# Exotic hadrons in s-wave chiral dynamics

**Tetsuo Hyodo[a,b],**

**Daisuke Jido[b], and Atsushi Hosaka[c]**

*TU München[a]  YITP, Kyoto[b]  RCNP, Osaka[c]*

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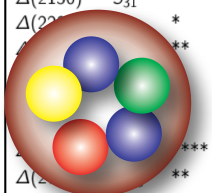
# Exotic hadrons in hadron spectrum

## Observed hadrons in experiments (PDG06) :

| | | |
|---|---|---|
| $p$ | $P_{11}$ | **** |
| $n$ | $P_{11}$ | **** |
| $N(1440)$ | $P_{11}$ | **** |
| $N(1520)$ | $D_{13}$ | **** |
| $N(1535)$ | $S_{11}$ | **** |
| $N(1650)$ | $S_{11}$ | **** |
| $N(1675)$ | $D_{15}$ | **** |
| $N(1680)$ | $F_{15}$ | **** |
| $N(1700)$ | $D_{13}$ | *** |
| $N(1710)$ | $P_{11}$ | *** |
| $N(1720)$ | $P_{13}$ | **** |
| $N(1900)$ | $P_{13}$ | ** |
| $N(1990)$ | $F_{17}$ | ** |
| $N(2000)$ | $F_{15}$ | ** |
| $N(2080)$ | $D_{13}$ | ** |
| $N(2090)$ | $S_{11}$ | * |
| $N(2100)$ | $P_{11}$ | * |
| $N(2190)$ | $G_{17}$ | **** |
| $N(2200)$ | $D_{15}$ | ** |
| $N(2220)$ | $H_{19}$ | **** |
| $N(2250)$ | $G_{19}$ | **** |
| $N(2600)$ | $I_{1,11}$ | *** |
| $N(2700)$ | $K_{1,13}$ | ** |

| | | |
|---|---|---|
| $\Delta(1232)$ | $P_{33}$ | **** |
| $\Delta(1600)$ | $P_{33}$ | *** |
| $\Delta(1620)$ | $S_{31}$ | **** |
| $\Delta(1700)$ | $D_{33}$ | **** |
| $\Delta(1750)$ | $P_{31}$ | * |
| $\Delta(1900)$ | $S_{31}$ | ** |
| $\Delta(1905)$ | $F_{35}$ | **** |
| $\Delta(1910)$ | $P_{31}$ | **** |
| $\Delta(1920)$ | $P_{33}$ | *** |
| $\Delta(1930)$ | $D_{35}$ | *** |
| $\Delta(1940)$ | $D_{33}$ | * |
| $\Delta(1950)$ | $F_{37}$ | **** |
| $\Delta(2000)$ | $F_{35}$ | ** |
| $\Delta(2150)$ | $S_{31}$ | * |
| $\Delta(22$[illegible] | [illegible] | * |
| [illegible] | [illegible] | ** |
| [illegible] | [illegible] | [illegible] |
| [illegible] | [illegible] | [illegible] |
| [illegible] | [illegible] | [illegible] |
| [illegible] | [illegible] | *** |
| $\Delta(2$[illegible] | [illegible] | ** |
| $\Delta(2950)$ | $K_{3,15}$ | ** |
| $\Theta(1540)^+$ | | * |



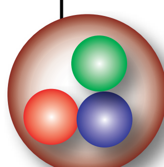
$\sim \frac{1}{300}$ !

| | | |
|---|---|---|
| $\Lambda$ | $P_{01}$ | **** |
| $\Lambda(1405)$ | $S_{01}$ | **** |
| $\Lambda(1520)$ | $D_{03}$ | **** |
| $\Lambda(1600)$ | $P_{01}$ | *** |
| $\Lambda(1670)$ | $S_{01}$ | **** |
| $\Lambda(1690)$ | $D_{03}$ | **** |
| $\Lambda(1800)$ | $S_{01}$ | *** |
| $\Lambda(1810)$ | $P_{01}$ | *** |
| $\Lambda(1820)$ | $F_{05}$ | **** |
| $\Lambda(1830)$ | $D_{05}$ | **** |
| $\Lambda(1890)$ | $P_{03}$ | **** |
| $\Lambda(2000)$ | | * |
| $\Lambda(2020)$ | $F_{07}$ | * |
| $\Lambda(2100)$ | $G_{07}$ | **** |
| $\Lambda(2110)$ | $F_{05}$ | *** |
| $\Lambda(2325)$ | $D_{03}$ | * |
| $\Lambda(2350)$ | $H_{09}$ | *** |
| $\Lambda(2585)$ | | ** |

| | | |
|---|---|---|
| $\Sigma^+$ | $P_{11}$ | **** |
| $\Sigma^0$ | $P_{11}$ | **** |
| $\Sigma^-$ | $P_{11}$ | **** |
| $\Sigma(1385)$ | $P_{13}$ | **** |
| $\Sigma(1480)$ | | * |
| $\Sigma(1560)$ | | ** |
| $\Sigma(1580)$ | $D_{13}$ | * |
| $\Sigma(1620)$ | $S_{11}$ | ** |
| $\Sigma(1660)$ | $P_{11}$ | *** |
| $\Sigma(1670)$ | $D_{13}$ | **** |
| $\Sigma(1690)$ | | ** |
| $\Sigma(1750)$ | $S_{11}$ | *** |
| $\Sigma(1770)$ | $P_{11}$ | * |
| $\Sigma(1775)$ | $D_{15}$ | **** |
| $\Sigma(1840)$ | $P_{13}$ | * |
| $\Sigma(1880)$ | $P_{11}$ | ** |
| $\Sigma(1915)$ | $F_{15}$ | **** |
| $\Sigma(1940)$ | $D_{13}$ | *** |
| $\Sigma(2000)$ | $S_{11}$ | * |
| $\Sigma(2030)$ | $F_{17}$ | **** |
| $\Sigma(2070)$ | $F_{15}$ | * |
| $\Sigma(2080)$ | $P_{13}$ | ** |
| $\Sigma(2100)$ | $G_{17}$ | * |
| $\Sigma(2250)$ | | *** |
| $\Sigma(2455)$ | | ** |
| $\Sigma(2620)$ | | ** |
| $\Sigma(3000)$ | | * |
| $\Sigma(3170)$ | | * |

