チャームバリオンの崩壊による
πΣ 散乱長の決定

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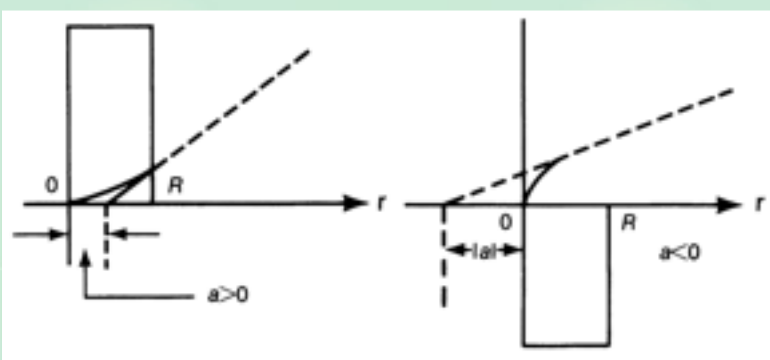
Scattering length

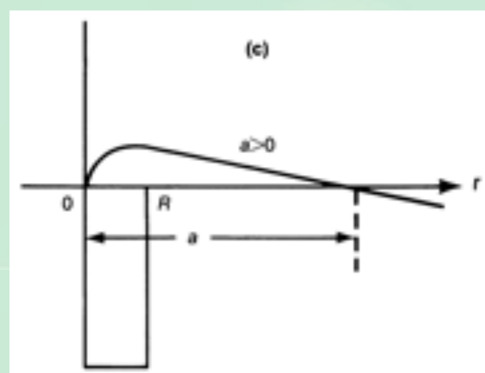
Scattering length: amplitude at threshold
- characterizes the low energy scattering
- changes the sign if there is a bound state

$$a = -f(k,\theta)|_{k\to 0}$$

$$\frac{d\sigma}{d\Omega} = |f(k,\theta)|^2, \quad \lim_{k\to 0}\sigma(k) = 4\pi a^2$$

$$f_{l=0}(k) = \frac{1}{k\cot\delta_0 - ik}, \quad k\cot\delta_0 = -\frac{1}{a} + r_e\frac{k^2}{2} + \dots$$





J.J. Sakurai, Modern Quantum Mechanics, p. 415

(In hadron physics we usually adopt the opposite sign
--> positive for attraction, negative for repulsion)

Importance of the πΣ scattering length

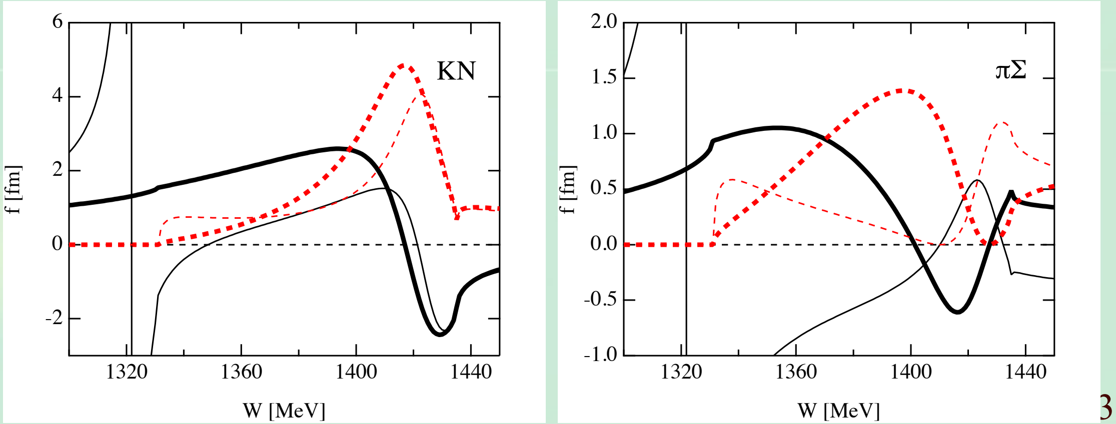
Structure of Λ(1405) and threshold behavior of πΣ scattering

