

Exotic hadrons in s-wave chiral dynamics



Tetsuo Hyodo^a,

Daisuke Jido^a, and Atsushi Hosaka^b

YITP, Kyoto^a RCNP, Osaka^b

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Exotic hadrons

Observed hadrons in experiments (PDG06) :

BARYONS				MESONS				LIGHT UNFLAVORED		STRANGE		BOTTOM						
								$(S=C=B=0)$		$(S=\pm 1, C=B=0)$		$(B=\pm 1)$						
								$J^P(J^{PC})$		$J^P(J^{PC})$		$J^P(J^{PC})$						
p	P_{11}	****	$\Delta(1232)$	P_{33}	****	Λ	P_{01}	****	Σ^+	P_{11}	****	Ξ^0	P_{11}	****	K^+	$1/2(0^-)$	B^+	$1/2(0^-)$
n	P_{11}	****	$\Delta(1600)$	P_{33}	***	$\Lambda(1405)$	S_{01}	****	Σ^0	P_{11}	****	Ξ^-	P_{11}	****	K^0	$1/2(0^-)$	B^0	$1/2(0^-)$
$N(1440)$	P_{11}	****	$\Delta(1620)$	S_{31}	****	$\Lambda(1520)$	D_{03}	****	Σ^-	P_{11}	****	$\Xi(1530)$	P_{13}	****	K_S^0	$1/2(0^-)$	B_S^0	$1/2(0^-)$
$N(1520)$	D_{13}	****	$\Delta(1700)$	D_{33}	****	$\Lambda(1600)$	P_{01}	***	$\Sigma(1385)$	P_{13}	****	$\Xi(1620)$	*		K_L^0	$1/2(0^-)$	B^{\pm}/B_S^0	AD MIXTURE
$N(1535)$	S_{11}	****	$\Delta(1750)$	P_{31}	*	$\Lambda(1670)$	S_{01}	****	$\Sigma(1480)$	*		$\Xi(1690)$	***		K_1^0	$1/2(0^-)$	B^{\pm}/B_S^0	b -baryon AD-MIXTURE
$N(1650)$	S_{11}	****	$\Delta(1900)$	S_{31}	**	$\Lambda(1690)$	D_{03}	****	$\Sigma(1560)$	**		$\Xi(1820)$	D_{13}	***	$K_2^0(800)$	$1/2(0^+)$	V_{ub}	and V_{cb} CKM Matrix Elements
$N(1675)$	D_{15}	****	$\Delta(1905)$	F_{35}	****	$\Lambda(1800)$	S_{01}	***	$\Sigma(1580)$	D_{13}	*	$\Xi(1950)$	*		$K^*(892)$	$1/2(1^-)$	B^*	$1/2(1^-)$
$N(1680)$	F_{15}	****	$\Delta(1910)$	P_{31}	****	$\Lambda(1810)$	P_{01}	***	$\Sigma(1620)$	S_{11}	**	$\Xi(2030)$	***		$K_1(1270)$	$1/2(1^+)$	B_5^*	(5732) ?(??)
$N(1700)$	D_{13}	***	$\Delta(1920)$	P_{33}	***	$\Lambda(1820)$	F_{05}	****	$\Sigma(1660)$	P_{11}	***	$\Xi(2120)$	*		$K_1(1400)$	$1/2(1^+)$	BOTTOM, STRANGE	
$N(1710)$	P_{11}	***	$\Delta(1930)$	D_{35}	***	$\Lambda(1830)$	F_{05}	****	$\Sigma(1670)$	D_{13}	****	$\Xi(2250)$	**		$K_2^*(1430)$	$1/2(2^+)$	$(B = \pm 1, S = \mp 1)$	
$N(1720)$	P_{13}	****	$\Delta(1940)$	D_{33}	*	$\Lambda(1890)$	P_{03}	****	$\Sigma(1690)$	*		$\Xi(2370)$	**		$K(1460)$	$1/2(0^-)$	B_c^0	$0(0^-)$
$N(1900)$	P_{13}	**	$\Delta(1950)$	F_{37}	****	$\Lambda(2000)$	*		$\Sigma(1750)$	S_{11}	**	$\Xi(2500)$	*		$K_2(1580)$	$1/2(2^-)$	B_c^+	$0(1^-)$
$N(1990)$	F_{17}	**	$\Delta(2000)$	F_{35}	**	$\Lambda(2020)$	F_{07}	*	$\Sigma(1770)$	P_{11}	*				$K(1630)$	$1/2(2^?)$	BOTTOM, CHARMED	
$N(2000)$	F_{15}	**	$\Delta(2150)$	S_{31}	*	$\Lambda(2100)$	G_{07}	****	$\Sigma(1775)$	D_{15}	****	Ω^-	****		$K_1(1650)$	$1/2(1^+)$	$(B = C = \pm 1)$	
