# Compositeness of bound states and resonances in chiral unitary approach

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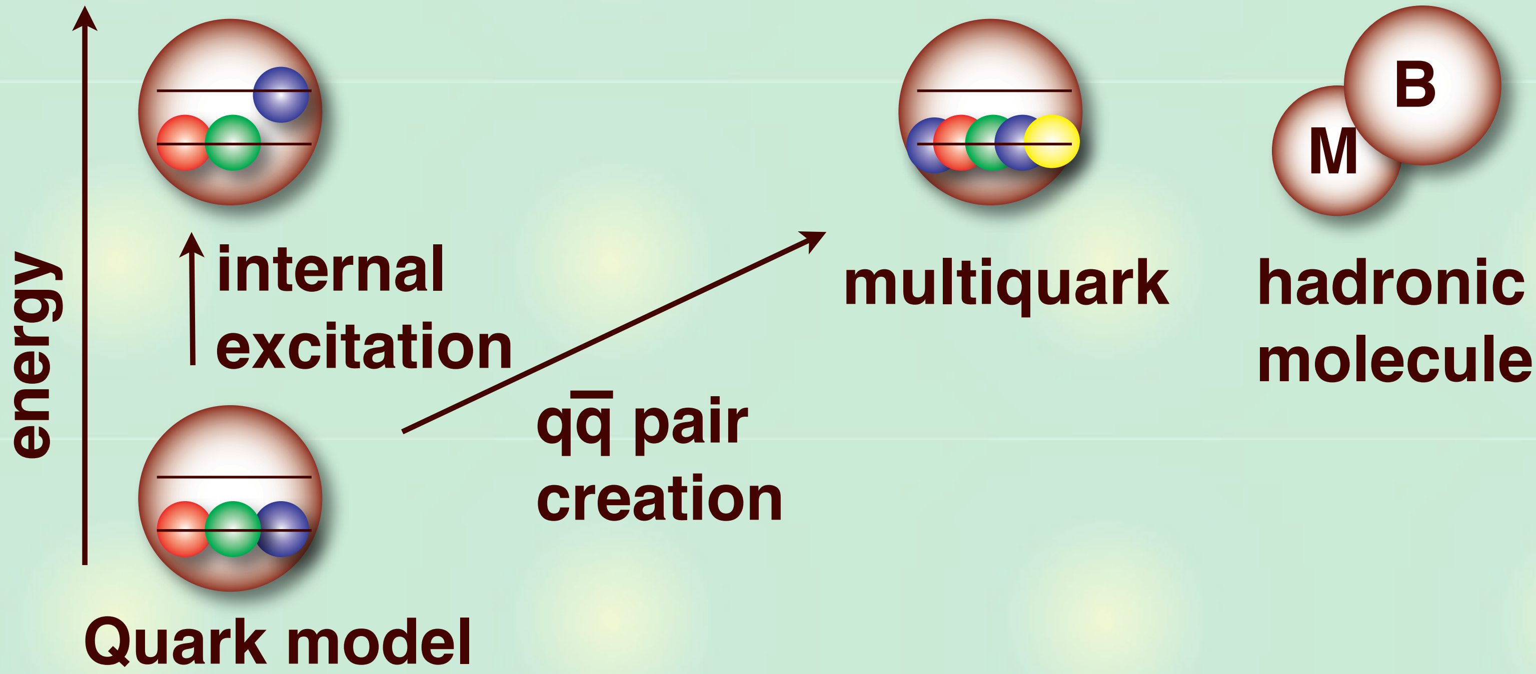
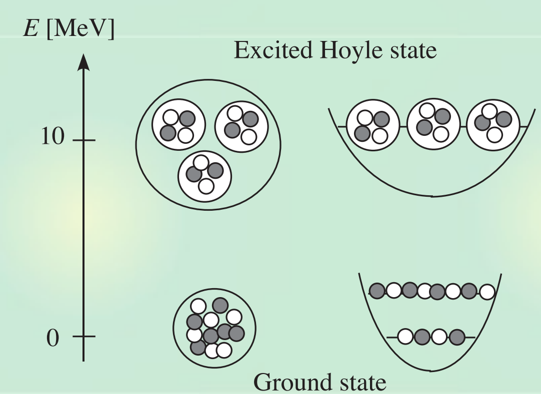
# Structure of hadron resonances

## Example) baryon excited state




**Excited states**
**= resonances in hadron scattering**

**Exotic structure** **near threshold?**
**c.f. $^{12}$C Hoyle state**



# Study of the internal structure

How to investigate the internal structure?

- Comparison of model calculation with experiments
  (mass, width, decay properties, etc.)

  : Any model can describe data with appropriate corrections
  : Model-dependent result

- Extrapolation to the ideal world, change the environment
  (large Nc, symmetry restoration, etc.)

  : Structure may change during the extrapolation
  : Qualitative discussion only

--> model-independent and quantitative study?