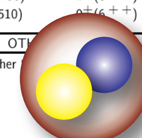
| | | |
|---|---|---|
| $\Xi^0$ | $P_{11}$ | **** |
| $\Xi^-$ | $P_{11}$ | **** |
| $\Xi(1530)$ | $P_{13}$ | **** |
| $\Xi(1620)$ | | * |
| $\Xi(1690)$ | | *** |
| $\Xi(1820)$ | $D_{13}$ | *** |
| $\Xi(1950)$ | | *** |
| $\Xi(2030)$ | | *** |
| $\Xi(2120)$ | | * |
| $\Xi(2250)$ | | ** |
| $\Xi(2370)$ | | ** |
| $\Xi(2500)$ | | * |
| $\Omega^-$ | | **** |
| $\Omega(2250)^-$ | | *** |
| $\Omega(2380)^-$ | | ** |
| $\Omega(2470)^-$ | | ** |
| $\Lambda_c^+$ | | **** |
| $\Lambda_c(2593)^+$ | | *** |
| $\Lambda_c(2625)^+$ | | *** |
| $\Lambda_c(2765)^+$ | | * |
| $\Lambda_c(2880)^+$ | | ** |
| $\Sigma_c(2455)$ | | **** |
| $\Sigma_c(2520)$ | | *** |
| $\Sigma_c(2800)$ | | *** |
| $\Xi_c^+$ | | *** |
| $\Xi_c^0$ | | *** |
| $\Xi_c'^+$ | | *** |
| $\Xi_c'^0$ | | *** |
| $\Xi_c(2645)$ | | *** |
| $\Xi_c(2790)$ | | *** |
| $\Xi_c(2815)$ | | *** |
| $\Omega_c^0$ | | *** |
| $\Xi_{cc}^+$ | | * |
| $\Lambda_b^0$ | | *** |

**~130 baryons**



LIGHT UNFLAVORED ($S = C = B = 0$)

| | $I^G(J^{PC})$ | | $I^G(J^{PC})$ |
|---|---|---|---|
| • $\pi^\pm$ | $1^-(0^-)$ | • $\pi_2(1670)$ | $1^-(2^{-+})$ |
| • $\pi^0$ | $1^-(0^{-+})$ | • $\phi(1680)$ | $0^-(1^{--})$ |
| • $\eta$ | $0^+(0^{-+})$ | • $\rho_3(1690)$ | $1^+(3^{--})$ |
| • $f_0(600)$ | $0^+(0^{++})$ | • $\rho(1700)$ | $1^+(1^{--})$ |
| • $\rho(770)$ | $1^+(1^{--})$ | $a_2(1700)$ | $1^-(2^{++})$ |
| • $\omega(782)$ | $0^-(1^{--})$ | • $f_0(1710)$ | $0^+(0^{++})$ |
| • $\eta'(958)$ | $0^+(0^{-+})$ | $\eta(1760)$ | $0^+(0^{-+})$ |
| • $f_0(980)$ | $0^+(0^{++})$ | • $\pi(1800)$ | $1^-(0^{-+})$ |
| • $a_0(980)$ | $1^-(0^{++})$ | $f_2(1810)$ | $0^+(2^{++})$ |
| • $\phi(1020)$ | $0^-(1^{--})$ | $X(1835)$ | $?^?(?^{-+})$ |
| • $h_1(1170)$ | $0^-(1^{+-})$ | • $\phi_3(1850)$ | $0^-(3^{--})$ |
| • $b_1(1235)$ | $1^+(1^{+-})$ | $\eta_2(1870)$ | $0^+(2^{-+})$ |
| • $a_1(1260)$ | $1^-(1^{++})$ | $\rho(1900)$ | $1^+(1^{--})$ |
| • $f_2(1270)$ | $0^+(2^{++})$ | $f_2(1910)$ | $0^+(2^{++})$ |
| • $f_1(1285)$ | $0^+(1^{++})$ | • $f_2(1950)$ | $0^+(2^{++})$ |
| • $\eta(1295)$ | $0^+(0^{-+})$ | $\rho_3(1990)$ | $1^+(3^{--})$ |
| • $\pi(1300)$ | $1^-(0^{-+})$ | • $f_2(2010)$ | $0^+(2^{++})$ |
| • $a_2(1320)$ | $1^-(2^{++})$ | $f_0(2020)$ | $0^+(0^{++})$ |
| • $f_0(1370)$ | $0^+(0^{++})$ | • $a_4(2040)$ | $1^-(4^{++})$ |
| $h_1(1380)$ | $?^-(1^{+-})$ | • $f_4(2050)$ | $0^+(4^{++})$ |
| • $\pi_1(1400)$ | $1^-(1^{-+})$ | $\pi_2(2100)$ | $1^-(2^{-+})$ |
| • $\eta(1405)$ | $0^+(0^{-+})$ | $f_0(2100)$ | $0^+(0^{++})$ |
| • $f_1(1420)$ | $0^+(1^{++})$ | $f_2(2150)$ | $0^+(2^{++})$ |
| • $\omega(1420)$ | $0^-(1^{--})$ | $\rho(2150)$ | $1^+(1^{--})$ |
| $f_2(1430)$ | $0^+(2^{++})$ | $f_0(2200)$ | $0^+(0^{++})$ |
| • $a_0(1450)$ | $1^-(0^{++})$ | $f_J(2220)$ | $0^+(2\text{ or }4^{++})$ |
| • $\rho(1450)$ | $1^+(1^{--})$ | $\eta(2225)$ | $0^+(0^{-+})$ |
| • $\eta(1475)$ | $0^+(0^{-+})$ | $\rho_3(2250)$ | $1^+(3^{--})$ |
| • $f_0(1500)$ | $0^+(0^{++})$ | • $f_2(2300)$ | $0^+(2^{++})$ |
| $f_1(1510)$ | $0^+(1^{++})$ | $f_4(2300)$ | $0^+(4^{++})$ |
| • $f_2'(1525)$ | $0^+(2^{++})$ | • $f_2(2340)$ | $0^+(2^{++})$ |
| $f_2(1565)$ | $0^+(2^{++})$ | $\rho_5(2350)$ | $1^+(5^{--})$ |
| $h_1(1595)$ | $0^-(1^{+-})$ | $a_6(2450)$ | $1^-(6^{++})$ |
| • $\pi_1(1600)$ | $1^-(1^{-+})$ | $f_6(2510)$ | $0^+(6^{++})$ |
| $a_1(1640)$ | $1^-(1^{++})$ | OTH[illegible] | |
| $f_2(1640)$ | $0^+(2^{++})$ | Further [illegible] | |
| • $\eta_2(1645)$ | $0^+(2^{-+})$ | | |
| • $\omega(1650)$ | $0^-(1^{--})$ | | |
| • $\omega_3(1670)$ | $0^-(3^{--})$ | | |



