

Softening of the dynamical sigma meson



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The sigma meson

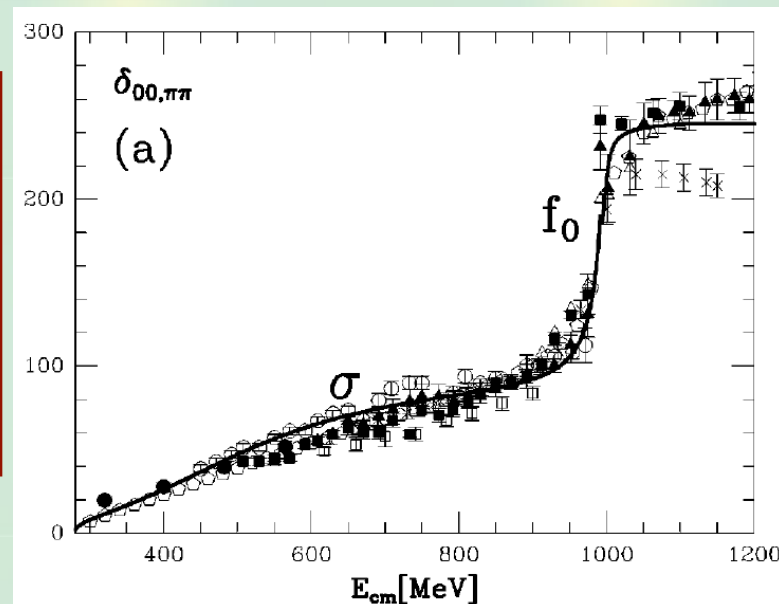
$f_0(600)$ or $\sigma : J^P = 0^+, I = 0$

Mass : 400-1200 MeV

Width : 600-1000 MeV

Decay modes : $\sigma \rightarrow \pi\pi$ dominant

$\sigma \rightarrow \gamma\gamma$ **seen**



σ meson

- is the lowest resonance in QCD
- plays an important role in hadron mass generation due to spontaneous chiral symmetry breaking
- provides attraction in phenomenological nuclear force

Recent progress in scattering theory + data precession

--> **determination of pole position** is now possible.

I. Caprini, G. Colangelo, H. Leutwyler, Phys. Rev. Lett. 96, 132001 (2006), ...

Structure of the sigma meson

Sigma meson in naive constituent quark model ($\sim \bar{q}q$) has some difficulties: **light mass** (v.s. p-wave excitation), **mass ordering** of scalar nonet (v.s. $\sigma > \kappa > f_0 \sim a_0$)

Alternative descriptions of the sigma meson

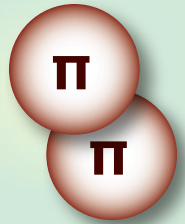
- **Chiral sigma**
(e.g. linear sigma model)

M. Gell-Mann, M. Levy, *Nuovo Cim.* 16, 705 (1960), ...



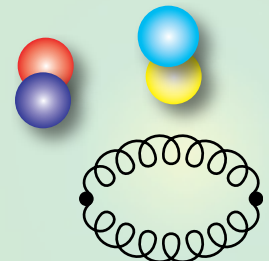
- **Dynamical sigma**
(e.g. mesonic molecule generated by π - π attraction)

J.A. Oller, E. Oset, J.R. Pelaez, *Phys. Rev. D* 59, 074001 (1999), ...



- **CDD pole contribution (pre-formed state)**
(e.g. constituent four-quark model, glueball, ...)

L.R. Jaffe, *Phys. Rev. D* 15, 267 (1977), ...



