On the compositeness of dynamically generated hadrons

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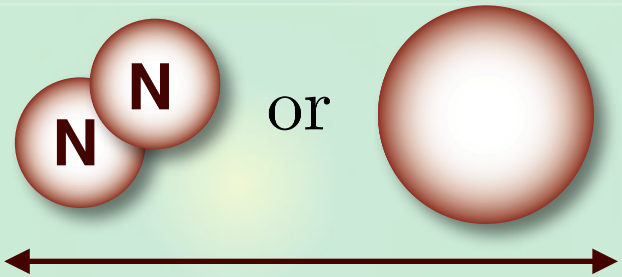
Compositeness of deuteron

What is the structure of the deuteron?

S. Weinberg, Phys. Rev. 137, B672 (1965)

$$|\text{deuteron}\rangle =$$



Z = 0 **Z = 1**

$\notin$ **NN model space**

~ elementary particle

$$Z \equiv |\langle B_0 | B \rangle|^2 \qquad 1 = | B_0 \rangle \langle B_0 | + \int d\boldsymbol{k} | \boldsymbol{k} \rangle \langle \boldsymbol{k} |$$

model independent relation for weakly bound state

$$\boxed{a_s} = \left[\frac{2(1-Z)}{2-Z} \right] \boxed{R} + \mathcal{O}(m_\pi^{-1}), \quad \boxed{r_e} = \left[\frac{-Z}{1-Z} \right] \boxed{R} + \mathcal{O}(m_\pi^{-1})$$

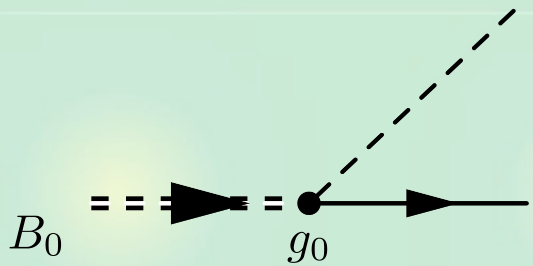
<-- Experiments

$$a_s = +5.41 \text{ [fm]}, \quad r_e = +1.75 \text{ [fm]}, \quad R \equiv (2\mu B)^{-1/2} = 4.31 \text{ [fm]}$$

$\Rightarrow Z \lesssim 0.2$ **--> deuteron is almost composite**

Outline of this talk

Definition of compositeness (Yukawa model)

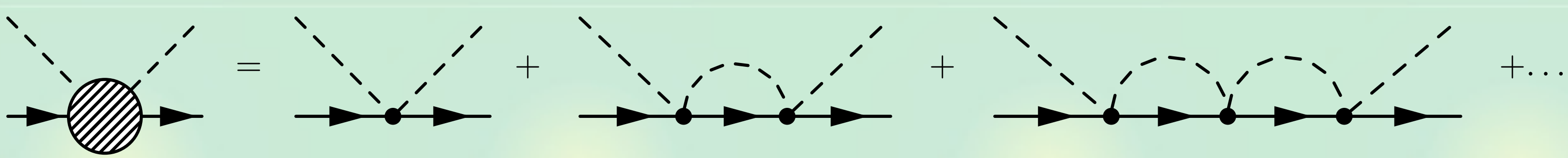


$(M_{B_0}, g_0, a) + 1$ cond. $\longleftrightarrow$ (M_B, g)

bare **physical**

Master formula of compositeness $1 - Z = f(M_B, g)$

Example of “physical” state (dynamical model with attractive contact interaction)



calculate (M_B, g) **Compositeness?**

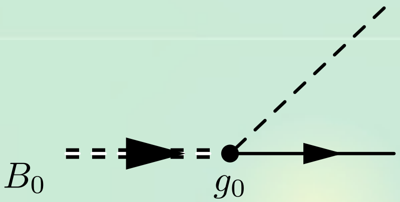
Z in Yukawa model

Field theory with Yukawa coupling (ψ,ϕ,B_0)

see D. Lurie and A. J. Macfarlane, Phys. Rev. 136, B816 (1963)

$$\mathcal{L}_0 = \bar{\psi}(i\partial\!\!\!/ - M)\psi + \frac{1}{2}(\partial_\mu\phi\partial^\mu\phi - m^2\phi^2) + \bar{B}_0(i\partial\!\!\!/ - M_{B_0})B_0$$

