

Exotic Hadrons in s-Wave Chiral Dynamics



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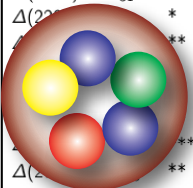
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2007, Mar. 28th 1

Exotic hadrons in hadron spectrum

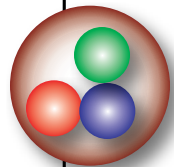
Observed hadrons in experiments (PDG06) :

BARYONS				MESONS																		
p	P_{11}	****	$\Delta(1232)$	P_{33}	****	Λ	P_{01}	****	Σ^+	P_{11}	****	Ξ^0	P_{11}	****	LIGHT UNFLAVORED ($S=C=B=0$)		STRANGE ($S=\pm 1, C=B=0$)		BOTTOM ($B=\pm 1$)			
n	P_{11}	****	$\Delta(1600)$	P_{33}	***	$\Lambda(1405)$	S_{01}	****	Σ^0	P_{11}	****	Ξ^-	P_{11}	****	ρ^0 (J^{PC})	ρ^0 (J^{PC})	K^+	$1/2(0^-)$	K^0	$1/2(0^-)$	B^+	$1/2(0^-)$
$N(1440)$	P_{11}	****	$\Delta(1620)$	S_{31}	****	$\Lambda(1520)$	D_{03}	****	Σ^-	P_{11}	****	$\Xi(1530)$	P_{13}	****	$\omega(782)$	$0^-(1^-)$	K^*	$1/2(1^-)$	K^*	$1/2(1^-)$	B^0	$1/2(0^-)$
$N(1520)$	D_{13}	****	$\Delta(1700)$	D_{33}	****	$\Lambda(1600)$	P_{01}	***	$\Sigma(1385)$	P_{13}	****	$\Xi(1620)$	*	*	$\eta(958)$	$0^+(0^+)$	K_S^0	$1/2(0^-)$	K_L^0	$1/2(0^-)$	B^{\pm}/B^0 ADMIXTURE	$1/2(0^-)$
$N(1535)$	S_{11}	****	$\Delta(1750)$	P_{31}	*	$\Lambda(1670)$	S_{01}	****	$\Sigma(1480)$	*	*	$\Xi(1690)$	***	***	$\eta(1760)$	$0^+(0^+)$	$K_1^*(892)$	$1/2(1^-)$	$K_2^*(892)$	$1/2(1^-)$	B^{\pm}/B^0 ADMIXTURE	$1/2(0^-)$
$N(1650)$	S_{11}	****	$\Delta(1900)$	S_{31}	**	$\Lambda(1690)$	D_{03}	****	$\Sigma(1560)$	**	**	$\Xi(1820)$	D_{13}	***	$\omega(1710)$	$0^+(0^+)$	$K_1^*(1270)$	$1/2(1^+)$	$K_2^*(1270)$	$1/2(1^+)$	B^+	$1/2(1^-)$
$N(1675)$	D_{15}	****	$\Delta(1905)$	F_{35}	****	$\Lambda(1800)$	S_{01}	***	$\Sigma(1580)$	D_{13}	*	$\Xi(1950)$	***	***	$\eta(1760)$	$0^+(0^+)$	$K_1^*(1400)$	$1/2(1^+)$	$K_2^*(1400)$	$1/2(1^+)$	B^+	$1/2(1^-)$
$N(1680)$	F_{15}	****	$\Delta(1910)$	P_{31}	****	$\Lambda(1810)$	P_{01}	***	$\Sigma(1620)$	S_{11}	**	$\Xi(2030)$	***	***	$\eta(1760)$	$0^+(0^+)$	$K_1^*(1410)$	$1/2(1^+)$	$K_2^*(1410)$	$1/2(1^+)$	B^+	$1/2(1^-)$