We want to clarify the **structure** <-- **softening**

Softening of the sigma meson

Softening of chiral sigma

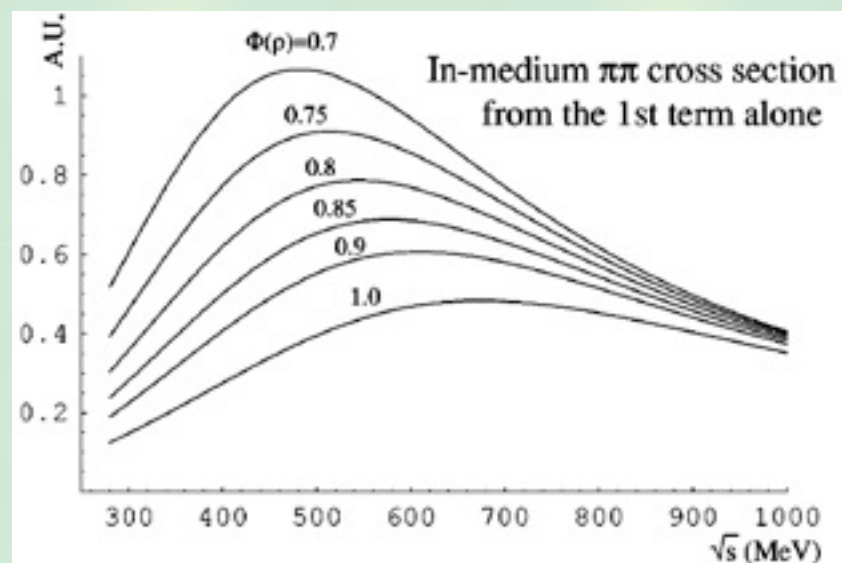
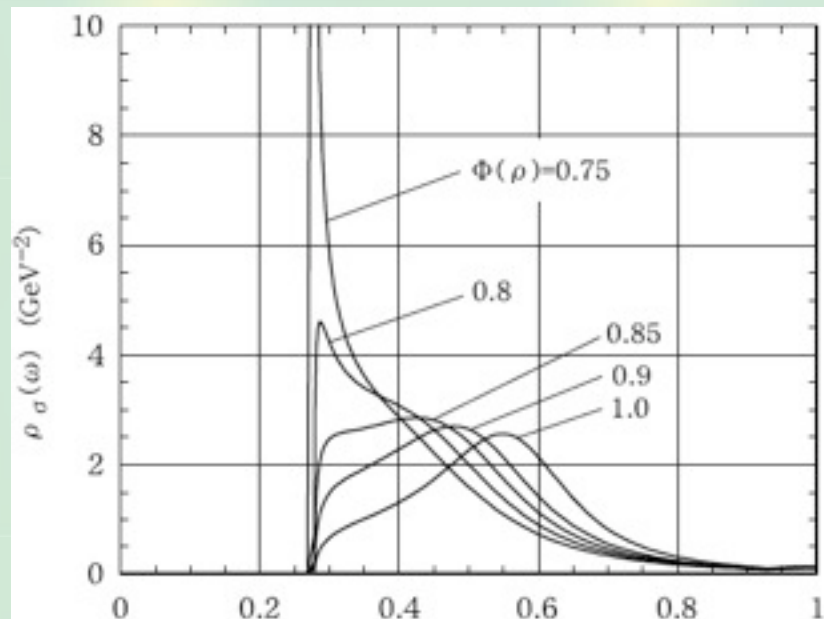
T. Hatsuda, T. Kunihiro, H. Shimizu
 Phys. Rev. Lett. 82, 2840 (1999)

Partial restoration of chiral sym.
 --> Spectral enhancement in
 $l=j=0$ channel near threshold

fluctuation of the order parameter
 of chiral phase transition

Threshold enhancement of π - π
 cross section, also for the
 dynamical sigma meson

D. Jido, T. Hatsuda, T. Kunihiro,
 Phys. Rev. D63, 011901 (2001)



Mechanism of the softening (chiral sigma)

In the previous studies, it seems that the softening takes place, irrespective to the structure of the sigma meson.

Mechanism of the softening?

