

Study of exotic hadrons in s-wave scatterings induced by chiral interaction in the flavor symmetric limit



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2006, Oct. 13th 1

Motivation : Exotic hadrons

Exotic hadrons : states other than $q\bar{q}$, qqq .

--> QCD **does not forbid exotic states,
effective models neither.**

**Experimentally, (almost?) completely absent
--> highly non-trivial fact**

Existence of exotic hadrons?

- **Resonance saturation + duality**

J.L. Rosner, Phys. Rev. Lett. 21, 950 (1968)

D.P. Roy and M. Suzuki, Phys. Lett. B28, 558 (1969)

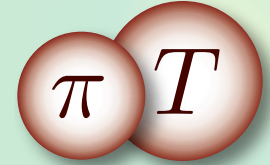
- **Large N_c + heavy quark**

T.D. Cohen, *et al.*, Phys. Rev. D72, 074010 (2005)

**--> Examine existence of exotic hadrons
in flavor SU(3) limit.**

Outline

Hadron-NG boson bound state



Chiral symmetry

s-wave low energy interaction

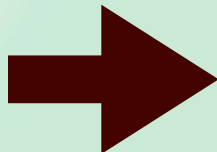
$$V_{\alpha} = -\frac{\omega}{2f^2} C_{\alpha,T} \quad C_{\text{exotic}} = 1$$

Scattering theory

Critical strength for a bound state

$$C_{\text{crit}} = \frac{2f^2}{m(-G(M_T + m))}$$

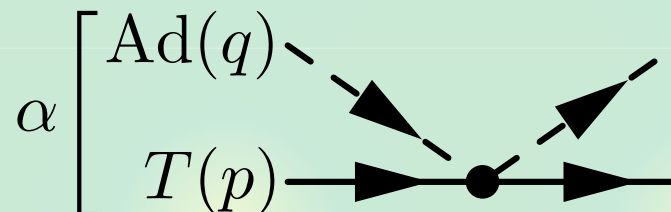
physical values : $C_{\text{exotic}} < C_{\text{crit}}$



No exotic state exists.

Low energy s-wave interaction

Scattering of a target (T) with the pion (Ad)

$$\alpha \left[\begin{array}{c} \text{Ad}(q) \\ T(p) \end{array} \right] = \frac{1}{f^2} \frac{p \cdot q}{2M_T} \langle \mathbf{F}_T \cdot \mathbf{F}_{\text{Ad}} \rangle_\alpha + \mathcal{O} \left((m/M_T)^2 \right)$$


In s-wave,

$$V_\alpha = -\frac{\omega}{2f^2} C_{\alpha,T}$$

- proportional to pion energy
- pion decay constant (No LEC)

Y. Tomozawa, *Nuovo Cim.* **46A**, 707 (1966)

S. Weinberg, *Phys. Rev. Lett.* **17**, 616 (1966)

$$C_{\alpha,T} \equiv -\langle 2\mathbf{F}_T \cdot \mathbf{F}_{\text{Ad}} \rangle_\alpha = C_2(T) - C_2(\alpha) + 3 \quad (\text{for } N_f = 3)$$

Coupling strengths : Examples

Examples of C_α : (positive is attractive)

$$C_{\alpha,T} = C_2(T) - C_2(\alpha) + 3$$

α	1	8	10	$\bar{10}$	27	35
T=8 (N, Λ, Σ, Ξ)	6	3	0	0	-2	
T=10(Δ, Σ^*, Ξ^*, Ω)		6	3		1	-3

α	$\bar{3}$	6	$\bar{15}$	24
T=$\bar{3}$ (Λ_c, Ξ_c)	3	1	-1	-2
T=6 (Σ_c, Ξ_c^*, Ω_c)	5	3	1	

- **Exotic channels** : mostly repulsive
- **Attractive interaction** : **C = 1**

Coupling strengths : General expression

$$T = [p, q] \quad \alpha \in [p, q] \otimes [1, 1]$$

α	$C_{\alpha, T}$	sign	ΔE
$[p + 1, q + 1]$	$-p - q$	repulsive	1 or 0
$[p + 2, q - 1]$	$1 - p$		1 or 0
$[p - 1, q + 2]$	$1 - q$		1 or 0
$[p, q]$	3	attractive	0
$[p, q]$	3	attractive	0
$[p + 1, q - 2]$	$3 + q$	attractive	0 or -1
$[p - 2, q + 1]$	$3 + p$	attractive	0 or -1
$[p - 1, q - 1]$	$4 + p + q$	attractive	0 or -1

- **C should be integer.**
- **Sign is determined for most cases.**

Exoticness : minimal number of extra $\bar{q}q$.

