Compositeness of resonances in chiral unitary approach

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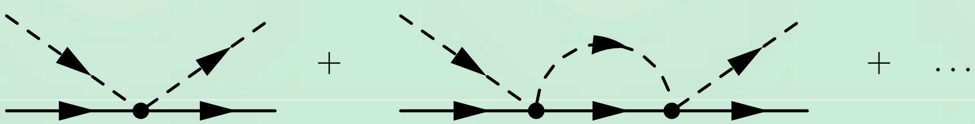
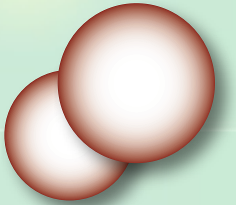
Classification of resonances

Resonances in two-body scattering

- Knowledge of interaction (potential)
- Experimental data (cross section, ...)

Dynamical state: two-body molecule, quasi-bound state, ...

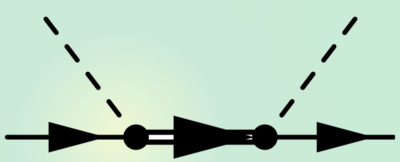


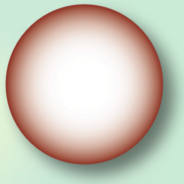


e.g.) Deuteron in NN, positronium in e^+e^-, ...

CDD pole: elementary particle, preformed state, ...

L. Castillejo, R.H. Dalitz, F.J. Dyson, Phys. Rev. 101, 453 (1956)





e.g.) J/Ψ in e^+e^-, ...

Chiral unitary approach

Description of S = -1, $\bar{K}N$ s-wave scattering: Λ(1405) in I=0

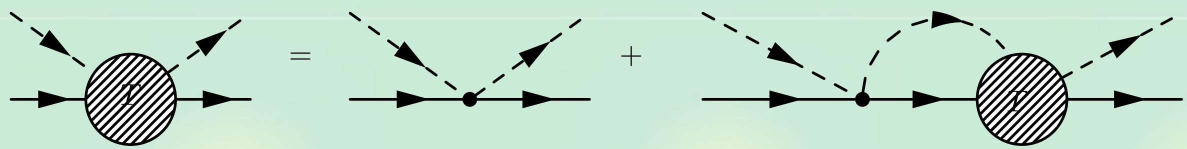
- Interaction <-- chiral symmetry

  Y. Tomozawa, Nuovo Cim. 46A, 707 (1966); S. Weinberg, Phys. Rev. Lett. 17, 616 (1966)

- Amplitude <-- unitarity in coupled channels

  R.H. Dalitz, T.C. Wong, G. Rajasekaran, Phys. Rev. 153, 1617 (1967)

$$T = \frac{1}{1 - VG} V$$



chiral

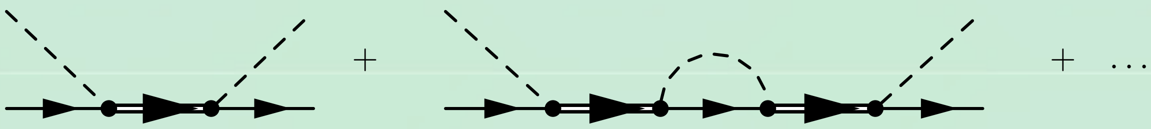
cutoff (subtraction constant)

N. Kaiser, P. B. Siegel, W. Weise, Nucl. Phys. A594, 325 (1995),
E. Oset, A. Ramos, Nucl. Phys. A635, 99 (1998),
J. A. Oller, U. G. Meissner, Phys. Lett. B500, 263 (2001),
M.F.M. Lutz, E. E. Kolomeitsev, Nucl. Phys. A700, 193 (2002), ... many others

It works successfully, also in S=0 sector, meson-meson scattering sectors, systems including heavy quarks, ...