$N(2080)$	D_{13}	**	$\Delta(2200)$	*		$\Lambda(2110)$	F_{05}	***	$\Sigma(1840)$	P_{13}	*	$\Omega(2250)^-$	***		$K_2(1770)$	$1/2(2^-)$	B_c^+	$0(0^-)$
$N(2090)$	S_{11}	*	$\Delta(2250)$	*		$\Lambda(2325)$	D_{03}	*	$\Sigma(1880)$	P_{11}	**	$\Omega(2380)^-$	**		$K_3^*(1780)$	$1/2(3^-)$	$c\bar{c}$	
$N(2100)$	P_{11}	*	$\Delta(2300)$	*		$\Lambda(2350)$	H_{09}	***	$\Sigma(1915)$	F_{15}	****	$\Omega(2470)^-$	**		$K_2(1820)$	$1/2(2^-)$	$\eta_c(1S)$	$0^+(0^-)$
$N(2190)$	G_{17}	****	$\Delta(2350)$	*		$\Lambda(2585)$	**		$\Sigma(1940)$	D_{13}	***	Λ_c^+	****		$K(1830)$	$1/2(0^-)$	$J/\psi(1S)$	$0^-(1^-)$
$N(2200)$	D_{15}	**	$\Delta(2400)$	*					$\Sigma(2000)$	S_{11}	*	$\Lambda_c(2593)^+$	***		$K_2^*(1950)$	$1/2(0^+)$	$\chi_{cc}(1P)$	$0^-(0^+)$
$N(2220)$	H_{19}	****	$\Delta(2450)$	*					$\Sigma(2030)$	F_{17}	****	$\Lambda_c(2625)^+$	***		$K_2^*(1980)$	$1/2(2^+)$	$\chi_{cc}(1P)$	$0^-(0^+)$
$N(2250)$	G_{19}	****	$\Delta(2500)$	*					$\Sigma(2070)$	F_{15}	*	$\Lambda_c(2765)^+$	***		$K_1^*(2045)$	$1/2(4^+)$	$\chi_{cc}(1P)$	$0^-(1^+)$
$N(2600)$	$h_{1,11}$	***	$\Delta(2950)$	$K_{3,15}$	**				$\Sigma(2080)$	P_{13}	**	$\Lambda_c(2880)^+$	**		$K_2(2320)$	$1/2(2^-)$	$\chi_{cc}(2P)$	$0^+(2^+)$
$N(2700)$	$K_{1,13}$	**	$\Theta(1540)^+$	*					$\Sigma(2100)$	G_{17}	***	$\Sigma_c(2455)$	****		$K_2(2350)$	$1/2(3^+)$	$\eta_c(2S)$	$0^+(0^-)$
									$\Sigma(2250)$	*		$\Sigma_c(2520)$	***		$K_4(2500)$	$1/2(4^-)$	$\psi(2S)$	$0^-(1^-)$
									$\Sigma(2455)$	**		$\Sigma_c(2800)$	***		$K(3100)$?(???)	$\psi(3770)$	$0^-(1^-)$
									$\Sigma(2620)$	**					CHARMED		$\chi(3872)$	$0^2(??^+)$
									$\Sigma(3000)$	*					$(C = \pm 1)$		$\chi_{cc}(2P)$	$0^+(2^+)$
									$\Sigma(3170)$	*							$\psi(3940)$?(???)
																	$\psi(4040)$	$0^-(1^-)$
																	$\psi(4160)$	$0^-(1^-)$
																	$\psi(4260)$?(1^-)
																	$\psi(4415)$	$0^-(1^-)$
																	$b\bar{b}$	
																	$\eta_b(1S)$	$0^+(0^-)$
																	$\Upsilon(1S)$	$0^-(1^-)$
																	$\chi_{bb}(1P)$	$0^+(0^+)$
																	$\chi_{bb}(1P)$	$0^+(1^+)$
																	$\chi_{bb}(2P)$	$0^+(2^+)$
																	$\Upsilon(2S)$	$0^-(1^-)$
																	$\Upsilon(1D)$	$0^-(2^-)$
																	$\chi_{bb}(2P)$	$0^+(0^+)$
																	$\chi_{bb}(2P)$	$0^+(1^+)$
																	$\chi_{bb}(2P)$	$0^+(2^+)$
																	$\Upsilon(3S)$	$0^-(1^-)$
																	$\Upsilon(4S)$	$0^-(1^-)$
																	$\Upsilon(10860)$	$0^-(1^-)$
																	$\Upsilon(11020)$	$0^-(1^-)$