$N(1700)$	D_{13}	***	$\Delta(1920)$	P_{33}	***	$\Lambda(1820)$	F_{05}	****	$\Sigma(1660)$	P_{11}	***	$\Xi(2120)$	*	*	$\eta(1760)$	$0^+(0^+)$	$K_1^*(1430)$	$1/2(1^+)$	$K_2^*(1430)$	$1/2(1^+)$	B^+	$1/2(1^-)$
$N(1710)$	P_{11}	***	$\Delta(1930)$	D_{35}	***	$\Lambda(1830)$	F_{05}	****	$\Sigma(1670)$	D_{13}	****	$\Xi(2250)$	**	**	$\eta(1760)$	$0^+(0^+)$	$K_1^*(1430)$	$1/2(1^+)$	$K_2^*(1430)$	$1/2(1^+)$	B^+	$1/2(1^-)$
$N(1720)$	P_{13}	****	$\Delta(1940)$	D_{33}	*	$\Lambda(1890)$	P_{03}	****	$\Sigma(1690)$	**	**	$\Xi(2370)$	**	**	$\eta(1760)$	$0^+(0^+)$	$K_1^*(1430)$	$1/2(1^+)$	$K_2^*(1430)$	$1/2(1^+)$	B^+	$1/2(1^-)$
$N(1900)$	P_{13}	**	$\Delta(1950)$	F_{37}	****	$\Lambda(2000)$	*	*	$\Sigma(1750)$	S_{11}	**	$\Xi(2500)$	*	*	$\eta(1760)$	$0^+(0^+)$	$K_1^*(1430)$	$1/2(1^+)$	$K_2^*(1430)$	$1/2(1^+)$	B^+	$1/2(1^-)$
$N(1990)$	F_{17}	**	$\Delta(2000)$	F_{35}	**	$\Lambda(2020)$	F_{07}	*	$\Sigma(1770)$	P_{11}	**	Ω^-	****	****	$\eta(1760)$	$0^+(0^+)$	$K_1^*(1430)$	$1/2(1^+)$	$K_2^*(1430)$	$1/2(1^+)$	B^+	$1/2(1^-)$
$N(2000)$	F_{15}	**	$\Delta(2150)$	S_{31}	*	$\Lambda(2100)$	G_{07}	****	$\Sigma(1775)$	D_{15}	****	$\Omega(2250)^-$	***	***	$\eta(1760)$	$0^+(0^+)$	$K_1^*(1430)$	$1/2(1^+)$	$K_2^*(1430)$	$1/2(1^+)$	B^+	$1/2(1^-)$
$N(2080)$	D_{13}	**	$\Delta(2200)$	*	*	$\Lambda(2110)$	F_{05}	***	$\Sigma(1840)$	P_{13}	*	$\Omega(2380)^-$	**	**	$\eta(1760)$	$0^+(0^+)$	$K_1^*(1430)$	$1/2(1^+)$	$K_2^*(1430)$	$1/2(1^+)$	B^+	$1/2(1^-)$
$N(2090)$	S_{11}	*	$\Delta(2250)$	*	*	$\Lambda(2325)$	D_{03}	*	$\Sigma(1880)$	P_{11}	**	$\Omega(2470)^-$	**	**	$\eta(1760)$	$0^+(0^+)$	$K_1^*(1430)$	$1/2(1^+)$	$K_2^*(1430)$	$1/2(1^+)$	B^+	$1/2(1^-)$
$N(2100)$	P_{11}	*	$\Delta(2300)$	*	*	$\Lambda(2350)$	H_{09}	***	$\Sigma(1915)$	F_{15}	****	$\Omega(2470)^-$	**	**	$\eta(1760)$	$0^+(0^+)$	$K_1^*(1430)$	$1/2(1^+)$	$K_2^*(1430)$	$1/2(1^+)$	B^+	$1/2(1^-)$