# Contents

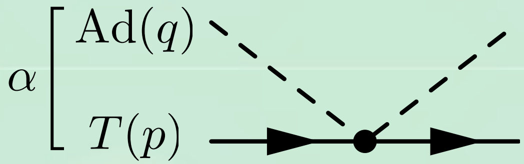
- Introduction
- Chiral SU(3) dynamics
- Origin of resonances in chiral dynamics
  - Natural renormalization condition

    T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C78, 025203 (2008)
- Compositeness of bound states
  - Field renormalization constant Z

    T. Hyodo, D. Jido, A. Hosaka, arXiv:1009.5754 [nucl-th]; in preparation
- Summary + future plan

# s-wave low energy interaction

## Low energy NG boson (Ad) + target hadron (T) scattering




$$= \frac{1}{f^2}\frac{p\cdot q}{2M_T}\langle \boldsymbol{F}_T\cdot\boldsymbol{F}_{\mathrm{Ad}}\rangle_\alpha + \mathcal{O}\left(\left(\frac{m}{M_T}\right)^2\right)$$

## Projection onto s-wave: Weinberg-Tomozawa (WT) term

Y. Tomozawa, Nuovo Cim. 46A, 707 (1966); S. Weinberg, Phys. Rev. Lett. 17, 616 (1966)

$$V_{ij} = -\frac{C_{ij}}{4f^2}(\omega_i+\omega_j)$$

**energy dependence (derivative coupling)**

**decay constant of π ($g_V$=1)**

$$C_{ij} = \sum_\alpha C_{\alpha,T}\begin{pmatrix} 8 & T & \| & \alpha \\ I_{M_i},Y_{M_i} & I_{T_i},Y_{T_i} & \| & I,Y\end{pmatrix}\begin{pmatrix} 8 & T & \| & \alpha \\ I_{M_j},Y_{M_j} & I_{T_j},Y_{T_j} & \| & I,Y\end{pmatrix}$$

$$C_{\alpha,T} = \langle 2\boldsymbol{F}_T\cdot\boldsymbol{F}_{\mathrm{Ad}}\rangle_\alpha = C_2(T) - C_2(\alpha) + 3$$

**Group theoretical structure and flavor SU(3) symmetry determines the sign and the strength of the interaction**

## Low energy theorem: leading order term in ChPT

# Scattering amplitude and unitarity

## Unitarity of S-matrix: Optical theorem

$$\mathrm{Im}[T^{-1}(s)] = \frac{\rho(s)}{2}$$ phase space of two-body state

## General amplitude by dispersion relation

$$T^{-1}(\sqrt{s}) = \sum_i \frac{R_i}{\sqrt{s}-W_i} + \tilde{a}(s_0) + \frac{s-s_0}{2\pi}\int_{s^+}^{\infty} ds' \frac{\rho(s')}{(s'-s)(s'-s_0)}$$

$R_i$, $W_i$, a: to be determined by chiral interaction

## Identify dispersion integral = loop function G, the rest = $V^{-1}$

$$T(\sqrt{s}) = \frac{1}{V^{-1}(\sqrt{s}) - G(\sqrt{s};a)}$$

Scattering amplitude

## V? chiral expansion of T, (conceptual) matching with ChPT

$$T^{(1)} = V^{(1)}, \quad T^{(2)} = V^{(2)}, \quad T^{(3)} = V^{(3)} - V^{(1)}GV^{(1)}, \quad \dots$$

## Amplitude T: consistent with chiral symmetry + unitarity

# Chiral unitary approach

## Meson-baryon scattering amplitude

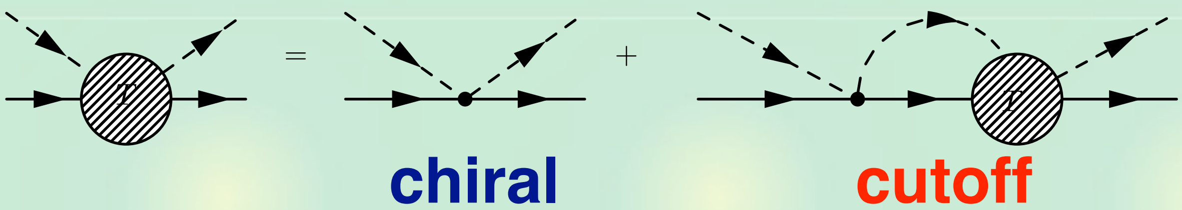
- Interaction <-- chiral symmetry

Y. Tomozawa, Nuovo Cim. 46A, 707 (1966); S. Weinberg, Phys. Rev. Lett. 17, 616 (1966)

- Amplitude <-- unitarity in coupled channels

R.H. Dalitz, T.C. Wong, G. Rajasekaran, Phys. Rev. 153, 1617 (1967)

$$T = \frac{1}{1 - VG} V$$




N. Kaiser, P. B. Siegel, W. Weise, Nucl. Phys. A594, 325 (1995),
E. Oset, A. Ramos, Nucl. Phys. A635, 99 (1998),
J. A. Oller, U. G. Meissner, Phys. Lett. B500, 263 (2001),
M.F.M. Lutz, E. E. Kolomeitsev, Nucl. Phys. A700, 193 (2002), .... many others

