

# Phenomenological study for the $\Theta^+$ and two-meson coupling



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## Introduction : Flavor SU(3) symmetry

Existence of  $\Theta^+$  + Flavor SU(3) symmetry

➡ Existence of **flavor partners** of  $\Theta^+$

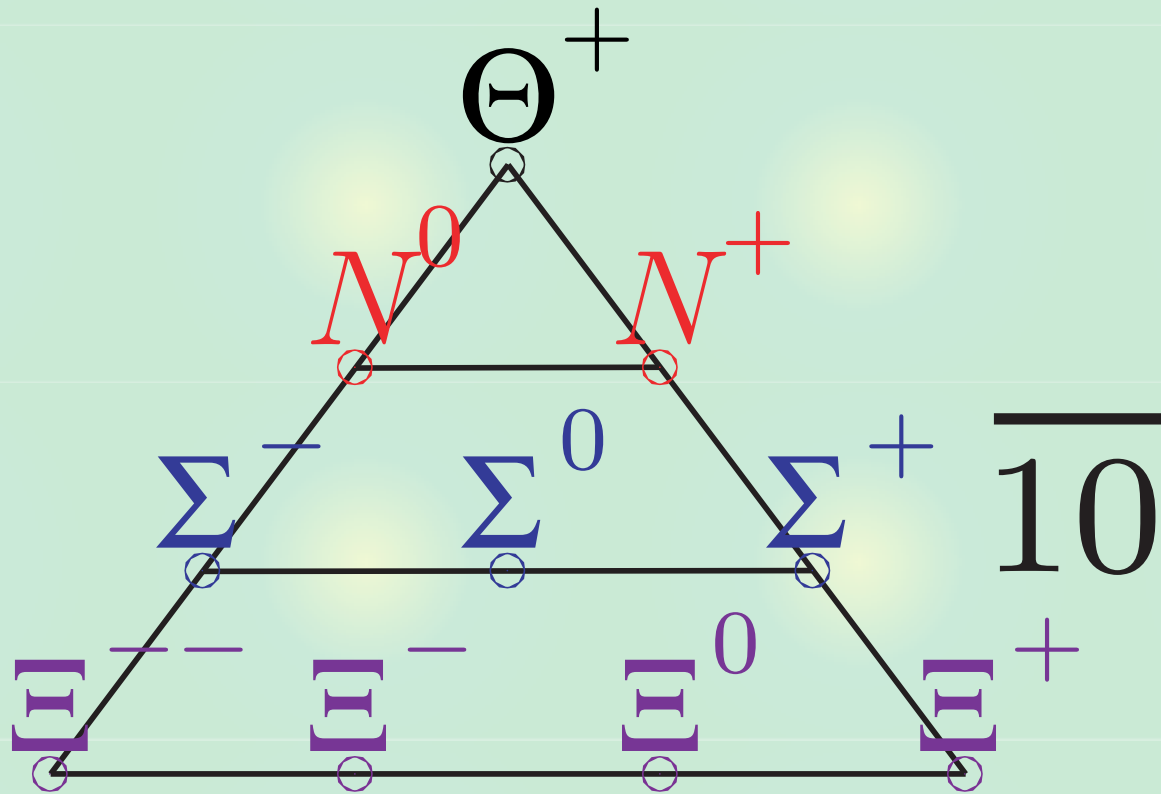
Assuming the flavor multiplet that  $\Theta^+$  belongs to, we examine its properties by symmetry relation, in connection with known baryon resonances.

➡ to determine the  $J^P$  of  $\Theta^+$

Phenomenological but model independent analysis up to  $O(m_s)$

# Pure antidecuplet case

## Simplest assignment for $\Theta^+$



Test the masses and widths of partners  
via flavor SU(3) symmetry relations

## Pure antidecuplet case

**Mass : Gell-Mann—Okubo formula**

$$M(\overline{10}; Y) = M_{\overline{10}} - aY$$

**Two parameters ← Mass of  $\Theta$  and  $N^*$**

**Width : SU(3) symmetric coupling**

$$g_{\Theta KN} = \sqrt{6}g_{N^* \pi N}$$

$$\Gamma_R = g_R^2 F_I \frac{p^{2l+1}}{M_R^{2l}}$$

**One parameter ← Width of  $N^*$**

## Pure antidecuplet case

# Mass and width [MeV]

$$M(\overline{\mathbf{10}}; Y) = M_{\overline{\mathbf{10}}} - aY, \quad g_{\Theta KN} = \sqrt{6}g_{N^*\pi N}, \quad \Gamma_R = g_R^2 F_I \frac{p^{2l+1}}{M_R^{2l}}$$