$N(2190)$	G_{17}	****	$\Delta(2350)$	*	*	$\Lambda(2585)$	**	**	$\Sigma(1940)$	D_{13}	****	Λ_c^+	****	****	$\eta(1760)$	$0^+(0^+)$	$K_1^*(1430)$	$1/2(1^+)$	$K_2^*(1430)$	$1/2(1^+)$	B^+	$1/2(1^-)$
$N(2200)$	D_{15}	**	$\Delta(2400)$	*	*		**	**	$\Sigma(2000)$	S_{11}	*	$\Lambda_c^+(2593)^+$	***	***	$\eta(1760)$	$0^+(0^+)$	$K_1^*(1430)$	$1/2(1^+)$	$K_2^*(1430)$	$1/2(1^+)$	B^+	$1/2(1^-)$
$N(2220)$	H_{19}	****	$\Delta(2450)$	*	*		**	**	$\Sigma(2030)$	F_{17}	****	$\Lambda_c^+(2625)^+$	***	***	$\eta(1760)$	$0^+(0^+)$	$K_1^*(1430)$	$1/2(1^+)$	$K_2^*(1430)$	$1/2(1^+)$	B^+	$1/2(1^-)$
$N(2250)$	G_{19}	****	$\Delta(2500)$	*	*		**	**	$\Sigma(2070)$	F_{15}	*	$\Lambda_c^+(2675)^+$	***	***	$\eta(1760)$	$0^+(0^+)$	$K_1^*(1430)$	$1/2(1^+)$	$K_2^*(1430)$	$1/2(1^+)$	B^+	$1/2(1^-)$
$N(2600)$	$h_{1,11}$	***	$\Delta(2950)$	$K_{3,15}$	**		**	**	$\Sigma(2080)$	P_{13}	**	$\Lambda_c^+(2765)^+$	*	*	$\eta(1760)$	$0^+(0^+)$	$K_1^*(1430)$	$1/2(1^+)$	$K_2^*(1430)$	$1/2(1^+)$	B^+	$1/2(1^-)$
$N(2700)$	$K_{1,13}$	**	$\Theta(1540)^+$	*	*		**	**	$\Sigma(2100)$	G_{17}	****	$\Lambda_c^+(2880)^+$	**	**	$\eta(1760)$	$0^+(0^+)$	$K_1^*(1430)$	$1/2(1^+)$	$K_2^*(1430)$	$1/2(1^+)$	B^+	$1/2(1^-)$
									$\Sigma(2250)$		****	$\Sigma_c(2455)$	****	****	$\eta(1760)$	$0^+(0^+)$	$K_1^*(1430)$	$1/2(1^+)$	$K_2^*(1430)$	$1/2(1^+)$	B^+	$1/2(1^-)$
									$\Sigma(2455)$		**	$\Sigma_c(2520)$	***	***	$\eta(1760)$	$0^+(0^+)$	$K_1^*(1430)$	$1/2(1^+)$	$K_2^*(1430)$	$1/2(1^+)$	B^+	$1/2(1^-)$
									$\Sigma(2620)$		**	$\Sigma_c(2800)$	***	***	$\eta(1760)$	$0^+(0^+)$	$K_1^*(1430)$	$1/2(1^+)$	$K_2^*(1430)$	$1/2(1^+)$	B^+	$1/2(1^-)$
									$\Sigma(3000)$		*	Λ_c^+	****	****	$\eta(1760)$	$0^+(0^+)$	$K_1^*(1430)$	$1/2(1^+)$	$K_2^*(1430)$	$1/2(1^+)$	B^+	$1/2(1^-)$
									$\Sigma(3170)$		*	Λ_c^0	****	****	$\eta(1760)$	$0^+(0^+)$	$K_1^*(1430)$	$1/2(1^+)$	$K_2^*(1430)$	$1/2(1^+)$	B^+	$1/2(1^-)$
												Λ_c^0	****	****	$\eta(1760)$	$0^+(0^+)$	$K_1^*(1430)$	$1/2(1^+)$	$K_2^*(1430)$	$1/2(1^+)$	B^+	$1/2(1^-)$



