Exotic hadrons and hadronic molecules in s-wave chiral dynamics

Tetsuo Hyodo[a],

Daisuke Jido[b], and Atsushi Hosaka[c]

Tokyo Institute of Technology[a] *YITP, Kyoto[b]* *RCNP, Osaka[c]*

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Contents

- Introduction to s-wave chiral dynamics
- Exotic hadrons (manifestly-exotic states)
  - chiral interaction in exotic channels
  - critical coupling strength

    Phys. Rev. Lett. 97, 192002 (2006) + Phys. Rev. D75, 034002 (2007)
- Hadronic molecules (crypto-exotic states)
  - natural renormalization scheme
  - field renormalization Z as “compositeness”

    Phys. Rev. C78, 025203 (2008) + in preparation
- Summary

Chiral symmetry breaking in hadron physics

Chiral symmetry: QCD with massless quarks

Consequence of chiral symmetry breaking in hadron physics

- **appearance of the Nambu-Goldstone (NG) boson**

  $m_\pi \sim 140 \text{ MeV}$

- **dynamical generation of hadron masses**

  $M_p \sim 1 \text{ GeV} \sim 3M_q, \quad M_q \sim 300 \text{ MeV} \quad v.s. \quad 3-7 \text{ MeV}$

- **constraints on the NG-boson--hadron interaction**
  low energy theorems <-- current algebra
  systematic low energy (m,p/4πf$_\pi$) expansion: ChPT

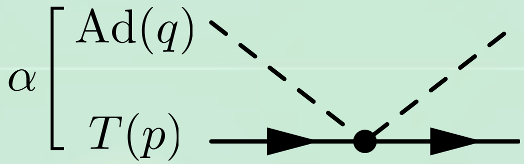
Chiral symmetry and its breaking

$$SU(3)_R \otimes SU(3)_L \to SU(3)_V$$

Underlying QCD <==> observed hadron phenomena

s-wave low energy interaction

Low energy NG boson (Ad) + target hadron (T) scattering



$$= \frac{1}{f^2}\frac{p\cdot q}{2M_T}\langle \boldsymbol{F}_T\cdot \boldsymbol{F}_{\rm Ad}\rangle_\alpha + \mathcal{O}\left(\left(\frac{m}{M_T}\right)^2\right)$$

Projection onto s-wave: Weinberg-Tomozawa (WT) term

Y. Tomozawa, Nuovo Cim. 46A, 707 (1966); S. Weinberg, Phys. Rev. Lett. 17, 616 (1966)

$$V_{ij} = -\frac{C_{ij}}{4f^2}(\omega_i+\omega_j)$$ energy dependence (derivative coupling)

decay constant of π (g_V=1)

$$C_{ij} = \sum_\alpha C_{\alpha,T}\begin{pmatrix} 8 & T & \| & \alpha \\ I_{M_i},Y_{M_i} & I_{T_i},Y_{T_i} & \| & I,Y\end{pmatrix}\begin{pmatrix} 8 & T & \| & \alpha \\ I_{M_j},Y_{M_j} & I_{T_j},Y_{T_j} & \| & I,Y\end{pmatrix}$$

$$C_{\alpha,T} = \langle 2\boldsymbol{F}_T\cdot\boldsymbol{F}_{\rm Ad}\rangle_\alpha = C_2(T) - C_2(\alpha) + 3$$

Group theoretical structure and flavor **SU(3) symmetry** determines **the sign and the strength** of the interaction

Low energy theorem: leading order term in ChPT

Scattering amplitude and unitarity

Unitarity of S-matrix: Optical theorem

$$\mathrm{Im}\,[T^{-1}(s)] = \frac{\rho(s)}{2}$$ phase space of two-body state

General amplitude by dispersion relation

$$T^{-1}(\sqrt{s}) = \sum_i \frac{R_i}{\sqrt{s}-W_i} + \tilde{a}(s_0) + \frac{s-s_0}{2\pi}\int_{s^+}^{\infty} ds' \frac{\rho(s')}{(s'-s)(s'-s_0)}$$

R_i, W_i, a: to be determined by chiral interaction

Identify dispersion integral = loop function G, the rest = V^{-1}

$$T(\sqrt{s}) = \frac{1}{V^{-1}(\sqrt{s}) - G(\sqrt{s};a)}$$ Scattering amplitude

V? chiral expansion of T, (conceptual) matching with ChPT

$$T^{(1)} = V^{(1)}, \quad T^{(2)} = V^{(2)}, \quad T^{(3)} = V^{(3)} - V^{(1)}GV^{(1)}, \ \ldots$$

Amplitude T: consistent with chiral symmetry + unitarity

Chiral unitary approach

Description of S = -1, $\bar{K}N$ s-wave scattering: Λ(1405) in I=0

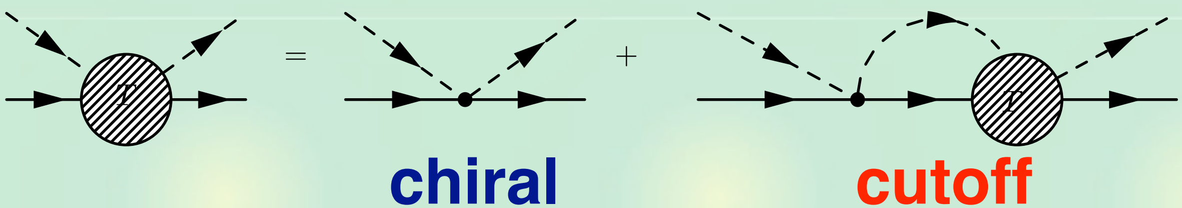
- Interaction <-- chiral symmetry

  Y. Tomozawa, Nuovo Cim. 46A, 707 (1966); S. Weinberg, Phys. Rev. Lett. 17, 616 (1966)

- Amplitude <-- unitarity in coupled channels

  R.H. Dalitz, T.C. Wong, G. Rajasekaran, Phys. Rev. 153, 1617 (1967)

