# Softening of the dynamical sigma meson

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# Contents

# The sigma meson

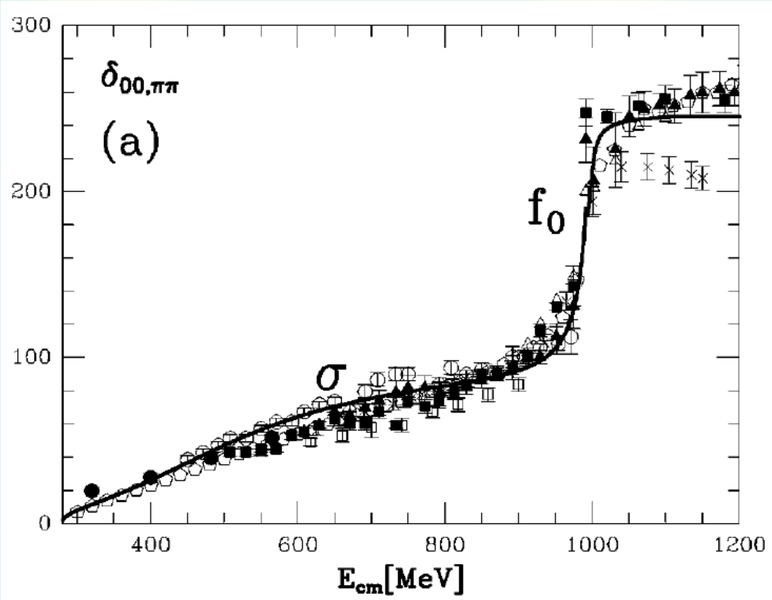
$f_0(600)$ or $\sigma : J^P = 0^+, I = 0$

**Mass : 400-1200 MeV**

**Width : 600-1000 MeV**

**Decay modes :** $\sigma \to \pi\pi$ **dominant**

$\sigma \to \gamma\gamma$ **seen**




**σ meson**

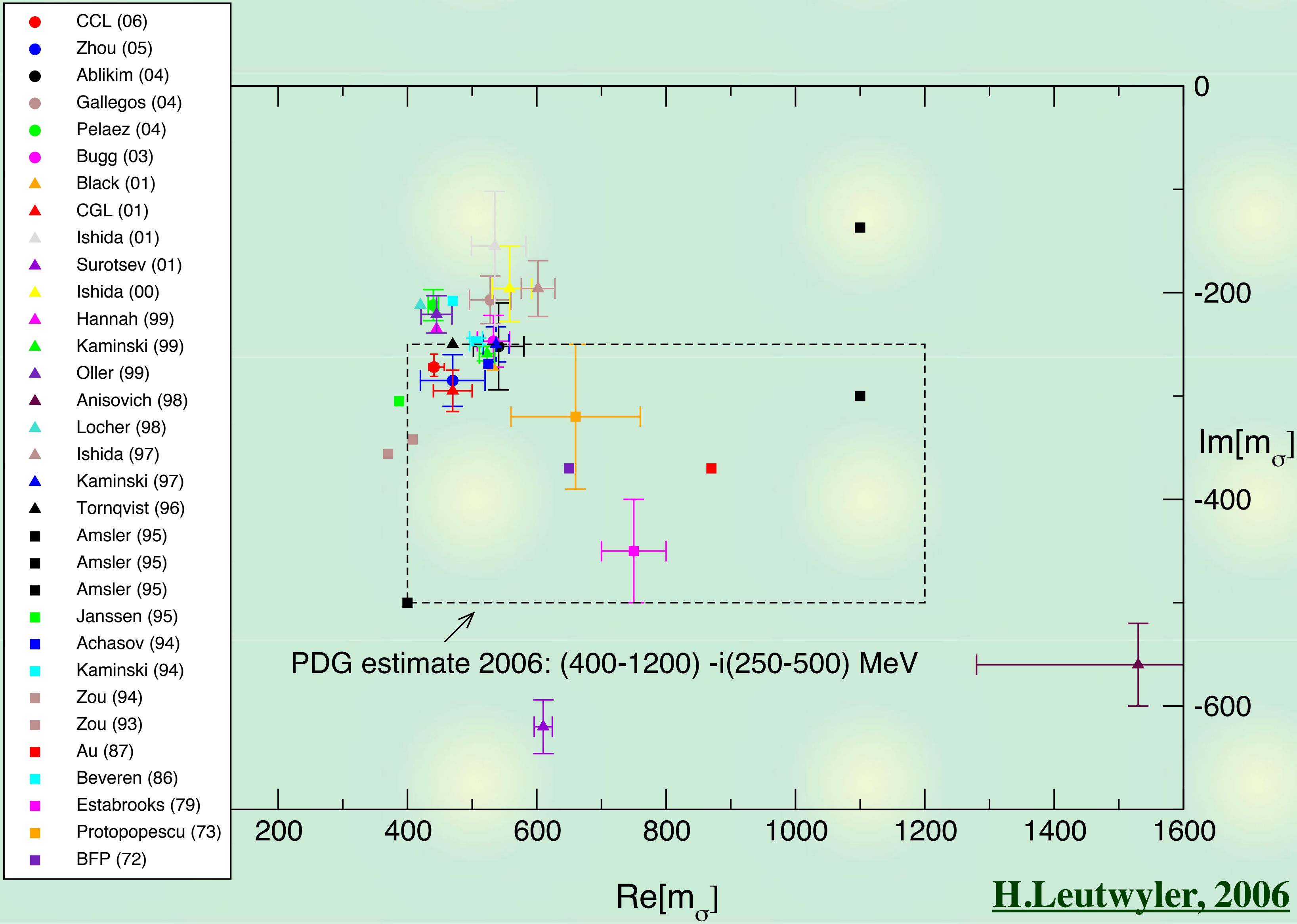
- is the lowest resonance in QCD
- plays an important role in hadron mass generation due to spontaneous chiral symmetry breaking
- provides attraction in phenomenological nuclear force

**Recent development**

: **precise pole position** is now available.

# Existence of the sigma pole

## Development of scattering theory + experimental data




H.Leutwyler, 2006

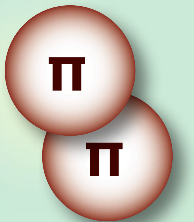
# Structure of the sigma meson

Sigma meson in naive constituent quark model ($\sim\bar{q}q$) has some difficulties: light mass (v.s. p-wave excitation), mass ordering of scalar nonet (v.s. $\sigma > \kappa > f_0 \sim a_0$)

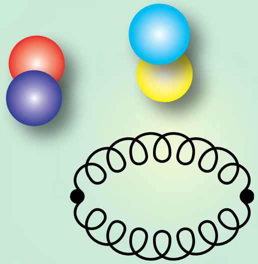
Alternative descriptions of the sigma meson

- Chiral sigma
  (e.g. linear sigma model)
  M. Gell-Mann, M. Levy, Nuovo Cim. 16, 705 (1960), ...


- Dynamical sigma
  (e.g. mesonic molecule generated by π-π attraction)
  J.A. Oller, E. Oset, J.R. Pelaez, Phys. Rev. D59, 074001 (1999), ...




- CDD pole contribution
  (e.g. constituent four-quark model, glueball, ...)
  L.R. Jaffe, Phys. Rev. D15, 267 (1977), ...