STRANGE ($S = \pm1$, $C = B = 0$)

| | $I(J^P)$ |
|---|---|
| • $K^\pm$ | $1/2(0^-)$ |
| • $K^0$ | $1/2(0^-)$ |
| • $K^0_S$ | $1/2(0^-)$ |
| • $K^0_L$ | $1/2(0^-)$ |
| $K_0^*(800)$ | $1/2(0^+)$ |
| • $K^*(892)$ | $1/2(1^-)$ |
| • $K_1(1270)$ | $1/2(1^+)$ |
| • $K_1(1400)$ | $1/2(1^+)$ |
| • $K^*(1410)$ | $1/2(1^-)$ |
| • $K_0^*(1430)$ | $1/2(0^+)$ |
| • $K_2^*(1430)$ | $1/2(2^+)$ |
| $K(1460)$ | $1/2(0^-)$ |
| $K_2(1580)$ | $1/2(2^-)$ |
| $K(1630)$ | $1/2(?^?)$ |
| $K_1(1650)$ | $1/2(1^+)$ |
| • $K^*(1680)$ | $1/2(1^-)$ |
| • $K_2(1770)$ | $1/2(2^-)$ |
| • $K_3^*(1780)$ | $1/2(3^-)$ |
| • $K_2(1820)$ | $1/2(2^-)$ |
| $K(1830)$ | $1/2(0^-)$ |
| $K_0^*(1950)$ | $1/2(0^+)$ |
| $K_2^*(1980)$ | $1/2(2^+)$ |
| • $K_4^*(2045)$ | $1/2(4^+)$ |
| $K_2(2250)$ | $1/2(2^-)$ |
| $K_3(2320)$ | $1/2(3^+)$ |
| $K_5^*(2380)$ | $1/2(5^-)$ |
| $K_4(2500)$ | $1/2(4^-)$ |
| $K(3100)$ | $?^?(?^{??})$ |

CHARMED ($C = \pm1$)

| | |
|---|---|
| • $D^\pm$ | $1/2(0^-)$ |
| • $D^0$ | $1/2(0^-)$ |
| • $D^*(2007)^0$ | $1/2(1^-)$ |
| • $D^*(2010)^\pm$ | $1/2(1^-)$ |
| $D_0^*(2400)^0$ | $1/2(0^+)$ |
| $D_0^*(2400)^\pm$ | $1/2(0^+)$ |
| • $D_1(2420)^0$ | $1/2(1^+)$ |
| $D_1(2420)^\pm$ | $1/2(?^?)$ |
| $D_1(2430)^0$ | $1/2(1^+)$ |
| • $D_2^*(2460)^0$ | $1/2(2^+)$ |
| • $D_2^*(2460)^\pm$ | $1/2(2^+)$ |
| $D^*(2640)^\pm$ | $1/2(?^?)$ |

CHARME[illegible], STRA[illegible]GE ([illegible] $S = \pm1$)

| | |
|---|---|
| • $D_s^\pm$ | $0(0^-)$ |
| • $D_s^{*\pm}$ | $0(?^?)$ |
| • $D_{s0}^*(2317)^\pm$ | $0(0^+)$ |
| • $D_{s1}(2460)^\pm$ | $0(1^+)$ |
| • $D_{s1}(2536)^\pm$ | $0(1^+)$ |

BOTTOM ($B = \pm1$)

| | $I^G(J^{PC})$ |
|---|---|
| • $B^\pm$ | $1/2(0^-)$ |
| • $B^0$ | $1/2(0^-)$ |
| • $B^\pm/B^0$ ADMIXTURE | |
| • $B^\pm/B^0/B^0_s/b$-baryon ADMIXTURE | |
| $V_{cb}$ and $V_{ub}$ CKM Matrix Elements | |
| • $B^*$ | $1/2(1^-)$ |
| $B_J^*(5732)$ | $?(?^?)$ |

BOTTOM, STRANGE ($B = \pm1$, $S = \mp1$)

| | |
|---|---|
| • $B_s^0$ | $0(0^-)$ |
| $B_s^*$ | $0(1^-)$ |
| $B_{sJ}^*(5850)$ | $?(?^?)$ |

BOTTOM, CHARMED ($B = C = \pm1$)

| | |
|---|---|
| • $B_c^\pm$ | $0(0^-)$ |

$c\overline{c}$

| | |
|---|---|
| • $\eta_c(1S)$ | $0^+(0^{-+})$ |
| • $J/\psi(1S)$ | $0^-(1^{--})$ |
| • $\chi_{c0}(1P)$ | $0^+(0^{++})$ |
| • $\chi_{c1}(1P)$ | $0^+(1^{++})$ |
| $h_c(1P)$ | $?^?(?^{??})$ |
| • $\chi_{c2}(1P)$ | $0^+(2^{++})$ |
| • $\eta_c(2S)$ | $0^+(0^{-+})$ |
| • $\psi(2S)$ | $0^-(1^{--})$ |
| • $\psi(3770)$ | $0^-(1^{--})$ |
| • $X(3872)$ | $0^?(?^{?+})$ |
| • $\chi_{c2}(2P)$ | $0^+(2^{++})$ |
| $Y(3940)$ | $?^?(?^{??})$ |
| • $\psi(4040)$ | $0^-(1^{--})$ |
| • $\psi(4160)$ | $0^-(1^{--})$ |
| $Y(4260)$ | $?^?(1^{--})$ |
| • $\psi(4415)$ | $0^-(1^{--})$ |

$b\overline{b}$

| | |
|---|---|
| $\eta_b(1S)$ | $0^+(0^{-+})$ |
| • $\Upsilon(1S)$ | $0^-(1^{--})$ |
| • $\chi_{b0}(1P)$ | $0^+(0^{++})$ |
| • $\chi_{b1}(1P)$ | $0^+(1^{++})$ |
| • $\chi_{b2}(1P)$ | $0^+(2^{++})$ |
| • $\Upsilon(2S)$ | $0^-(1^{--})$ |
| $\Upsilon(1D)$ | $0^-(2^{--})$ |
| • $\chi_{b0}(2P)$ | $0^+(0^{++})$ |
| • $\chi_{b1}(2P)$ | $0^+(1^{++})$ |
| • $\chi_{b2}(2P)$ | $0^+(2^{++})$ |
| • $\Upsilon(3S)$ | $0^-(1^{--})$ |
| • $\Upsilon(4S)$ | $0^-(1^{--})$ |
| • $\Upsilon(10860)$ | $0^-(1^{--})$ |
| • $\Upsilon(11020)$ | $0^-(1^{--})$ |

NON-$q\overline{q}$ CANDIDATES

**~160 mesons**

**Exotic hadrons are indeed exotic !!**

# Exotic hadrons : experiment vs theory

Exotic hadrons : valence quark-antiquark(s)

non-exotic $uds, \quad u\bar{d}, \quad uds\, u\bar{u}, \quad u\bar{d}\, u\bar{u}, \ldots$

exotic $uudd\bar{s}, \quad ud\bar{s}\bar{s}, \ldots$

Experimentally, they are exotic ~ 1/300.

Theoretically, are they exotic?

--> There is no simple way to forbid exotic states in QCD, effective models, ...

--> Evidences of multiquark components in non-exotic hadrons.