~130 baryons

$\sim \frac{1}{300}!$



~160 mesons

Exotic hadrons are indeed exotic !!

Exotic hadrons : experiment vs theory

Exotic hadrons : valence quark-antiquark(s)

non-exotic

$uds, u\bar{d}, uds u\bar{u}, u\bar{d} u\bar{u}, \dots$

exotic

$uudd\bar{s}, ud\bar{s}\bar{s}, \dots$

Experimentally, they are exotic $\sim 1/300$.

Theoretically, are they exotic?

--> There is no simple way to forbid exotic states in QCD, effective models, ...

--> Importance of multiquark components in non-exotic hadrons.

Why aren't the exotics observed??

Non-exotic hadrons in s-wave chiral dynamics

Hadron excited states $\sim \pi T$

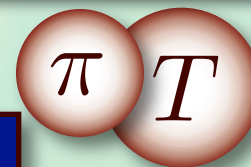
- Interaction \leftarrow chiral symmetry
- Amplitude \leftarrow unitarity

light baryon	$J^P = 1/2^-$	$\Lambda(1405)$	$\Lambda(1670)$	$\Sigma(1670)$	
		$N(1535)$	$\Xi(1620)$	$\Xi(1690)$	
	$J^P = 3/2^-$	$\Lambda(1520)$	$\Xi(1820)$	$\Sigma(1670)$	
heavy		$\Lambda_c(2880)$	$\Lambda_c(2593)$	$D_s(2317)$	
light meson	$J^P = 1^+$	$b_1(1235)$	$h_1(1170)$	$h_1(1380)$	$a_1(1260)$
		$f_1(1285)$	$K_1(1270)$	$K_1(1440)$	
	$J^P = 0^+$	$\sigma(600)$	$\kappa(900)$	$f_0(980)$	$a_0(980)$

What about exotic hadrons?

Exotic hadrons in s-wave chiral dynamics

Hadron-NG boson bound state



Chiral Symmetry

s-wave low energy interaction

$$V_{\alpha} = -\frac{\omega}{2f^2} C_{\alpha,T} \quad C_{\text{exotic}} = 1$$

Scattering theory

Critical strength for a bound state

$$C_{\text{crit}} = \frac{2f^2}{m[-G(M_T + m)]}$$


physical values : $C_{\text{exotic}} < C_{\text{crit}}$

➔ **No exotic state exists in SU(3) limit.**


T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. Lett. 97, 192002 (2006)

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. D 75, 034002 (2007)

Topics



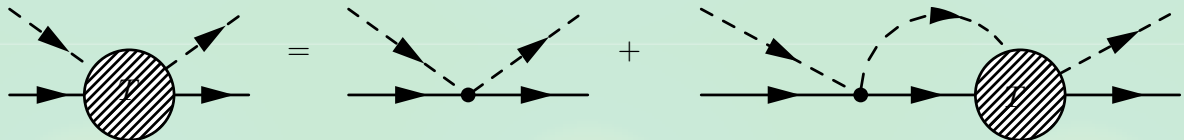
Physical meaning of the renormalization condition.



N_c dependence of the interaction and bound states in large N_c limit

Renormalization condition

Scattering amplitude

$$T = \frac{1}{1 - VG} V$$


$$V_\alpha = -\frac{\omega}{2f^2} C_{\alpha,T} \quad \text{tree}$$

$$G(\sqrt{s}) = \frac{2M_T}{(4\pi)^2} \left(a(\mu) + \ln \frac{M_T^2}{\mu^2} + \dots \right) \quad \text{loop}$$

Subtraction constant $a(\mu)$: condition

$$G(\mu) = 0, \quad \Leftrightarrow \quad T(\mu) = V(\mu) \quad \text{at} \quad \mu = M_T$$