**It works successfully, also in S=0 sector, meson-meson scattering sectors, systems including heavy quarks, ...**

# Hadron excited states

## Resonances are "dynamically generated"

| | | |
|---|---|---|
| light baryon | $J^P = 1/2^-$ | $\Lambda(1405)$ $\Lambda(1670)$ $\Sigma(1670)$ <br> $N(1535)$ $\Xi(1620)$ $\Xi(1690)$ |
| | $J^P = 3/2^-$ | $\Lambda(1520)$ $\Xi(1820)$ $\Sigma(1670)$ |
| heavy | | $\Lambda_c(2880)$ $\Lambda_c(2593)$ $D_s(2317)$ |
| light meson | $J^P = 1^+$ | $b_1(1235)$ $h_1(1170)$ $h_1(1380)$ $a_1(1260)$ <br> $f_1(1285)$ $K_1(1270)$ $K_1(1440)$ |
| | $J^P = 0^+$ | $\sigma(600)$ $\kappa(900)$ $f_0(980)$ $a_0(980)$ |

**No states with exotic quantum number**

- **No attraction in exotic channel**

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. Lett. 97, 192002 (2006); Phys. Rev. D75, 034002 (2007)

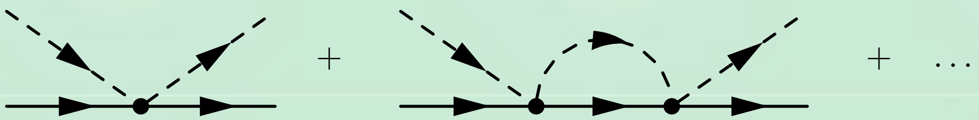
**--> Structure of these resonances?**

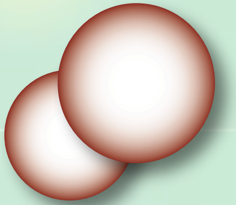
# Classification of resonances

## Resonances in two-body scattering

- Knowledge of interaction (potential)
- Experimental data (cross section, ...)

**Dynamical state**: composite particle, two-body molecule, ...

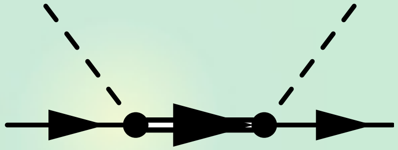


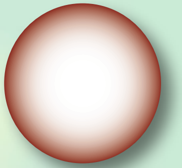


e.g.) Deuteron in NN, positronium in $e^+e^-$, ...

**CDD pole**: elementary particle, preformed state, ...

L. Castillejo, R.H. Dalitz, F.J. Dyson, Phys. Rev. 101, 453 (1956)



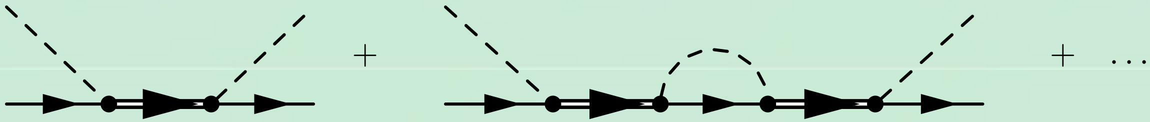


~ pole term in V

e.g.) J/Ψ in $e^+e^-$, ...

# (Known) CDD poles in chiral unitary approach

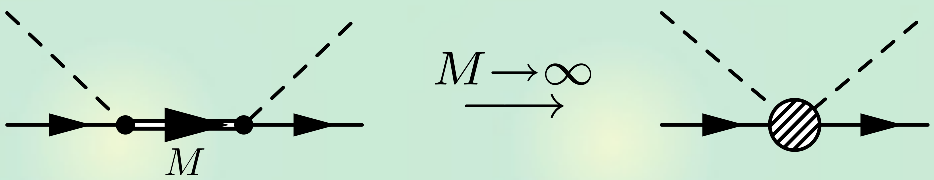
## Explicit resonance field in V (interaction): Δ(1232), Σ(1385),...



U.G. Meissner, J.A. Oller, Nucl. Phys. A673, 311 (2000)

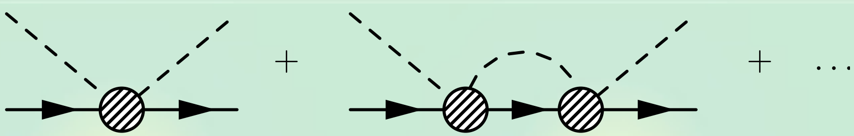
D. Jido, E. Oset, A. Ramos, Phys. Rev. C66, 055203 (2002)

## Contracted resonance propagator in higher order V




G. Ecker, J. Gasser, A. Pich, E. de Rafael, Nucl. Phys. B321, 311 (1989)

V. Bernard, N. Kaiser, U.G. Meissner, Nucl. Phys. A615, 483 (1997)



J.A. Oller, E. Oset, J.R. Pelaez, Phys. Rev. D59, 074001 (1999)

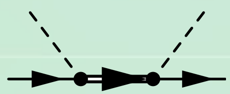
## Is that all? subtraction constant?