$$\mathcal{L}_{\rm int} = g_0\bar{\psi}\phi B_0 + ({\rm h.c.})$$



Physical bound state B at total energy W=M_B

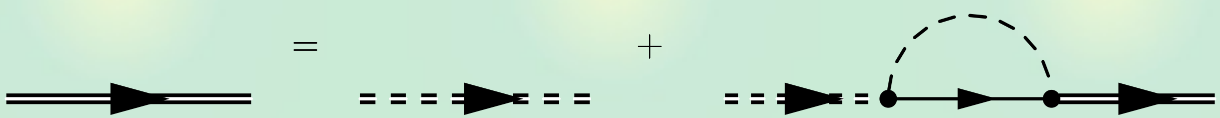
Free (full) propagator of B_0 (B) field (positive energy part)

$$\Delta_0(W) = \frac{1}{W - M_{B_0}}, \quad \Delta(W) = \frac{Z}{W - M_B}$$

Z: field renormalization constant

Dyson equation: relation between full and free propagators

$$\Delta(W) = \Delta_0(W) + \Delta_0(W)g_0G(W)g_0\Delta(W)$$



Master formula of compositeness

Solution of Dyson equation and renormalization

$$\Delta(W) = \frac{1}{W - M_{B_0} - g_0^2 G(W)} \to \frac{1}{W - g_0^2 G(W;a)}$$

Renormalization condition, pole at W=M$_B$: $M_B = g_0^2 G(M_B;a)$

The field renormalization constant: residue of the propagator

$$Z = \lim_{W \to M_B} \frac{W - M_B}{W - g_0^2 G(W;a)} = \frac{1}{1 - g_0^2 G'(M_B)}$$

Physical coupling constant: residue of T-matrix

$$g^2 = g_0^2 Z$$

Compositeness in Yukawa theory

$$1 - Z = -g^2 G'(M_B)$$

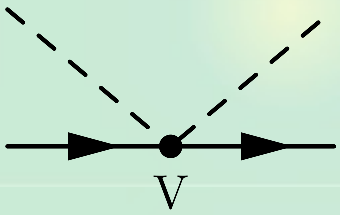
Physical coupling constant

Single-channel scattering of meson m and baryon M

$$T(W) = \frac{1}{1 - V(W)G(W;a)} V(W)$$

V: 4-point interaction, attractive

$$V(W) = \begin{cases} V^{(\mathrm{const})} = Cm & \text{constant interaction} \\ V^{(\mathrm{WT})}(W) = C(W - M) & \text{WT interaction} \end{cases}$$



Bound state condition: pole at W=M_B

$$1 - V(M_B)G(M_B;a) = 0$$

Coupling constant: residue of the pole

$$g^2 = \lim_{W \to M_B} (W - M_B)T(W) = \begin{cases} -[G'(M_B)]^{-1} & \text{constant interaction} \\ -\left[G'(M_B) + \dfrac{G(M_B;a)}{M_B - M}\right]^{-1} & \text{WT interaction} \end{cases}$$

Coupling g <-- mass (and cutoff)

Compositeness of bound states

Coupling g --> master formula

$$1 - Z = -g^2 G'(M_B) = \begin{cases} 1 & \text{constant interaction} \\ \left[1 + \dfrac{G(M_B; a)}{(M_B - M)G'(M_B)}\right]^{-1} & \text{WT interaction} \end{cases}$$

Constant interaction : purely composite bound state

- equivalence between ϕ^4 and Yukawa for $M_0 \to \infty$

WT interaction : mixture of composite and elementary

- Compositeness is normalized

$$0 \leq 1 - Z \leq 1$$

- pole term through renormalization

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C78, 025203 (2008).