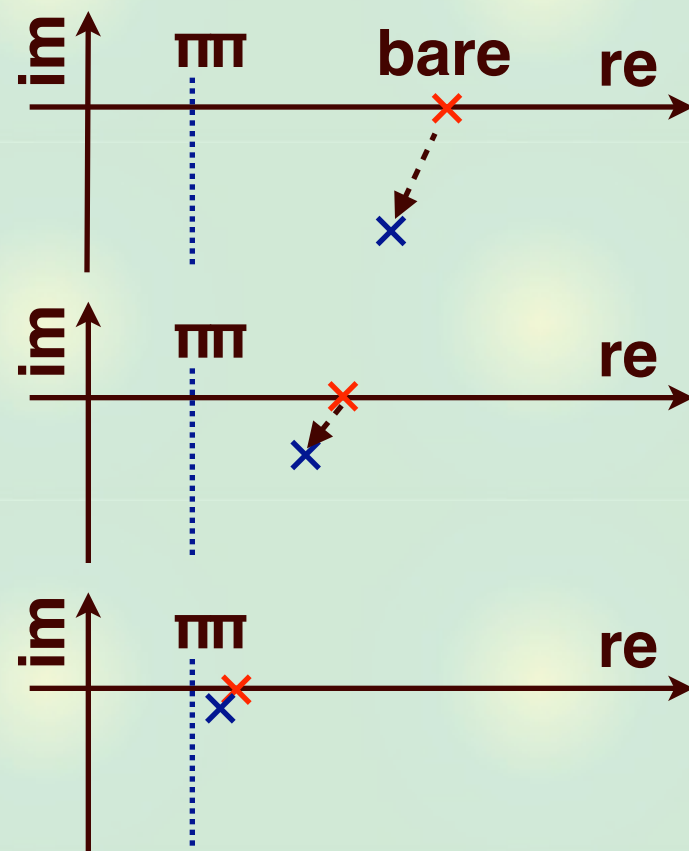
Softening of the **chiral sigma** (linear sigma model)

Sigma meson:

bare sigma pole acquires finite width through the coupling to π - π

Chiral symmetry restoration:

- > **lowering bare sigma mass**
- > reduction of the phase space
- > narrow spectrum



Mechanism of the softening (dynamical sigma)

Softening of the **dynamical sigma** (ChPT + unitarization)

Sigma meson: dynamically generated by π - π attraction

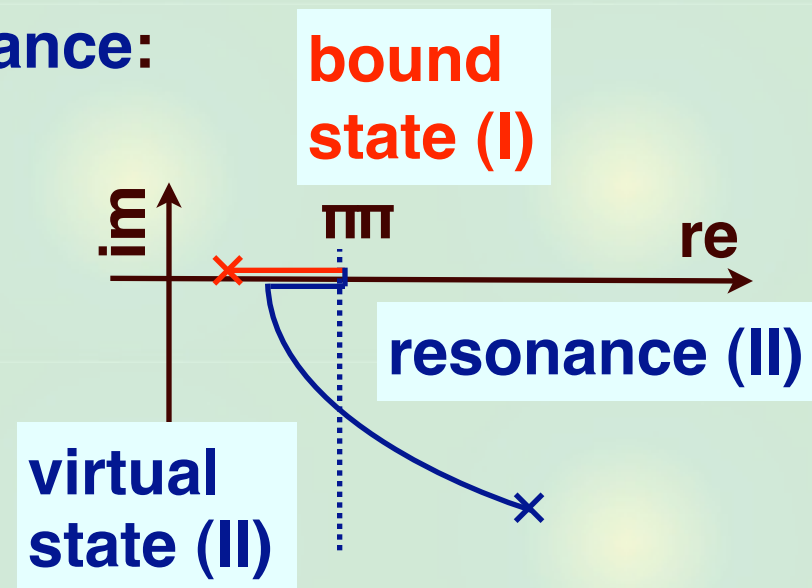
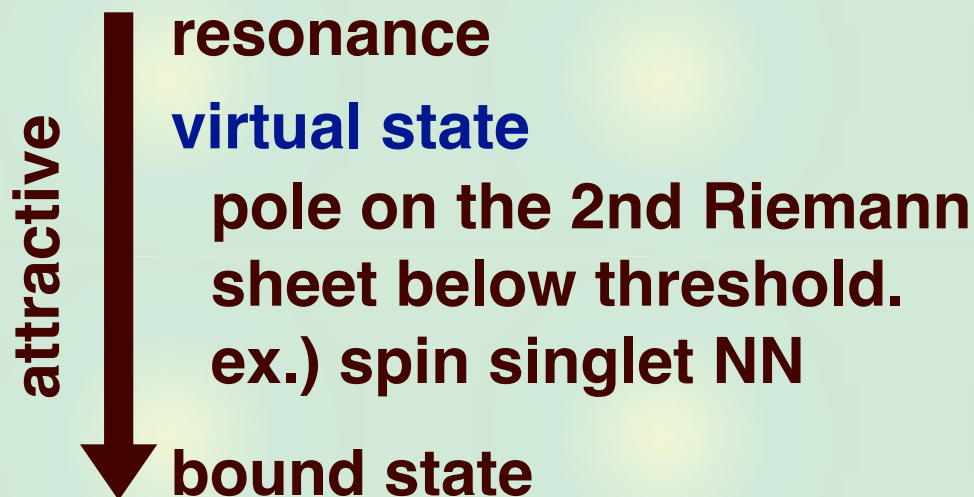
Chiral symmetry restoration:

--> $f_\pi \sim \langle \sigma \rangle$ decreases

--> **(attractive) interaction** $\sim (f_\pi)^{-2}$ increases

--> resonance turns into bound state, spectrum gets narrow

Special nature of the **s-wave resonance**:



--> novel softening pattern?

Dynamical chiral models

Tree-level π - π scattering amplitude in 2-flavor sigma model

$$A(s, t, u) = \frac{s - m_\pi^2}{\langle \sigma \rangle^2} - \frac{(s - m_\pi^2)^2}{\langle \sigma \rangle^2} \frac{1}{s - m_\sigma^2}$$

leading order term of ChPT

chiral sigma (model A) : both terms

dynamical sigma (model B) : 1st term only ($m_\sigma \rightarrow \infty$)

Projection to $l=j=0$ + unitarization (N/D method)

J. A. Oller, E. Oset, Phys. Rev. D60, 074023 (1999)

$$T(s; x) = \frac{1}{T_{\text{tree}}^{-1}(s; x) + G(s)}$$

$$G(s) = \frac{1}{2} \frac{1}{(4\pi)^2} \left\{ a(\mu) + \ln \frac{m_\pi^2}{\mu^2} + \sqrt{1 - \frac{4m_\pi^2}{s}} \left[\ln \frac{\sqrt{1 - \frac{4m_\pi^2}{s}} + 1}{\sqrt{1 - \frac{4m_\pi^2}{s}} - 1} \right] \right\}$$

$$a(m_\pi) = -\pi/\sqrt{3}$$

Exclude the CDD pole contribution from the loop function

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C78, 025203 (2008)

Prescription for symmetry restoration

We introduce the effect of chiral symmetry restoration from the outside of the model, by modifying m_π , m_σ , $\langle\sigma\rangle$.