~130 baryons

$\sim \frac{1}{300}!$

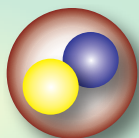
~160 mesons

Exotic hadrons are indeed exotic !!

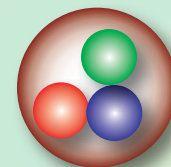
Motivation 1 : Exotic hadrons

Exotic hadrons : states other than $q\bar{q}$, qqq .
Experimentally, they are **exotic**.

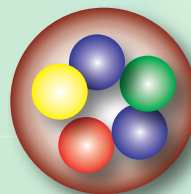
~160 mesons



~130 baryons



1 pentaquark with *

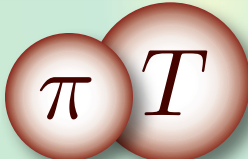


Theoretically, are they exotic?

--> There is no simple way to forbid exotic states in QCD, effective models, ...

Why aren't the exotics observed??

Motivation 2 : Chiral unitary approaches

Hadron excited states \sim 

- Interaction \leftarrow chiral symmetry
- Amplitude \leftarrow unitarity

R.H. Dalitz, and S.F. Tuan, *Ann. Phys. (N.Y.)* 10, 307 (1960)

J.H.W. Wyld, *Phys. Rev.* 155, 1649 (1967)

N. Kaiser, P. B. Siegel and W. Weise, *Nucl. Phys.* A594, 325 (1995)

E. Oset and A. Ramos, *Nucl. Phys.* A635, 99 (1998)

J. A. Oller and U. G. Meissner, *Phys. Lett.* B500, 263 (2001)

M.F.M. Lutz and E. E. Kolomeitsev, *Nucl. Phys.* A700, 193 (2002)

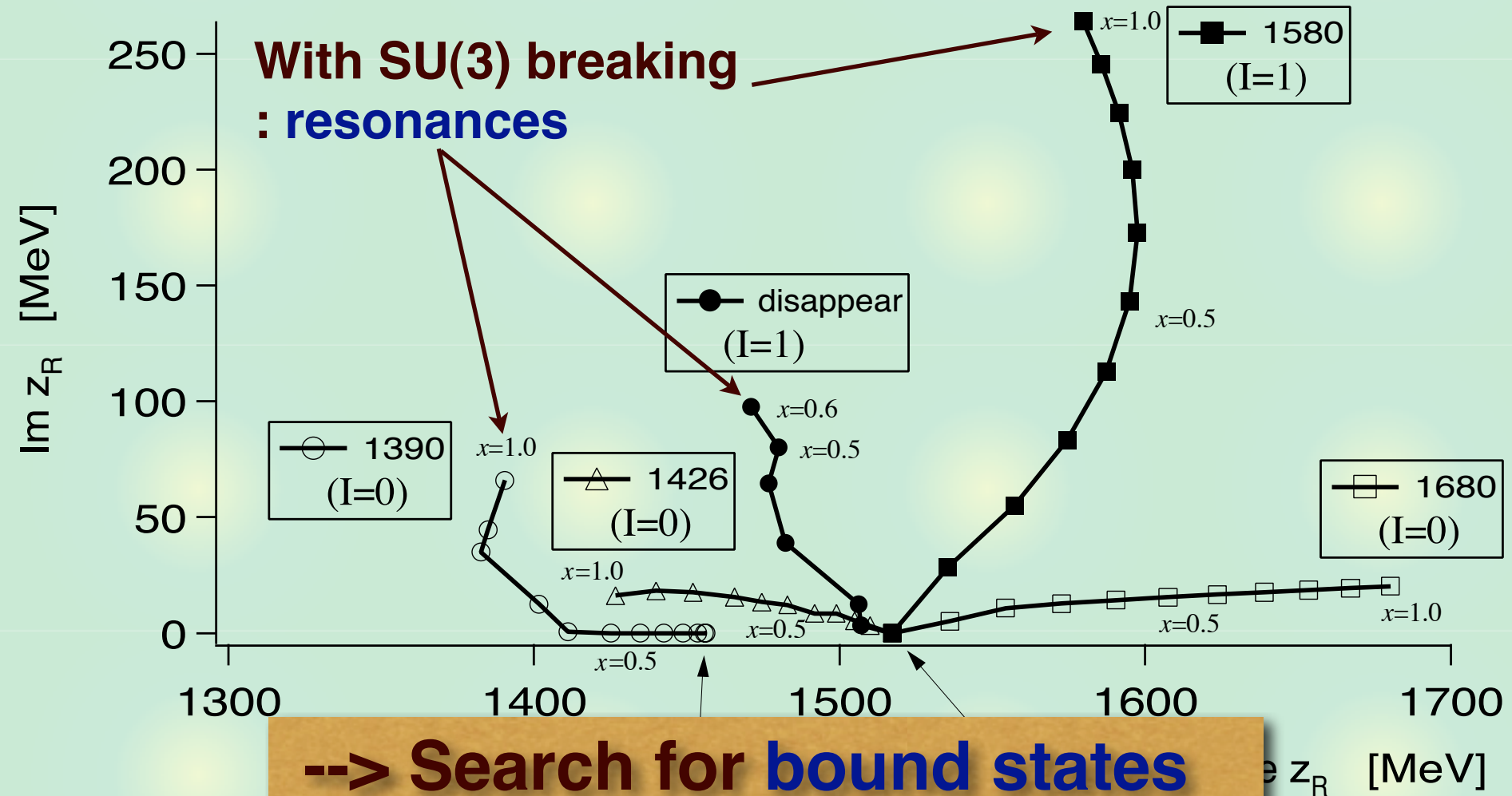
Many hadron resonances ($\Lambda(1405)$, $N(1535)$, $\Lambda(1520)$, $D_s(2317)$,...) are well described.