$J^P$	$M_\Theta$	$M_N$	$M_\Sigma$	$M_\Xi$	$\Gamma_\Theta$
$1/2^-$ exp.	1540 $\Theta(1540)$	1647 N(1650)	1753 $\Sigma(1750)$	1860 $\Xi(1860)$	156.1
$1/2^+$ exp.					
$3/2^+$ exp.					
$3/2^-$ exp.					

## Pure antidecuplet case

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$1/2^+$ exp.	1540 $\Theta(1540)$	1710 N(1710)	1880 $\Sigma(1880)$	2050 $\Xi(2030)$	7.2
$3/2^+$ exp.					
$3/2^-$ exp.					

## Pure antidecuplet case

# Mass and width [MeV]

$$M(\overline{10}; Y) = M_{\overline{10}} - aY, \quad g_{\Theta KN} = \sqrt{6}g_{N^*\pi N}, \quad \Gamma_R = g_R^2 F_I \frac{p^{2l+1}}{M_R^{2l}}$$

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$3/2^+$ exp.	1540 $\Theta(1540)$	1720 N(1720)	<b>1900</b>	<b>2080</b>	<b>10.6</b>
$3/2^-$ exp.					



## Pure antidecuplet case

# Mass and width [MeV]

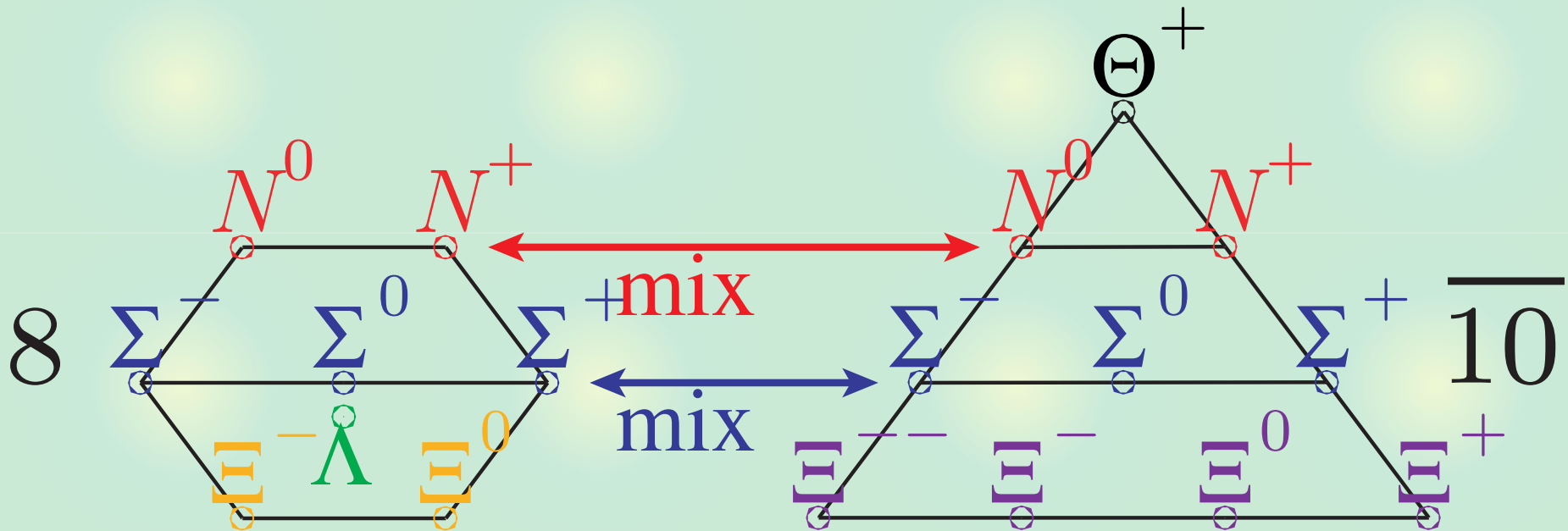
$$M(\overline{10}; Y) = M_{\overline{10}} - aY, \quad g_{\Theta KN} = \sqrt{6}g_{N^*\pi N}, \quad \Gamma_R = g_R^2 F_I \frac{p^{2l+1}}{M_R^{2l}}$$

