ nuclei and $\Lambda(1405)$ physics

  Y. Ikeda, T. Hyodo, D. Jido, H. Kamano, T. Sato, K. Yazaki, Prog. Theor. Phys. 125, 1205 (2011)

- **Threshold cusp** : kinematical effect

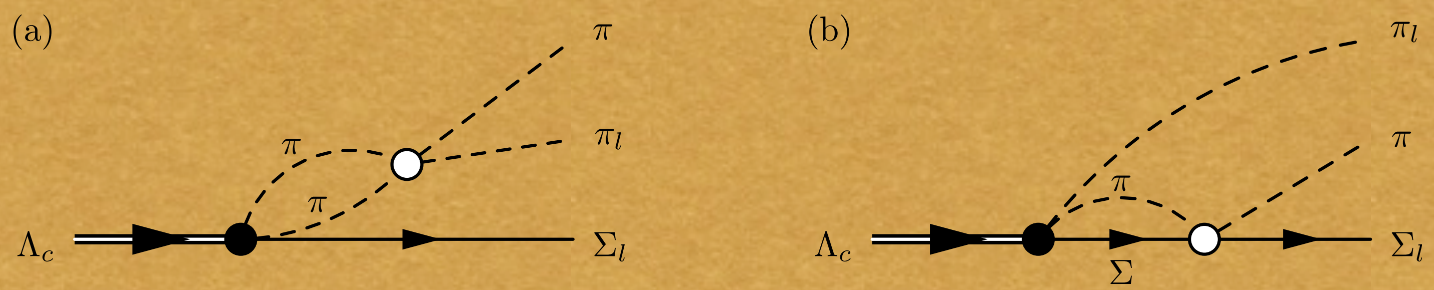
- Cusp in **Λ_c --> $\pi\pi\Sigma$ decay** is related with the $\pi\Sigma$ scattering length.

  T. Hyodo, M. Oka, Phys. Rev. C 83, 055202 (2011)

Summary

Future plans: estimate of amplitude

- **πΣ scattering length** final state interaction



- relative phase between M_0 and M_1

- Cusp in **$\pi\bar{K}N$** channel

$$a(\bar{K}^0 n \to K^- p) = \frac{1}{2}(a^{I=0} - a^{I=1})$$

c.f. Kaonic hydrogen: $a(K^- p) = \frac{1}{2}(a^{I=0} + a^{I=1})$