What about exotic hadrons?

Origin of the resonances

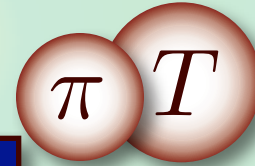
Trajectory of poles

D. Jido, *et al.*, Nucl. Phys. A 723, 205 (2003)



Outline

Hadron-NG boson bound state



Chiral Symmetry

s-wave low energy interaction

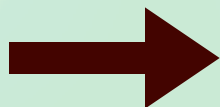
$$V_{\alpha} = -\frac{\omega}{2f^2} C_{\alpha,T} \quad C_{\text{exotic}} = 1$$

Scattering theory

Critical strength for a bound state

$$C_{\text{crit}} = \frac{2f^2}{m[-G(M_T + m)]}$$

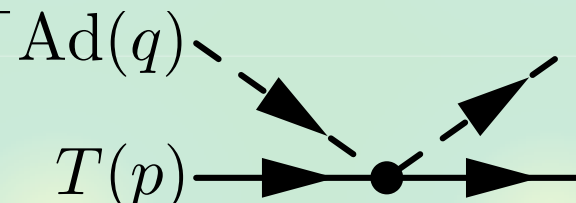
physical values : $C_{\text{exotic}} < C_{\text{crit}}$



No exotic state exists.

Low energy s-wave interaction

Scattering of a target (T) with the pion (Ad)

$$\alpha \left[\begin{array}{c} \text{Ad}(q) \\ T(p) \end{array} \right] \begin{array}{c} \diagdown \\ \bullet \\ \diagup \end{array} = \frac{1}{f^2} \frac{p \cdot q}{2M_T} \langle \mathbf{F}_T \cdot \mathbf{F}_{\text{Ad}} \rangle_\alpha + \mathcal{O}((m/M_T)^2)$$


s-wave : Weinberg-Tomozawa term

$$V_\alpha = -\frac{\omega}{2f^2} C_{\alpha,T}$$

$$C_{\alpha,T} \equiv -\langle 2\mathbf{F}_T \cdot \mathbf{F}_{\text{Ad}} \rangle_\alpha = C_2(T) - C_2(\alpha) + 3 \quad (\text{for } N_f = 3)$$

Coupling : pion decay constant model-independent interaction at low energy

Y. Tomozawa, *Nuovo Cim.* 46A, 707 (1966)

S. Weinberg, *Phys. Rev. Lett.* 17, 616 (1966)

Coupling strengths : Examples

Coupling strengths : (positive is attractive)

$$C_{\alpha,T} = C_2(T) - C_2(\alpha) + 3$$

α	1	8	10	$\overline{10}$	27	35
$T = \mathbf{8}(N, \Lambda, \Sigma, \Xi)$	6	3	0	0	-2	
$T = \mathbf{10}(\Delta, \Sigma^*, \Xi^*, \Omega)$		6	3		1	-3

α	$\overline{3}$	6	$\overline{15}$	24
$T = \overline{\mathbf{3}}(\Lambda_c, \Xi_c)$	3	1	-1	
$T = \mathbf{6}(\Sigma_c, \Xi_c^*, \Omega_c)$	5	3	1	-2

- **Exotic channels** : mostly repulsive
- **Attractive interaction** : **C = 1**

Coupling strengths : General expression

For a general target $T = [p, q]$

$\alpha \in [p, q] \otimes [1, 1]$	$C_{\alpha, T}$	sign
$[p + 1, q + 1]$	$-p - q$	repulsive
$[p + 2, q - 1]$	$1 - p$	
$[p - 1, q + 2]$	$1 - q$	
$[p, q]$	3	attractive
$[p, q]$	3	attractive
$[p + 1, q - 2]$	$3 + q$	attractive
$[p - 2, q + 1]$	$3 + p$	attractive
$[p - 1, q - 1]$	$4 + p + q$	attractive

- **Strength should be integer.**
- **Sign is determined for most cases.**

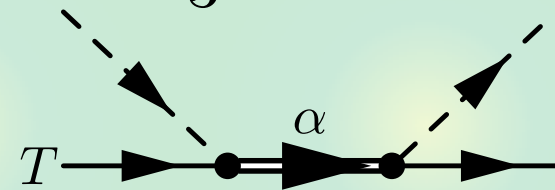
Exotic channels

Exoticness : minimal number of extra $\bar{q}q$.

$$E = \epsilon\theta(\epsilon) + \nu\theta(\nu) \quad \epsilon \equiv \frac{p+2q}{3} - B, \quad \nu \equiv \frac{p-q}{3} - B$$

$\Delta E = E_\alpha - E_T = +1$ is realized when

○ $\alpha = [p+1, q+1] : C_{\alpha,T} = -p - q$
repulsive



○ $\alpha = [p+2, q-1] : C_{\alpha,T} = 1 - p$

attraction : $p = 0$ then $\nu_T \geq 0 \rightarrow B \geq -q/3$
not considered here

○ $\alpha = [p-1, q+2] : C_{\alpha,T} = 1 - q$

attraction : $q = 0$ then $\nu_T \leq 0 \rightarrow B \geq p/3$ OK!