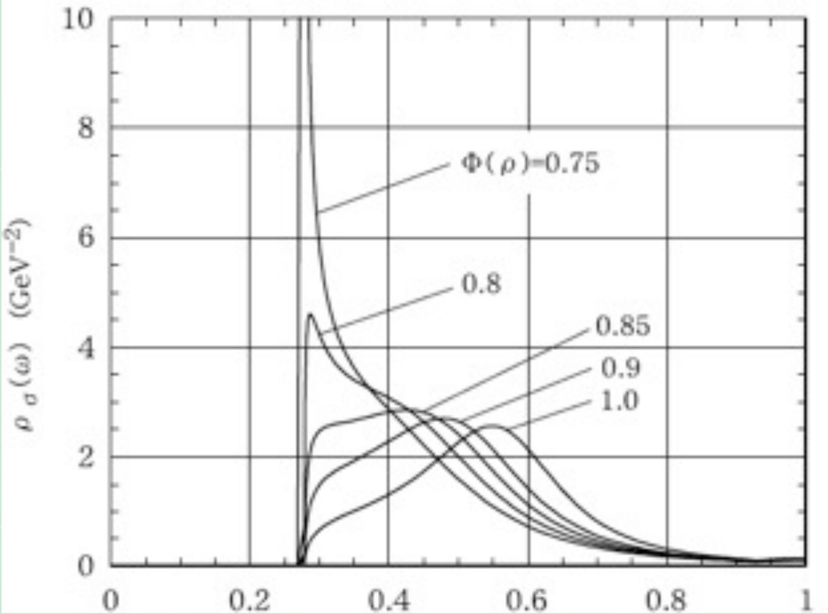


We want to clarify the structure <-- softening

# Softening of the sigma meson

## Softening of chiral sigma

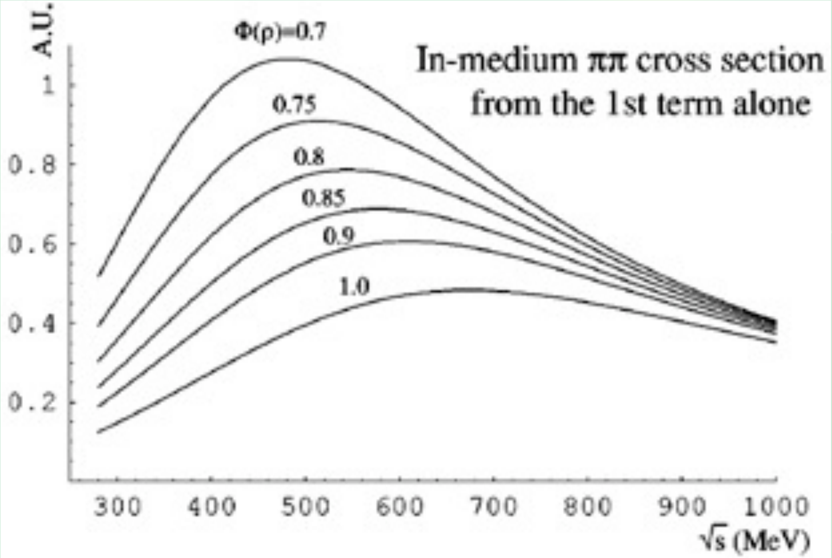
T. Hatsuda, T. Kunihiro, H. Shimizu
Phys. Rev. Lett. 82, 2840 (1999)




Spectral enhancement in I=J=0 channel near threshold, when the chiral sym. is partially restored.

sigma: fluctuation of the order parameter of chiral phase transition

Threshold enhancement of π-π cross section, also for the dynamical sigma meson




D. Jido, T. Hatsuda, T. Kunihiro,
Phys. Rev. D63, 011901 (2001)

# Softening of the sigma meson

## Systematic study up to restoration limit.

K. Yokokawa, T. Hatsuda, A. Hayashigaki, T. Kunihiro, Phys. Rev. C66, 022201 (2002)

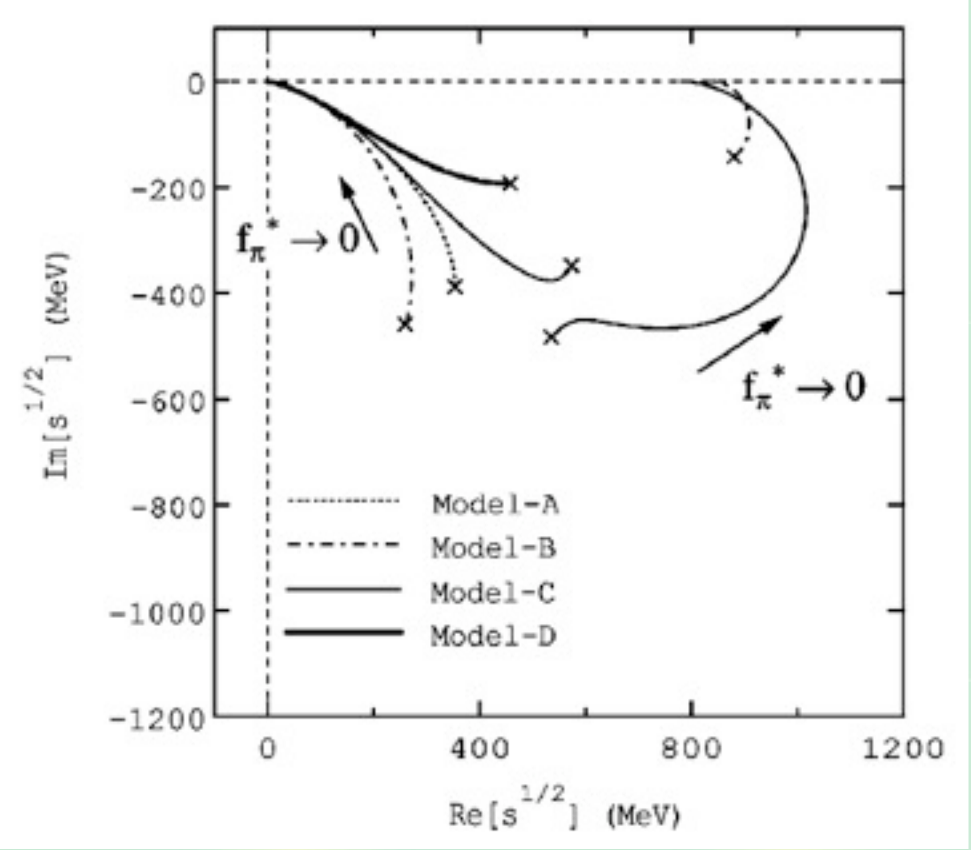
- dynamical model with chiral symmetry, unitarity, analyticity, (crossing)

  K. Igi, K. Hikasa, Phys. Rev. D59, 034005 (1999)

- 4 cases: σ pole on/off ⊗ ρ pole on/off

- roughly corresponds to dynamical sigma and/or CDD pole

- "universal softening" at $f_\pi^*/f_\pi << 1$



# Mechanism of the softening

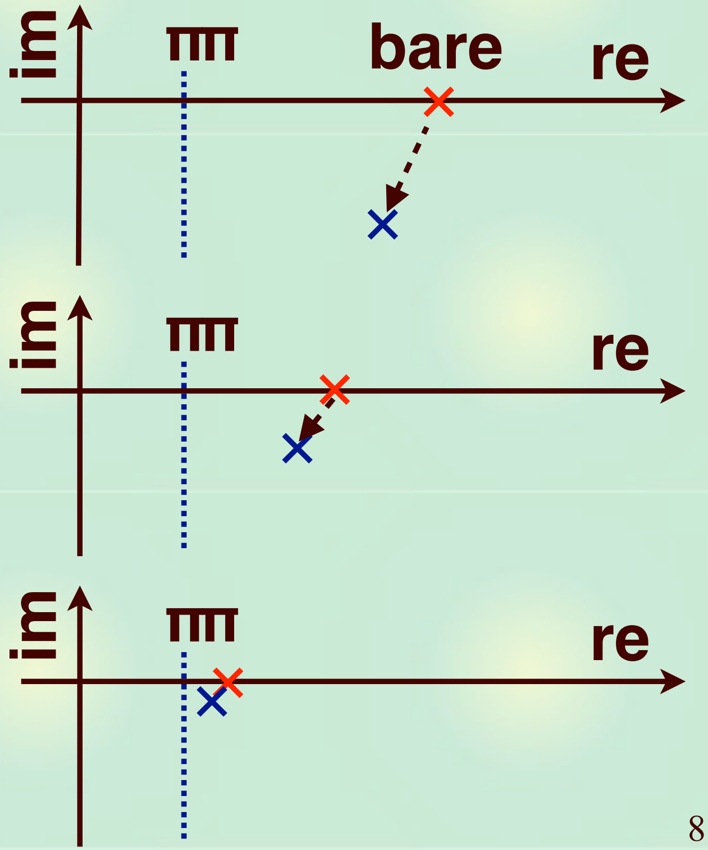
**In the previous studies, it seems that the softening takes place, irrespective to the structure of the sigma meson.**