**Why aren't the exotics observed??**

# Chiral dynamics for non-exotic hadrons

## Hadron excited states ~ $\pi T$

- ○ Interaction <-- chiral symmetry
- ○ Amplitude <-- unitarity (coupled channel)

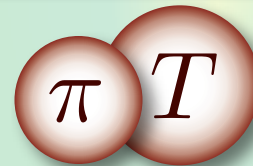
## With phenomenological vector meson exchange interaction

R.H. Dalitz, and S.F. Tuan, Ann. Phys. (N.Y.) 10, 307 (1960)
J.H.W. Wyld, Phys. Rev. 155, 1649 (1967)

## Chiral perturbation theory for interaction

N. Kaiser, P. B. Siegel and W. Weise, Nucl. Phys. A594, 325 (1995)
E. Oset and A. Ramos, Nucl. Phys. A635, 99 (1998)
J. A. Oller and U. G. Meissner, Phys. Lett. B500, 263 (2001)
M.F.M. Lutz and E. E. Kolomeitsev, Nucl. Phys. A700, 193 (2002)
....

# Chiral dynamics for non-exotic hadrons

## Hadron excited states ~ 


## Many hadron resonances are well described.

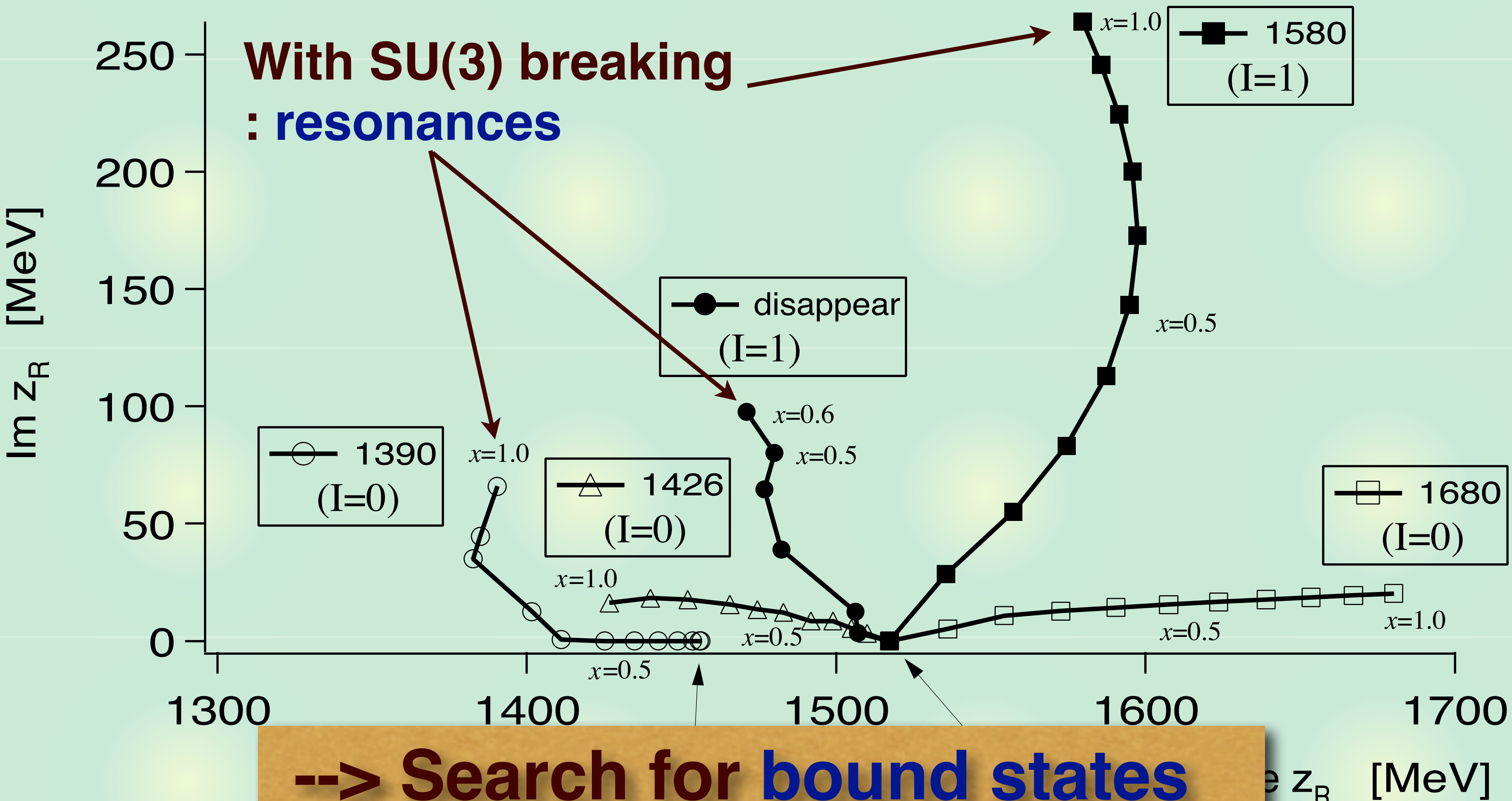
| | | |
|---|---|---|
| light baryon | $J^P = 1/2^-$ | $\Lambda(1405)$ $\Lambda(1670)$ $\Sigma(1670)$ |
| | | $N(1535)$ $\Xi(1620)$ $\Xi(1690)$ |
| | $J^P = 3/2^-$ | $\Lambda(1520)$ $\Xi(1820)$ $\Sigma(1670)$ |
| heavy | | $\Lambda_c(2880)$ $\Lambda_c(2593)$ $D_s(2317)$ |
| light meson | $J^P = 1^+$ | $b_1(1235)$ $h_1(1170)$ $h_1(1380)$ $a_1(1260)$ |
| | | $f_1(1285)$ $K_1(1270)$ $K_1(1440)$ |
| | $J^P = 0^+$ | $\sigma(600)$ $\kappa(900)$ $f_0(980)$ $a_0(980)$ |

**What about exotic hadrons?**

# Origin of the resonances

## Trajectory of poles

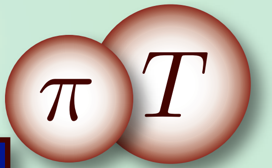
D. Jido, *et al.*, Nucl. Phys. A 723, 205 (2003)




**--> Search for bound states in SU(3) symmetric limit.**

# Outline

## Hadron-NG boson bound state




### Chiral Symmetry

**s-wave low energy interaction**

$$V_\alpha = -\frac{\omega}{2f^2} C_{\alpha,T} \qquad C_{\text{exotic}} = 1$$

### Scattering theory

**Critical strength for a bound state**

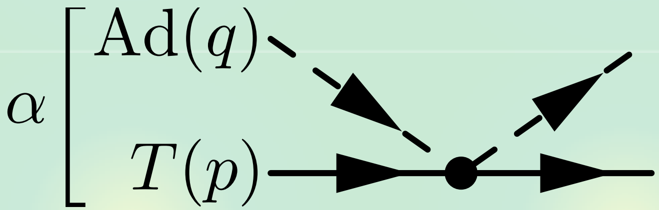
$$C_{\text{crit}} = \frac{2f^2}{m[-G(M_T + m)]}$$