# CDD pole in subtraction constant?

**Phenomenological (standard) scheme**
**--> V is given, “a” is determined by data**

$$T = \frac{1}{(V^{(1)})^{-1} - G(\underline{a})}$$ **leading order**

$$T = \frac{1}{(V^{(1)} + V^{(2)})^{-1} - G(\underline{a'})}$$ **next to leading order**

↑pole  ?

**“a” represents the effect which is not included in V.**
**CDD pole contribution in G?**

**Natural renormalization scheme**
**--> fix “a” first, then determine V**

**to exclude CDD pole contribution from G,**
**based on theoretical argument.**

# Natural renormalization condition

## Conditions for the subtraction constant

1) Loop function G should be negative below threshold.
<--> no states below threshold

$$G(\sqrt{s}) \sim \sum_n \frac{|\langle \dots \rangle|^2}{\sqrt{s} - E_n} \leq 0 \quad \text{for} \quad \sqrt{s} \leq E_0$$ --> upper limit for "a"

2) T matches with the chiral interaction V at low energy.

$$T(\mu_m; a) = V(\mu_m) \quad \text{for} \quad M_T \leq \mu_m \leq M_T + m$$ --> lower limit for "a"

To satisfy 1) and 2), "a" is uniquely determined as

$$G(\sqrt{s} = M_T) = 0 \quad \Leftrightarrow \quad T(M_T) = V(M_T)$$

- subtraction constant: $a_{\text{natural}}$

We regard this condition as the exclusion of the CDD pole contribution from G.

# Pole in the effective interaction: single channel

## Leading order V: Weinberg-Tomozawa term

$$V_{\mathrm{WT}} = -\frac{C}{2f^2}(\sqrt{s} - M_T) \qquad G(\sqrt{s}; a) = \frac{2M_T}{(4\pi)^2}\Big\{a + \dots$$

$$T^{-1} = V_{\mathrm{WT}}^{-1} - G(a_{\mathrm{pheno}}) = V_{\mathrm{natural}}^{-1} - G(a_{\mathrm{natural}})$$

↑ChPT ↑data fit ↑given

## Effective interaction in natural scheme

$$V_{\mathrm{natural}} = -\frac{C}{2f^2}(\sqrt{s} - M_T) + \boxed{\frac{C}{2f^2}\frac{(\sqrt{s} - M_T)^2}{\sqrt{s} - M_{\mathrm{eff}}}}$$

pole!
a seed of resonance?

$$M_{\mathrm{eff}} = M_T - \frac{16\pi^2 f^2}{C M_T \Delta a}, \quad \Delta a = a_{\mathrm{pheno}} - a_{\mathrm{natural}}$$

**There is always a pole for** $a_{\mathrm{pheno}} \neq a_{\mathrm{natural}}$

- **small deviation <=> pole at irrelevant energy scale**
- **large deviation <=> pole at relevant energy scale**

# Pole in the effective interaction

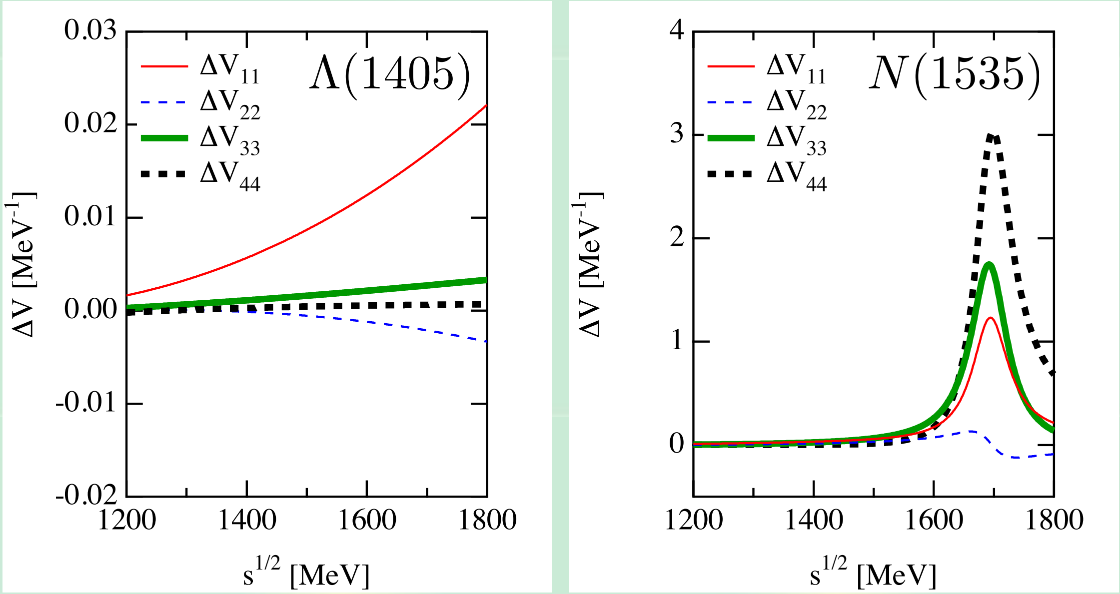
## Pole in the effective interaction ($M_{\mathrm{eff}}$) : pure CDD pole

$$T^{-1} = V_{\mathrm{WT}}^{-1} - G(a_{\mathrm{pheno}}) = \boxed{V_{\mathrm{natural}}^{-1}} - G(a_{\mathrm{natural}})$$