$$T = \frac{1}{1 - VG} V$$

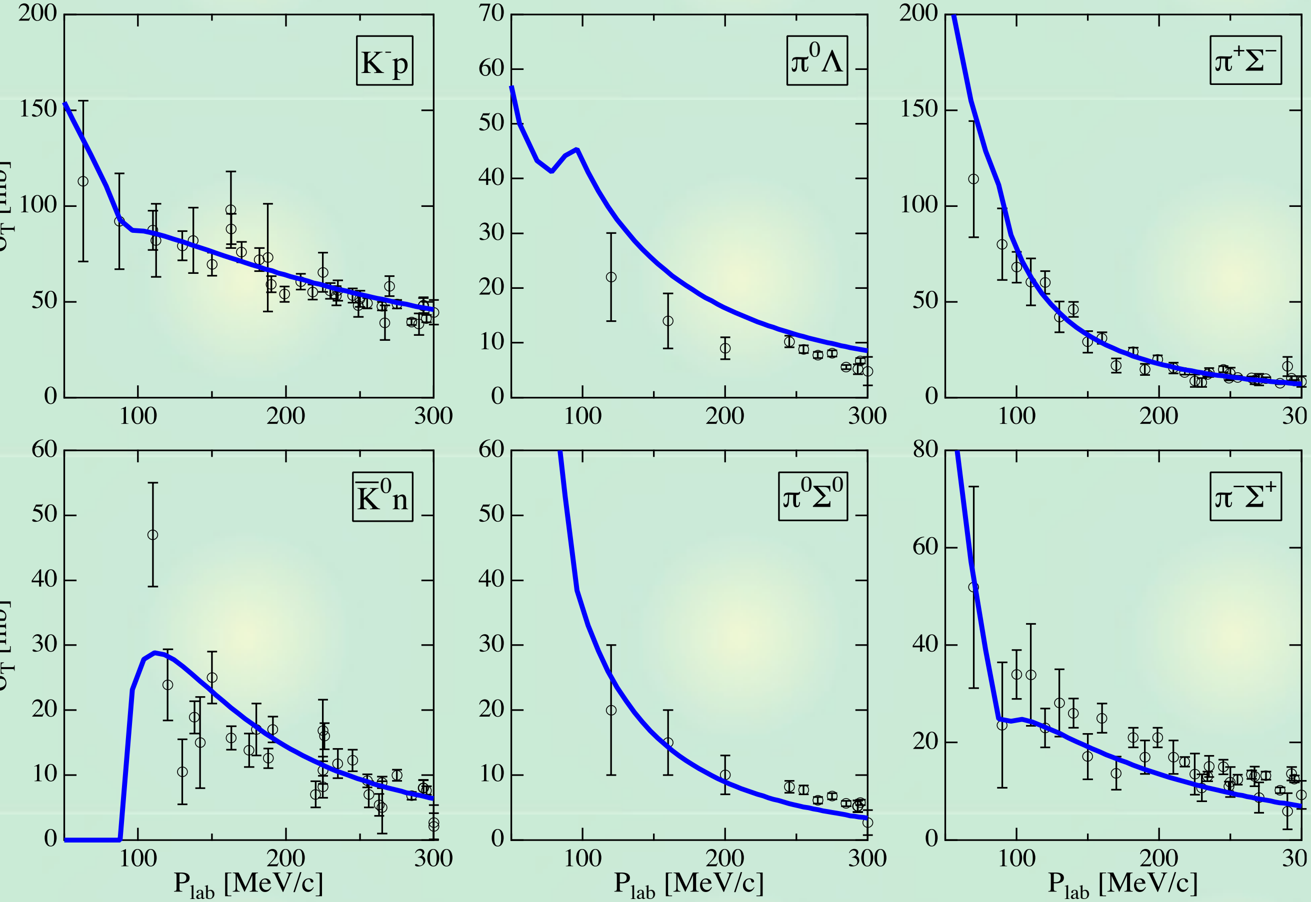


N. Kaiser, P. B. Siegel, W. Weise, Nucl. Phys. A594, 325 (1995),
E. Oset, A. Ramos, Nucl. Phys. A635, 99 (1998),
J. A. Oller, U. G. Meissner, Phys. Lett. B500, 263 (2001),
M.F.M. Lutz, E. E. Kolomeitsev, Nucl. Phys. A700, 193 (2002), ... many others

It works successfully, also in S=0 sector, meson-meson scattering sectors, systems including heavy quarks, ...

The simplest model (1 parameter) v.s. experimental data

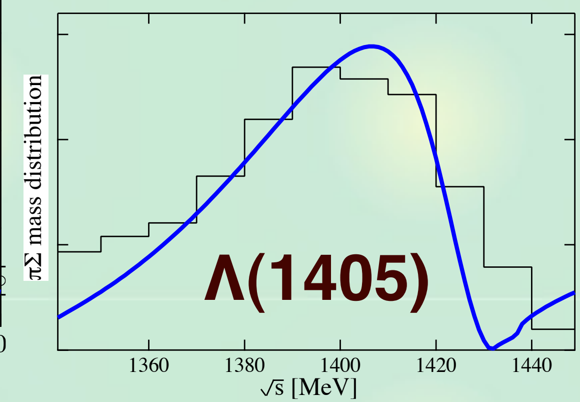
Total cross section of K-p scattering



Branching ratio

	γ	R_c	R_n
exp.	2.36	0.664	0.189
theo.	1.80	0.624	0.225

πΣ spectrum



T. Hyodo, S.I. Nam, D. Jido, A. Hosaka, PRC68, 018201 (2003); PTP 112, 73 (2004)

Good agreement with data above, at, and below $\bar{K}N$ threshold

more quantitatively --> fine tuning, higher order terms,...

Chiral dynamics for non-exotic hadrons

Hadron excited states in NG boson-hadron scattering

light baryon	$J^P = 1/2^-$	$\Lambda(1405)$ $\Lambda(1670)$ $\Sigma(1670)$ $N(1535)$ $\Xi(1620)$ $\Xi(1690)$
	$J^P = 3/2^-$	$\Lambda(1520)$ $\Xi(1820)$ $\Sigma(1670)$
heavy		$\Lambda_c(2880)$ $\Lambda_c(2593)$ $D_s(2317)$
light meson	$J^P = 1^+$	$b_1(1235)$ $h_1(1170)$ $h_1(1380)$ $a_1(1260)$ $f_1(1285)$ $K_1(1270)$ $K_1(1440)$
	$J^P = 0^+$	$\sigma(600)$ $\kappa(900)$ $f_0(980)$ $a_0(980)$

many references....

Questions

- No particle with exotic quantum number. Why?
- Are they all hadronic molecule?

Exotic hadrons in hadron spectrum

Observed hadrons in experiments (PDG06):

p	P_{11}	****
n	P_{11}	****
$N(1440)$	P_{11}	****
$N(1520)$	D_{13}	****
$N(1535)$	S_{11}	****
$N(1650)$	S_{11}	****
$N(1675)$	D_{15}	****
$N(1680)$	F_{15}	****
$N(1700)$	D_{13}	***
$N(1710)$	P_{11}	***
$N(1720)$	P_{13}	****
$N(1900)$	P_{13}	**
$N(1990)$	F_{17}	**
$N(2000)$	F_{15}	**
$N(2080)$	D_{13}	**
$N(2090)$	S_{11}	*
$N(2100)$	P_{11}	*
$N(2190)$	G_{17}	****
$N(2200)$	D_{15}	**
$N(2220)$	H_{19}	****
$N(2250)$	G_{19}	****
$N(2600)$	$I_{1,11}$	***
$N(2700)$	$K_{1,13}$	**

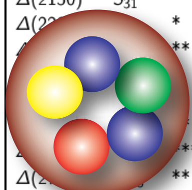
$\Delta(1232)$	P_{33}	****
$\Delta(1600)$	P_{33}	***
$\Delta(1620)$	S_{31}	****
$\Delta(1700)$	D_{33}	****
$\Delta(1750)$	P_{31}	*
$\Delta(1900)$	S_{31}	**
$\Delta(1905)$	F_{35}	****
$\Delta(1910)$	P_{31}	****
$\Delta(1920)$	P_{33}	***
$\Delta(1930)$	D_{35}	***
$\Delta(1940)$	D_{33}	*
$\Delta(1950)$	F_{37}	****
$\Delta(2000)$	F_{35}	**
$\Delta(2150)$	S_{31}	*
$\Delta(22$[illegible]	[illegible]	*
[illegible]	[illegible]	**
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	****
$\Delta(2$[illegible]	[illegible]	**
$\Delta(2950)$	$K_{3,15}$	**
$\Theta(1540)^+$		*

Λ	P_{01}	****
$\Lambda(1405)$	S_{01}	****
$\Lambda(1520)$	D_{03}	****
$\Lambda(1600)$	P_{01}	***
$\Lambda(1670)$	S_{01}	****
$\Lambda(1690)$	D_{03}	****
$\Lambda(1800)$	S_{01}	***
$\Lambda(1810)$	P_{01}	***
$\Lambda(1820)$	F_{05}	****
$\Lambda(1830)$	D_{05}	****
$\Lambda(1890)$	P_{03}	****
$\Lambda(2000)$		*
$\Lambda(2020)$	F_{07}	*
$\Lambda(2100)$	G_{07}	****
$\Lambda(2110)$	F_{05}	***
$\Lambda(2325)$	D_{03}	*
$\Lambda(2350)$	H_{09}	***
$\Lambda(2585)$		**