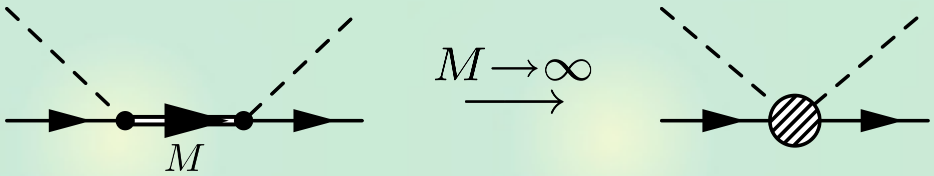
(Known) CDD poles in chiral unitary approach

Explicit resonance field in V (interaction)

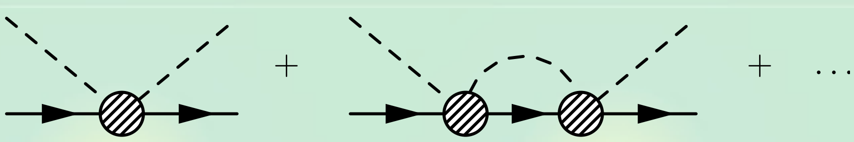


U.G. Meissner, J.A. Oller, Nucl. Phys. A673, 311 (2000)
D. Jido, E. Oset, A. Ramos, Phys. Rev. C66, 055203 (2002)

Contracted resonance propagator in higher order V



G. Ecker, J. Gasser, A. Pich, E. de Rafael, Nucl. Phys. B321, 311 (1989)
V. Bernard, N. Kaiser, U.G. Meissner, Nucl. Phys. A615, 483 (1997)



J.A. Oller, E. Oset, J.R. Pelaez, Phys. Rev. D59, 074001 (1999)

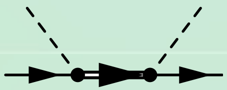
Is that all? subtraction constant?

CDD pole in subtraction constant?

Phenomenological (standard) scheme
--> V is given, "a" is determined by data

$$T = \frac{1}{(V^{(1)})^{-1} - G(\underline{a})}$$ **leading order**

$$T = \frac{1}{(V^{(1)} + V^{(2)})^{-1} - G(\underline{a'})}$$ **next to leading order**

↑pole  **?**

"a" represents the effect which is not included in V.
CDD pole contribution in G?

Natural renormalization scheme
--> fix "a" first, then determine V

to exclude CDD pole contribution from G,
based on theoretical argument.

Natural renormalization condition

Conditions for natural renormalization

- Loop function G should be negative below threshold.
- T matches with V at low energy scale.

"a" is uniquely determined such that

$$G(\sqrt{s} = M_T) = 0 \quad \Leftrightarrow \quad T(M_T) = V(M_T)$$

subtraction constant: a_{natural}

matching with low energy interaction

K. Igi, K. Hikasa, Phys. Rev. D59, 034005 (1999)
U.G. Meissner, J.A. Oller, Nucl. Phys. A673, 311 (2000)

crossing symmetry (matching with u-channel amplitude)

M.F.M. Lutz, E. Kolomeitsev, Nucl. Phys. A700, 193 (2002)

We regard this condition as the **exclusion of the CDD pole contribution from G**.

Pole in the effective interaction: single channel

Leading order V : Weinberg-Tomozawa term

$$V_{\mathrm{WT}} = -\frac{C}{2f^2}(\sqrt{s} - M_T)$$

C/f² : coupling constant
no s-wave resonance

$$T^{-1} = V_{\mathrm{WT}}^{-1} - G(a_{\mathrm{pheno}}) = V_{\mathrm{natural}}^{-1} - G(a_{\mathrm{natural}})$$

↑ChPT ↑data fit ↑given

Effective interaction in natural scheme

$$V_{\mathrm{natural}} = -\frac{C}{2f^2}(\sqrt{s} - M_T) + \boxed{\frac{C}{2f^2}\frac{(\sqrt{s} - M_T)^2}{\sqrt{s} - M_{\mathrm{eff}}}}$$

pole!

$$M_{\mathrm{eff}} = M_T - \frac{16\pi^2 f^2}{C M_T \Delta a}, \quad a_{\mathrm{pheno}} - a_{\mathrm{natural}}$$