**Mechanism of the softening?**

**Softening of the chiral sigma (linear sigma model)**

**Sigma meson:**
**bare sigma pole acquires finite width through the coupling to π-π**

**Chiral symmetry restoration:**
**--> lowering bare sigma mass**
**--> reduction of the phase space**
**--> narrow spectrum**



# Mechanism of the softening

## Softening of the dynamical sigma (ChPT + unitarization)

Sigma meson: dynamically generated by π-π attraction

Chiral symmetry restoration:

--> $f_\pi \sim \langle\sigma\rangle$ decreases

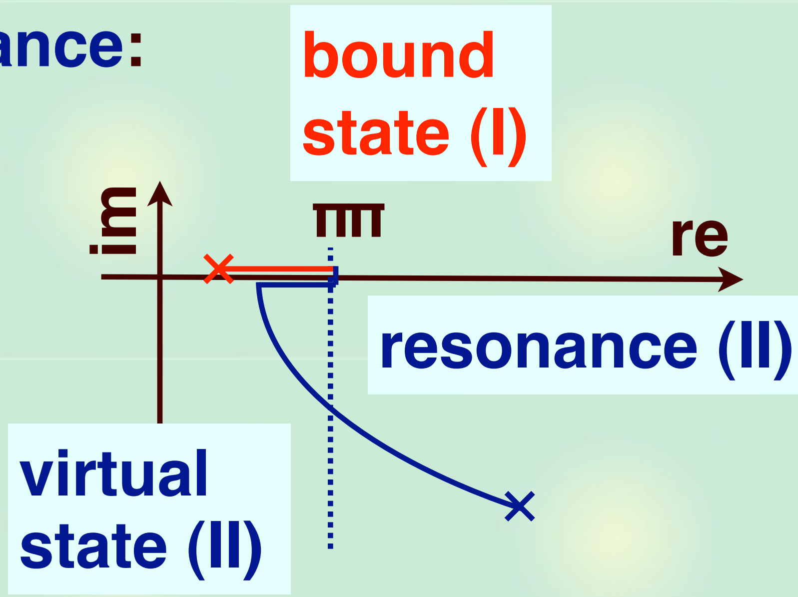
--> (attractive) interaction ~ $(f_\pi)^{-2}$ increases

--> resonance turns into bound state, spectrum gets narrow

## Special nature of the s-wave resonance:

attractive (↓)

- resonance
- virtual state
  - pole on the 2nd Riemann sheet below threshold.
  - ex) spin singlet deuteron
- bound state




--> novel softening pattern?

# The aim of this study

We want to study the **structure of the "sigma meson"** through the behavior in the **softening phenomena**.

For this, we use a schematic model of chiral dynamics.

Comparison with previous studies

| | $m_\pi$ | chiral restoration | sigma meson |
|---|---|---|---|
| Jido, Hatsuda, Kunihiro | finite | phi -> 0.7 | chiral, dynamical |
| Yokokawa, et al. | 0 | phi -> 0 | dynamical, CDD, mixture |
| This work | finite | phi -> 0 | chiral, dynamical, CDD, mixture |

It is important to keep $m_\pi$ finite and to take restoration limit.

# Tree level interaction

## Lagrangian of 2-flavor linear sigma model

$$\mathcal{L} = \frac{1}{4}\mathrm{Tr}\left[\partial M \partial M^\dagger - \mu^2 M M^\dagger - \frac{2\lambda}{4!}(MM^\dagger)^2 + h(M + M^\dagger)\right], M = \sigma + i\boldsymbol{\tau}\cdot\boldsymbol{\pi}$$

## 3 parameters <-- $m_\pi$, $m_\sigma$, $\langle\sigma\rangle$ at mean field level.

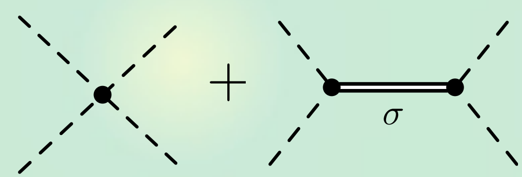
## π-π scattering amplitude in general (crossing symmetry)

$$T_{\mathrm{tree}}(s,t,u) = A(s,t,u)\delta_{ab}\delta_{cd} + A(t,s,u)\delta_{ac}\delta_{bd} + A(u,t,s)\delta_{ad}\delta_{bc}$$

## Tree-level π-π scattering amplitude

$$A(s,t,u) = -\frac{\lambda}{3} - \frac{\lambda^2\langle\sigma\rangle^2}{9}\frac{1}{s-m_\sigma^2}$$

$$= \boxed{\frac{s-m_\pi^2}{\langle\sigma\rangle^2}} - \frac{(s-m_\pi^2)^2}{\langle\sigma\rangle^2}\frac{1}{s-m_\sigma^2}$$




**leading order term of ChPT**

- low energy expansion
- 1st and 2nd terms are chiral invariant

# Tree level interaction

**Introduce a parameter using chiral invariant decomposition**

$$A(s;x) = \frac{s-m_\pi^2}{\langle\sigma\rangle^2} - \boxed{x}\frac{(s-m_\pi^2)^2}{\langle\sigma\rangle^2}\frac{1}{s-m_\sigma^2}$$

$x \to 1$ **: linear sigma model**

$x \to 0$ **: leading order term in ChPT**

$x \to 1/2$ **: model C in Yokokawa et al. (σ-ρ degeneracy, KSRF relation, and duality)**

**Parameter $x$ is useful to extrapolate models.**
**The origin of the resonance can be investigated (later).**

**Projecting the amplitude onto I=J=0, we obtain**

$$T_{\text{tree}}(s;x) = \frac{m_\sigma^2-m_\pi^2}{\langle\sigma\rangle^2}\left[\frac{2s-m_\pi^2}{m_\sigma^2-m_\pi^2}(1-x)-5x -3x\frac{m_\sigma^2-m_\pi^2}{s-m_\sigma^2} - 2x\frac{m_\sigma^2-m_\pi^2}{s-4m_\pi^2}\ln\left(\frac{m_\sigma^2}{m_\sigma^2+s-4m_\pi^2}\right)\right]$$

# Unitarization

**Unitarity of S-matrix : conservation of probability.**
**Tree-level amplitude violates unitarity at certain energy.**

**Optical theorem :**

$$\mathrm{Im}\ T^{-1}(s) = -\frac{\Theta(s)}{2} \quad \text{for} \quad s > 4m_\pi^2 \qquad \Theta(s) = (16\pi)^{-1}\sqrt{1 - \frac{4m_\pi^2}{s}}$$