**physical values :** $C_{\text{exotic}} < C_{\text{crit}}$

→ **No exotic state exists in SU(3) limit.**

# Low energy s-wave interaction

## Scattering of a target (T) with the pion (Ad)




$$= \frac{1}{f^2}\frac{p\cdot q}{2M_T}\langle \boldsymbol{F}_T\cdot \boldsymbol{F}_{\mathrm{Ad}}\rangle_\alpha + \mathcal{O}\left(\left(\frac{m}{M_T}\right)^2\right)$$

## s-wave : Weinberg-Tomozawa term

$$V_\alpha = -\frac{\omega}{2f^2}C_{\alpha,T}$$

$$C_{\alpha,T} \equiv -\langle 2\boldsymbol{F}_T\cdot \boldsymbol{F}_{\mathrm{Ad}}\rangle_\alpha = C_2(T) - C_2(\alpha) + 3 \quad (\text{for } N_f = 3)$$

## Coupling : pion decay constant

## model-independent interaction at low energy

Y. Tomozawa, Nuovo Cim. 46A, 707 (1966)

S. Weinberg, Phys. Rev. Lett. 17, 616 (1966)

# Coupling strengths : Examples

## Coupling strengths : (positive is attractive)

$$C_{\alpha,T} = C_2(T) - C_2(\alpha) + 3$$

| $\alpha$ | $\mathbf{1}$ | $\mathbf{8}$ | $\mathbf{10}$ | $\overline{\mathbf{10}}$ | $\mathbf{27}$ | $\mathbf{35}$ |
|---|---|---|---|---|---|---|
| $T = \mathbf{8}(N, \Lambda, \Sigma, \Xi)$ | 6 | 3 | 0 | 0 | $-2$ | |
| $T = \mathbf{10}(\Delta, \Sigma^*, \Xi^*, \Omega)$ | | 6 | 3 | | 1 | $-3$ |

| $\alpha$ | $\overline{\mathbf{3}}$ | $\mathbf{6}$ | $\overline{\mathbf{15}}$ | $\mathbf{24}$ |
|---|---|---|---|---|
| $T = \overline{\mathbf{3}}(\Lambda_c, \Xi_c)$ | 3 | 1 | $-1$ | |
| $T = \mathbf{6}(\Sigma_c, \Xi_c^*, \Omega_c)$ | 5 | 3 | 1 | $-2$ |

- **Exotic channels : mostly repulsive**
- **Attractive interaction : C = 1**

# Coupling strengths : General expression

## For a general target $T = [p, q]$

| $\alpha \in [p, q] \otimes [1, 1]$ | $C_{\alpha,T}$ | sign |
|---|---|---|
| $[p+1, q+1]$ | $-p-q$ | repulsive |
| $[p+2, q-1]$ | $1-p$ | |
| $[p-1, q+2]$ | $1-q$ | |
| $[p, q]$ | $3$ | attractive |
| $[p, q]$ | $3$ | attractive |
| $[p+1, q-2]$ | $3+q$ | attractive |
| $[p-2, q+1]$ | $3+p$ | attractive |
| $[p-1, q-1]$ | $4+p+q$ | attractive |

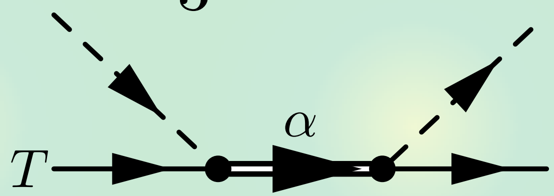
- Strength should be integer.
- Sign is determined for most cases.

# Exotic channels

## Exoticness : minimal number of extra $\bar{q}q$.

$$E = \epsilon\theta(\epsilon) + \nu\theta(\nu) \qquad \epsilon \equiv \frac{p+2q}{3} - B, \quad \nu \equiv \frac{p-q}{3} - B$$

$\Delta E = E_\alpha - E_T = +1$ is realized when




- $\alpha = [p+1, q+1] : C_{\alpha,T} = -p - q$

  **repulsive**

- $\alpha = [p+2, q-1] : C_{\alpha,T} = 1 - p$

  attraction : $p = 0$ then $\nu_T \geq 0 \to B \geq -q/3$

  **not considered here**

- $\alpha = [p-1, q+2] : C_{\alpha,T} = 1 - q$

  attraction : $q = 0$ then $\nu_T \leq 0 \to B \geq p/3$ **OK!**

## Universal attraction for more "exotic" channel

$$C_{\text{exotic}} = 1 \quad \text{for} \quad T = [p, 0], \quad \alpha = [p-1, 2]$$

# Unitarization : N/D method

## Unitarity cut --> N, unphysical cut --> D

$$T(s) = N(s)/D(s)$$

$$\mathrm{Im}D(s) = \mathrm{Im}[T^{-1}(s)]N(s) = -\rho(s)N(s) \quad \text{for } s > s_+$$

$$\mathrm{Im}N(s) = \mathrm{Im}[T(s)]D(s) \quad \text{for } s < s_-$$

## Neglect unphysical cut, set N=1

$$T^{-1}(s) = \left( a(s_0) + \frac{s - s_0}{2\pi} \int_{s^+}^{\infty} ds' \frac{\rho(s')}{(s' - s)(s' - s_0)} \right) + \mathcal{T}^{-1}(s)$$

**loop function** $\rightarrow G(s)$

**Interaction (tree level)**

**subtraction constant (regularization parameter of the loop)**

# Renormalization and bound states

## Identifying the interaction as $V_\alpha = -\frac{\omega}{2f^2} C_{\alpha,T}$

$$T_\alpha(\sqrt{s}) = \frac{1}{1 - V_\alpha(\sqrt{s}) G(\sqrt{s})} V_\alpha(\sqrt{s})$$

## Renormalization parameter : condition

$$G(\mu) = 0, \quad \Leftrightarrow \quad T(\mu) = V(\mu) \quad \text{at} \quad \mu = M_T$$

K. Igi, and K. Hikasa, Phys. Rev. D59, 034005 (1999)

M.F.M. Lutz, and E. Kolomeitsev, Nucl. Phys. A700, 193-308 (2002)