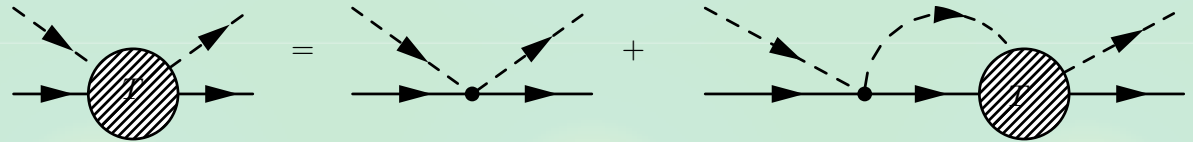
Universal attraction for more “exotic” channel

$$C_{\text{exotic}} = 1 \quad \text{for} \quad T = [p, 0], \quad \alpha = [p-1, 2]$$

Renormalization and bound states

Solve the scattering problem with $V_\alpha = -\frac{\omega}{2f^2} C_{\alpha,T}$

$$T = \frac{1}{1 - VG} V$$



Unitarity : OK

Renormalization parameter : condition

$$G(\mu) = 0, \quad \Leftrightarrow \quad T(\mu) = V(\mu) \quad \text{at} \quad \mu = M_T$$

K. Igi, and K. Hikasa, Phys. Rev. D59, 034005 (1999)

M.F.M. Lutz, and E. Kolomeitsev, Nucl. Phys. A700, 193-308 (2002)

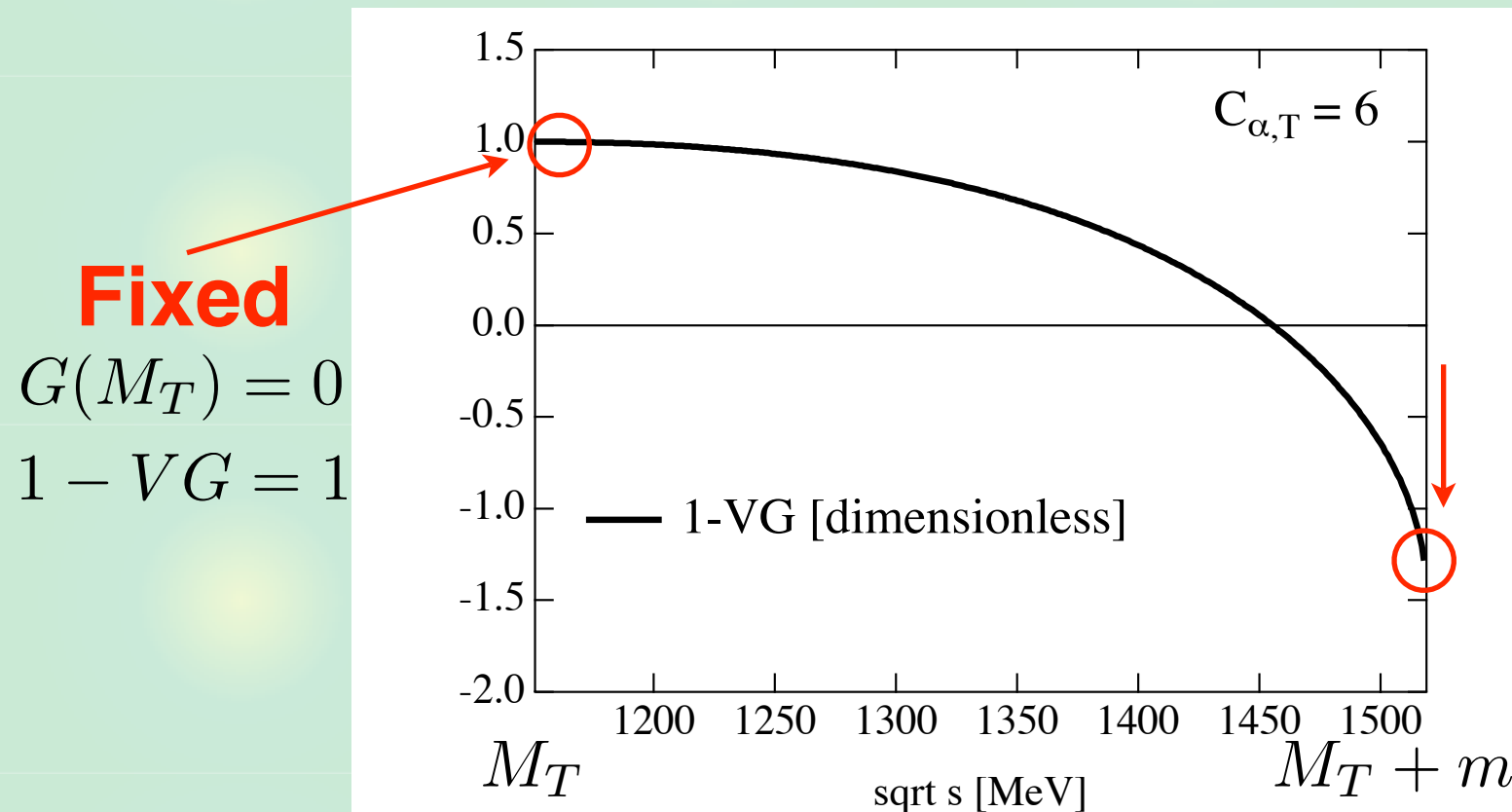
Matching with the u-channel amplitude : OK