**For Λ(1405):** $z_{\mathrm{eff}}^{\Lambda^*} \sim 7.9$ GeV **irrelevant!**

**For N(1535):** $z_{\mathrm{eff}}^{N^*} = 1693 \pm 37i$ MeV **relevant?**

## Difference of interactions $\Delta V \equiv V_{\mathrm{natural}} - V_{\mathrm{WT}}$




**==> Important CDD pole contribution in N(1535)**

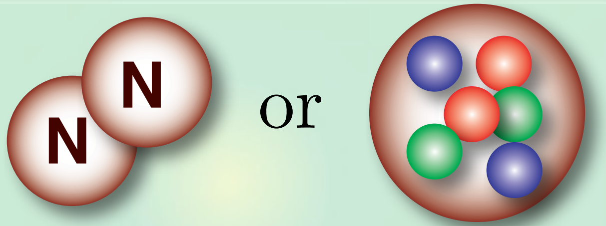
**Next question: quantitative measure for compositeness?**

# Structure of the deuteron

## Deuteron: elementary or NN bound state?

S. Weinberg, Phys. Rev. 137 B672-B678 (1965)

$$|\mathrm{deuteron}\rangle =$$




$\notin$ NN model space

~ elementary particle

Z = 0 ⟷ Z = 1

**model independent result for a weakly bound state:**

$$\boxed{a_s} = \left[\frac{2(1-Z)}{2-Z}\right]\boxed{R} + \mathcal{O}(m_\pi^{-1}), \quad \boxed{r_e} = \left[\frac{-Z}{1-Z}\right]\boxed{R} + \mathcal{O}(m_\pi^{-1})$$

$a_s$: scattering length

$r_e$: effective range

R: deuteron radius (binding energy)

<-- Experiments

$$a_s = +5.41 \text{ [fm]}, \quad r_e = +1.75 \text{ [fm]}, \quad R \equiv (2\mu B)^{-1/2} = 4.31 \text{ [fm]}$$

$\Rightarrow Z \lesssim 0.2$ --> deuteron is almost composite!

# Definition of the compositeness 1-Z

## Hamiltonian of two-body system: free + interaction V

$$\mathcal{H} = \mathcal{H}_0 + V$$

## Complete set for free Hamiltonian: bare $|B_0\rangle$ + continuum

$$1 = |\, B_0 \,\rangle\langle\, B_0 \,| + \int d\boldsymbol{k} |\, \boldsymbol{k} \,\rangle\langle\, \boldsymbol{k} \,|$$

$$\mathcal{H}_0 |\, B_0 \,\rangle = E_0 |\, B_0 \,\rangle, \quad \mathcal{H}_0 |\, \boldsymbol{k} \,\rangle = E(\boldsymbol{k}) |\, \boldsymbol{k} \,\rangle$$

## Physical bound state $|B\rangle$ : eigenstate of full Hamiltonian

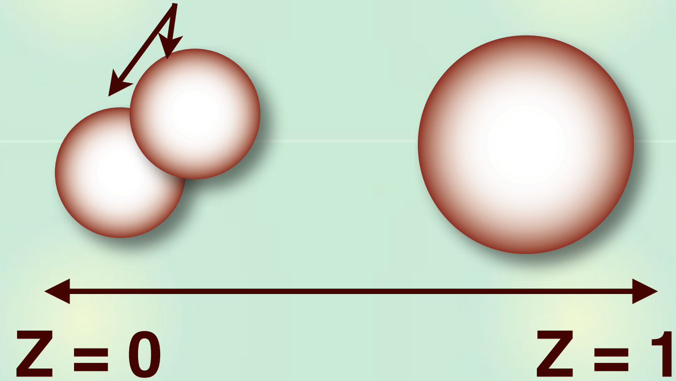
$$(\mathcal{H}_0 + V) |\, B \,\rangle = -B |\, B \,\rangle$$

**B: binding energy**

**Define Z as the overlap of B and $B_0$**
**: probability of finding the physical bound state in the bare state $|B\rangle$**

$$Z \equiv |\langle\, B_0 \,|\, B \,\rangle|^2$$

**They are assumed to be elementary**

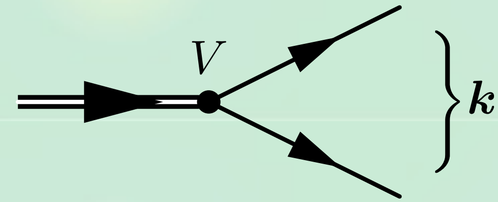



**1 - Z : Compositeness of the bound state**

# Model-independent but approximated method

**With the Schrödinger equation, we obtain**

$$1 - Z = \int d\boldsymbol{k} \frac{|\langle \boldsymbol{k} | V | B \rangle|^2}{[E(\boldsymbol{k}) + B]^2} \qquad \langle \boldsymbol{k} | V | B \rangle : \quad B$$




$$= 4\pi \sqrt{2\mu^3} \int_0^\infty dE \frac{\sqrt{E} |G_W(E)|^2}{(E+B)^2} \qquad \langle \boldsymbol{k} | V | B \rangle \equiv G_W[E(\boldsymbol{k})]$$ **for s-wave**