**Scattering amplitude (N/D method + matching with $T_{tree}$)**

J. A. Oller, E. Oset, Phys. Rev. D60, 074023 (1999)

$$T(s;x) = \frac{1}{T_{\mathrm{tree}}^{-1}(s;x) + G(s)}$$

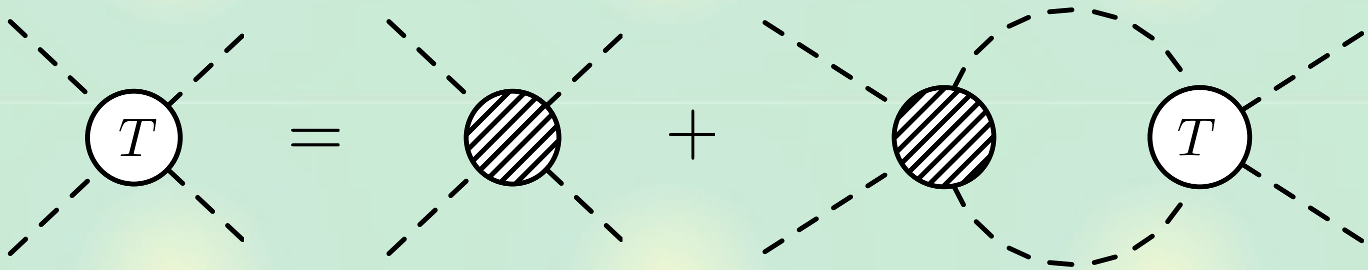
$$G(s) = \frac{1}{2}\frac{1}{(4\pi)^2}\left\{ \boxed{a(\mu)} + \ln\frac{m_\pi^2}{\mu^2} + \sqrt{1 - \frac{4m_\pi^2}{s}}\left[\ln\frac{\sqrt{1-\frac{4m_\pi^2}{s}}+1}{\sqrt{1-\frac{4m_\pi^2}{s}}-1}\right]\right\}$$

- **Left hand cut (crossed diagrams) is neglected.**
- **zeroth N/D iteration (N=1); c.f. single iteration (N=$T_{tree}$).**

K. Igi, K. Hikasa, Phys. Rev. D59, 034005 (1999)

# Renormalization

## Dynamical model with chiral symmetry.




**With sufficient attraction, a resonance can be generated.**

**Single-subtraction <=> log divergence of loop function**
**We determine the cutoff degree of freedom as**

$$G(s) = 0 \quad \text{at} \quad s = m_\pi^2 \qquad\qquad a(m_\pi) = -\frac{\pi}{\sqrt{3}}$$

**Exclude the CDD pole contribution from the loop function**

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C78, 025203 (2008)

**Consistency of the amplitude with chiral low energy theorem**

K. Igi, K. Hikasa, Phys. Rev. D59, 034005 (1999)

**Crossing symmetry (matching with u-channel amplitude)**

M.F.M. Lutz, E. Kolomeitsev, Nucl. Phys. A700, 193 (2002)

# Prescription for symmetry restoration

**We introduce the effect of chiral symmetry restoration from the outside of the model, by modifying $m_\pi$, $m_\sigma$, $\langle\sigma\rangle$.**

**1) chiral condensate (pion decay constant) : decreases**

$$\langle\sigma\rangle = \Phi\langle\sigma\rangle_0, \quad 0 \le \Phi \le 1$$

**2) mass of pion : no change**

$$\frac{\partial m_\pi}{\partial \Phi} = 0$$

**3) mass of sigma -- two possibilities**

**- case I (chiral sigma : decreases)**

$$m_\sigma\Big|_{\Phi\to 0} = m_\pi \quad (\text{case I}) \qquad m_\sigma = \sqrt{\lambda\frac{\langle\sigma\rangle^2}{3} + m_\pi^2}$$

**- case II (CDD pole : no change)**

$$\frac{\partial m_\sigma}{\partial \Phi} = 0 \quad (\text{case II})$$

# Restoration limit and chiral partner

**Properties of the chiral partner in the restoration limit**

- mass degeneracy with pion
- coupling to π-π scattering state vanishes

**Model for chiral sigma (x=1, case I ~ linear sigma model)**

- pole term in the tree-level interaction

$$T_{\text{tree}}(s;1) = -\frac{\lambda^2\langle\sigma\rangle^2}{3}\frac{1}{s - m_\pi^2 - \frac{\lambda}{3}\langle\sigma\rangle^2} + \dots$$

- renormalization condition

$$G(s) = 0 \quad \text{at} \quad s = m_\pi^2$$

$$T(m_\pi^2;1)|_{\Phi\to 0} = T_{\text{tree}}(m_\pi^2;1)|_{\Phi\to 0} \equiv -\frac{g^2}{s - M_{\text{pole}}^2}$$

**Thus, the pole in the π-π amplitude behaves as**

$g \to 0, \quad M_{\text{pole}} \to m_\pi \quad \text{for} \quad \Phi \to 0$ **like the chiral partner**

# Restoration limit and chiral partner

## Model for dynamical sigma and/or CDD pole (case II)

$$T_{\text{tree}}(s;x) \propto \frac{1}{\langle\sigma\rangle^2}$$

**The amplitude in the restoration limit is solely determined by the loop function G, irrespective to x:**

$$T(s;x) = \frac{1}{T_{\text{tree}}^{-1}(s,x) + G(s)} \to \frac{1}{G(s)} \quad \text{for} \quad \Phi \to 0$$

**- renormalization condition requires a pole at $m_\pi$**

$$G(s) = 0 \quad \text{at} \quad s = m_\pi^2$$

**- coupling can be calculated : proportional to $m_\pi$**

$$g^2\big|_{\Phi\to 0} = -(s - m_\pi^2)T(s)\big|_{s\to m_\pi^2, \Phi\to 0}$$

$$= -\left.\frac{s - m_\pi^2}{G(s)}\right|_{s\to m_\pi^2} = (4\pi)^2\left(\frac{\pi}{3\sqrt{3}} - \frac{1}{2}\right)^{-1} m_\pi^2$$

**How to interpret this result?**

# Restoration limit and chiral partner

## Properties of chiral sigma

$$g \to 0, \quad M_{\rm pole} \to m_\pi \quad \text{for} \quad \Phi \to 0$$

## Properties of dynamical sigma

$$g^2 \to (4\pi)^2 \left( \frac{\pi}{3\sqrt{3}} - \frac{1}{2} \right)^{-1} m_\pi^2, \quad M_{\rm pole} \to m_\pi \quad \text{for} \quad \Phi \to 0$$

- mass degenerates with pion
- coupling to π-π : vanishes in the chiral limit

## Dynamical sigma as chiral partner of pion?

- Renormalization condition plays an important role.
  <-- consistency with chiral theorem. Generally,

$G(\mu^2) = 0 \quad \text{at} \quad 0 \le \mu \le 2m_\pi$ --> deviation ~ $m_\pi$

- dynamical resonance as chiral partner (mass degeneracy)

J.A. Oller, hep-ph/0007349

S. Leupold, M.F.M. Lutz, M. Wagner, 0811.2398 [nucl-th]

# Model setup

**We numerically analyze four models:**

| | x | $m_\sigma$ | sigma origin |
|---|---|---|---|
| model A | 1 | case I | chiral |
| model B | 0 | - | dynamical |
| model C | 1 | case II | CDD |
| model D | 1/2 | case II | CDD + dynamical |

**model C : same with model A, but $m_\sigma$ unchanged.**
**pole term + repulsion (c.f. linear sigma model)**

$$T_{\rm tree}(s;x) \equiv T^{\rm (contact)}_{\rm tree}(s;x) + T^{\rm (pole)}_{\rm tree}(s;x)$$