$$\tilde{V}(W) = C(W - M) - C\frac{(W - M)^2}{(W - M_{\text{eff}})}$$

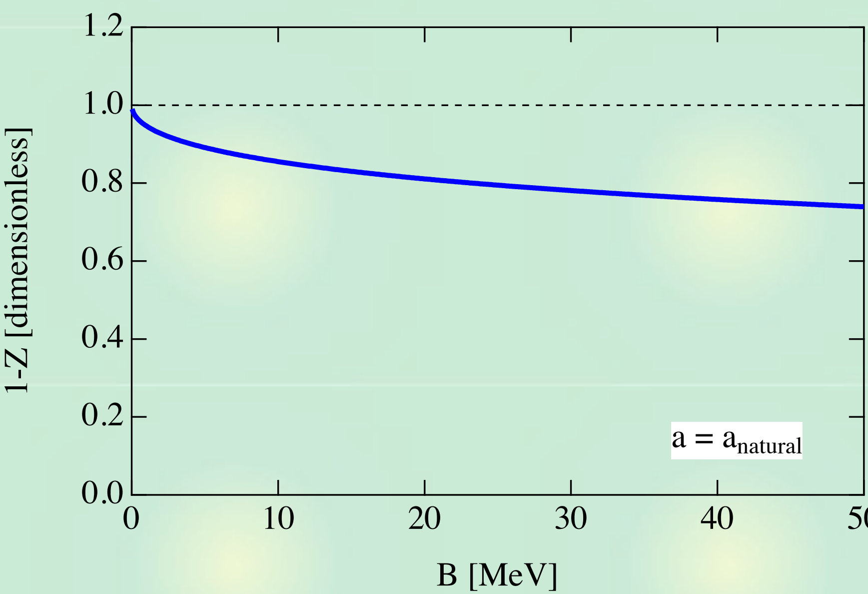
Check of the natural renormalization scheme

Natural renormalization condition
<-- to exclude elementary contribution from the loop function

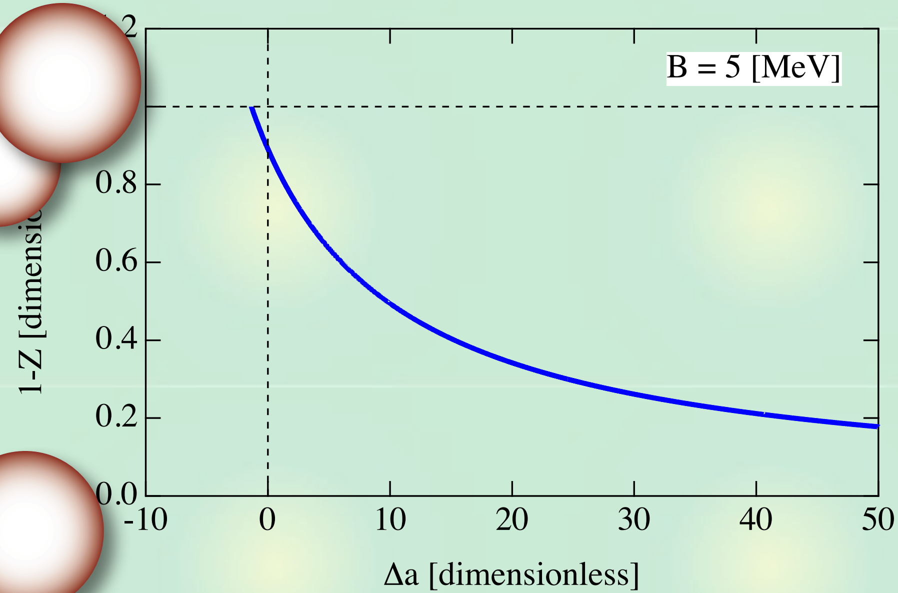
T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C78, 025203 (2008)

$$G(W = M; a_{\text{natural}}) = 0$$

1) a = a_{natural}, vary B = M_B-M-m



2) B = 5 MeV, vary a



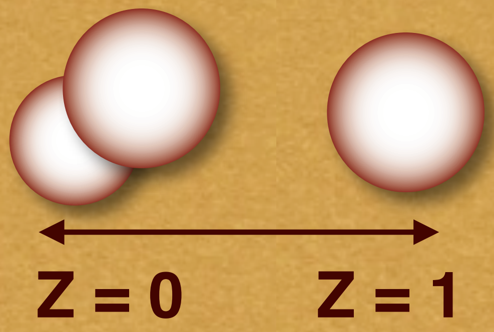
natural scheme --> Z ~ 0

large deviation --> Z ~ 1

Summary

We study compositeness of bound states

- **Field renormalization constant Z: compositeness**



- **Master formula**

  coupling: $g^2 = g_0^2 Z$

  compositeness: $1 - Z = -g^2 G'(M_B)$

- **Example of dynamical bound state**

  constant int. --> purely composite

  WT int. --> mixture

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C 85, 015201 (2012)