There is always a pole for $a_{\mathrm{pheno}} \neq a_{\mathrm{natural}}$

- **small deviation <=> pole at irrelevant energy scale**
- **large deviation <=> pole at relevant energy scale**

Pole in the effective interaction

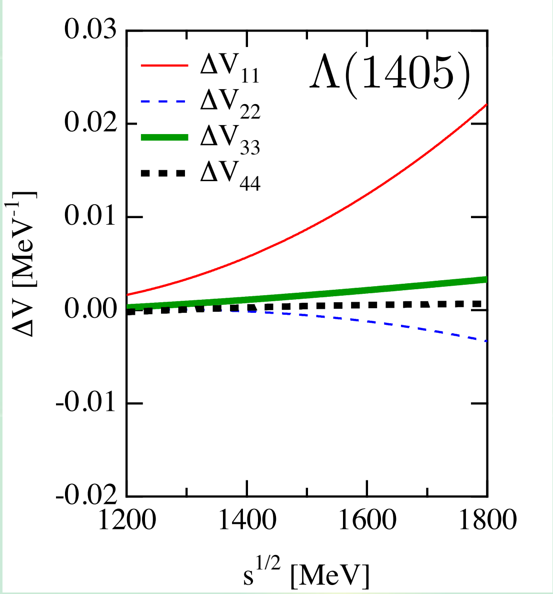
Pole in the effective interaction (M_{eff}) : pure CDD pole

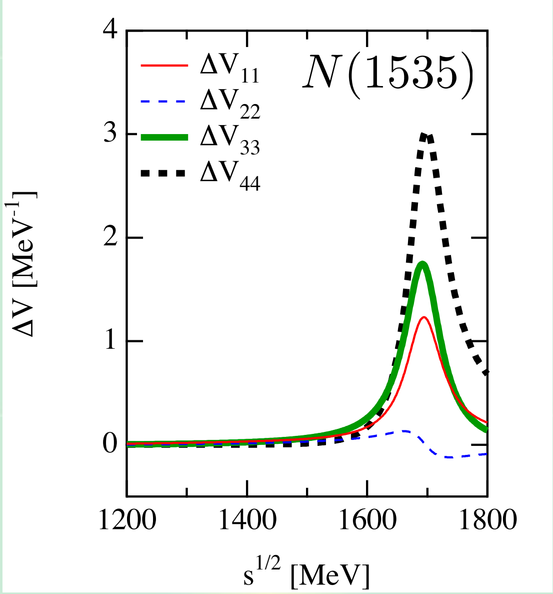
$$T^{-1} = V_{\rm WT}^{-1} - G(a_{\rm pheno}) = V_{\rm natural}^{-1} - G(a_{\rm natural})$$

For Λ(1405): $z_{\rm eff}^{\Lambda^*} \sim 7.9$ GeV **irrelevant!**

For N(1535): $z_{\rm eff}^{N^*} = 1693 \pm 37i$ MeV **relevant?**

Difference of interactions $\Delta V \equiv V_{\rm natural} - V_{\rm WT}$





==> Important CDD pole contribution in N(1535)

Next question: quantitative measure for compositeness?

Weinberg's theorem for deuteron

"Evidence That the Deuteron Is Not an Elementary Particle"

S. Weinberg, Phys. Rev. 137 B672-B678, (1965)

Z: probability of finding deuteron in a bare elementary state

$$|\, d \,\rangle = \sqrt{Z} |\, d_0 \,\rangle + \sqrt{1-Z} \int d\boldsymbol{k} |\, \boldsymbol{k} \,\rangle$$

$$1 = |d_0\rangle\langle d_0| + \int d\boldsymbol{k} |\boldsymbol{k}\rangle\langle\boldsymbol{k}|$$ **: eigenstates of free Hamiltonian**

For a bound state with small binding energy, the following equation should be satisfied model independently:

$$a_s = \left[\frac{2(1-Z)}{2-Z}\right] R + \mathcal{O}(m_\pi^{-1}), \quad r_e = \left[\frac{-Z}{1-Z}\right] R + \mathcal{O}(m_\pi^{-1})$$