K. Igi, and K. Hikasa, Phys. Rev. D59, 034005 (1999)

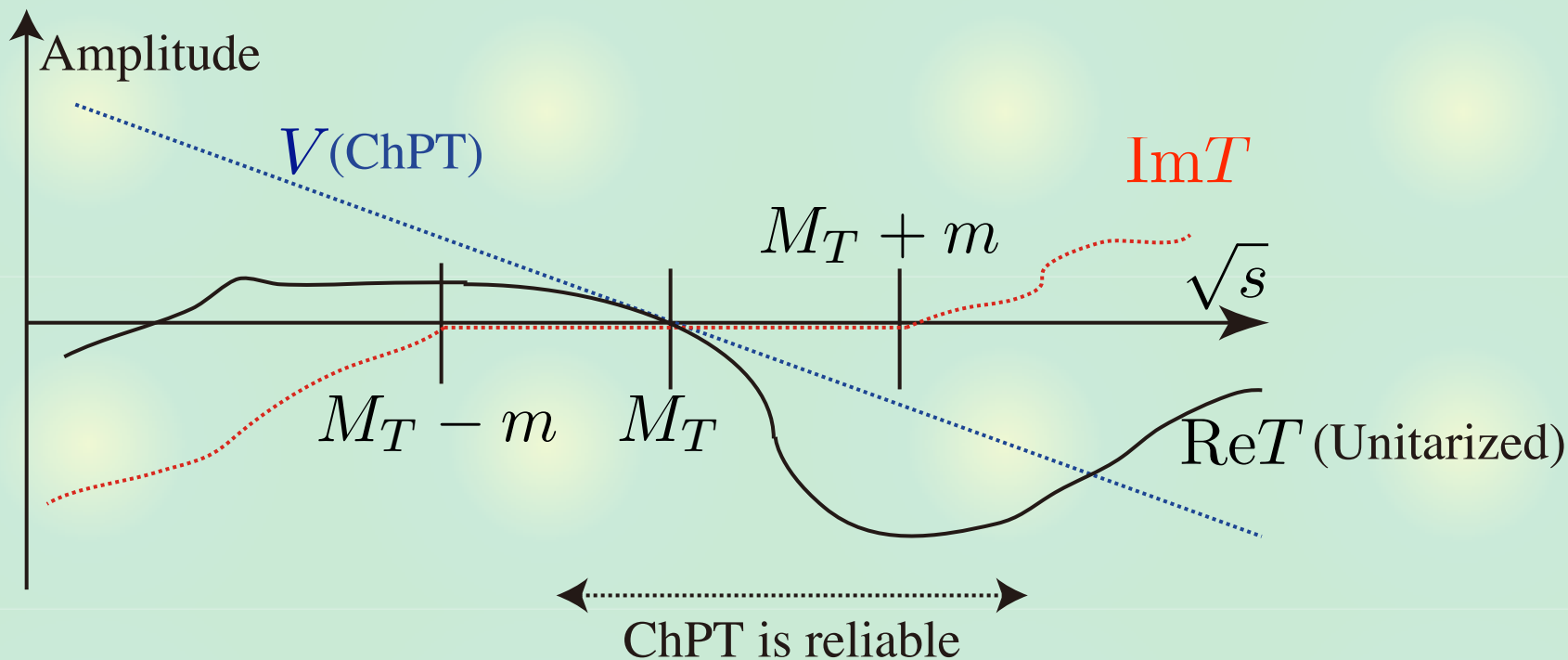
M.F.M. Lutz, and E. Kolomeitsev, Nucl. Phys. A700, 193-308 (2002)

Physical meaning of the condition?

Matching with ChPT

Match the unitarized amplitude with ChPT

$$G(\mu_m) = 0, \quad \Leftrightarrow \quad T(\mu_m) = V(\mu_m)$$



subtraction constant : real

$$\Rightarrow \quad M_T - m \leq \mu_m \leq M_T + m$$

Natural value for the subtraction constant

Single-channel case

$$G(\mu_m) = 0, \quad \Leftrightarrow \quad T(\mu_m) = V(\mu_m)$$

$$M_T - m \leq \mu_m \leq M_T + m$$

Coupled-channel case

$$G_i(\mu_m) = 0, \quad \Leftrightarrow \quad T_{ij}(\mu_m) = V_{ij}(\mu_m)$$

$$\text{Max}(M_T - m)_i \leq \mu_m \leq \text{Min}(M_T + m)_i$$

Natural value for the subtraction constant
: consistent with the 3d cutoff ~ 630 MeV

$$a(630\text{MeV}) \sim -2$$

J.A. Oller, U.G. Meissner, Phys. Lett. B500, 263 (2001)

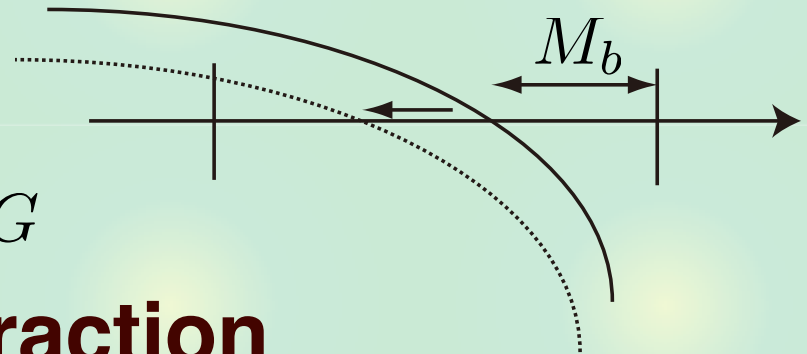
In practice : fitted to the data

Unnatural value \rightarrow seed of resonance?