**Scale at which ChPT works.**

**Matching with the u-channel amplitude : OK**

## Bound state:

$$1 - V(M_b) G(M_b) = 0 \qquad\qquad M_T < M_b < M_T + m$$

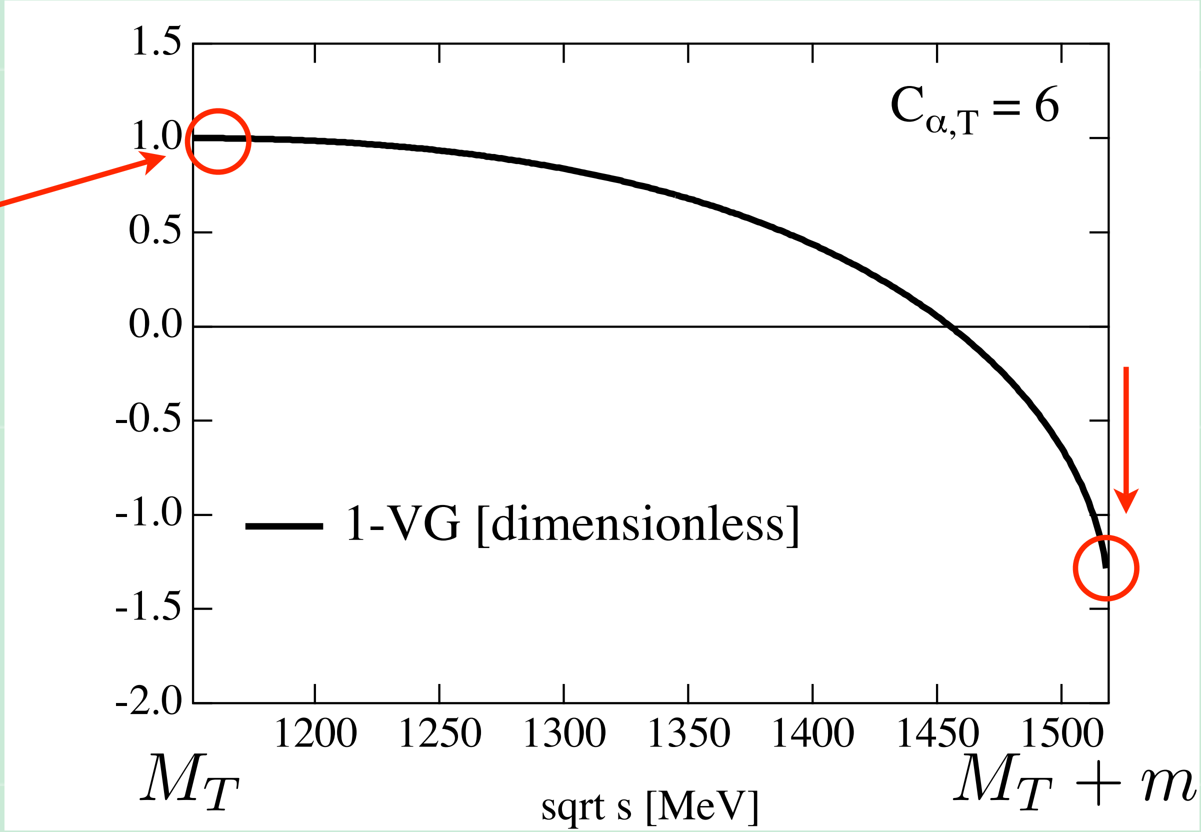
# Critical attraction

$1 - V(\sqrt{s})G(\sqrt{s})$ **: monotonically decreasing.**

**Fixed**

$G(M_T) = 0$

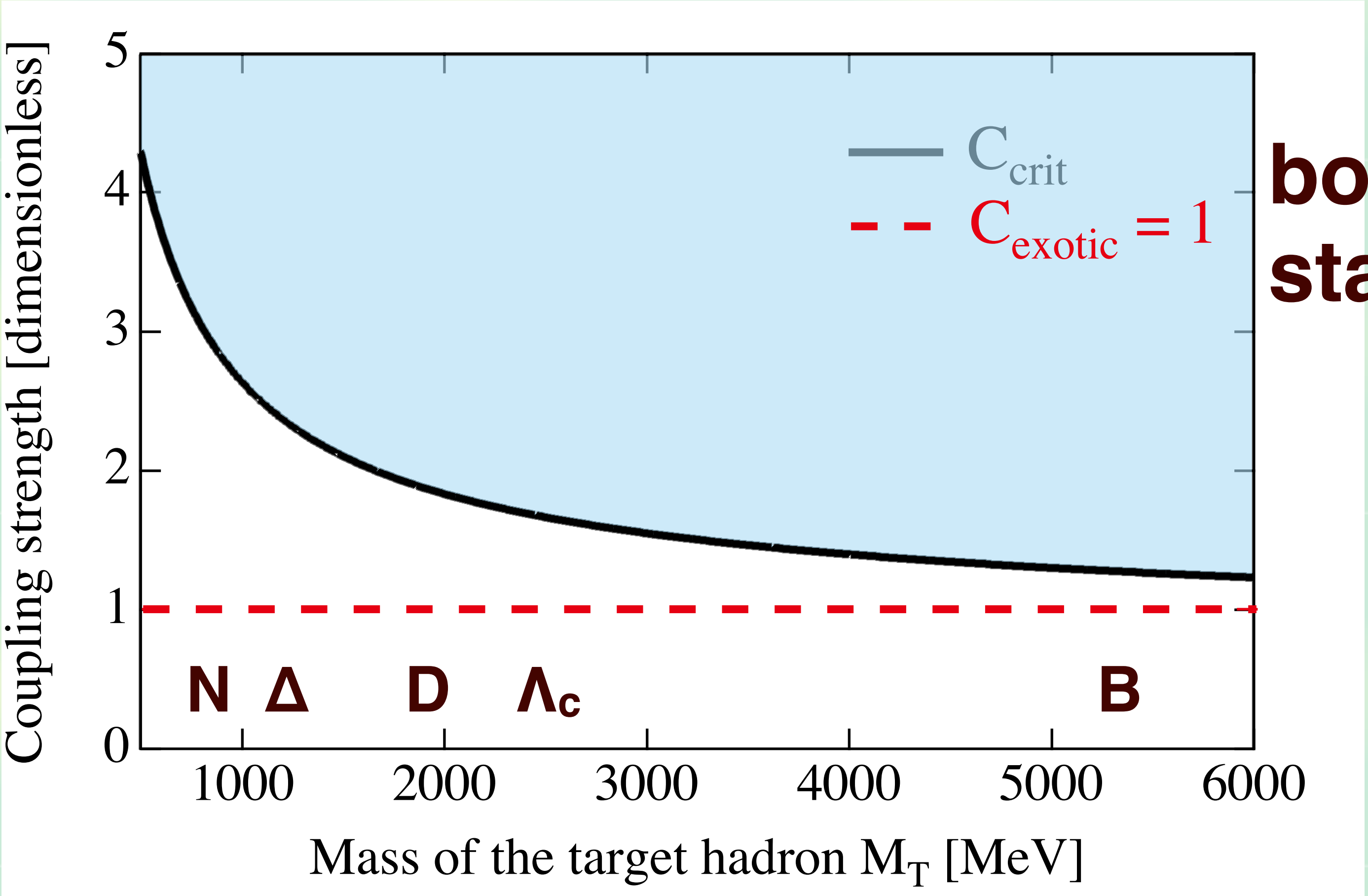
$1 - VG = 1$




**Critical attraction :** $1 - VG = 0$ at $\sqrt{s} = M_T + m$

$$\Rightarrow \quad C_{\text{crit}} = \frac{2f^2}{m[-G(M_T + m)]}$$

# Critical attraction and exotic channel




**bound state**

$$m = 368 \text{ MeV and } f = 93 \text{ MeV}$$

→ **Strength is not enough.**

# Summary 1 : SU(3) limit

We study the exotic bound states in s-wave chiral dynamics in flavor SU(3) limit.