For $[p, q]$ and baryon number B ,

$$E = \epsilon\theta(\epsilon) + \nu\theta(\nu) \quad \epsilon \equiv \frac{p+2q}{3} - B, \quad \nu \equiv \frac{p-q}{3} - B$$

$\Delta E = E_\alpha - E_T = +1$ is realized when

- $\Delta\epsilon = 1, \Delta\nu = 0, \epsilon_T \geq 0,$
 $\alpha = [p+1, q+1] : C_{\alpha,T} = -p - q$ **repulsive**
- $\Delta\epsilon = 0, \Delta\nu = 1, \nu_T \geq 0,$
 $\alpha = [p+2, q-1] : C_{\alpha,T} = 1 - p$
 attraction : $p = 0$ then $\nu_T \geq 0 \rightarrow B \leq -q/3$ **not considered here**
- $\Delta\epsilon = 1, \Delta\nu = -1, \nu_T \leq 0,$
 $\alpha = [p-1, q+2] : C_{\alpha,T} = 1 - q$
 attraction : $q = 0$ then $\nu_T \leq 0 \rightarrow B \geq p/3$ **OK!**

Universal attraction for more “exotic” channel

$$C_{\text{exotic}} = 1 \quad \text{for} \quad T = [p, 0], \quad \alpha = [p-1, 2]_7$$

Coupling strengths : large N_c behavior

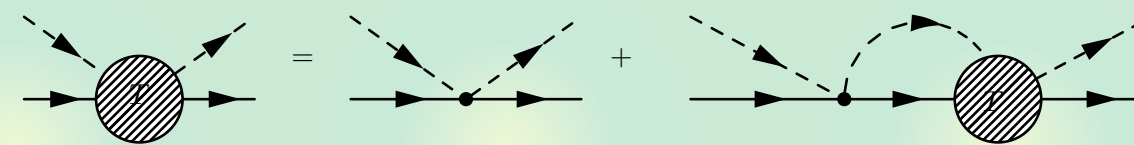
For arbitrary N_c , $[p, q] \rightarrow \left[p, q + \frac{3 - N_c}{2} \right] \quad V \propto -\frac{1}{f^2} C \sim \frac{1}{N_c} C(N_c)$

α	$C^{\text{"}\alpha\text{"}, \text{"}T\text{"}}(N_c)$	$V(N_c \rightarrow \infty)$	ΔE
$[p + 1, q + 1]$	$(3 - N_c)/2 - p - q$	repulsive	1 or 0
$[p + 2, q - 1]$	$1 - p$	0	1 or 0
$[p - 1, q + 2]$	$(5 - N_c)/2 - q$	repulsive	1 or 0
$[p, q]$	3	0	0
$[p, q]$	3	0	0
$[p + 1, q - 2]$	$(3 + N_c)/2 + q$	attractive	0 or -1
$[p - 2, q + 1]$	$3 + p$	0	0 or -1
$[p - 1, q - 1]$	$(5 + N_c)/2 + p + q$	attractive	0 or -1

- **Exotic attraction --> repulsion**
- **No attraction in exotic channels.**

Renormalization and bound states

Solve the scattering problem with $V_\alpha = -\frac{\omega}{2f^2} C_{\alpha,T}$

$$T = \frac{1}{1 - VG} V$$


Unitarity : OK

Renormalization parameter : condition

$$G(\mu) = 0, \quad \Leftrightarrow \quad T(\mu) = V(\mu) \quad \text{at} \quad \mu = M_T$$