$J^P$	$M_\Theta$	$M_N$	$M_\Sigma$	$M_\Xi$	$\Gamma_\Theta$
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$1/2^+$ exp.	1540 $\Theta(1540)$	1710 N(1710)	1880 $\Sigma(1880)$	2050 $\Xi(2030)$	7.2
$3/2^+$ exp.	1540 $\Theta(1540)$	1720 N(1720)	1900	2080	10.6
$3/2^-$ exp.	1540 $\Theta(1540)$	1700 N(1700)	1860	2020 $\Xi(2030)$	1.3

are not reproduced simultaneously.

# Octet-antidecuplet mixing

## Second simplest assignment for $\Theta^+$



Mixing is induced by the  $SU(3)$  breaking in mass term.

# Octet-antidecuplet mixing

## Mass formulae : GMO + mixing (N,Σ)

$$M_{\Theta} = M_{\overline{10}} - 2a$$

$$M_{\Xi_{\overline{10}}} = M_{\overline{10}} + a$$

$$M_{\Lambda} = M_{\mathbf{8}}$$

$$M_{\Xi_{\mathbf{8}}} = M_{\mathbf{8}} + b + \frac{1}{2}c$$

$$M_{N_1} = \left( M_{\mathbf{8}} - b + \frac{1}{2}c \right) \cos^2 \theta_N + (M_{\overline{10}} - a) \sin^2 \theta_N - \delta \sin 2\theta_N$$

$$M_{N_2} = \left( M_{\mathbf{8}} - b + \frac{1}{2}c \right) \sin^2 \theta_N + (M_{\overline{10}} - a) \cos^2 \theta_N + \delta \sin 2\theta_N$$

$$M_{\Sigma_1} = (M_{\mathbf{8}} + 2c) \cos^2 \theta_{\Sigma} + M_{\overline{10}} \sin^2 \theta_{\Sigma} - \delta \sin 2\theta_{\Sigma}$$