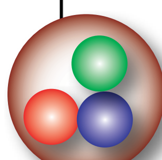
Σ^+	P_{11}	****
Σ^0	P_{11}	****
Σ^-	P_{11}	****
$\Sigma(1385)$	P_{13}	****
$\Sigma(1480)$		*
$\Sigma(1560)$		**
$\Sigma(1580)$	D_{13}	*
$\Sigma(1620)$	S_{11}	**
$\Sigma(1660)$	P_{11}	***
$\Sigma(1670)$	D_{13}	****
$\Sigma(1690)$		**
$\Sigma(1750)$	S_{11}	***
$\Sigma(1770)$	P_{11}	*
$\Sigma(1775)$	D_{15}	****
$\Sigma(1840)$	P_{13}	*
$\Sigma(1880)$	P_{11}	**
$\Sigma(1915)$	F_{15}	****
$\Sigma(1940)$	D_{13}	***
$\Sigma(2000)$	S_{11}	*
$\Sigma(2030)$	F_{17}	****
$\Sigma(2070)$	F_{15}	*
$\Sigma(2080)$	P_{13}	**
$\Sigma(2100)$	G_{17}	*
$\Sigma(2250)$		***
$\Sigma(2455)$		**
$\Sigma(2620)$		**
$\Sigma(3000)$		*
$\Sigma(3170)$		*

Ξ^0	P_{11}	****
Ξ^-	P_{11}	****
$\Xi(1530)$	P_{13}	****
$\Xi(1620)$		*
$\Xi(1690)$		***
$\Xi(1820)$	D_{13}	***
$\Xi(1950)$		***
$\Xi(2030)$		***
$\Xi(2120)$		*
$\Xi(2250)$		**
$\Xi(2370)$		**
$\Xi(2500)$		*
Ω^-		****
$\Omega(2250)^-$		***
$\Omega(2380)^-$		**
$\Omega(2470)^-$		**
Λ_c^+		****
$\Lambda_c(2593)^+$		***
$\Lambda_c(2625)^+$		***
$\Lambda_c(2765)^+$		*
$\Lambda_c(2880)^+$		**
$\Sigma_c(2455)$		****
$\Sigma_c(2520)$		***
$\Sigma_c(2800)$		***
Ξ_c^+		***
Ξ_c^0		***
$\Xi_c'^+$		***
$\Xi_c'^0$		***
$\Xi_c(2645)$		***
$\Xi_c(2790)$		***
$\Xi_c(2815)$		***
Ω_c^0		***
Ξ_{cc}^+		*
Λ_b^0		***
Ξ_b^0, Ξ_b^-		*

$\sim \dfrac{1}{300}$!



~130 baryons



LIGHT UNFLAVORED ($S = C = B = 0$)

	$I^G(J^{PC})$		$I^G(J^{PC})$
• $\pi^\pm$	$1^-(0^-)$	• $\pi_2(1670)$	$1^-(2^{-+})$
• π^0	$1^-(0^{-+})$	• $\phi(1680)$	$0^-(1^{--})$
• η	$0^+(0^{-+})$	• $\rho_3(1690)$	$1^+(3^{--})$
• $f_0(600)$	$0^+(0^{++})$	• $\rho(1700)$	$1^+(1^{--})$
• $\rho(770)$	$1^+(1^{--})$	$a_2(1700)$	$1^-(2^{++})$
• $\omega(782)$	$0^-(1^{--})$	• $f_0(1710)$	$0^+(0^{++})$
• $\eta'(958)$	$0^+(0^{-+})$	$\eta(1760)$	$0^+(0^{-+})$
• $f_0(980)$	$0^+(0^{++})$	• $\pi(1800)$	$1^-(0^{-+})$
• $a_0(980)$	$1^-(0^{++})$	$f_2(1810)$	$0^+(2^{++})$
• $\phi(1020)$	$0^-(1^{--})$	$X(1835)$	$?^?(?^{-+})$
• $h_1(1170)$	$0^-(1^{+-})$	• $\phi_3(1850)$	$0^-(3^{--})$
• $b_1(1235)$	$1^+(1^{+-})$	$\eta_2(1870)$	$0^+(2^{-+})$
• $a_1(1260)$	$1^-(1^{++})$	$\rho(1900)$	$1^+(1^{--})$
• $f_2(1270)$	$0^+(2^{++})$	$f_2(1910)$	$0^+(2^{++})$
• $f_1(1285)$	$0^+(1^{++})$	• $f_2(1950)$	$0^+(2^{++})$
• $\eta(1295)$	$0^+(0^{-+})$	$\rho_3(1990)$	$1^+(3^{--})$
• $\pi(1300)$	$1^-(0^{-+})$	• $f_2(2010)$	$0^+(2^{++})$
• $a_2(1320)$	$1^-(2^{++})$	$f_0(2020)$	$0^+(0^{++})$
• $f_0(1370)$	$0^+(0^{++})$	• $a_4(2040)$	$1^-(4^{++})$
$h_1(1380)$	$?^-(1^{+-})$	• $f_4(2050)$	$0^+(4^{++})$
• $\pi_1(1400)$	$1^-(1^{-+})$	$\pi_2(2100)$	$1^-(2^{-+})$
• $\eta(1405)$	$0^+(0^{-+})$	$f_0(2100)$	$0^+(0^{++})$
• $f_1(1420)$	$0^+(1^{++})$	$f_2(2150)$	$0^+(2^{++})$
• $\omega(1420)$	$0^-(1^{--})$	$\rho(2150)$	$1^+(1^{--})$
$f_2(1430)$	$0^+(2^{++})$	$f_0(2200)$	$0^+(0^{++})$
• $a_0(1450)$	$1^-(0^{++})$	$f_J(2220)$	$0^+(2 \text{ or } 4^{++})$
• $\rho(1450)$	$1^+(1^{--})$	$\eta(2225)$	$0^+(0^{-+})$
• $\eta(1475)$	$0^+(0^{-+})$	$\rho_3(2250)$	$1^+(3^{--})$
• $f_0(1500)$	$0^+(0^{++})$	• $f_2(2300)$	$0^+(2^{++})$
$f_1(1510)$	$0^+(1^{++})$	$f_4(2300)$	$0^+(4^{++})$
• $f_2'(1525)$	$0^+(2^{++})$	• $f_2(2340)$	$0^+(2^{++})$
$f_2(1565)$	$0^+(2^{++})$	$\rho_5(2350)$	$1^+(5^{--})$
$h_1(1595)$	$0^-(1^{+-})$	$a_6(2450)$	$1^-(6^{++})$
• $\pi_1(1600)$	$1^-(1^{-+})$	$f_6(2510)$	$0^+(6^{++})$
$a_1(1640)$	$1^-(1^{++})$	OTH[illegible]	
$f_2(1640)$	$0^+(2^{++})$	Further [illegible]	
• $\eta_2(1645)$	$0^+(2^{-+})$		
• $\omega(1650)$	$0^-(1^{--})$		
• $\omega_3(1670)$	$0^-(3^{--})$		

STRANGE ($S = \pm1$, $C = B = 0$)