**model D : pole term + attraction**

**existence of dynamical state <--> sign of the contact term**

# Results in vacuum

## Input: $m_\pi = 140$ MeV, $m_\sigma = 550$ MeV, $\langle\sigma\rangle = 93$ MeV

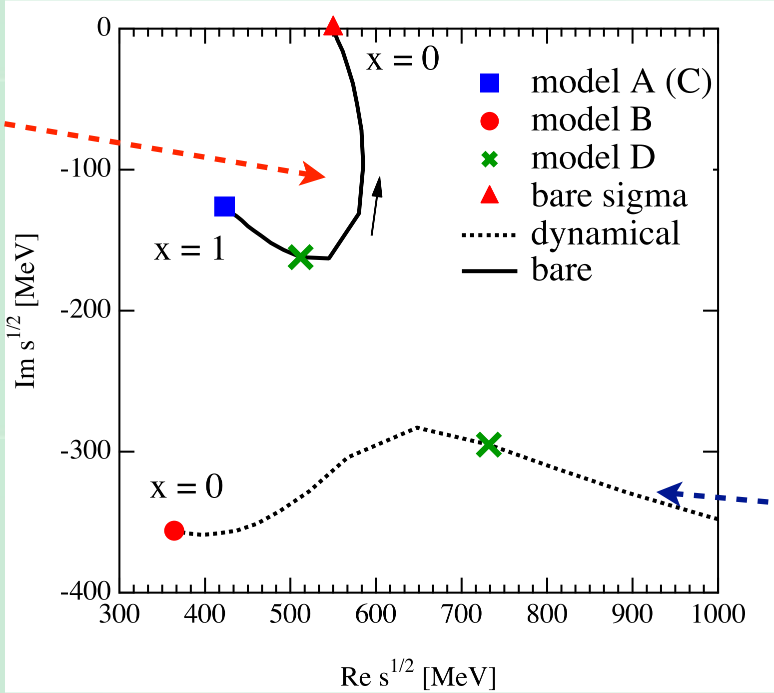
| | scattering length $(m_\pi)^{-1}$ | pole position [MeV] |
|---|---|---|
| model A, C | 0.244 | 423 - 126 i |
| model B | 0.174 | 364 - 356 i |
| model D | 0.208 | 512 - 162 i, 732 - 295 i |
| (experiment) | 0.216 [1] | 441 - 272 i [2] |

[1] S. Pislak *et al.*, Phys. Rev. D67, 072004 (2003)
[2] I. Caprini, G. Colangelo, H. Leutwyler, Phys. Rev. Lett. 96, 132001 (2006)

# Model extrapolations and origin of the pole

## Poles in the complex energy plane

bare pole




dynamical

Trace pole position of model A (x=1) with x -> 0

: approaches to bare pole, through one pole in model D

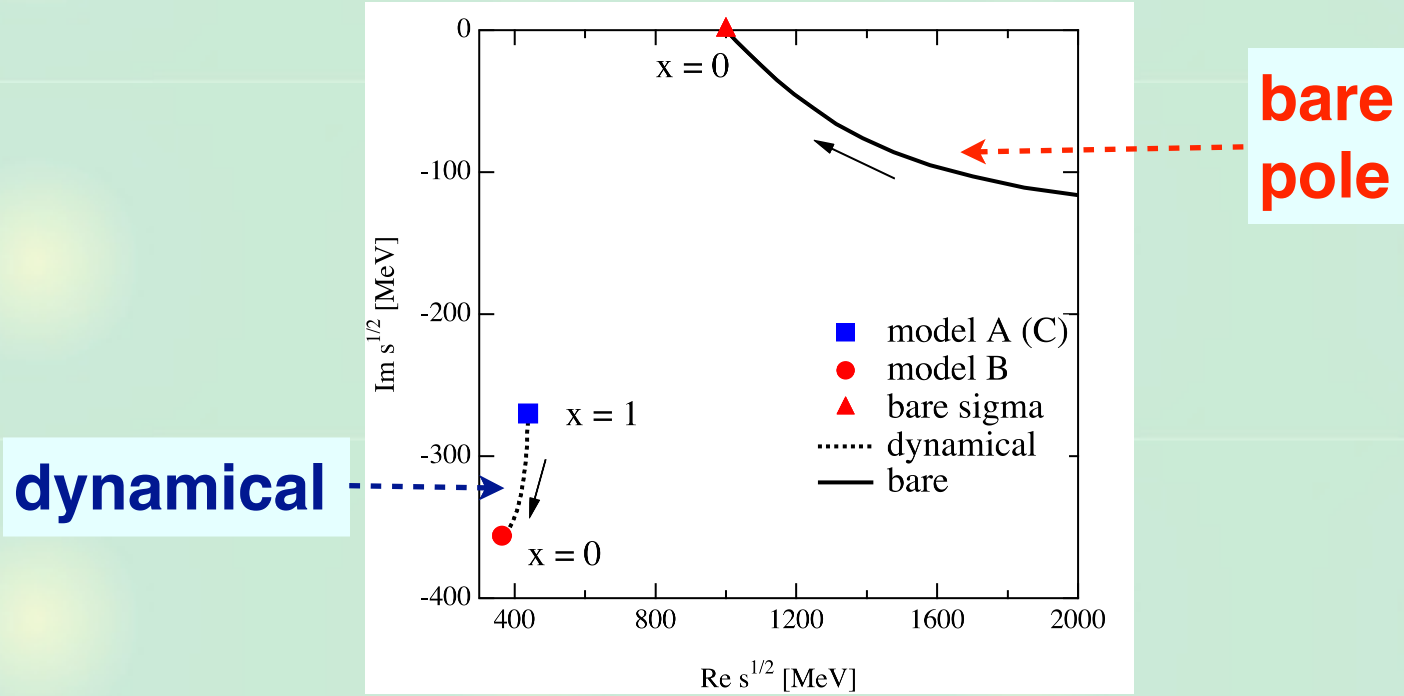
Trace pole position of model B (x=0) with x -> 1

: dissolves into continuum, through one pole in model D

Poles in model D: one bare pole origin, one dynamical.