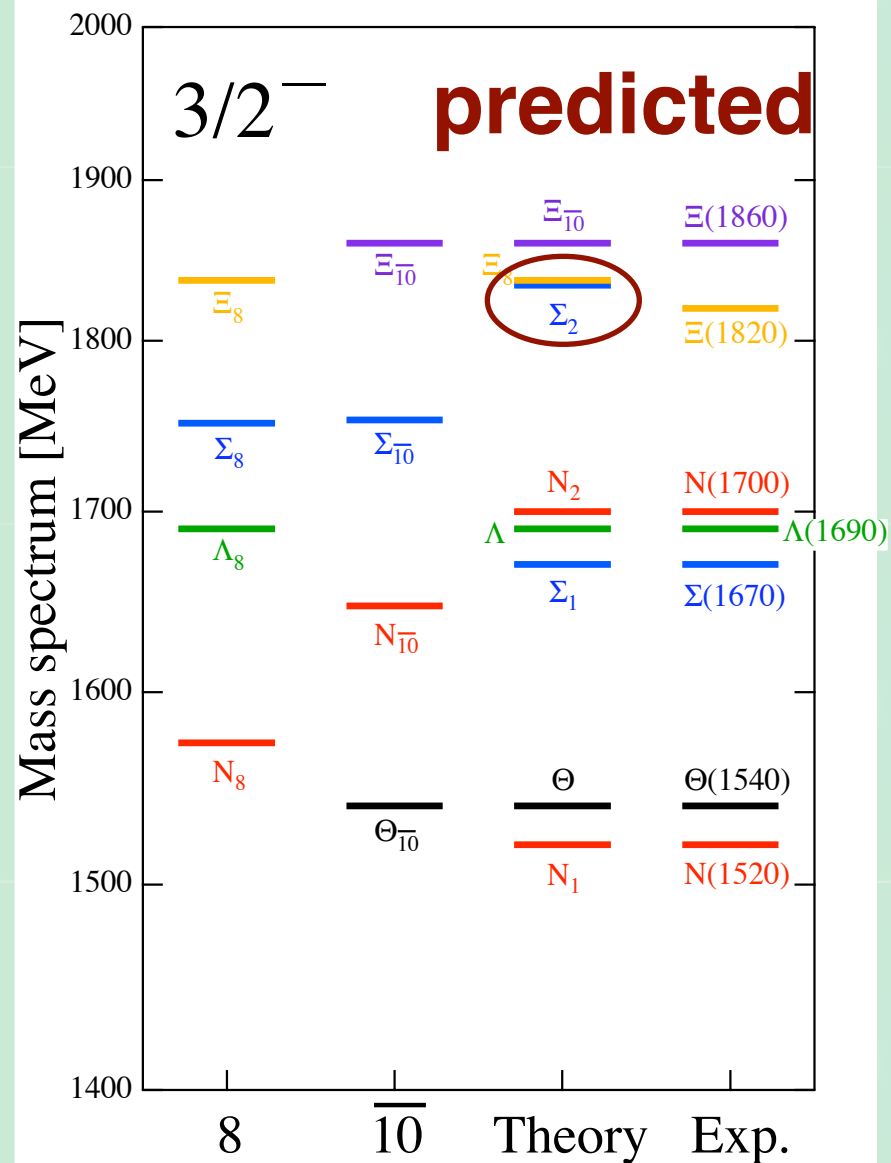
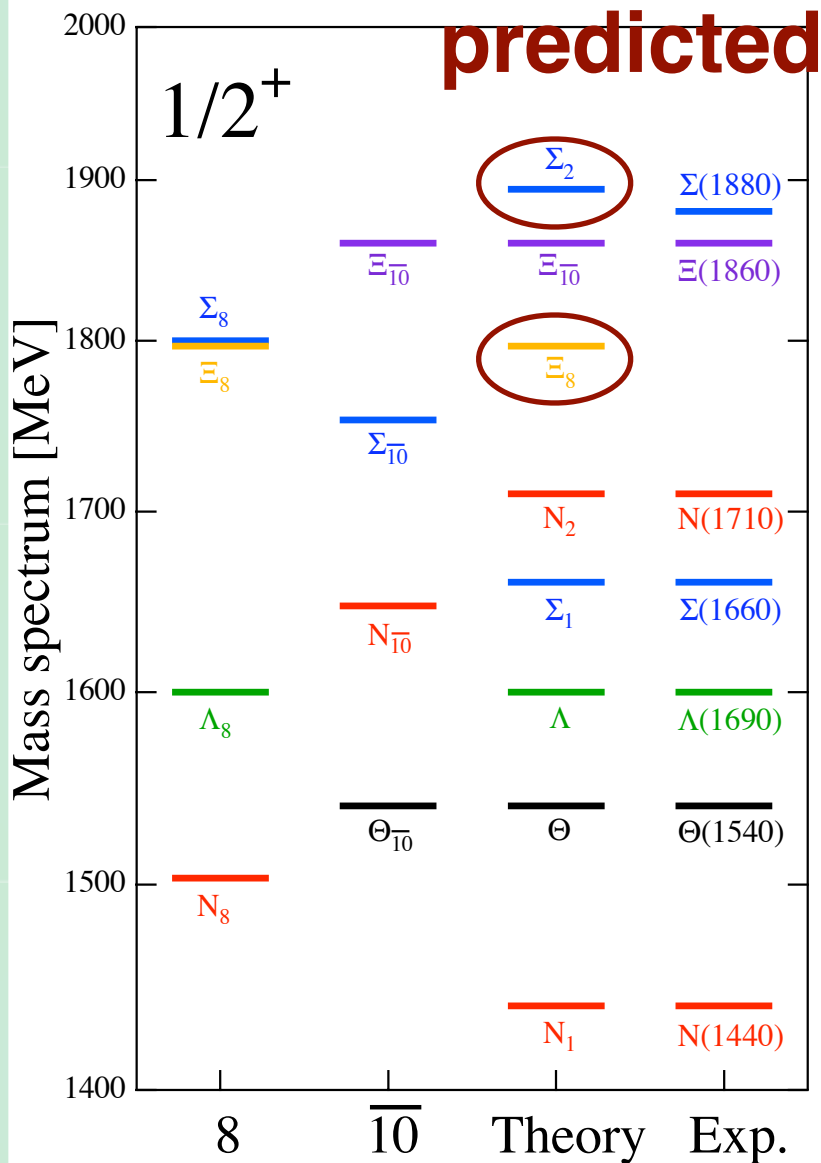
$$M_{\Sigma_2} = (M_{\mathbf{8}} + 2c) \sin^2 \theta_{\Sigma} + M_{\overline{10}} \cos^2 \theta_{\Sigma} + \delta \sin 2\theta_{\Sigma}$$