K. Igi, and K. Hikasa, PRD59, 034005 (1999)

M.F.M. Lutz, and E. Kolomeitsev, NPA 700, 193-308 (2002)

Approximate crossing symmetry : OK

Bound state:

$$1 - V(M_b)G(M_b) = 0 \quad M_T < M_b < M_T + m,$$

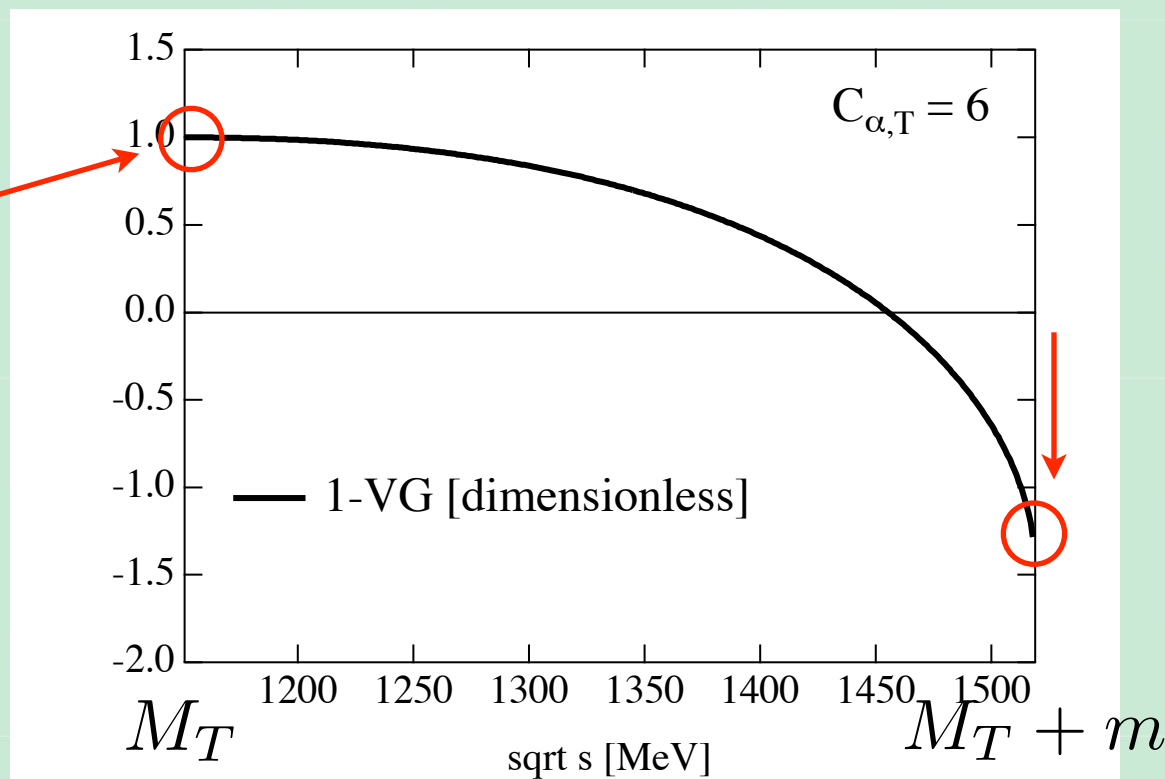
Critical attraction

$1 - V(\sqrt{s})G(\sqrt{s})$: monotonically decreasing.