1) chiral condensate (pion decay constant) : **decreases**

$$\langle\sigma\rangle = \Phi\langle\sigma\rangle_0, \quad 0 \leq \Phi \leq 1$$

2) mass of pion : **no change**

$$\frac{\partial m_\pi}{\partial \Phi} = 0$$

3) mass of chiral sigma : **decreases**

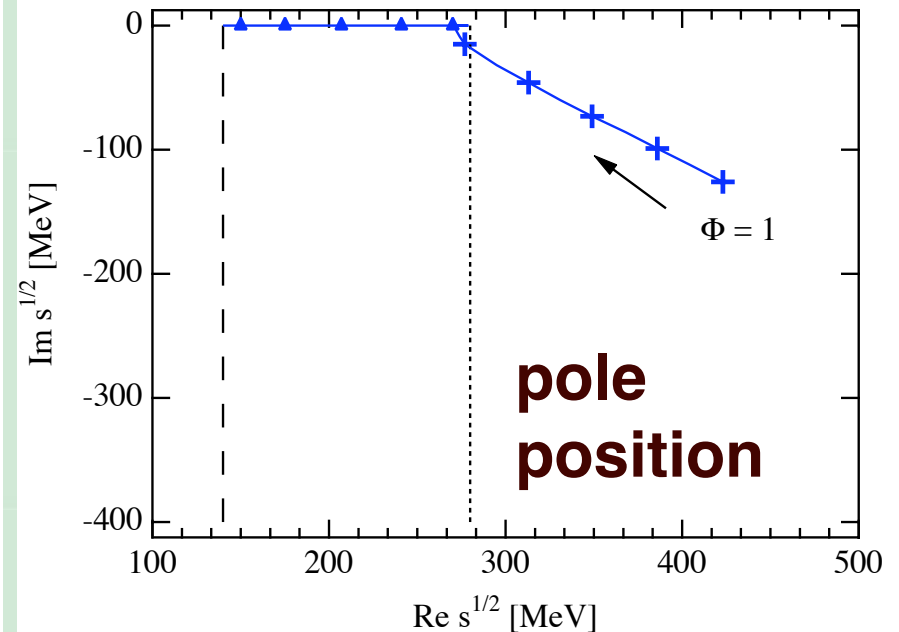
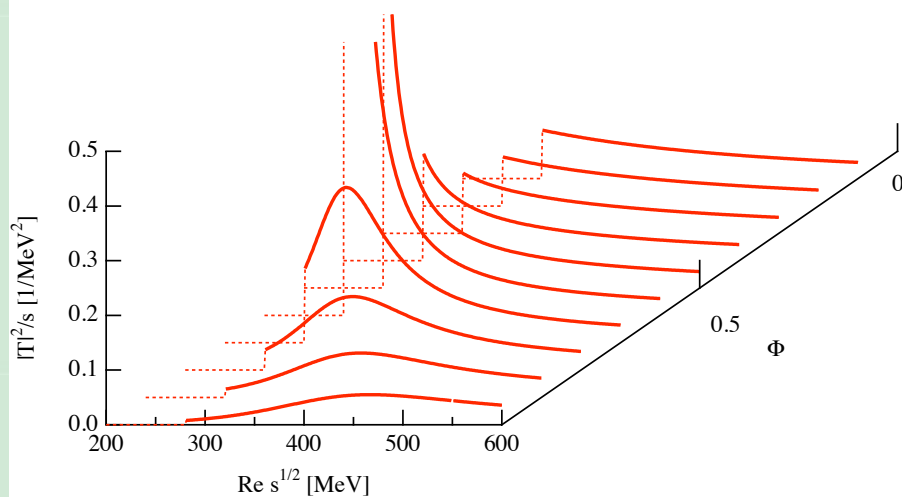
$$m_\sigma|_{\Phi \rightarrow 0} = m_\pi \quad (\text{case I}) \quad m_\sigma = \sqrt{\lambda \frac{\langle\sigma\rangle^2}{3} + m_\pi^2}$$

with λ and m_π being fixed.

The effect of symmetry restoration is modeled by the **change of Φ** .