	$I(J^P)$
• $K^\pm$	$1/2(0^-)$
• K^0	$1/2(0^-)$
• K_S^0	$1/2(0^-)$
• K_L^0	$1/2(0^-)$
$K_0^*(800)$	$1/2(0^+)$
• $K^*(892)$	$1/2(1^-)$
• $K_1(1270)$	$1/2(1^+)$
• $K_1(1400)$	$1/2(1^+)$
• $K^*(1410)$	$1/2(1^-)$
• $K_0^*(1430)$	$1/2(0^+)$
• $K_2^*(1430)$	$1/2(2^+)$
$K(1460)$	$1/2(0^-)$
$K_2(1580)$	$1/2(2^-)$
$K(1630)$	$1/2(?^?)$
$K_1(1650)$	$1/2(1^+)$
• $K^*(1680)$	$1/2(1^-)$
• $K_2(1770)$	$1/2(2^-)$
• $K_3^*(1780)$	$1/2(3^-)$
• $K_2(1820)$	$1/2(2^-)$
$K(1830)$	$1/2(0^-)$
$K_0^*(1950)$	$1/2(0^+)$
$K_2^*(1980)$	$1/2(2^+)$
• $K_4^*(2045)$	$1/2(4^+)$
$K_2(2250)$	$1/2(2^-)$
$K_3(2320)$	$1/2(3^+)$
$K_5^*(2380)$	$1/2(5^-)$
$K_4(2500)$	$1/2(4^-)$
$K(3100)$	$?^?(?^{??})$

CHARMED ($C = \pm1$)

• $D^\pm$	$1/2(0^-)$
• D^0	$1/2(0^-)$
• $D^*(2007)^0$	$1/2(1^-)$
• $D^*(2010)^\pm$	$1/2(1^-)$
$D_0^*(2400)^0$	$1/2(0^+)$
$D_0^*(2400)^\pm$	$1/2(0^+)$
• $D_1(2420)^0$	$1/2(1^+)$
$D_1(2420)^\pm$	$1/2(?^?)$
$D_1(2430)^0$	$1/2(1^+)$
• $D_2^*(2460)^0$	$1/2(2^+)$
• $D_2^*(2460)^\pm$	$1/2(2^+)$
$D^*(2640)^\pm$	$1/2(?^?)$

CHARMED, STRANGE ($C = S = \pm1$)

• $D_s^\pm$	$0(0^-)$
• $D_s^{*\pm}$	$0(?^?)$
• $D_{s0}^*(2317)^\pm$	$0(0^+)$
• $D_{s1}(2460)^\pm$	$0(1^+)$
• $D_{s1}(2536)^\pm$	$0(1^+)$
• $D_{s2}(2573)^\pm$	$0(?^?)$

BOTTOM ($B = \pm1$)

	$I^G(J^{PC})$
• $B^\pm$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^\pm/B^0$ ADMIXTURE	
• $B^\pm/B^0/B_s^0/b$-baryon ADMIXTURE	
V_{cb} and V_{ub} CKM Matrix Elements	
• B^*	$1/2(1^-)$
$B_J^*(5732)$	$?(?^?)$

BOTTOM, STRANGE ($B = \pm1$, $S = \mp1$)

• B_s^0	$0(0^-)$
B_s^*	$0(1^-)$
$B_{sJ}^*(5850)$	$?(?^?)$

BOTTOM, CHARMED ($B = C = \pm1$)

• $B_c^\pm$	$0(0^-)$

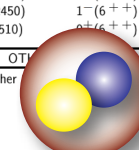
$c\bar{c}$

• $\eta_c(1S)$	$0^+(0^{-+})$
• $J/\psi(1S)$	$0^-(1^{--})$
• $\chi_{c0}(1P)$	$0^+(0^{++})$
• $\chi_{c1}(1P)$	$0^+(1^{++})$
$h_c(1P)$	$?^?(?^{??})$
• $\chi_{c2}(1P)$	$0^+(2^{++})$
• $\eta_c(2S)$	$0^+(0^{-+})$
• $\psi(2S)$	$0^-(1^{--})$
• $\psi(3770)$	$0^-(1^{--})$
• $X(3872)$	$0^?(?^{?+})$
• $\chi_{c2}(2P)$	$0^+(2^{++})$
$Y(3940)$	$?^?(?^{??})$
• $\psi(4040)$	$0^-(1^{--})$
• $\psi(4160)$	$0^-(1^{--})$
$Y(4260)$	$?^?(1^{--})$
• $\psi(4415)$	$0^-(1^{--})$

$b\bar{b}$

$\eta_b(1S)$	$0^+(0^{-+})$
• $\Upsilon(1S)$	$0^-(1^{--})$
• $\chi_{b0}(1P)$	$0^+(0^{++})$
• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $\chi_{b2}(1P)$	$0^+(2^{++})$
• $\Upsilon(2S)$	$0^-(1^{--})$
$\Upsilon(1D)$	$0^-(2^{--})$
• $\chi_{b0}(2P)$	$0^+(0^{++})$
• $\chi_{b1}(2P)$	$0^+(1^{++})$
• $\chi_{b2}(2P)$	$0^+(2^{++})$
• $\Upsilon(3S)$	$0^-(1^{--})$
• $\Upsilon(4S)$	$0^-(1^{--})$
• $\Upsilon(10860)$	$0^-(1^{--})$
• $\Upsilon(11020)$	$0^-(1^{--})$

NON-$q\bar{q}$ CANDIDATES



~160 mesons

Exotic hadrons are indeed exotic !!

Exotic hadrons in chiral dynamics

Exotic hadrons: more than four valence quarks

ex) Θ^+, $\Xi(\Phi)^{--}$, Θ_c($\bar{D}N$ bound state), T_{cc}, H_c, H,...

- hard to observe experimentally
- easy to generate theoretically (in general)
  quark model, soliton model, ..., QCD?

s-wave chiral dynamics
--> many resonances but no exotic state

We should look at

1) property of the interaction in exotic channel
<-- exotic channel?

2) strength of the attraction to form a bound state
<-- critical coupling strength?

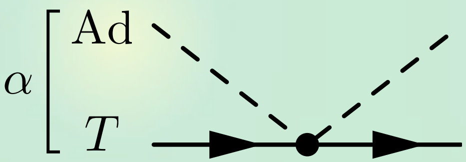
Let us study the SU(3) symmetric limit for simplicity

s-wave low energy interaction

Weinberg-Tomozawa interaction in SU(3) limit

$$V_{\alpha,T} = -C_{\alpha,T}\frac{\omega}{2f^2}$$

$$C_{\alpha,T} = \langle 2\boldsymbol{F}_T \cdot \boldsymbol{F}_{\mathrm{Ad}}\rangle_\alpha = C_2(T) - C_2(\alpha) + 3$$

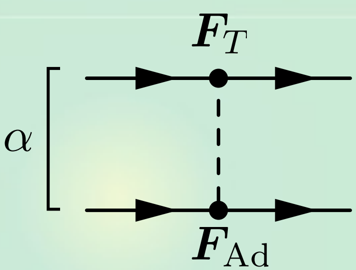


α: representation of total system ~ resonance.