# Model extrapolations and origin of the pole

**If $m_\sigma = 1$ GeV, then**

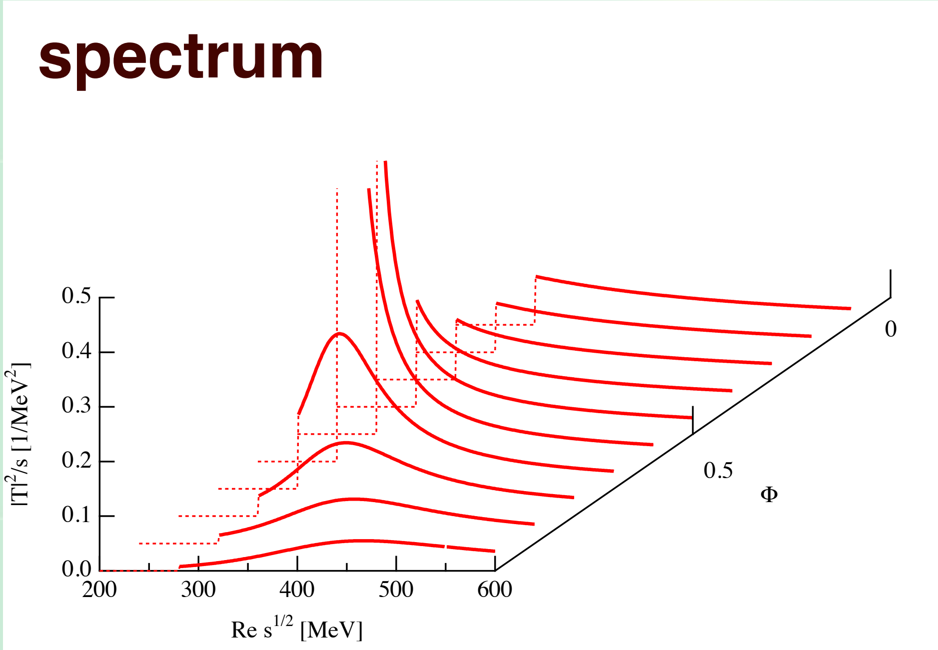


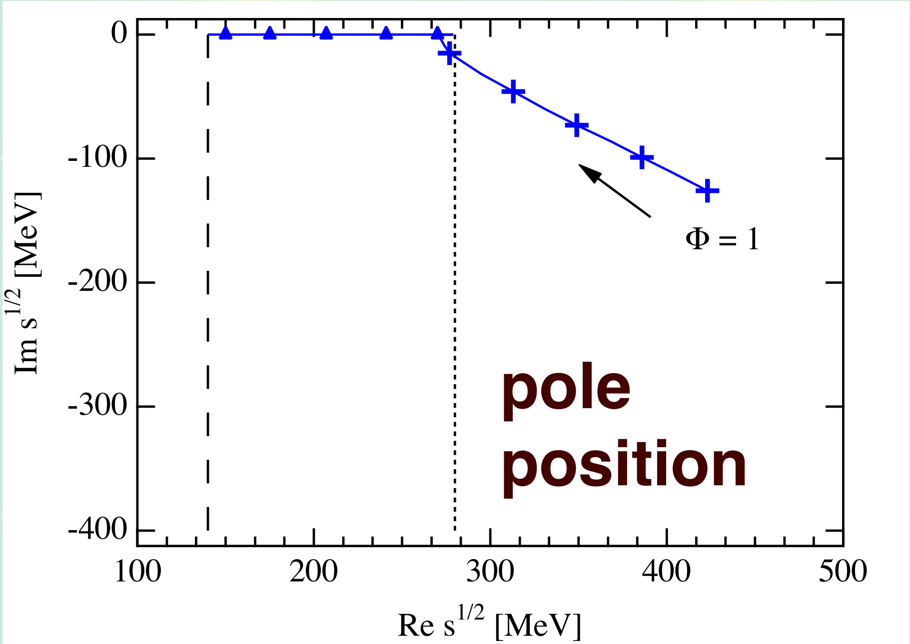

**A bare pole at sufficiently high energy than the energy region under consideration ==> effective attraction**

$$-G^2\frac{1}{s-m_\sigma^2} = G^2\frac{1}{m_\sigma^2}\left(1+\frac{s}{m_\sigma^2}+\cdots\right) \quad \text{for} \quad s \ll m_\sigma^2 .$$

**In this case the origin of the pole in model A(C) is dynamical.**

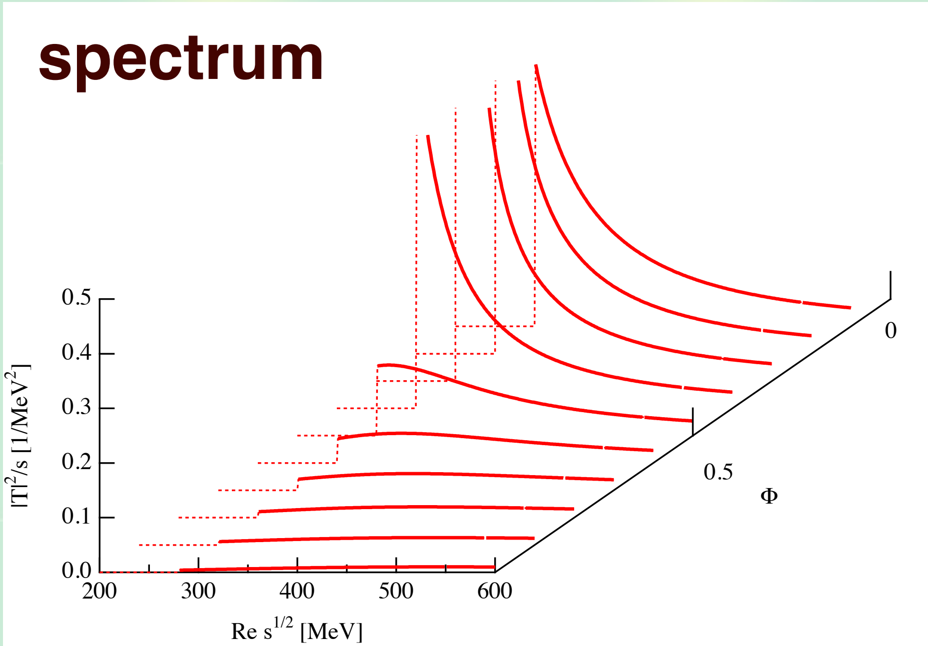
# Results in model A

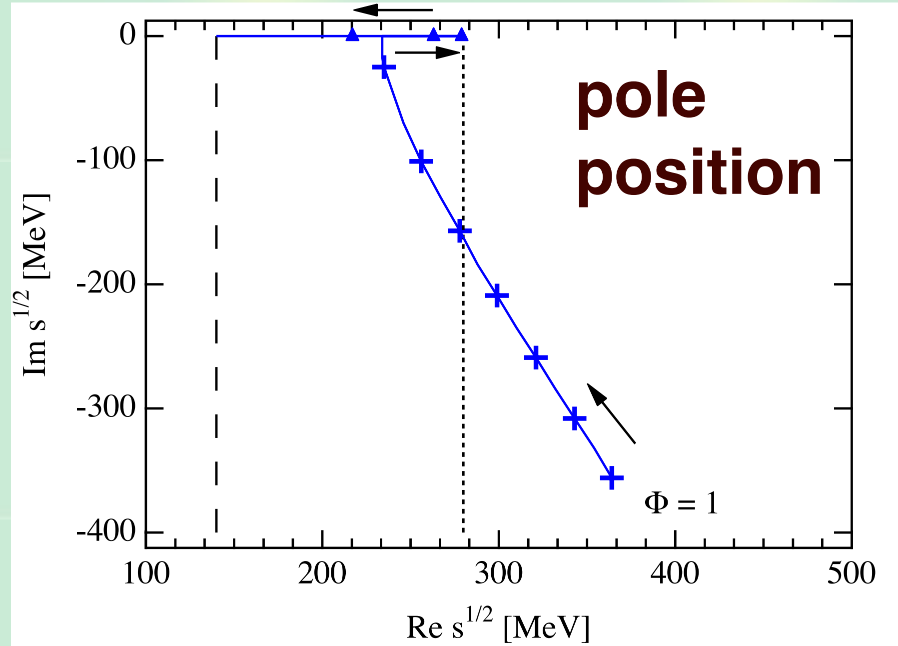



- Linear sigma model + unitarization : **chiral sigma**
- Softening takes place, as expected.
- peak at threshold : **$\Phi \sim 0.6$**
  <=> bare sigma pole moves below the threshold
- $M_{pole}$ -> $m_\pi$ as $\Phi$ -> 0