Effective attraction from the loop

Bound state energy M_b :

$$1 - V(M_b)G(M_b) = 0$$



To enlarge M_b : increase VG

$V < 0$ for attractive interaction

$$G(\sqrt{s}) = \frac{2M_T}{(4\pi)^2} \left(a(\mu) + \ln \frac{M_T^2}{\mu^2} + \dots \right)$$

**larger $(-a)$: smaller μ_m : larger M_b
--> effective attraction**

T. Hyodo, S.I. Nam, D. Jido, A. Hosaka, Phys. Rev. C. 68, 018201 (2003)

T. Hyodo, S.I. Nam, D. Jido, A. Hosaka, Prog. Thor. Phys. 112, 73 (2004)

The condition $\mu_m = M_T$: largest effective attraction in s-channel scattering region

Large N_c limit : introduction

$1/N_c$: a possible expansion parameter

G. 't Hooft, Nucl. Phys. B72, 461 (1974)

E. Witten, Nucl. Phys. B160, 57 (1979)

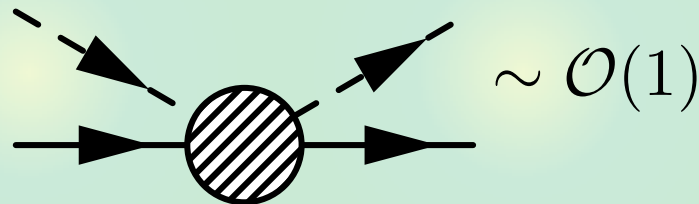
Scaling of the physical quantities
 ← N_c^2 gluons and N_c quarks.

Meson mass : $m \sim \mathcal{O}(1)$

Baryon mass : $M \sim \mathcal{O}(N_c)$

Decay constant : $f \sim \mathcal{O}(\sqrt{N_c})$

MB scattering :



Coupling strengths in large Nc limit

WT interaction in large Nc limit

$$V_\alpha = -\frac{\omega}{2f^2} C_{\alpha,T} \sim \frac{1}{N_c} \times C_{\alpha,T}$$

Flavor representation of baryons

$$[p, q] \rightarrow \text{“}[p, q]\text{”} = \left[p, q + \frac{N_c - 3}{2} \right]$$

Coupling strength has **linear Nc dependence**

$$\underline{C_{\text{“}\alpha\text{”}, \text{“}T\text{”}}(N_c)} = C_2(\text{“}T\text{”}) - C_2(\text{“}\alpha\text{”}) + 3$$

$$C(\text{“}[p, q]\text{”}) = \frac{1}{3} \left(-\frac{9}{4} + p^2 + \frac{3p}{2} + pq + q^2 \right) + \frac{1}{3} \left(\frac{p}{2} + q \right) \underline{N_c} + \frac{N_c^2}{12}$$

Coupling strengths for the general target

For arbitrary N_c , $V \propto -\frac{C}{f^2} \sim -\frac{C(N_c)}{N_c}$

$\alpha \in [p, q] \otimes [1, 1]$	$C_{\alpha, T}(N_c)$	$V(N_c \rightarrow \infty)$
$[p + 1, q + 1]$	$(3 - N_c)/2 - p - q$	repulsive
$[p + 2, q - 1]$	$1 - p$	
$[p - 1, q + 2]$	$(5 - N_c)/2 - q$	repulsive
$[p, q]$	3	
$[p, q]$	3	
$[p + 1, q - 2]$	$(3 + N_c)/2 + q$	attractive
$[p - 2, q + 1]$	$3 + p$	
$[p - 1, q - 1]$	$(5 + N_c)/2 + p + q$	attractive