Bound state:

$$1 - V(M_b)G(M_b) = 0 \quad M_T < M_b < M_T + m$$

Critical attraction

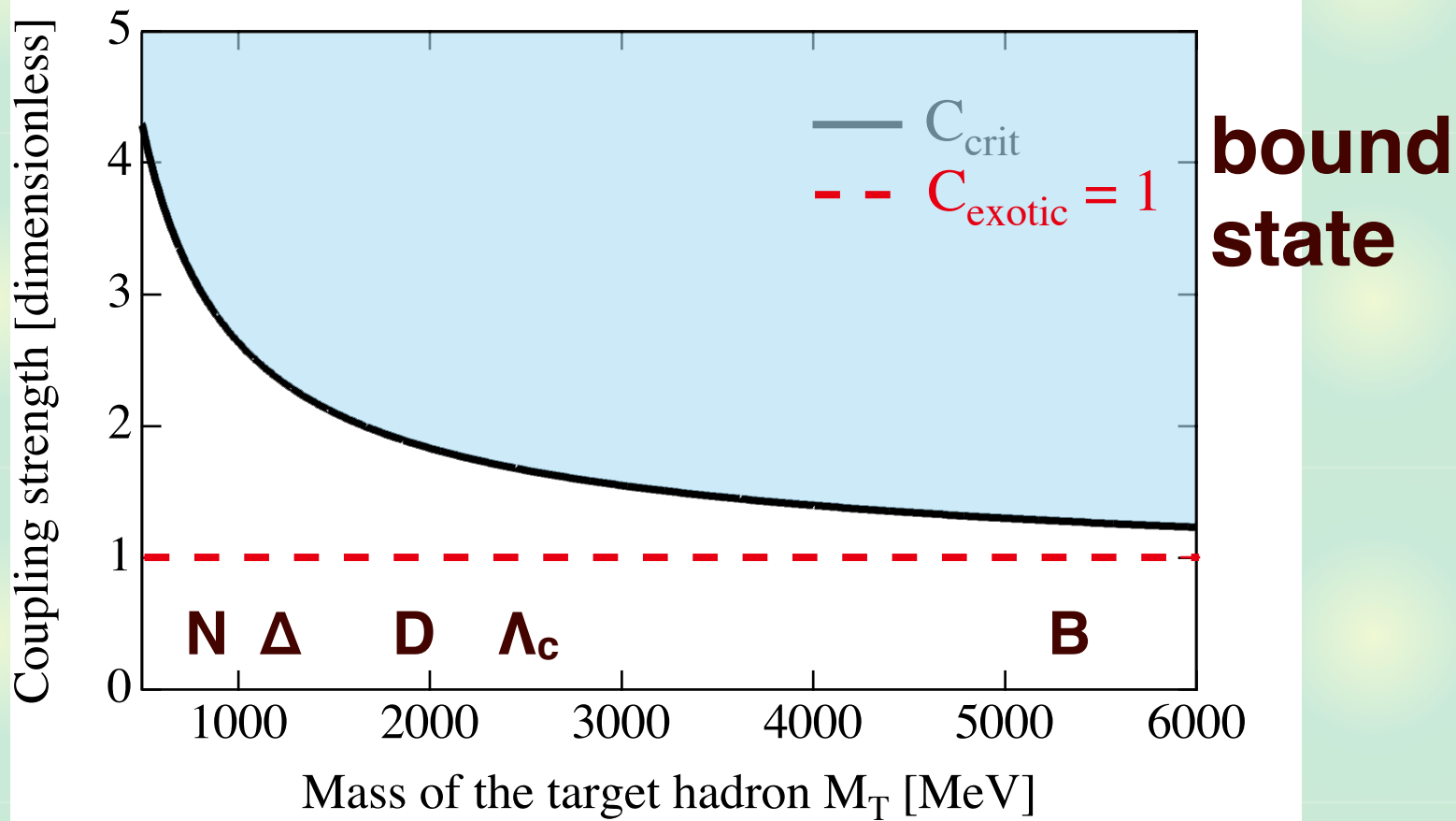
$1 - V(\sqrt{s})G(\sqrt{s})$: monotonically decreasing.