If the Casimir for α is large, the interaction is repulsive.

exotic channel --> large representation --> large Casimir

c.f. Vector meson exchange



$\propto \langle \boldsymbol{F}_T \cdot \boldsymbol{F}_{\mathrm{Ad}}\rangle_\alpha$

--> Result is generic for flavor current exchange interaction

Coupling strengths: Examples

Coupling strengths (positive --> attractive interaction)

$$C_{\alpha,T} = \langle 2\boldsymbol{F}_T \cdot \boldsymbol{F}_{\mathrm{Ad}} \rangle = C_2(T) - C_2(\alpha) + 3$$

α	$\mathbf{1}$	$\mathbf{8}$	$\mathbf{10}$	$\overline{\mathbf{10}}$	$\mathbf{27}$	$\mathbf{35}$
$T = \mathbf{8}(N, \Lambda, \Sigma, \Xi)$	6	3	0	0	-2	
$T = \mathbf{10}(\Delta, \Sigma^*, \Xi^*, \Omega)$		6	3		1	-3

α	$\overline{\mathbf{3}}$	$\mathbf{6}$	$\overline{\mathbf{15}}$	$\mathbf{24}$
$T = \overline{\mathbf{3}}(\Lambda_c, \Xi_c)$	3	1	-1	
$T = \mathbf{6}(\Sigma_c, \Xi_c^*, \Omega_c)$	5	3	1	-2

- Exotic channels: **mostly repulsive**
- Attractive interaction: **C = 1**

Next question: what do we have in general case?

Coupling strengths : General expression

For a general target $T = [p, q]$

$\alpha \in [p,q] \otimes [1,1]$	$C_{\alpha,T}$	sign
$[p+1, q+1]$	$-p-q$	repulsive
$[p+2, q-1]$	$1-p$	
$[p-1, q+2]$	$1-q$	
$[p, q]$	3	attractive
$[p, q]$	3	attractive
$[p+1, q-2]$	$3+q$	attractive
$[p-2, q+1]$	$3+p$	attractive
$[p-1, q-1]$	$4+p+q$	attractive

- Strength should be an integer.
- Sign is determined for most cases.

Next question: which is the exotic channel?

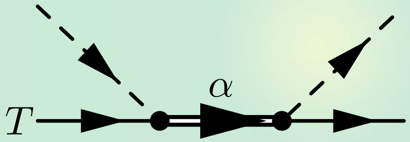
Exoticness and exotic channel

Exoticness E: minimal number of extra $q\bar{q}$ for [p,q] representation with baryon number B

$$E = \epsilon\theta(\epsilon) + \nu\theta(\nu), \quad \epsilon \equiv \frac{p+2q}{3} - B, \quad \nu \equiv \frac{p-q}{3} - B$$

Exotic channel: $\Delta E = E_\alpha - E_T = +1$

<-- resonance is more exotic than the target



$$\alpha = [p+1, q+1] : C_{\alpha,T} = -p - q$$ **repulsive**

$$\alpha = [p+2, q-1] : C_{\alpha,T} = 1 - p$$

attraction: $p = 0 \;\oplus\; \nu_T \geq 0 \;\Rightarrow\; B_T \leq -q/3$ **not considered here**

$$\alpha = [p-1, q+2] : C_{\alpha,T} = 1 - q$$

attraction: $q = 0 \;\oplus\; \nu_T \leq 0 \;\Rightarrow\; B_T \geq p/3$ **OK!**

Universal attraction for the exotic channel

$$C_{\text{exotic}} = 1 \quad T = [p, 0], \quad \alpha = [p-1, 2]$$

Renormalization and bound states

Scattering amplitude

$$V_\alpha = -\frac{\omega}{2f^2} C_{\alpha,T}$$

$$T_\alpha(\sqrt{s}) = \frac{1}{1 - V_\alpha(\sqrt{s})G(\sqrt{s})} V_\alpha(\sqrt{s})$$

Cutoff parameter in the loop function
<-- renormalization condition

$$G(\mu) = 0 \quad \Leftrightarrow \quad T(\mu) = V(\mu) \quad \text{at} \quad \mu = M_T$$

exclusion of the genuine quark states from the loop function (more detail in latter part of this seminar)

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C78, 025203 (2008)

Condition to have a bound state

$$1 - V(M_b)G(M_b) = 0 \quad M_T < M_b < M_T + m$$

--> critical value of the coupling strength C

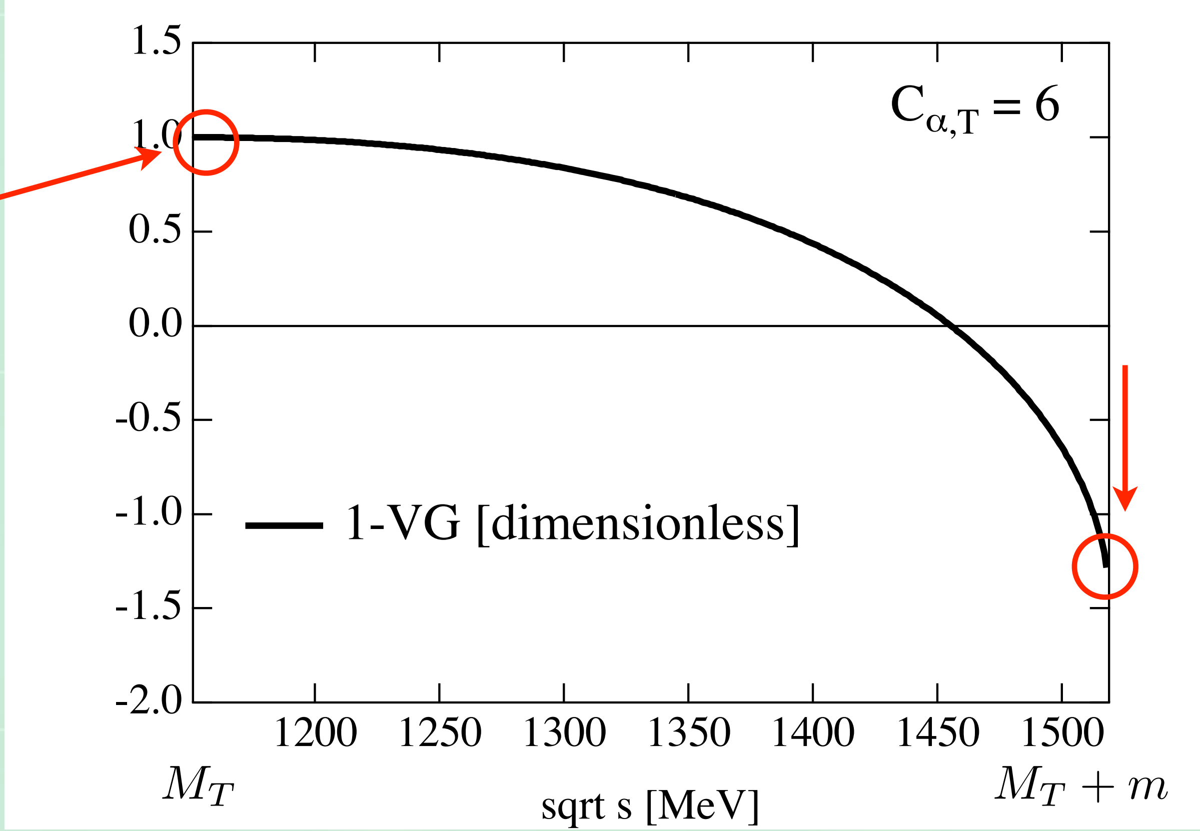
Critical coupling strength

Behavior of the function $1 - V(\sqrt{s})G(\sqrt{s})$
--> monotonically decreasing