## 8 masses v.s. 6 parameters

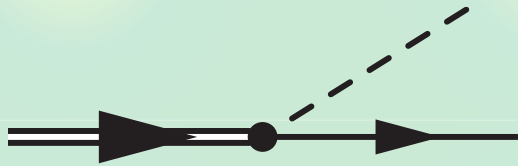
$J^P = 1/2^-$  : too wide width

$J^P = 3/2^+$  : states are not well established

# Mass spectra



# Decay width of $\Theta$



**N\* decay**

$$g_{\Theta} = \sqrt{6} (g_{N_2} \cos \theta_N - g_{N_1} \sin \theta_N)$$

**from masses**

$$\Gamma_R = g_R^2 F_I \frac{p^{2l+1}}{M_R^{2l}}$$

$J^P$	$\theta_N$ [deg]	$\Gamma_{\Theta}$ [MeV]
$1/2^+$	29	29.1
$3/2^-$	33	3.1

**Narrow width**

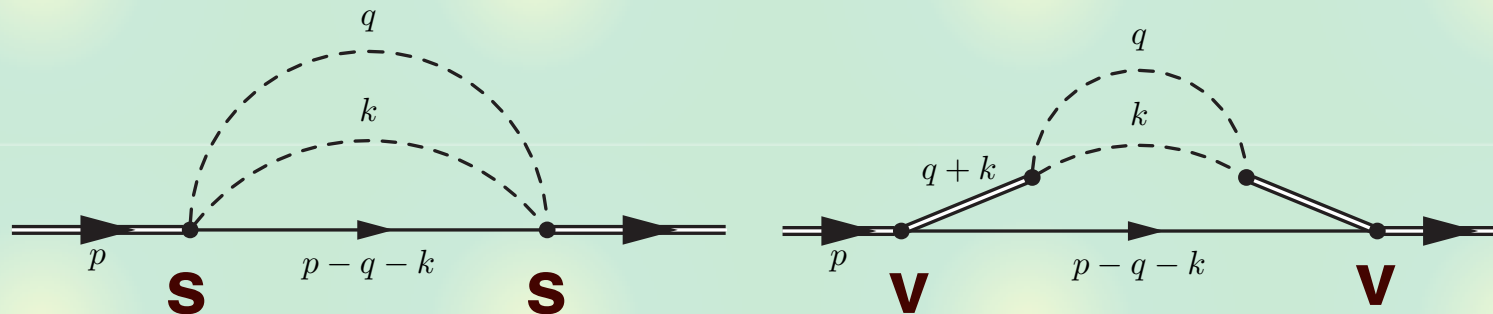


# Two-meson coupling

## The structure of the two-meson coupling

Hosaka, Hyodo, Estrada, Oset, Peláez, Vacas, PRC71, 074021 (2005).

- The effect of the two-meson coupling was studied by evaluating the self-energy.
- We examined possible structures, and found that **two types** of the interaction Lagrangians were important.