Critical attraction : $1 - VG = 0$ at $\sqrt{s} = M_T + m$

$$\longrightarrow C_{\text{crit}} = \frac{2f^2}{m[-G(M_T + m)]}$$

Critical attraction and exotic channel



$$m = 368 \text{ MeV and } f = 93 \text{ MeV}$$

➔ Strength is not enough.

Large N_c limit : introduction

$1/N_c$: a possible expansion parameter

G. 't Hooft, Nucl. Phys. B72, 461 (1974)

E. Witten, Nucl. Phys. B160, 57 (1979)

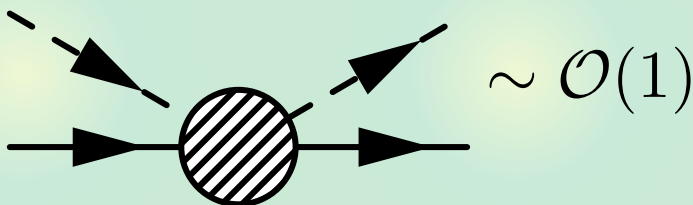
Scaling of the physical quantities ← N_c^2 gluons and N_c quarks.

Meson mass : $m \sim \mathcal{O}(1)$

Baryon mass : $M \sim \mathcal{O}(N_c)$

Decay constant : $f \sim \mathcal{O}(\sqrt{N_c})$

MB scattering :

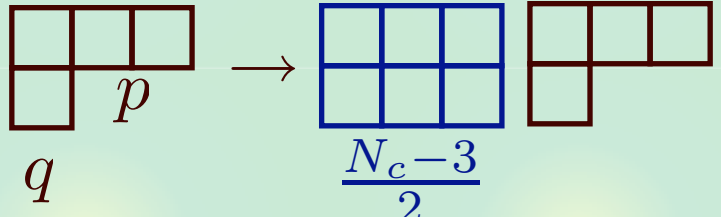


Coupling strengths in large Nc limit

WT interaction in large Nc limit

$$V_\alpha = -\frac{\omega}{2f^2} C_{\alpha,T} \sim \frac{1}{N_c} \times C_{\alpha,T}$$

Flavor representation of baryons

$$[p, q] \rightarrow \text{“}[p, q]\text{”} = \left[p, q + \frac{N_c - 3}{2} \right]$$


Coupling strength has **linear Nc dependence**

$$\underline{C_{\text{“}\alpha\text{”}, \text{“}T\text{”}}(N_c)} = C_2(\text{“}T\text{”}) - C_2(\text{“}\alpha\text{”}) + 3$$

$$C(\text{“}[p, q]\text{”}) = \frac{1}{3} \left(-\frac{9}{4} + p^2 + \frac{3p}{2} + pq + q^2 \right) + \frac{1}{3} \left(\frac{p}{2} + q \right) \underline{N_c} + \frac{N_c^2}{12}$$

Coupling strengths for the general target

For arbitrary N_c , $V \propto -\frac{C}{f^2} \sim -\frac{C(N_c)}{N_c}$

$\alpha \in [p, q] \otimes [1, 1]$	$C_{\alpha, T}(N_c)$	$V(N_c \rightarrow \infty)$
$[p + 1, q + 1]$	$(3 - N_c)/2 - p - q$	repulsive
$[p + 2, q - 1]$	$1 - p$	
$[p - 1, q + 2]$	$(5 - N_c)/2 - q$	repulsive
$[p, q]$	3	
$[p, q]$	3	
$[p + 1, q - 2]$	$(3 + N_c)/2 + q$	attractive
$[p - 2, q + 1]$	$3 + p$	
$[p - 1, q - 1]$	$(5 + N_c)/2 + p + q$	attractive

- No attraction in **exotic channels.**

Coupling strengths : Examples

Coupling strengths with arbitrary N_c

$$C_{\alpha, T}(N_c) = C_2(T) - C_2(\alpha) + 3$$

α	“1”	“8”	“10”	“ $\overline{10}$ ”	“27”	“35”
$T = \text{“8”}$	$\frac{9+N_c}{2}$	3	0	$\frac{3-N_c}{2}$	$\frac{-2-N_c}{2}$	
$T = \text{“10”}$		6	3		$\frac{5-N_c}{2}$	$\frac{-3-N_c}{2}$

α	“ $\overline{3}$ ”	“6”	“ $\overline{15}$ ”	“24”
$T = \text{“\overline{3}”}$	3	1	$\frac{1-N_c}{2}$	
$T = \text{“6”}$	5	3	$\frac{5-N_c}{2}$	$\frac{-2-N_c}{2}$