<-- Experiments (observables)

$$a_s = +5.41[\mathrm{fm}], \quad r_e = +1.75[\mathrm{fm}], \quad R \equiv (2\mu B)^{-1/2} = 4.31[\mathrm{fm}]$$

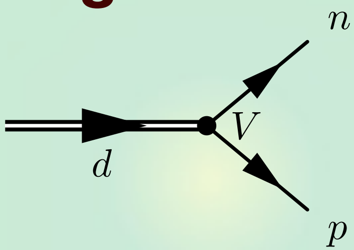
$\Rightarrow Z \lesssim 0.2$ **--> deuteron is composite!**

Derivation of the theorem

The theorem is derived in two steps:

Step 1 (Sec. II): Z --> p-n-d coupling constant

$$g^2 = \frac{2\sqrt{B}(1-Z)}{\pi\rho} \qquad \rho = 4\pi\sqrt{2\mu^3}$$



Step 2 (Sec. III): coupling constant --> a_s, r_e

$$a_s = 2R\left[1 + \frac{2\sqrt{B}}{\pi\rho g^2}\right] \qquad r_e = R\left[1 - \frac{2\sqrt{B}}{\pi\rho g^2}\right]$$

uncertainty for order $R=(2\mu B)^{1/2}$ quantity: m_π^{-1}

The coupling constant g^2 can be calculated by the residue of the pole in chiral unitary approach ==> Z?

--> study Z in natural renormalization scheme

Field renormalization constant

Single-channel problem: M_T and m

$$T = \frac{1}{1 - VG(a)} V$$

$$V = -\frac{C}{2f^2}(\sqrt{s} - M_T) = \tilde{C}(\sqrt{s} - M_T)$$

2 parameters: $(\tilde{C}, a)$

For the system with a bound state

$$1 - VG|_{\sqrt{s}=M_B} = 1 - \tilde{C}(M_B - M_T)G(M_B; a) = 0$$

:relation among $\tilde{C}, a, M_B$

--> system can be characterized by $(\tilde{C}, a)$ **or** (a, M_B)

Check the Z factor in natural renormalization scheme from the residue of the pole

$$g^2(M_B; a) = \lim_{\sqrt{s} \to M_B} (\sqrt{s} - M_B) T(\sqrt{s})$$

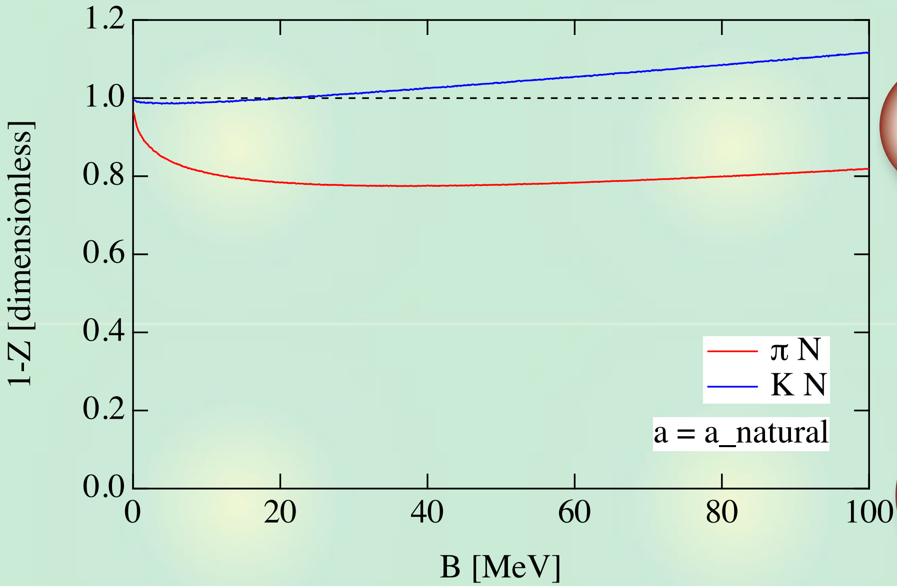
Field renormalization constant

The residue can be calculated analytically:

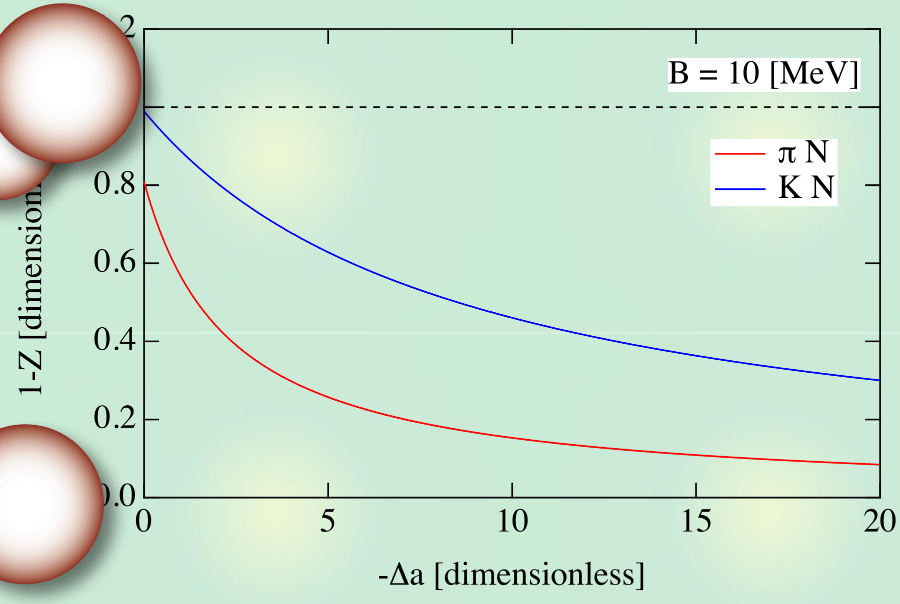
$$g^2(M_B;a) = -\frac{M_B - M_T}{G(M_B;a) + (M_B - M_T)G'(M_B)} \longleftarrow (a, M_B)$$

$$1 - Z = \sqrt{\frac{2mM_T}{(M_T+m)(M_T+m-M_B)}}\frac{M_T}{8\pi M_B}g^2(M_B;a)$$ **valid for small M_B**

1) $a = a_{natural}$, vary M_B



2) $M_B = 10$ MeV, vary a



natural scheme --> Z ~ 0

large deviation --> Z ~ 1

Summary: Origin of resonances

We study the origin (dynamical/CDD) of the resonances in the chiral unitary approach

- **Natural renormalization scheme**

  Exclude CDD pole contribution from the loop function, consistent with N/D.

- **Comparison with phenomenology**
  --> **Pole** in the effective interaction

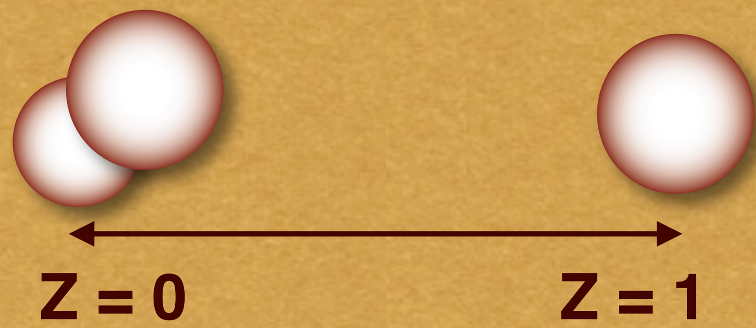
  Λ(1405) : predominantly dynamical
  N(1535) : dynamical + CDD pole

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C78, 025203 (2008)

Summary: Compositeness of resonances

We consider a single-channel problem with a bound state to study the compositeness

- **Field renormalization constant Z: quantitative measure of compositeness**



- **Residue of the pole --> coupling constant**

natural scheme corresponds to Z ~ 0

T. Hyodo, D. Jido, A. Hosaka, in preparation