- No attraction in **exotic channels.**

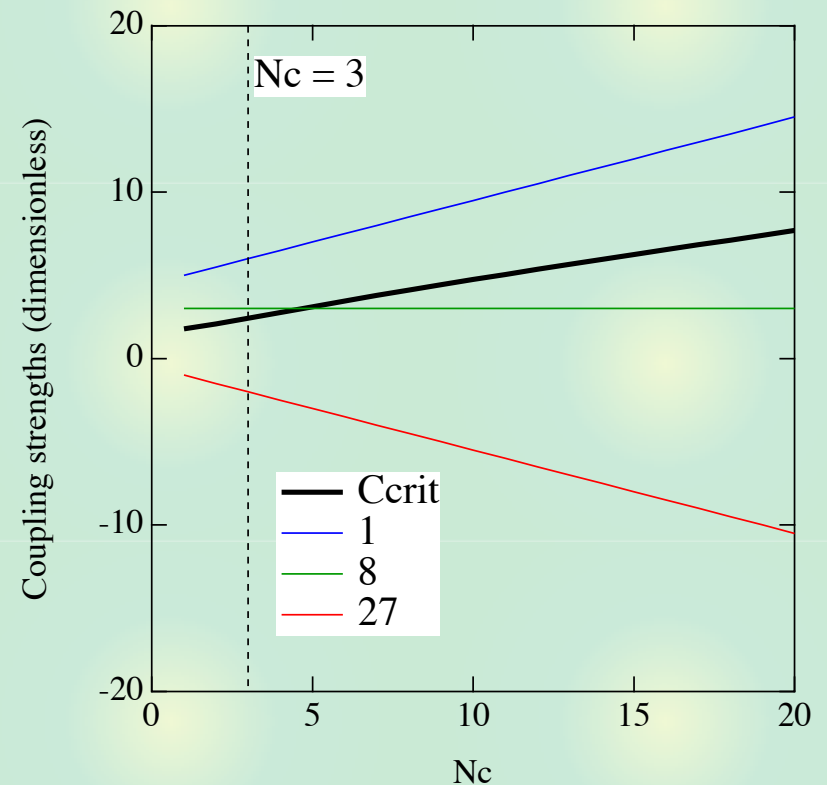
$S = -1$ $I = 0$ channel in $SU(3)$ basis

α	“1”	“ 8_s ”	“ 8_a ”	“27”
$T = \text{“8”}$	$\frac{9+N_c}{2}$	3	3	$\frac{-2-N_c}{2}$

$$C_{\text{crit}}(N_c) = \frac{2[f(N_c)]^2}{m[-G(M_T(N_c) + m)]}$$

$$M_T(N_c) = M_0 \times \frac{N_c}{3}$$

$$f(N_c) = f_0 \times \sqrt{\frac{N_c}{3}}$$



○ **Bound state in “1” in the large N_c limit.**

S = -1 I = 0 channel in Isospin basis

Basis transformation via CG Coef. with Nc

T.D. Cohen, and R.F. Lebed, Phys. Rev. D 70, 096015 (2004)

$$C_{ij}(N_c) = \begin{pmatrix} \bar{K}N & \pi\Sigma & \eta\Lambda & K\Xi \\ \frac{1}{2}(3 + N_c) & -\frac{\sqrt{3}}{2}\sqrt{-1 + N_c} & \frac{\sqrt{3}}{2}\sqrt{3 + N_c} & 0 \\ -\frac{\sqrt{3}}{2}\sqrt{-1 + N_c} & 4 & 0 & \frac{\sqrt{3 + N_c}}{2} \\ \frac{\sqrt{3}}{2}\sqrt{3 + N_c} & 0 & 0 & -\frac{\sqrt{3}}{2}\sqrt{-1 + N_c} \\ 0 & \frac{\sqrt{3 + N_c}}{2} & -\frac{\sqrt{3}}{2}\sqrt{-1 + N_c} & \frac{1}{2}(9 - N_c) \end{pmatrix}$$

Combining with the 1/Nc factor of 1/f²,

- $\bar{K}N \rightarrow \bar{K}N$: **attractive** at large Nc
- $\bar{K}N \rightarrow \pi\Sigma$: $\mathcal{O}(1/\sqrt{N_c})$
- $\pi\Sigma \rightarrow \pi\Sigma$: $\mathcal{O}(1/N_c)$

Summary 1 : Exotic hadrons

We study the exotic bound states in s-wave chiral dynamics in flavor SU(3) limit.

- The interaction in exotic channels is in most cases **repulsive**.
- There are **attractive interactions** in exotic channels, with **universal** and the smallest strength : $C_{\text{exotic}} = 1$
- The strength is **not enough** to generate a bound state : $C_{\text{exotic}} < C_{\text{crit}}$

The result is **model independent** as far as we respect chiral symmetry.

Summary 2 : Topics



Physical meaning of the renormalization condition.

The renormalization condition provides the **largest strength of the effective attraction** in the s-channel region.



N_c dependence of the interaction and bound states in large N_c limit

Attraction in “1” and $\bar{K}N$ channels is strong enough to provide a **bound state in the large N_c limit.**