- **Exotic attraction** -> repulsive
- **Two poles of $\Lambda(1405)$?**

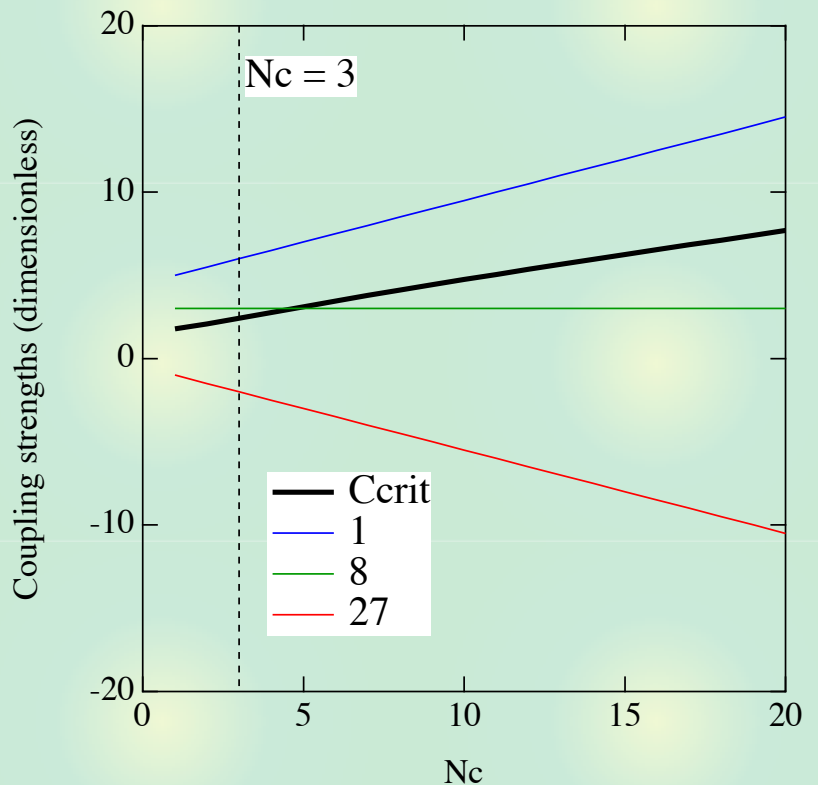
$S = -1$ $I = 0$ channel in $SU(3)$ basis

α	“1”	“ 8_s ”	“ 8_a ”	“27”
$T = \text{“8”}$	$\frac{9+N_c}{2}$	3	3	$\frac{-2-N_c}{2}$

$$C_{\text{crit}}(N_c) = \frac{2[f(N_c)]^2}{m[-G(M_T(N_c) + m)]}$$

$$M_T(N_c) = M_0 \times \frac{N_c}{3}$$

$$f(N_c) = f_0 \times \sqrt{\frac{N_c}{3}}$$



○ **Bound state in “1” in the large N_c limit.**

S = -1 I = 0 channel in Isospin basis

Basis transformation via CG Coef. with N_c

T.D. Cohen, and R.F. Lebed, Phys. Rev. D 70, 096015 (2004)

$$C_{ij}(N_c) = \begin{pmatrix} \bar{K}N & \pi\Sigma & \eta\Lambda & K\Xi \\ \frac{1}{2}(3 + N_c) & -\frac{\sqrt{3}}{2}\sqrt{-1 + N_c} & \frac{\sqrt{3}}{2}\sqrt{3 + N_c} & 0 \\ -\frac{\sqrt{3}}{2}\sqrt{-1 + N_c} & 4 & 0 & \frac{\sqrt{3 + N_c}}{2} \\ \frac{\sqrt{3}}{2}\sqrt{3 + N_c} & 0 & 0 & -\frac{\sqrt{3}}{2}\sqrt{-1 + N_c} \\ 0 & \frac{\sqrt{3 + N_c}}{2} & -\frac{\sqrt{3}}{2}\sqrt{-1 + N_c} & \frac{1}{2}(9 - N_c) \end{pmatrix}$$

Combining with the $1/N_c$ factor of $1/f^2$,

- $\bar{K}N \rightarrow \bar{K}N$: attractive at large N_c
- $\bar{K}N \rightarrow \pi\Sigma$: $\mathcal{O}(1/\sqrt{N_c})$
- $\pi\Sigma \rightarrow \pi\Sigma$: $\mathcal{O}(1/N_c)$

Summary 1 : SU(3) limit




We study the exotic bound states in s-wave chiral dynamics in flavor SU(3) limit.

- The interaction in exotic channels are in most cases **repulsive**.
- There are **attractive interactions** in exotic channels, with **universal** and the smallest strength : $C_{\text{exotic}} = 1$
- The strength is **not enough** to generate a bound state : $C_{\text{exotic}} < C_{\text{crit}}$

The result is **model-independent** as far as we respect chiral symmetry.

Summary 2 : Physical world

Caution!

-  The exotic hadrons here are the **s-wave** meson-hadron molecule states ($1/2^-$ for Θ^+).
-  We do not exclude the exotics which have **other origins** (genuine quark state, soliton rotation,...).
-  In practice, **SU(3) breaking** effect, **higher order** terms,...

In Nature, it is **difficult** to generate exotic hadrons as in the same way with $\Lambda(1405)$, $\Lambda(1520)$,... based on chiral dynamics.

[T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. Lett. 97, 192002 \(2006\)](#)

[T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. D 75, 034002 \(2007\)](#)