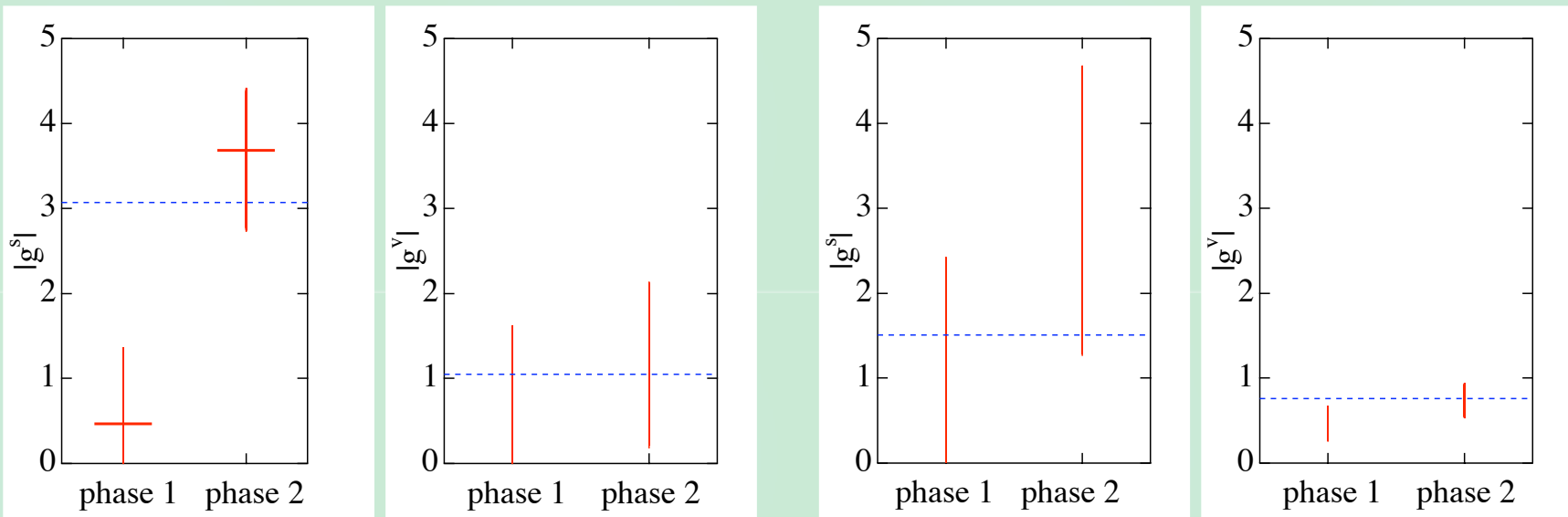


- These terms provided a sizable contribution.

# Two-meson coupling

## Branching fraction [%]

$J^P$	state	$\pi N$	$\pi\pi N(s)$	$\pi\pi N(v)$
$1/2^+$	N(1440)	65	7.5	<8
	N(1710)	15	25	15
$3/2^-$	N(1520)	55	25	20
	N(1700)	10	<85-95	<35



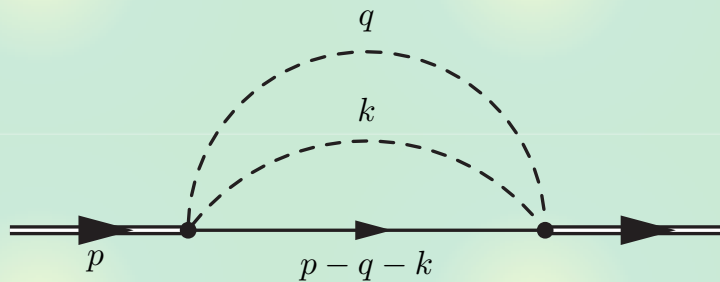
**Still large uncertainty**



# Constraints on the coupling

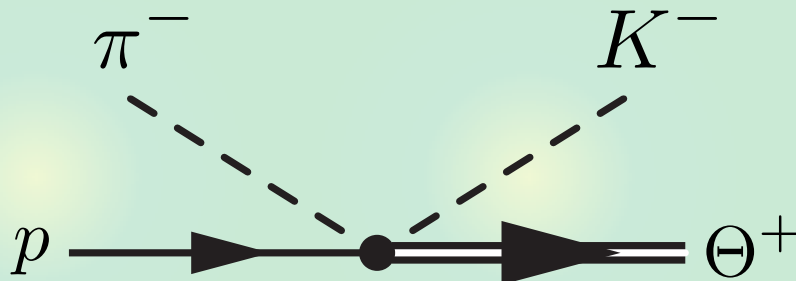
We impose phenomenological constraints.