**Approximation: For small binding energy B<<1, the vertex $G_W(E)$ can be regarded as a constant:** $G_W(E) \sim g_W$

**Then the integration can be done analytically, leading to**

$$1 - Z = 2\pi^2 \sqrt{2\mu^3} \frac{g_W^2}{\sqrt{B}}$$

**Compositeness <-- coupling g and binding energy B**

**S. Weinberg, Phys. Rev. 137 B672-B678 (1965)**

- **Model-independent: no information of V**
- **Approximated: valid only for small B**

# Exact but model-dependent method

## Formal solution of the Lippmann-Schwinger equation

$$T(E) = V + V\frac{1}{E-\mathcal{H}}V$$

## Insert complete set for full Hamiltonian (Low's equation)

$$1 = |\,B\,\rangle\langle\,B\,| + \int d\boldsymbol{k}|\,\boldsymbol{k},\mathrm{in}\,\rangle\langle\,\boldsymbol{k},\mathrm{in}\,| \qquad V|\,\boldsymbol{k},\mathrm{in}\,\rangle = T|\,\boldsymbol{k}\,\rangle$$

$$t(E) = v(E) + \boxed{\frac{|G_W(E)|^2}{E+B}} + 4\pi\sqrt{2\mu^3}\int_0^\infty dE'\frac{\sqrt{E'}|t(E)|^2}{E-E'+i\epsilon}$$

**(for s-wave)**

**--> integrand of the formula for 1-Z !**

## Exact expression of the compositeness 1-Z

$$1-Z = 4\pi\sqrt{2\mu^3}\int_0^\infty dE\frac{\sqrt{E}|G_W(E)|^2}{(E+B)^2}$$

$$= 4\pi\sqrt{2\mu^3}\int_0^\infty dE\frac{\sqrt{E}}{E+B}\left[t(E) - v(E) - 4\pi\sqrt{2\mu^3}\int_0^\infty dE'\frac{\sqrt{E'}|t(E')|^2}{E-E'+i\epsilon}\right]$$

T. Hyodo, D. Jido, A. Hosaka, arXiv:1009.5754 [nucl-th]; in preparation

## Short summary

**We have defined the compositeness of the bound state 1-Z.**

$$1 - Z = 1 - |\langle B_0 | B \rangle|^2 = \int d\boldsymbol{k} \frac{|\langle \boldsymbol{k} | V | B \rangle|^2}{[E(\boldsymbol{k}) + B]^2}$$

**Method 1: model independent but approximated**

$$1 - Z = 2\pi^2 \sqrt{2\mu^3} \frac{g_W^2}{\sqrt{B}}$$

**Method 2: exact (valid for any B) but model dependent**

$$1 - Z = 4\pi\sqrt{2\mu^3} \int_0^\infty dE \frac{\sqrt{E}}{E + B} \left[ t(E) - v(E) - 4\pi\sqrt{2\mu^3} \int_0^\infty dE' \frac{\sqrt{E'}|t(E')|^2}{E - E' + i\epsilon} \right]$$

- **Model dependent: interaction V has to be specified (c.f. potential + wave function --> observable)**
- **Imaginary part vanishes by the optical theorem**
- **RHS can be calculated by model (chiral unitary approach)**
- **Completeness of the full Hamiltonian <--> energy dep.?**

# Single-channel chiral unitary approach

To apply the argument on Z, we study the **bound state** with mass $M_B$ in the **single channel** chiral unitary approach.

- particle masses: M and m, bound state $M_B$
- Weinberg-Tomozawa interaction
- parameters: coupling strength $C/2f^2$, subtraction "a"

## Natural renormalization scheme formulae

$$T^{-1} = V_{\rm WT}^{-1} - G(a_{\rm pheno}) = V_{\rm natural}^{-1} - G(a_{\rm natural})$$

$$V_{\rm natural} = -\frac{C}{2f^2}(\sqrt{s} - M_T) + \frac{C}{2f^2}\frac{(\sqrt{s} - M_T)^2}{\sqrt{s} - M_{\rm eff}}$$

$$M_{\rm eff} = M_T - \frac{16\pi^2 f^2}{C M_T \Delta a}, \quad \Delta a = a_{\rm pheno} - a_{\rm natural}$$

If $a_{\rm natural}$ corresponds to the purely composite case, then $M_{\rm eff}$ for $a_{\rm pheno}$ corresponds to the bare mass $M_{B0}$