Fixed

$G(M_T) = 0$

$1 - VG = 1$

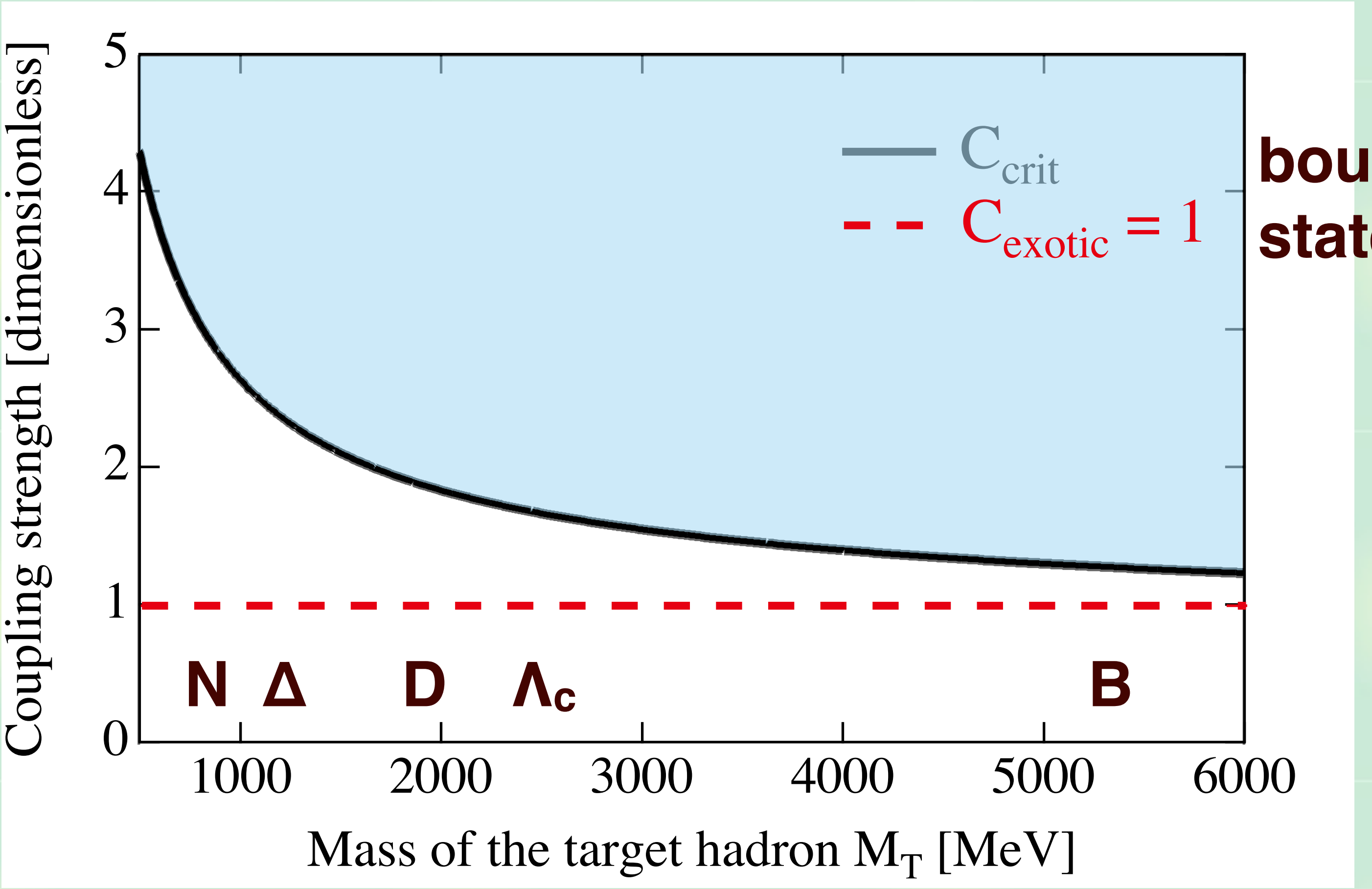


Critical attraction: $1 - VG = 0 \quad \text{at} \quad \sqrt{s} = M_T + m$

$$\Rightarrow \quad C_{\rm crit} = \frac{2f^2}{m[-G(M_T+m)]}$$

Critical attraction and exotic attraction

Critical coupling strength $m = 368$ MeV and $f = 93$ MeV



bound state

The attraction is not enough to generate a bound state.

$C_{\rm exotic} < C_{\rm crit}$

Summary 1: SU(3) limit

We study the exotic bound states in s-wave chiral dynamics in flavor SU(3) limit.

- The interactions in exotic channels are in most cases repulsive.
- There are attractive interactions in exotic channels, with universal and the smallest strength: $C_{\text{exotic}} = 1$
- The strength is not enough to generate a bound state: $C_{\text{exotic}} < C_{\text{crit}}$

The result is model independent as far as we respect chiral symmetry.

Summary 2: Physical world

Pro

- The repulsion in exotic channel is generic for flavor current exchange interaction.

Contra

- We do not exclude the exotics which have other origins (genuine quark state, soliton rotation,...).
- In practice, SU(3) breaking effect, higher order terms,...

In Nature, it is difficult to generate exotic hadrons as in the same way with Λ(1405),... based on chiral dynamics.

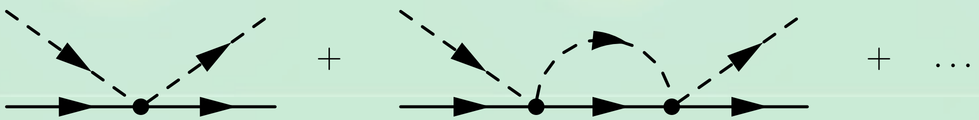
T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. Lett. 97, 192002 (2006);
T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. D75, 034002 (2007)

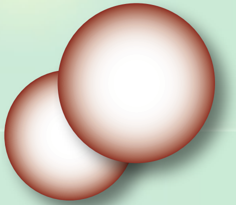
Classification of resonances

Resonances in two-body scattering

- Knowledge of interaction (potential)
- Experimental data (cross section, ...)

Dynamical state: two-body molecule, quasi-bound state, ...

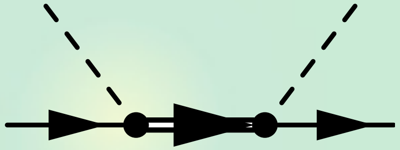


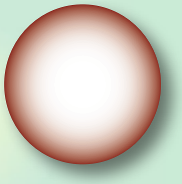


e.g.) Deuteron in NN, positronium in e^+e^-, ...

CDD pole: elementary particle, preformed state, ...

L. Castillejo, R.H. Dalitz, F.J. Dyson, Phys. Rev. 101, 453 (1956)

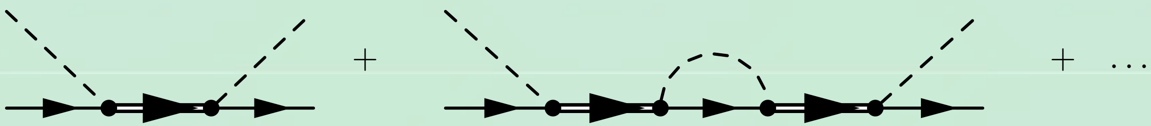




e.g.) J/Ψ in e^+e^-, ...