Y. Ikeda, T. Hyodo, D. Jido, H. Kamano, T. Sato, K. Yazaki, PTP 125, 1205 (2011)

Simple model extrapolation with $\bar{K}N(I=0)$ being fixed
--> large uncertainty at low energy

Model	A1	A2	B E-dep	B E-indep
parameter $(\pi\Sigma)$	$d_{\pi\Sigma} = -1.67$	$d_{\pi\Sigma} = -2.85$	$\Lambda_{\pi\Sigma} = 1005$ MeV	$\Lambda_{\pi\Sigma} = 1465$ MeV
parameter $(\bar{K}N)$	$d_{\bar{K}N} = -1.79$	$d_{\bar{K}N} = -2.05$	$\Lambda_{\bar{K}N} = 1188$ MeV	$\Lambda_{\bar{K}N} = 1086$ MeV
pole 1 [MeV]	$1422 - 16i$	$1425 - 11i$	$1422 - 22i$	$1423 - 29i$
pole 2 [MeV]	$1375 - 72i$ (R)	1321 (B)	$1349 - 54i$ (R)	1325 (V)
$a_{\pi\Sigma}$ [fm]	0.934	-2.30	1.44	5.50
r_e [fm]	5.02	5.89	3.96	0.458
$a_{\bar{K}N}$ [fm] (input)	$-1.70 + 0.68i$	$-1.70 + 0.68i$	$-1.70 + 0.68i$	$-1.70 + 0.68i$

- **πΣ threshold behavior**
  - **--> pole structure**
  - **--> K (Λ*) nuclei**



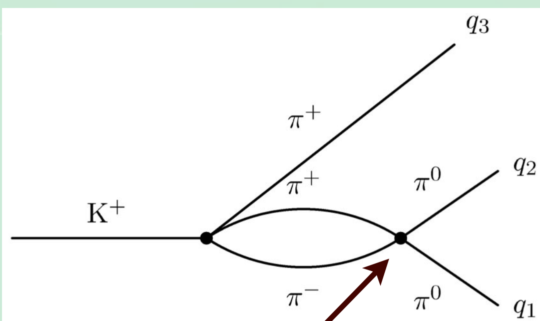
Determination of the scattering length

Extraction of hadron scattering length

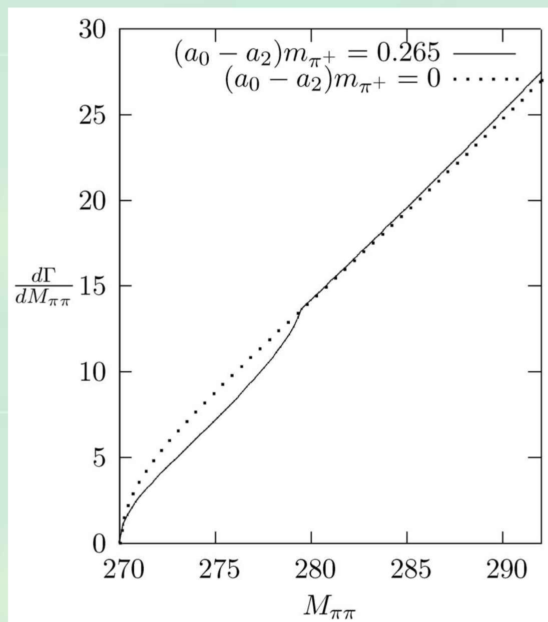
- shift and width of atomic state (c.f. Kaonic hydrogen)
- extrapolation of low energy phase shift
- final state interaction from heavy particle's decay