<-- to be checked in the followings

# Single-channel chiral unitary approach

**We use the model-independent formula**

$$1 - Z = 2\pi^2 \sqrt{2\mu^3} \frac{g_W^2}{\sqrt{B}}$$

**We need to calculate the coupling g and binding energy B**

**- condition for the bound state: $M_B = M + m - B$**

$$1 - \frac{C}{2f^2}(M_B - M)G(M_B; a) = 0$$

**--> parameter of the system: ($M_B$, a) or ($M_B$, $M_{eff} = M_{B0}$)**

**- coupling constant: residue of the pole at $M_B$**

$$[g(M_B; a)]^2 = \lim_{W \to M_B} (W - M_B) T(W) = -\frac{M_B - M}{G(M_B; a) + (M_B - M) G'(M_B)}$$

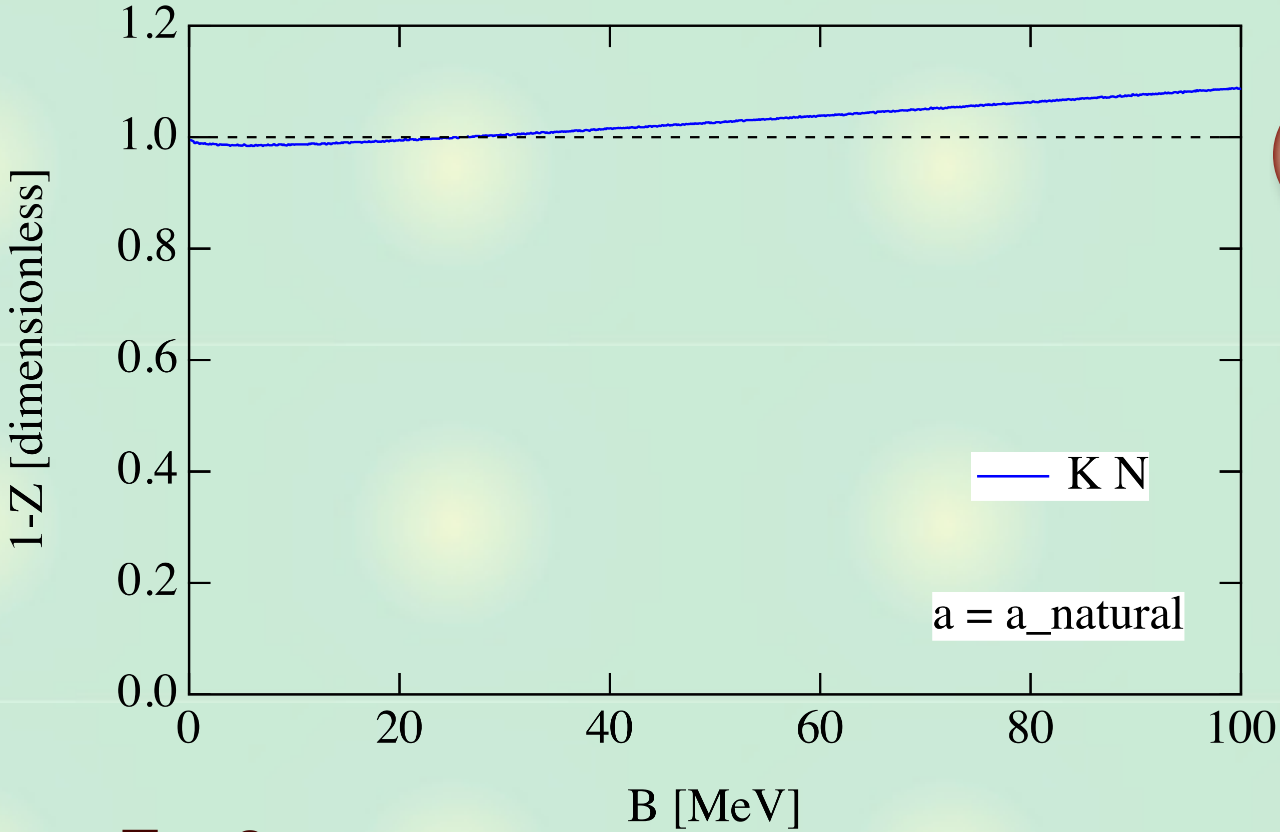
**- normalization of the amplitude (a kinematical factor)**

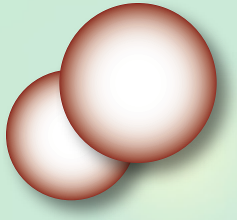
$$1 - Z = \frac{M|\bar{q}(M_B)|}{8\pi M_B (M + m - M_B)} [g(M_B; a)]^2$$ **(for small $B = M + m - M_B$)**