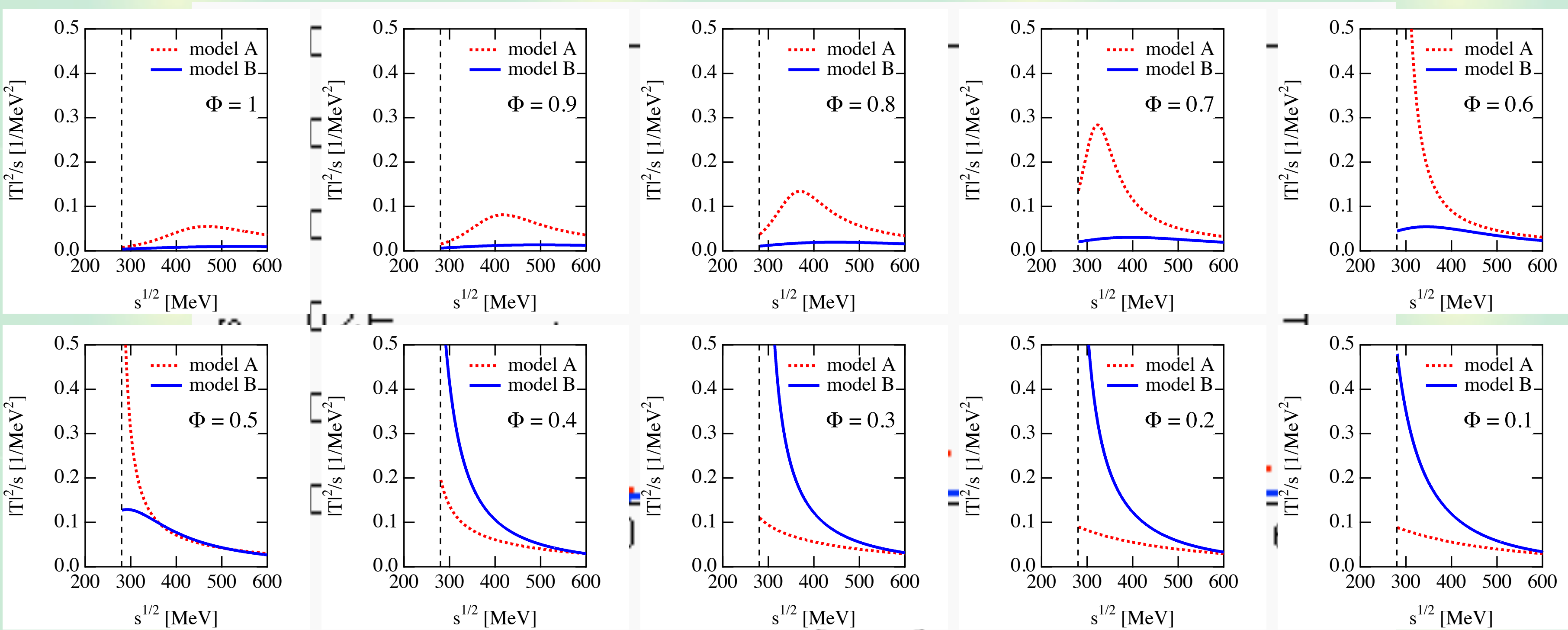
# Results in model B



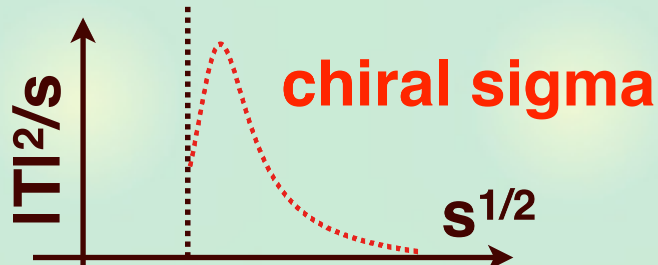



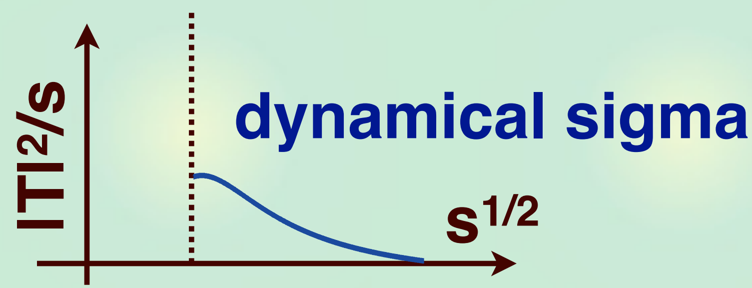

- ChPT + unitarization : **dynamical sigma**
- Softening takes place, but **virtual state** appears.
- at $\mathrm{Re}[M_{pole}] = 2m_\pi$ ($\Phi \sim 0.6$), due to finite width, spectrum does not show the peak structure
- peak at threshold : **$\Phi \sim 0.3$** <=> formation of bound state
- $M_{pole} \to m_\pi$ as $\Phi \to 0$

# Comparison of model A and model B

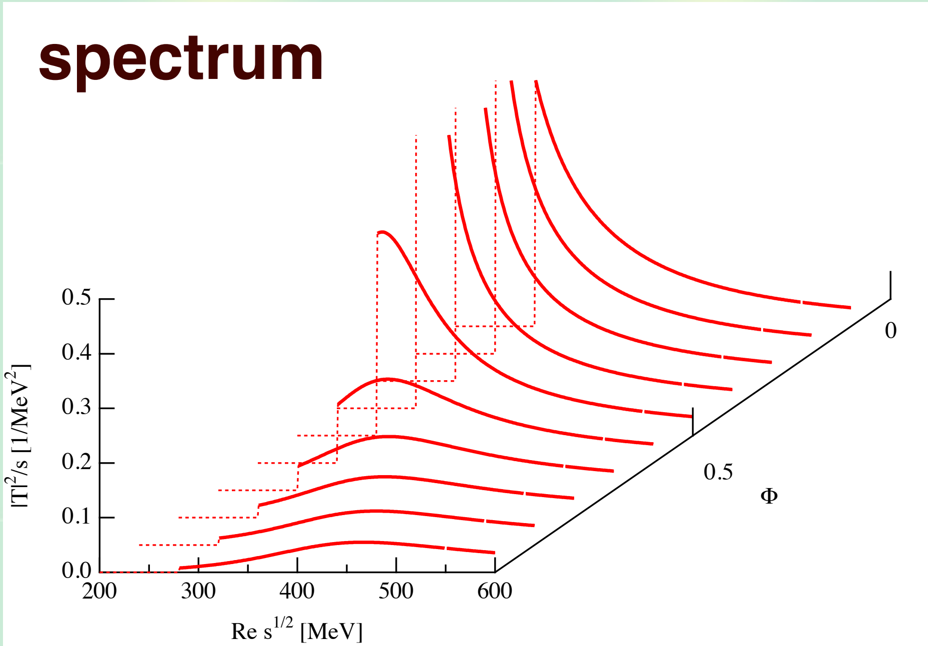


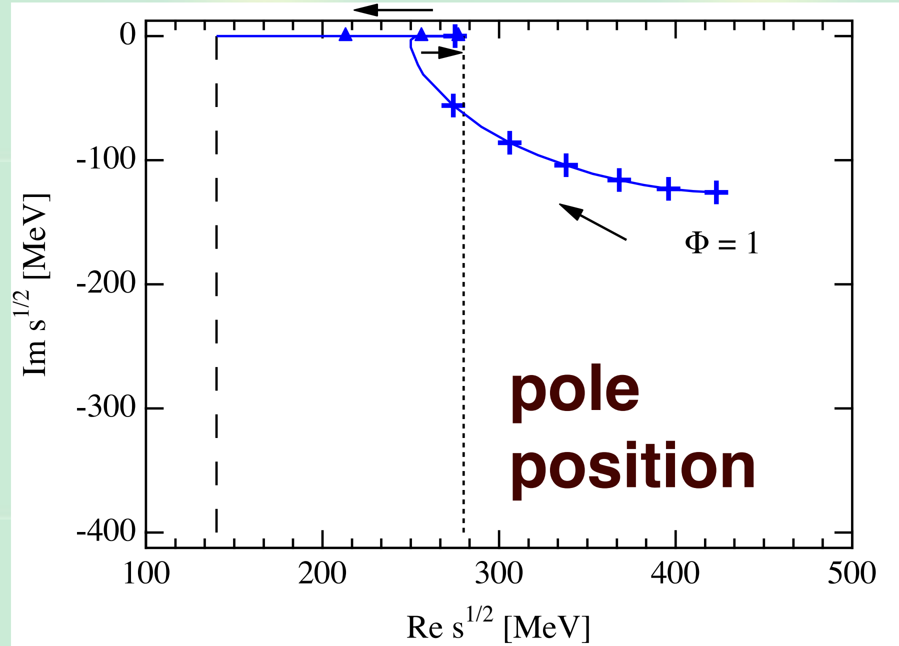

- **Strong threshold enhancement : different from each other.**
- **Shape of the spectrum?**