Results in model A

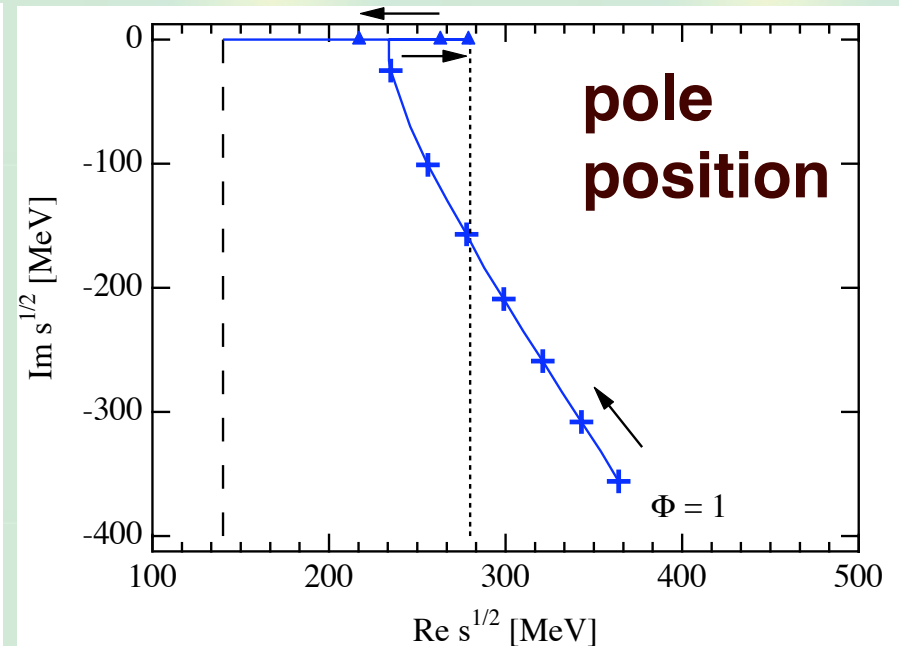
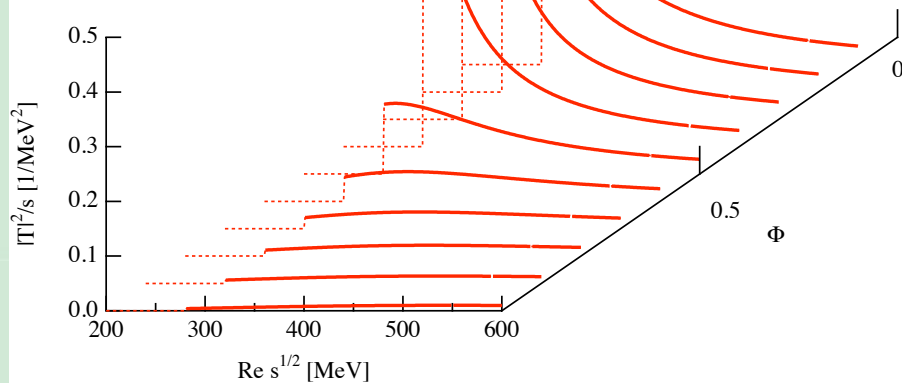
spectrum



- Linear sigma model + unitarization : **chiral sigma**
- Softening takes place, as expected.
- peak at threshold : **$\Phi \sim 0.6$**
 \Leftrightarrow bare sigma pole moves below the threshold