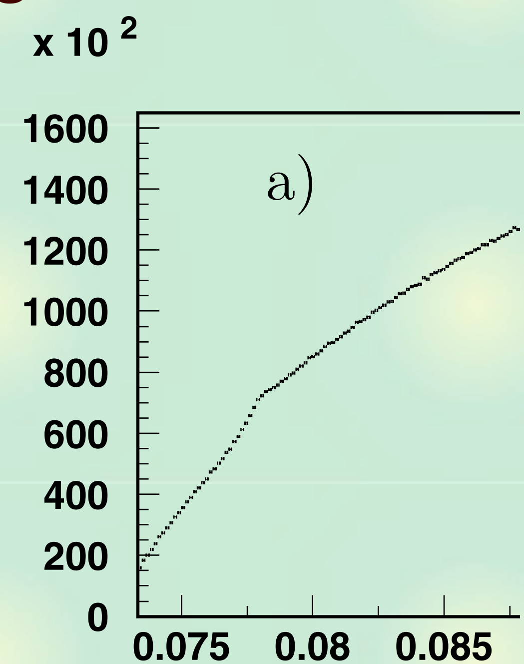
Cabibbo's method for π-π scattering length

N. Cabibbo, Phys. Rev. Lett. 93, 121801 (2004)



$a_{I=0} - a_{I=2}$



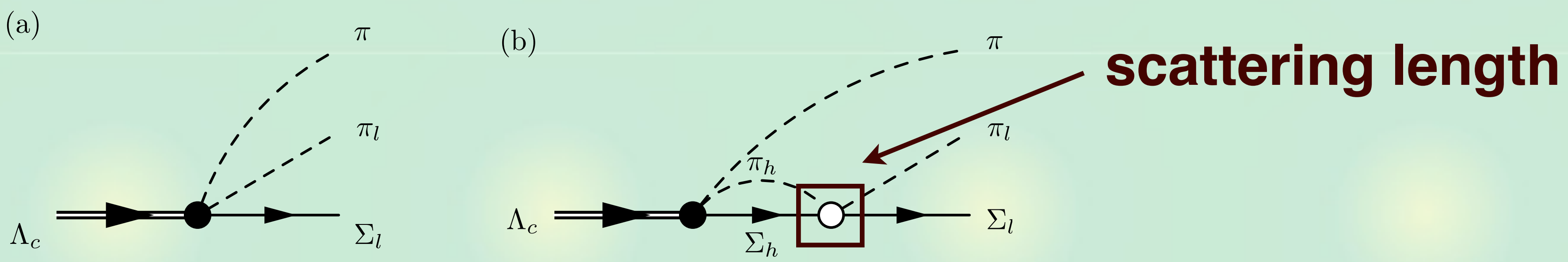


NA48/2, J.R. Batley et al, Phys. Lett. B686, 101 (2010)

isospin violation
+ threshold cusp
+ amplitude interference
--> extraction of $a_{I=0} - a_{I=2}$

Decay process and loop function

Decay diagrams for Λ_c --> $\pi\ \pi_l\ \Sigma_l$ process



Spectral representation of the loop function

$$G(W) = \frac{1}{2\pi}\int_{W_{th}}^{\infty} dW' \frac{\rho(W')}{W - W' + i\epsilon} + (\text{subtractions}) \qquad \rho(W) = 2M_h \frac{q(W)}{4\pi W}$$

ρ: phase space, q: three-momentum

Imaginary part of the loop function (on-shell part):

$$\mathrm{Im}\ G(W) = -\frac{\rho(W)}{2}\Theta(W - W_{th})$$

amplitude (a) : real

amplitude (b) : real ($W < W_{th}$), complex ($W > W_{th}$)