- The interactions in exotic channels are in most cases repulsive.
- There are attractive interactions in exotic channels, with universal and the smallest strength : $C_{\text{exotic}} = 1$
- The strength is not enough to generate a bound state : $C_{\text{exotic}} < C_{\text{crit}}$

The result is model independent as far as we respect chiral symmetry.

# Summary 2 : Physical world

## Caution!

- The exotic hadrons here are the s-wave meson-hadron molecule states ($1/2^-$ for $\Theta^+$).
- We do not exclude the exotics which have other origins (genuine quark state, soliton rotation,...).
- In practice, SU(3) breaking effect, higher order terms,...

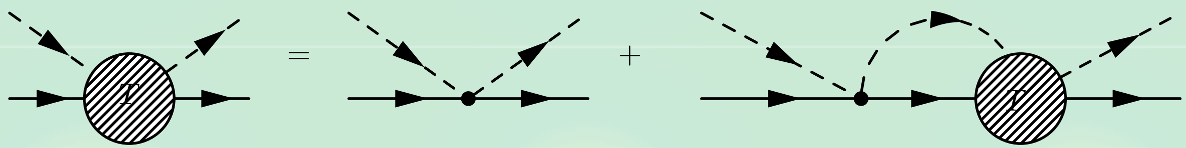
**In Nature, it is difficult to generate exotic hadrons as in the same way with Λ(1405), Λ(1520),... based on chiral interaction.**

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. Lett. 97, 192002 (2006)
T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. D 75, 034002 (2007)

# Renormalization condition

## Scattering amplitude

$$T = \frac{1}{1 - VG} V$$




$$V_\alpha = -\frac{\omega}{2f^2} C_{\alpha,T}$$ **tree (ChPT)**

$$G(\sqrt{s}) = \frac{2M_T}{(4\pi)^2} \left( a(\mu) + \ln \frac{M_T^2}{\mu^2} + \dots \right)$$ **loop**

## Subtraction constant $a(\mu)$ : condition

$$G(\mu) = 0, \quad \Leftrightarrow \quad T(\mu) = V(\mu) \quad \text{at} \quad \mu = M_T$$

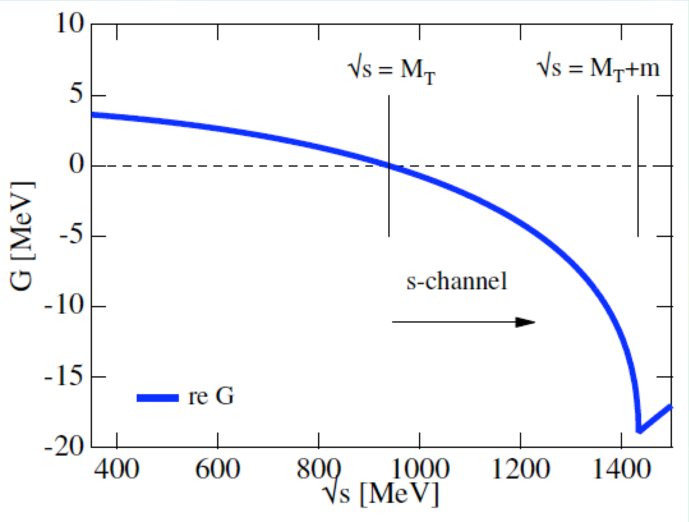
K. Igi, and K. Hikasa, Phys. Rev. D59, 034005 (1999)
M.F.M. Lutz, and E. Kolomeitsev, Nucl. Phys. A700, 193-308 (2002)

**Physical meaning of the condition?**

# Consistency with physical loop function

## Real part below threshold : negative
## <-- level repulsion

$$G(\sqrt{s}) = \frac{2M_T}{(4\pi)^2}\left(a(\mu) + \ln\frac{M_T^2}{\mu^2} + \dots\right)$$