Fixed

$$G(M_T) = 0$$

$$1 - VG = 1$$

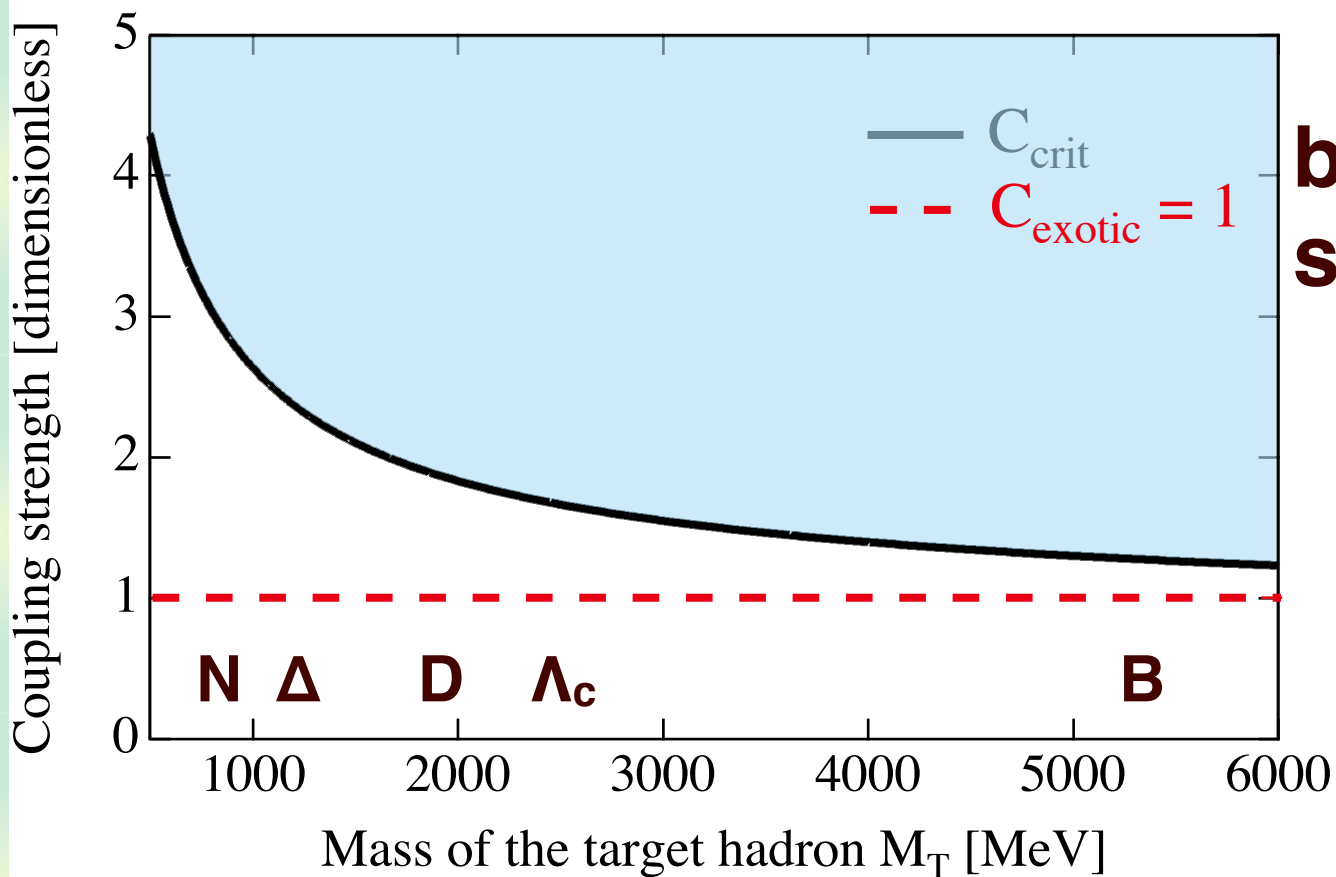


Critical attraction : $1 - VG = 0$ at $\sqrt{s} = M_T + m$

$$C_{\text{crit}} = \frac{2f^2}{m(-G(M_T + m))}$$

Critical attraction and exotic channel




$$m = 368 \text{ MeV} \text{ and } f = 93 \text{ MeV}$$



➔ Strength is not enough.

Summary

We study the exotic bound states in s-wave chiral dynamics in flavor SU(3) limit.

-  There are **attractions in exotic channels**, with **universal** and the **smallest strength** : $C_{\text{exotic}} = 1$
-  This is **not enough** to generate a bound state : $C_{\text{exotic}} < C_{\text{crit}}$
-  This does not exclude the exotic states with other origin.

T. Hyodo, D. Jido, A. Hosaka, hep-ph/0609014, Phys. Rev. Lett., in press