Threshold cusp in the spectrum

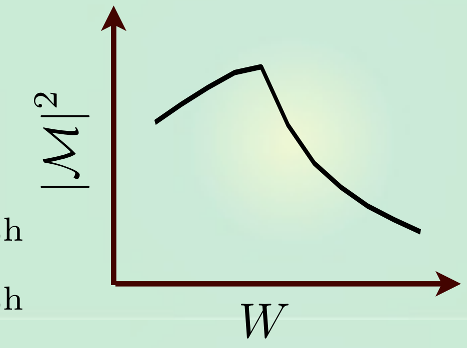
Real and imaginary part of the amplitude:

$$\mathcal{M}(W) = \mathcal{M}_0(W) + \tilde{\mathcal{M}}_1(W) m_h \delta$$

δ ~ velocity, vanish at threshold

The $\pi_l \Sigma_l$ invariant mass spectrum

$$|\mathcal{M}|^2 = \begin{cases} (\mathcal{M}_0)^2 + (\tilde{\mathcal{M}}_1 m_h)^2 |\delta|^2 & \text{for} \quad W > W_{\rm th} \\ (\mathcal{M}_0)^2 + \underline{2\mathcal{M}_0 \tilde{\mathcal{M}}_1 m_h \delta} + (\tilde{\mathcal{M}}_1 m_h)^2 \delta^2 & \text{for} \quad W < W_{\rm th} \end{cases}$$



- Spectrum is continuous (δ vanishes at threshold)

- Derivative of the spectrum is discontinuous

$$\left.\frac{d|\mathcal{M}|^2}{dW}\right|_{W \to W_{\rm th}-0} - \left.\frac{d|\mathcal{M}|^2}{dW}\right|_{W \to W_{\rm th}+0} \propto -\frac{2\mathcal{M}_0 \tilde{\mathcal{M}}_1 m_h M_h}{M_h + m_h} \frac{1}{\delta} + \mathcal{O}(\delta)$$

--> threshold cusp

It is purely kinematical effect. General phenomena.

M_1 amplitude: proportional to the scattering length

Determination of πΣ scattering length

Expansion of the decay spectrum --> scattering length

$$|\mathcal{M}|^2 = \begin{cases} A + C'|\delta|^2 + \mathcal{O}(|\delta|^4) & \text{for} \quad W > W_{\rm th} \\ A + B\delta + C\delta^2 + \mathcal{O}(\delta^3) & \text{for} \quad W < W_{\rm th} \end{cases}$$