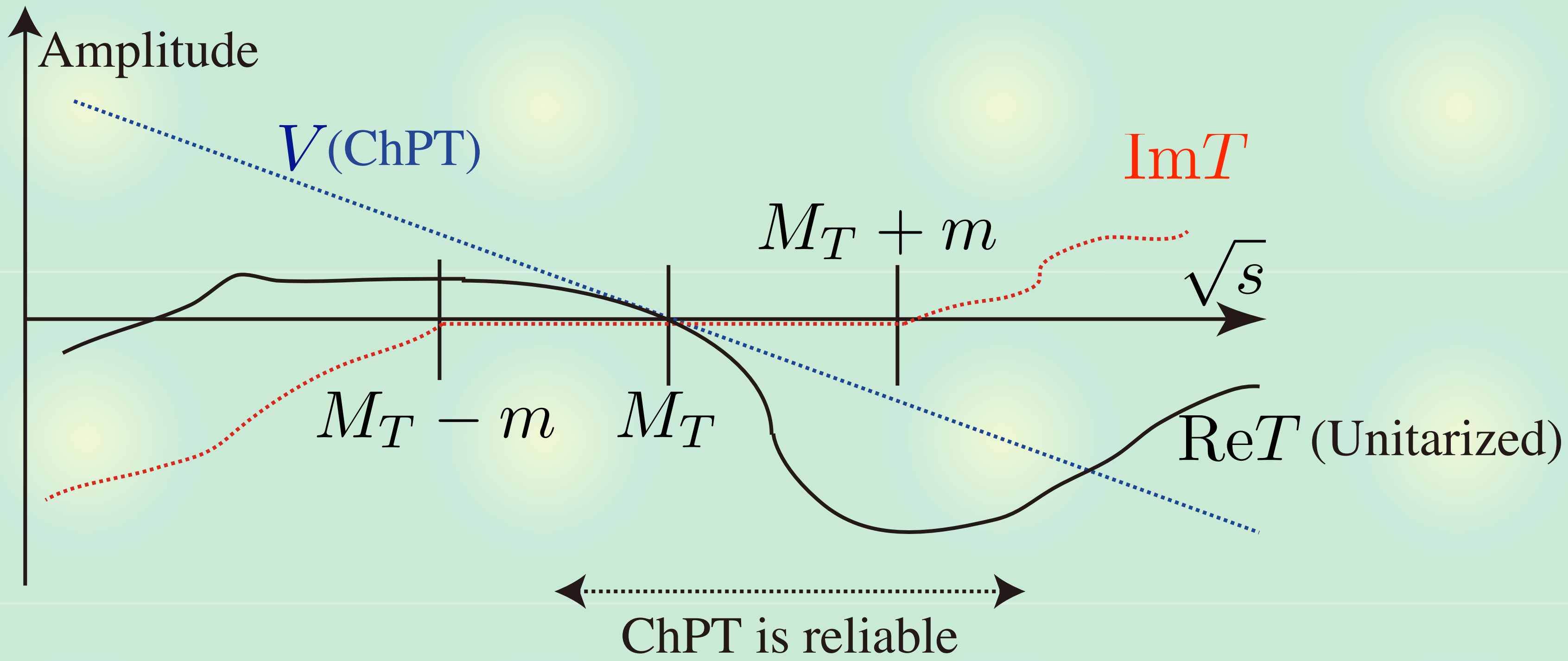

## subtraction constant has an upper limit

$$G(M_T) \leq 0, \quad a \leq a_{\max}$$

# Matching with ChPT

## Match the unitarized amplitude with ChPT

$$G(\mu_m) = 0, \quad \Leftrightarrow \quad T(\mu_m) = V(\mu_m)$$




## subtraction constant : real

$\Rightarrow \mu_m = M_T$ **natural unitarization**

# Effective attraction from the loop

$$T = (V^{-1} - G(a + \Delta a))^{-1} = ((V')^{-1} - G(a))^{-1}$$

$$V = -\frac{\omega}{2f^2}C \sim -\frac{\sqrt{s} - M_T}{2f^2}C$$

$$G(a) = \frac{2M_T}{(4\pi)^2}\left(a(\mu) + \ln\frac{M_T^2}{\mu^2} + \dots\right)$$

$$\Rightarrow (f')^2 = \frac{CM\Delta a}{16\pi^2}(\sqrt{s} - M) + f^2$$

**negative $\Delta a$ -> smaller $f^2$ -> effective attraction**

T. Hyodo, S.I. Nam, D. Jido, A. Hosaka, Phys. Rev. C. 68, 018201 (2003)
T. Hyodo, S.I. Nam, D. Jido, A. Hosaka, Prog. Thor. Phys. 112, 73 (2004)

**The condition $\mu_m = M_T$ : largest effective attraction in s-channel scattering region**

# Pole in the effective interaction

$$T = (V^{-1} - G(a + \Delta a))^{-1} = ((V')^{-1} - G(a))^{-1}$$

↑phenomonological ↑natural

## Effective interaction

$$V' = -\frac{8\pi^2}{M\Delta a}\frac{\sqrt{s} - M}{\sqrt{s} - M_{\text{eff.}}}$$

$$= -\frac{C}{2f^2}(\sqrt{s} - M_T) + \frac{C}{2f^2}\frac{(\sqrt{s} - M_T)^2}{\sqrt{s} - M_{\text{eff.}}}$$

pole!

$$M_{\text{eff.}} = M - \frac{16\pi^2 f^2}{CM\Delta a}$$

## Physically meaningful pole :

$$C > 0, \quad \Delta a < 0$$

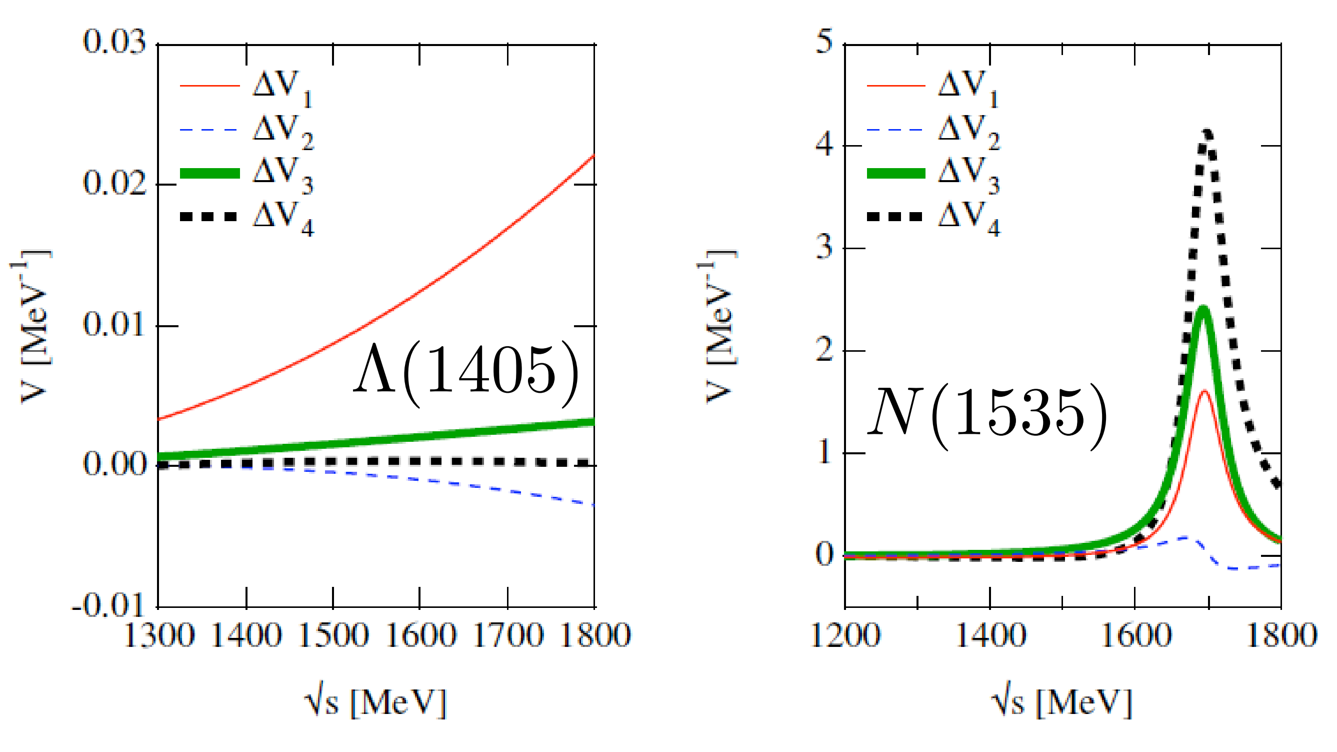
**Origin of dynamical pole?**

# Example : L(1405) and N(1535)

$$T = (V^{-1} - G(a + \Delta a))^{-1} = ((V')^{-1} - G(a))^{-1}$$

↑phenomonological ↑natural

## Absorb $\Delta a$ into effective interaction




**Origin of dynamical pole?**

# Summary 2 : Natural unitarization

- The renormalization condition in the study of exotic hadrons:

  The renomlization condition provides the most advantageous to generate a bound state in the s-channel region.

- Natural unitarization enables us to extract the low energy structure.

  Study the origin of the generated resonances using phenomenological subtraction constant.

T. Hyodo, D. Jido, A. Hosaka, in preparation...