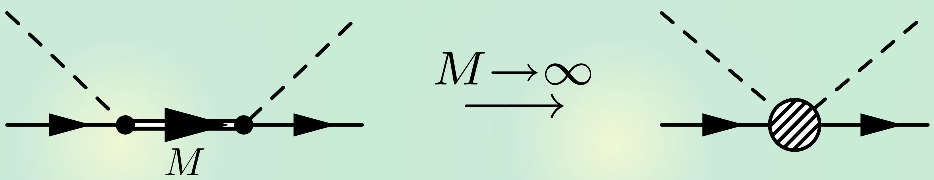
(Known) CDD poles in chiral unitary approach

Explicit resonance field in V (interaction)

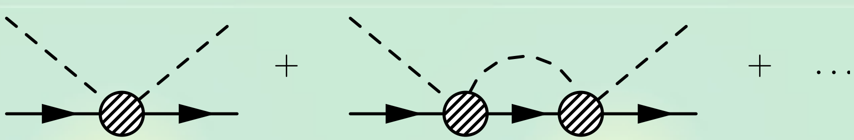


U.G. Meissner, J.A. Oller, Nucl. Phys. A673, 311 (2000)
D. Jido, E. Oset, A. Ramos, Phys. Rev. C66, 055203 (2002)

Contracted resonance propagator in higher order V



G. Ecker, J. Gasser, A. Pich, E. de Rafael, Nucl. Phys. B321, 311 (1989)
V. Bernard, N. Kaiser, U.G. Meissner, Nucl. Phys. A615, 483 (1997)



J.A. Oller, E. Oset, J.R. Pelaez, Phys. Rev. D59, 074001 (1999)

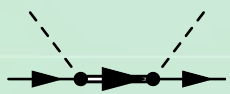
Is that all? subtraction constant?

CDD pole in subtraction constant?

Phenomenological (standard) scheme
--> V is given, “a” is determined by data

$$T = \frac{1}{(V^{(1)})^{-1} - G(\underline{a})}$$ leading order

$$T = \frac{1}{(V^{(1)} + V^{(2)})^{-1} - G(\underline{a'})}$$ next to leading order

↑pole  ?

“a” represents the effect which is not included in V.
CDD pole contribution in G?

Natural renormalization scheme
--> fix “a” first, then determine V

to exclude CDD pole contribution from G,
based on theoretical argument.

Natural renormalization condition

Conditions for natural renormalization

- Loop function G should be negative below threshold.
- T matches with V at low energy scale.

"a" is uniquely determined such that

$$G(\sqrt{s} = M_T) = 0 \quad \Leftrightarrow \quad T(M_T) = V(M_T)$$

subtraction constant: a_{natural}

matching with low energy interaction

K. Igi, K. Hikasa, Phys. Rev. D59, 034005 (1999)
U.G. Meissner, J.A. Oller, Nucl. Phys. A673, 311 (2000)

crossing symmetry (matching with u-channel amplitude)

M.F.M. Lutz, E. Kolomeitsev, Nucl. Phys. A700, 193 (2002)

We regard this condition as the **exclusion of the CDD pole contribution from G**.

Pole in the effective interaction: single channel

Leading order V : Weinberg-Tomozawa term

$$V_{\rm WT} = -\frac{C}{2f^2}(\sqrt{s} - M_T)$$

C/f² : coupling constant

no s-wave resonance

$$T^{-1} = V_{\rm WT}^{-1} - G(a_{\rm pheno}) = V_{\rm natural}^{-1} - G(a_{\rm natural})$$

↑ChPT ↑data fit ↑given

Effective interaction in natural scheme

$$V_{\rm natural} = -\frac{C}{2f^2}(\sqrt{s} - M_T) + \boxed{\frac{C}{2f^2}\frac{(\sqrt{s} - M_T)^2}{\sqrt{s} - M_{\rm eff}}}$$

pole!

$$M_{\rm eff} = M_T - \frac{16\pi^2 f^2}{C M_T \Delta a}, \quad a_{\rm pheno} - a_{\rm natural}$$

There is always a pole for $a_{\rm pheno} \neq a_{\rm natural}$

- small deviation <=> pole at irrelevant energy scale
- large deviation <=> pole at relevant energy scale

Pole in the effective interaction

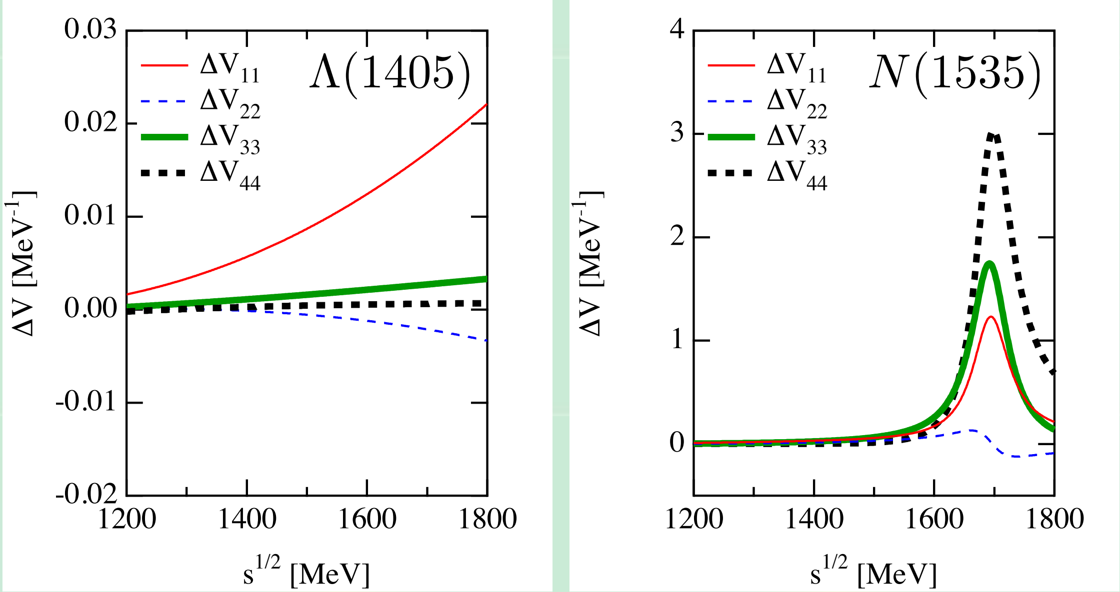
Pole in the effective interaction (M_{eff}) : pure CDD pole

$$T^{-1} = V_{\rm WT}^{-1} - G(a_{\rm pheno}) = \boxed{V_{\rm natural}^{-1}} - G(a_{\rm natural})$$

For Λ(1405): $z_{\rm eff}^{\Lambda^*} \sim 7.9$ GeV **irrelevant!**

For N(1535): $z_{\rm eff}^{N^*} = 1693 \pm 37i$ MeV **relevant?**

Difference of interactions $\Delta V \equiv V_{\rm natural} - V_{\rm WT}$



==> Important CDD pole contribution in N(1535)

Next question: quantitative measure for compositeness?

Weinberg's theorem for deuteron

"Evidence That the Deuteron Is Not an Elementary Particle"

S. Weinberg, Phys. Rev. 137 B672-B678, (1965)

Z: probability of finding deuteron in a bare elementary state

$$| d \rangle = \sqrt{Z} | d_0 \rangle + \sqrt{1-Z} \int d\boldsymbol{k} | \boldsymbol{k} \rangle$$

$$1 = |d_0\rangle\langle d_0| + \int d\boldsymbol{k} |\boldsymbol{k}\rangle\langle\boldsymbol{k}|$$ **: eigenstates of free Hamiltonian**

For a bound state with small binding energy, the following equation should be satisfied model independently:

$$\boxed{a_s} = \left[\frac{2(1-Z)}{2-Z}\right]\boxed{R} + \mathcal{O}(m_\pi^{-1}), \quad \boxed{r_e} = \left[\frac{-Z}{1-Z}\right]\boxed{R} + \mathcal{O}(m_\pi^{-1})$$

<-- Experiments (observables)

$$a_s = +5.41[\mathrm{fm}], \quad r_e = +1.75[\mathrm{fm}], \quad R \equiv (2\mu B)^{-1/2} = 4.31[\mathrm{fm}]$$

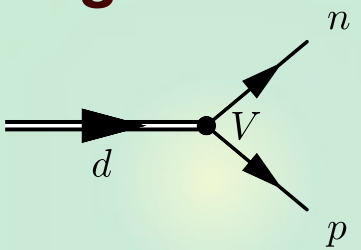
$\Rightarrow Z \lesssim 0.2$ **--> deuteron is composite!**

Derivation of the theorem

The theorem is derived in two steps:

Step 1 (Sec. II): Z --> p-n-d coupling constant

$$g^2 = \frac{2\sqrt{B}(1-Z)}{\pi\rho} \qquad \rho = 4\pi\sqrt{2\mu^3}$$



Step 2 (Sec. III): coupling constant --> a_s, r_e

$$a_s = 2R\left[1 + \frac{2\sqrt{B}}{\pi\rho g^2}\right] \qquad r_e = R\left[1 - \frac{2\sqrt{B}}{\pi\rho g^2}\right]$$

uncertainty for order $R=(2\mu B)^{1/2}$ quantity: m_π^{-1}

The coupling constant g^2 can be calculated by the residue of the pole in chiral unitary approach ==> Z?