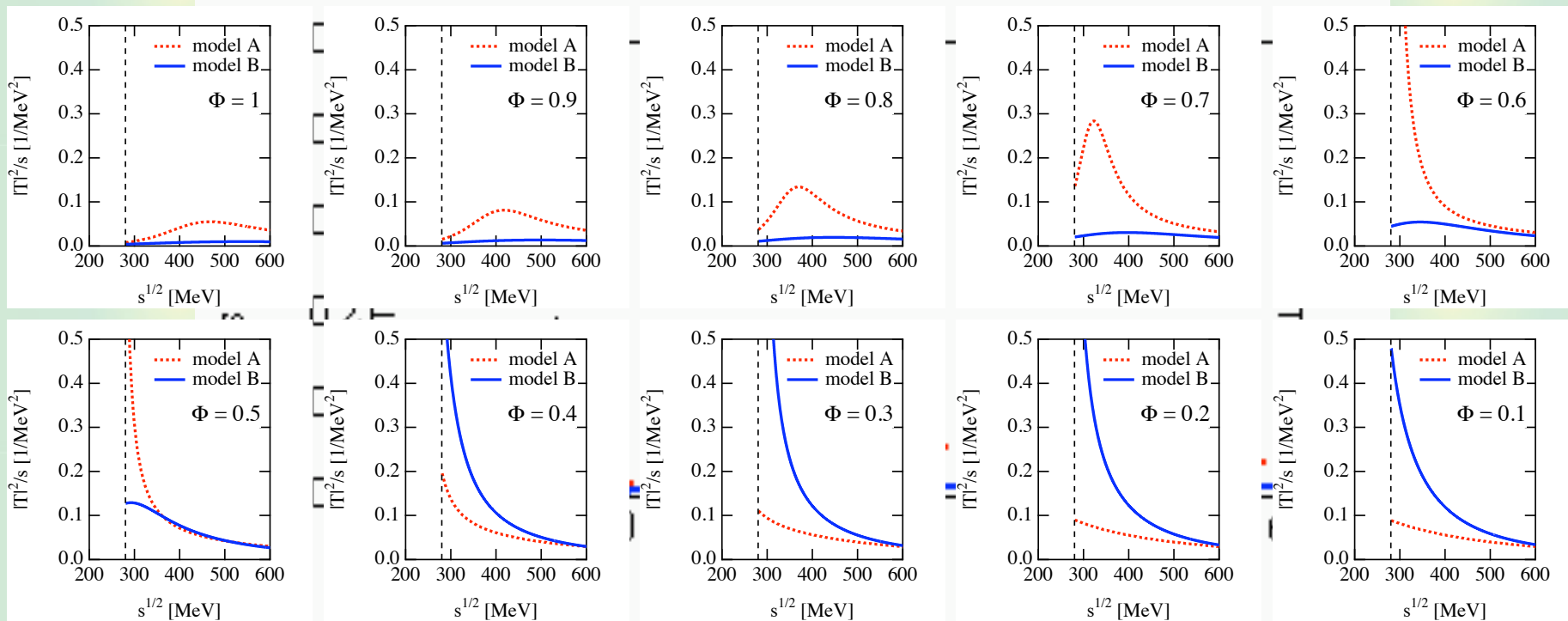
Results in model B

spectrum

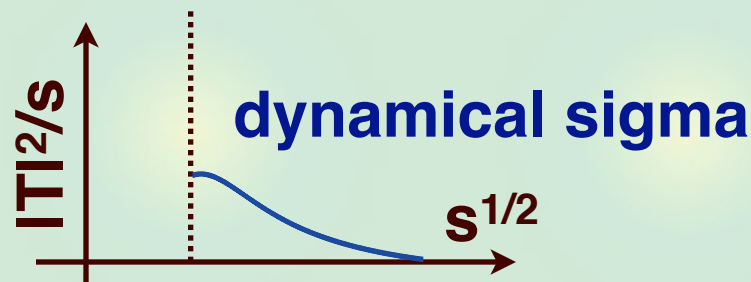
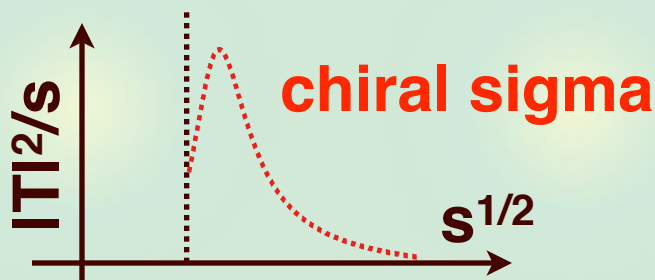


- ChPT + unitarization : **dynamical sigma**
- Softening takes place, but **virtual state** appears.
- at $\text{Re}[M_{\text{pole}}] = 2m_\pi$ ($\Phi \sim 0.6$), due to finite width, spectrum does not show the peak structure
- peak at threshold : $\Phi \sim 0.3 \iff$ formation of bound state

Comparison of model A and model B



- Strong threshold enhancement : different from each other.
- Shape of the spectrum?



Summary

We study the structure of the sigma meson with chiral symmetry restoration.



We construct dynamical chiral models where the sigma meson is realized as

(i) chiral partner of pion

(ii) dynamically generated molecule



Softening phenomena:

Dynamical sigma softens qualitatively differently from chiral sigma.

<-- virtual state (s-wave resonance)