# Results in model C




- Bare pole + unitarization : CDD pole

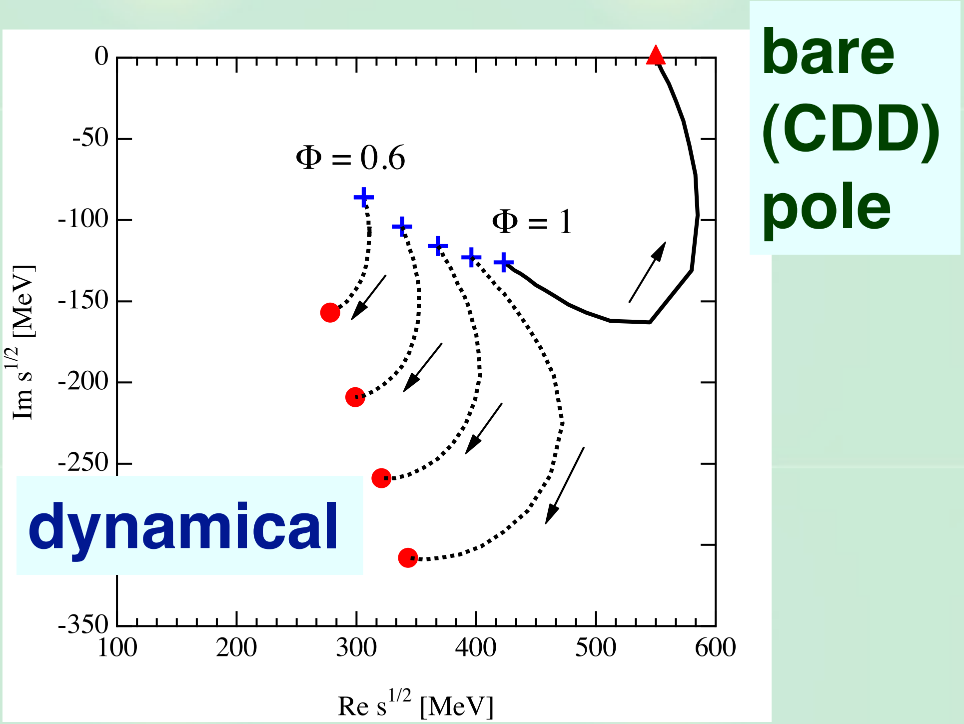
- Softening : similar to dynamical sigma (virtual state).
  <--> bare pole origin ??

- peak at threshold : $\Phi \sim 0.3$ <=> formation of bound state

- $M_{pole}$ -> $m_\pi$ as $\Phi$ -> 0

# Property change of the pole

## Extrapolation to x=0 for 1 ≧ Φ ≧ 0.6

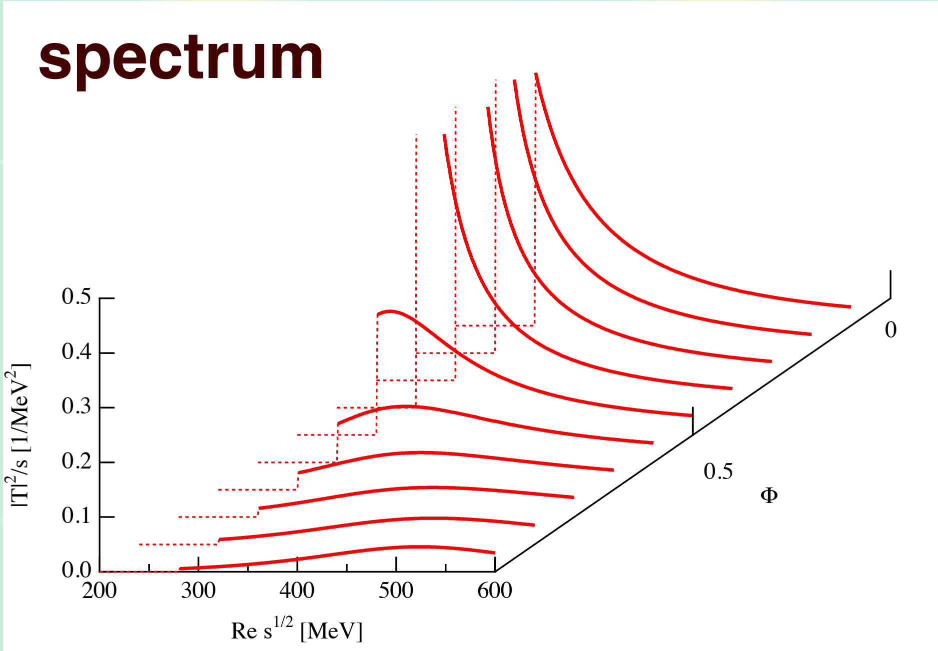


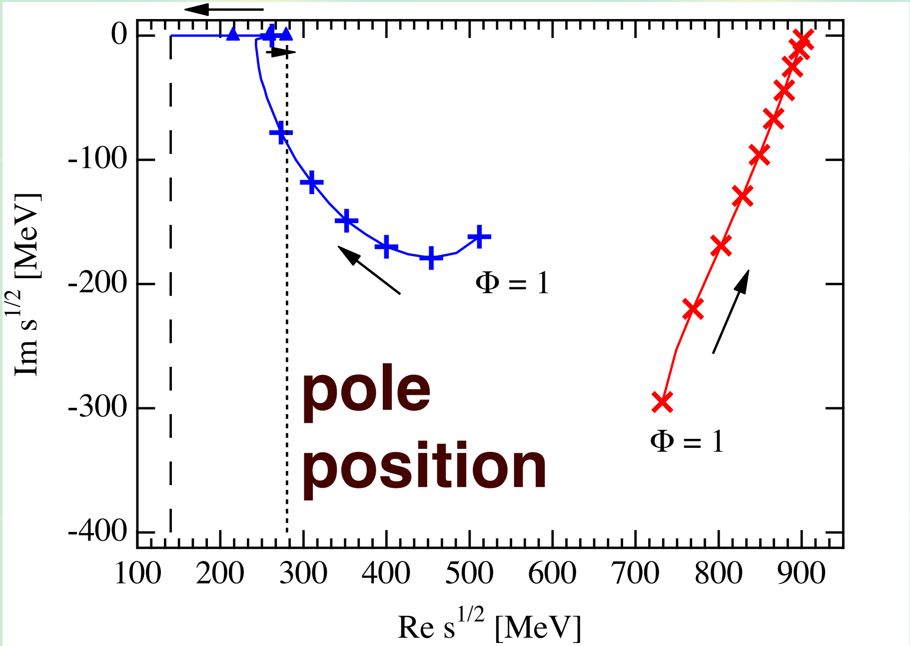

**Symmetry restoration --> pole moves to lower energy**

**Bare pole unchanged, so it behaves as an effective attraction.**

**Origin of the pole : CDD pole --> dynamical sigma.**

# Results in model D




- Pole + attraction + unitarization : CDD pole + dynamical
- Softening : similar to dynamical sigma
- the other pole : goes to a kinematical singularity, reducing residue (irrelevant for spectrum).
- $M_{pole}$ -> $m_\pi$ as Φ -> 0

# Summary : chiral dynamics and sigma meson

**We study the structure of the sigma meson with chiral symmetry restoration.**

- We classify the possible structure of the sigma meson into three classes:
  - (i) chiral sigma
  - (ii) dynamical sigma
  - (iii) CDD pole contribution
- We construct two-flavor dynamical chiral models which account for them.
- Dynamical sigma (s-wave resonance) is expected to behave differently.

# Summary : results

- **In the chiral restoration limit:**

  Mass and coupling of the dynamical sigma behave similarly with chiral sigma, in the chiral limit. Chiral partner?

- **Softening phenomena:**

  Dynamical sigma softens qualitatively differently from chiral sigma.
  <-- virtual state (s-wave resonance)

  CDD pole case reduces to dynamical case around threshold. Universality?