--> study Z in natural renormalization scheme

Field renormalization constant

Single-channel problem: M_T and m

$$T = \frac{1}{1 - VG(a)} V$$

$$V = -\frac{C}{2f^2}(\sqrt{s} - M_T) = \tilde{C}(\sqrt{s} - M_T)$$

2 parameters: $(\tilde{C}, a)$

For the system with a bound state

$$1 - VG|_{\sqrt{s}=M_B} = 1 - \tilde{C}(M_B - M_T)G(M_B; a) = 0$$

:relation among $\tilde{C}, a, M_B$

--> system can be characterized by $(\tilde{C}, a)$ **or** (a, M_B)

Check the Z factor in natural renormalization scheme from the residue of the pole

$$g^2(M_B; a) = \lim_{\sqrt{s} \to M_B} (\sqrt{s} - M_B) T(\sqrt{s})$$

Field renormalization constant

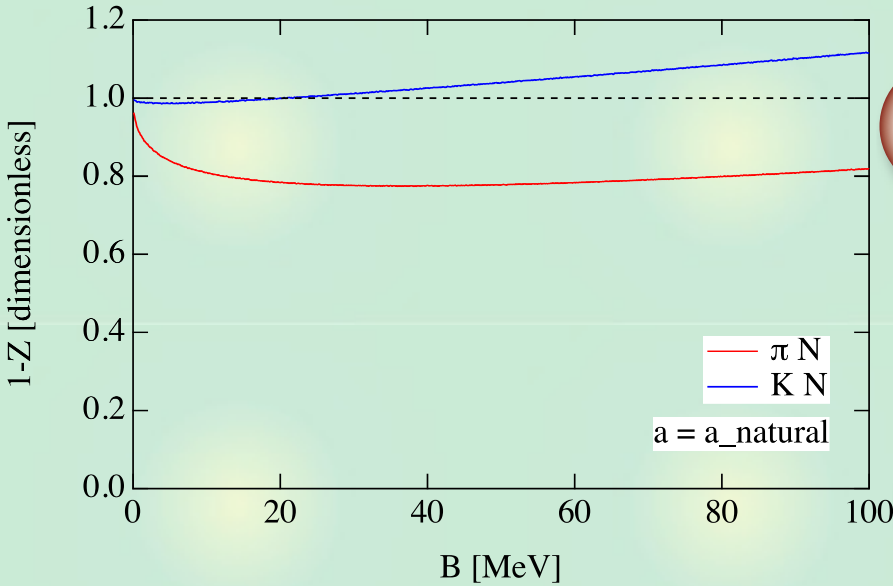
The residue can be calculated analytically:

$$g^2(M_B; a) = -\frac{M_B - M_T}{G(M_B; a) + (M_B - M_T)G'(M_B)} \longleftarrow (a, M_B)$$

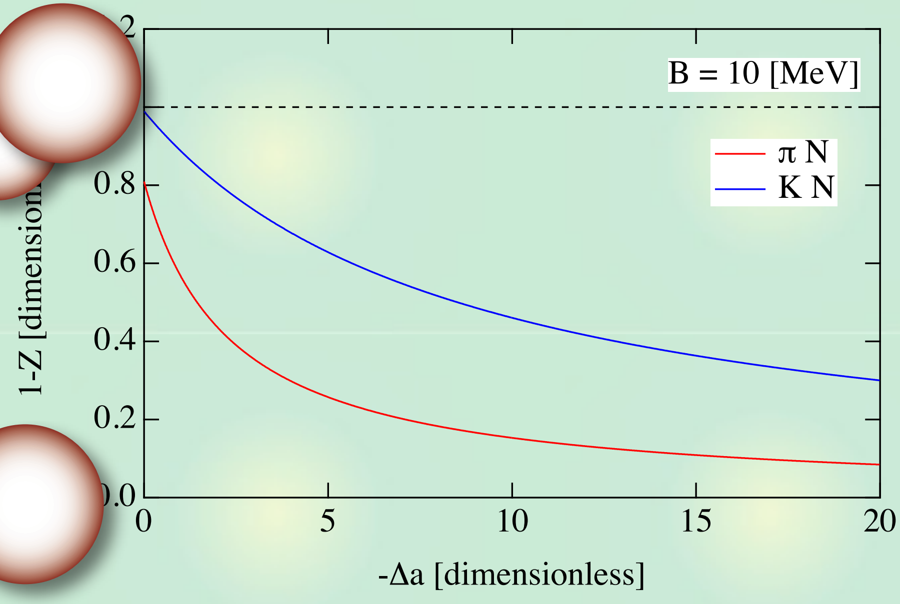
$$1 - Z = \sqrt{\frac{2mM_T}{(M_T + m)(M_T + m - M_B)}} \frac{M_T}{8\pi M_B} g^2(M_B; a)$$

valid for small
$B = (M_T + m) - M_B$

1) $a = a_{natural}$, vary B



2) B = 10 MeV, vary a



natural scheme --> Z ~ 0

large deviation --> Z ~ 1

Summary: Origin of resonances

We study the origin (dynamical/CDD) of the resonances in the chiral unitary approach

- **Natural renormalization scheme**

  Exclude CDD pole contribution from the loop function, consistent with N/D.

- **Comparison with phenomenology**
  --> Pole in the effective interaction

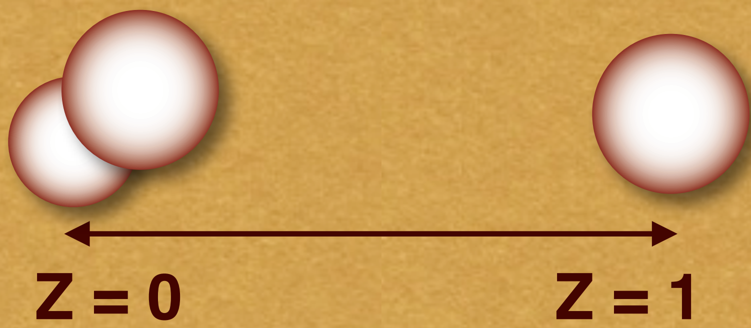
  Λ(1405) : predominantly dynamical
  N(1535) : dynamical + CDD pole

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C78, 025203 (2008)

Summary: Compositeness of resonances

We consider a single-channel problem with a bound state to study the compositeness

- **Field renormalization constant Z: quantitative measure of compositeness**



- **Residue of the pole --> coupling constant**

natural scheme corresponds to Z ~ 0

T. Hyodo, D. Jido, A. Hosaka, in preparation