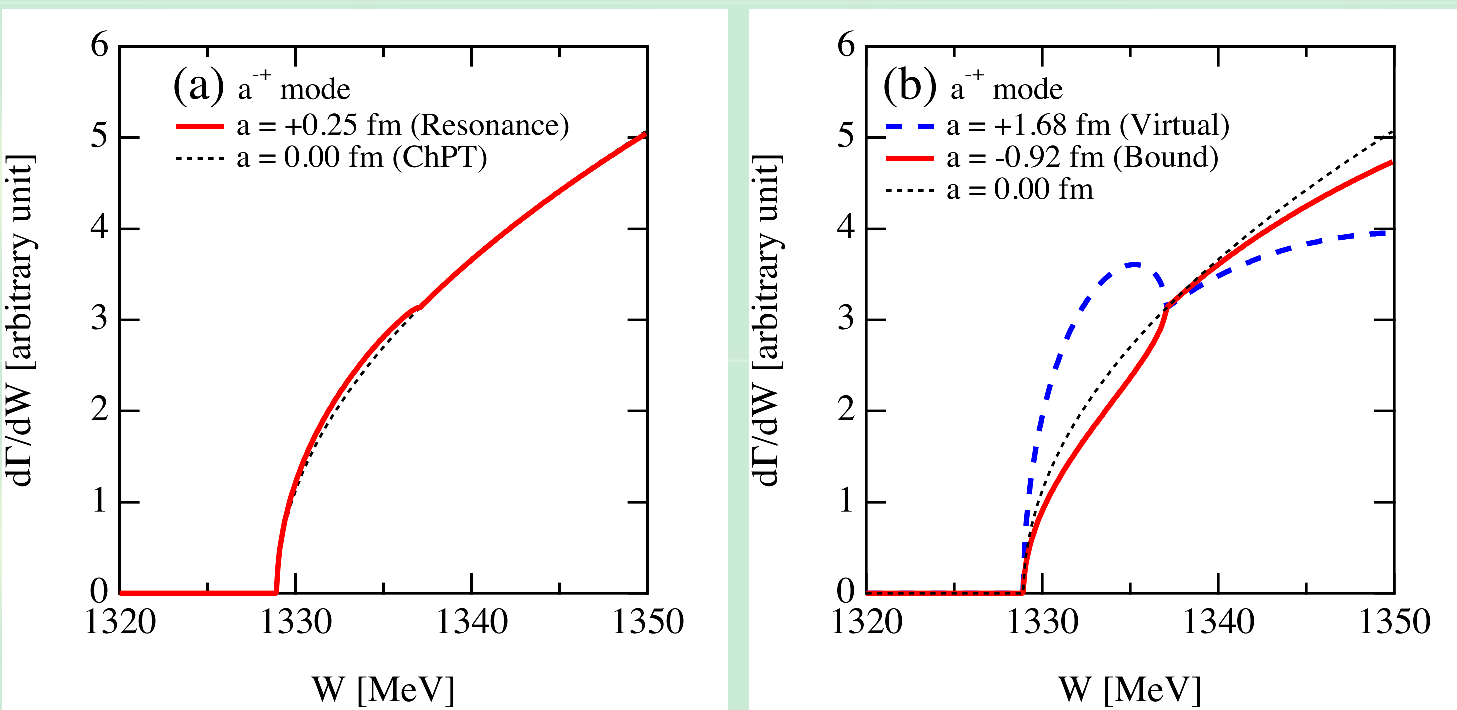
weak process

$M_0^{h(0)}, M_0^{(0)}$

$$|a_{h\to l}| = \frac{|B|}{2m_h\sqrt{A}|\mathcal{M}_0^{h(0)}|}, \qquad \frac{a_{h\to l}}{|a_{h\to l}|} = \frac{\mathcal{M}_0^{(0)}}{|\mathcal{M}_0^{(0)}|}\cdot\frac{\mathcal{M}_0^{h(0)}}{|\mathcal{M}_0^{h(0)}|}\cdot\frac{B}{|B|}$$