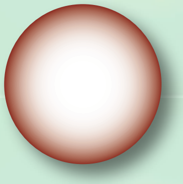
# Numerical analysis

## Compositeness of the bound state in chiral unitary approach

### 1) B dependence with $M_{B0}$ -> ∞ (a = $a_{natural}$)



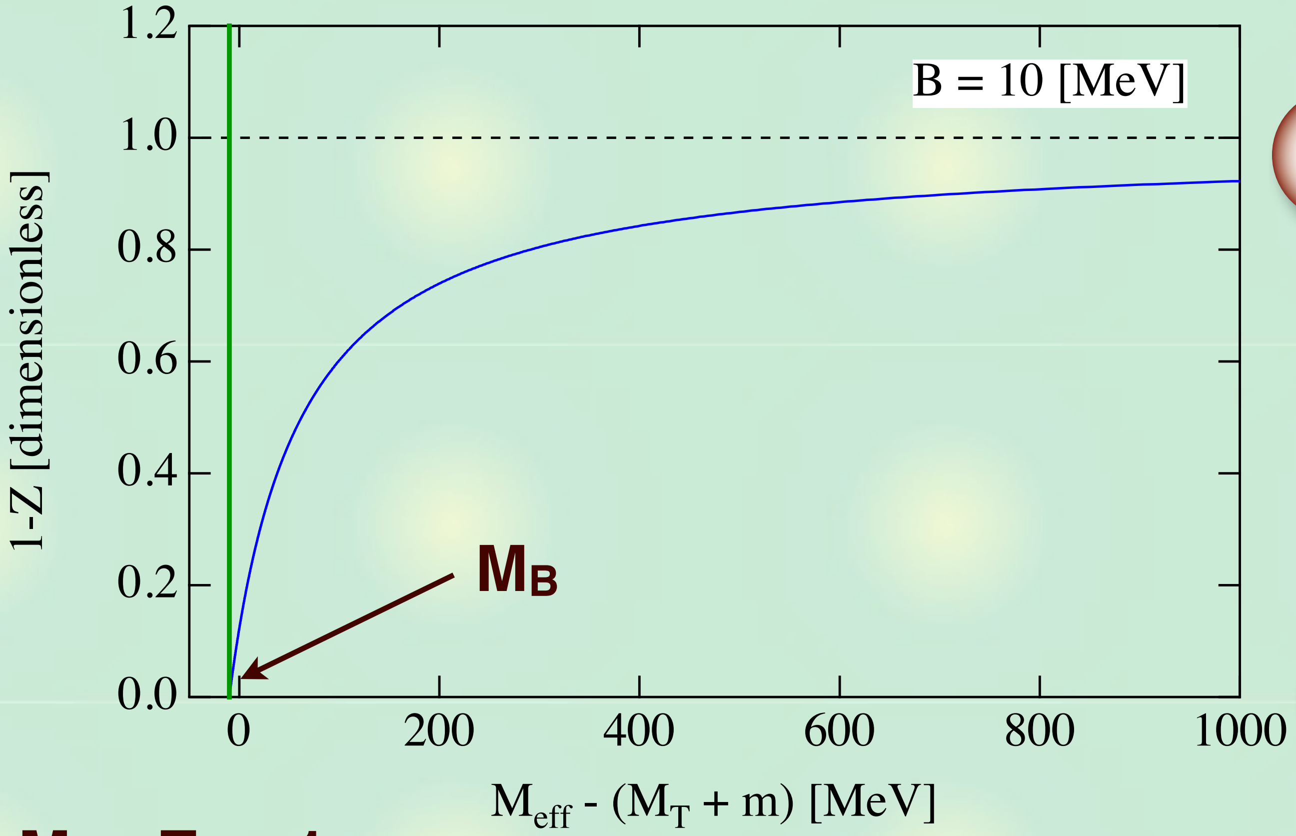


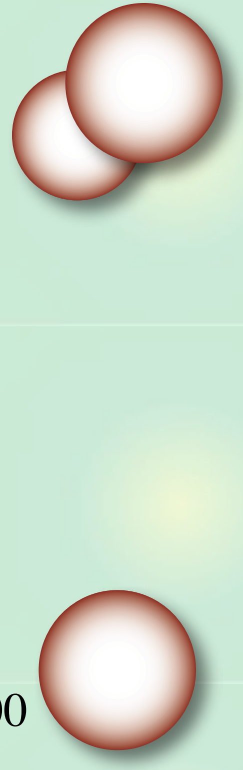
- $M_{B0}$ -> ∞ : Z ~ 0
- Z = 0 at B = 0
- large B behavior is not justified by the approximation

# Numerical analysis

## Compositeness of the bound state in chiral unitary approach

### 2) $M_{B0}$ dependence with B = 10 MeV



- $M_{B0}$ -> $M_B$ : Z -> 1
- Mass difference of $M_{B0}$ and $M_B$ : self-energy of bare state
  --> large if the composite component is large

# Summary

## Structure of resonances/bound states

- **Natural renormalization scheme**
  exclude CDD pole contribution from the loop function to generate **purely molecule resonance**
  T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C78, 025203 (2008)
- **Field renormalization constant Z:** quantitative measure of **compositeness**
- **Natural scheme corresponds to Z ~ 0**
  --> generated bound state: composite
  T. Hyodo, D. Jido, A. Hosaka, arXiv:1009.5754 [nucl-th]

# Future plan

## To apply hadron resonances, we should ...

- extend to the coupled-channel problem

  This may be straightforward,
  but technically complicated.

- extend to resonances

  Define Z in relativistic field theory
  (comparison with Yukawa theory)
  The composite condition seems to be
  $G(M_B)=0$
  c.f. natural scheme $G(M)=0$

T. Hyodo, D. Jido, A. Hosaka, in preparation