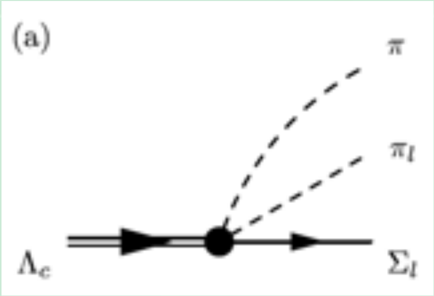
spectrum

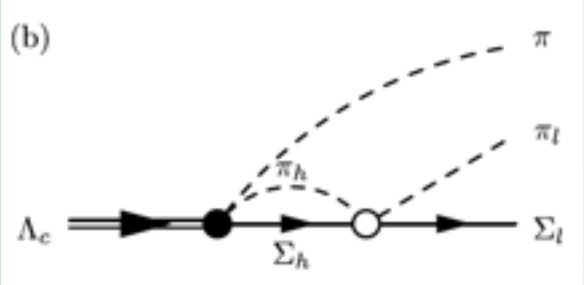
A, B, C

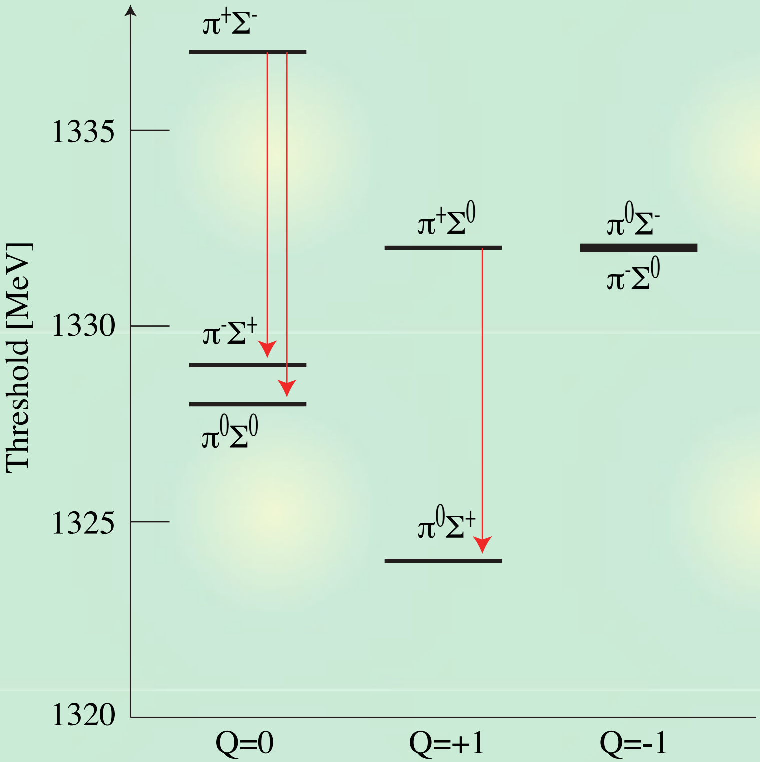


Determination of πΣ scattering length

Isospin violation in πΣ channels







Σ^+(~uus) < Σ^0(~uds) < Σ^-(~dds)
--> complicated spectrum

To utilize threshold cusp, appreciable mass difference between $(\pi\Sigma)_h$ and $(\pi\Sigma)_l$ is necessary.

$$\pi^+\Sigma^- \to \pi^-\Sigma^+, \quad \pi^+\Sigma^- \to \pi^0\Sigma^0, \quad \pi^+\Sigma^0 \to \pi^0\Sigma^+,$$

Determination of πΣ scattering length

Three decay channels

$$a^{-+} = \frac{1}{3}a^0 - \frac{1}{2}a^1 + \frac{1}{6}a^2 + \cdots ,$$
$$a^{00} = \frac{1}{3}a^0 - \frac{1}{3}a^2 + \cdots ,$$
$$a^{0+} = -\frac{1}{2}a^1 + \frac{1}{2}a^2 + \cdots ,$$

mode	$\Lambda_c \to \pi(\pi\Sigma)_h$	$\Lambda_c \to \pi(\pi\Sigma)_l$
a^{-+}	1.7 ± 0.5 %	3.6 ± 1.0 %
a^{00}	1.7 ± 0.5 %	1.8 ± 0.8 %
a^{0+}	1.8 ± 0.8 %	not known

A lot of Λ_c (Belle, Babar, LHC, ...) --> feasible?

Structure around the cusp in $(\pi\Sigma)_l$ + spectrum in $(\pi\Sigma)_h$
--> extraction of the scattering length

Three unknown scattering lengths, two constraints

$$a^{-+} - a^{00} = a^{0+} + \cdots$$

I=2 scattering length: lattice QCD (Y. Ikeda et al., 17pSG12)

Summary

πΣ scattering length from Λ_c decay

- **πΣ scattering length** is important for low energy KN-πΣ amplitude.
  --> K nuclei and Λ(1405) physics

  Y. Ikeda, T. Hyodo, D. Jido, H. Kamano, T. Sato, K. Yazaki, Prog. Theor. Phys. 125, 1205 (2011)

- **Threshold cusp** in Λ_c --> ππΣ decay is related with the **πΣ scattering length**.

  T. Hyodo, M. Oka, Phys. Rev. C 83, 055202 (2011)

- **3** isospin states vs. **2** decay modes: Lattice QCD can help to complete.