Self-energy : not too large, but not too small



$\sim 100 \text{ MeV}$

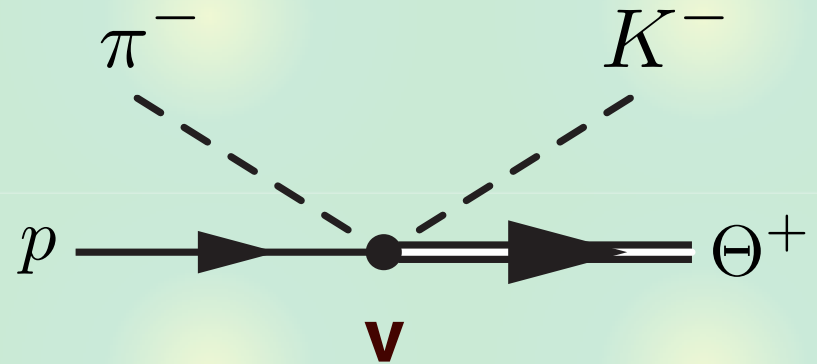
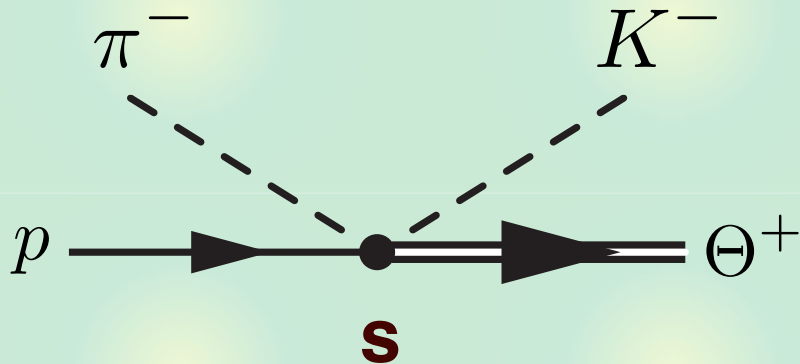
$\pi^- p \rightarrow K^- \Theta^+$  at KEK : upper limit is  $\sim 4.1 \mu\text{b}$



$< 4.1 \mu\text{b}$

## Constraints on the coupling

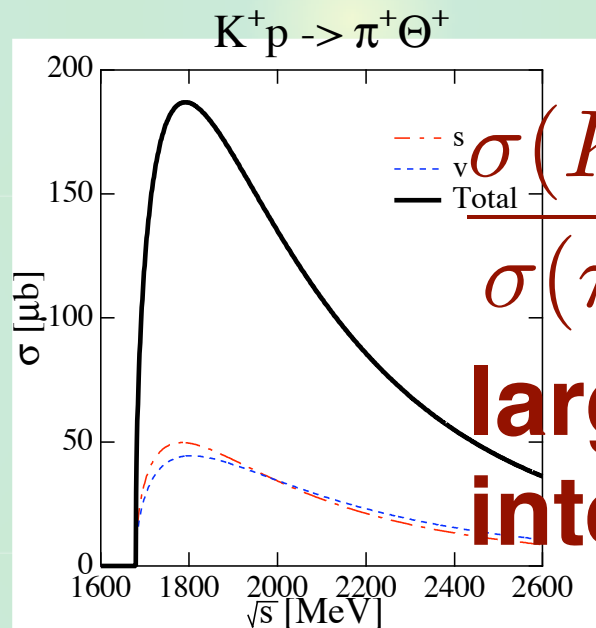
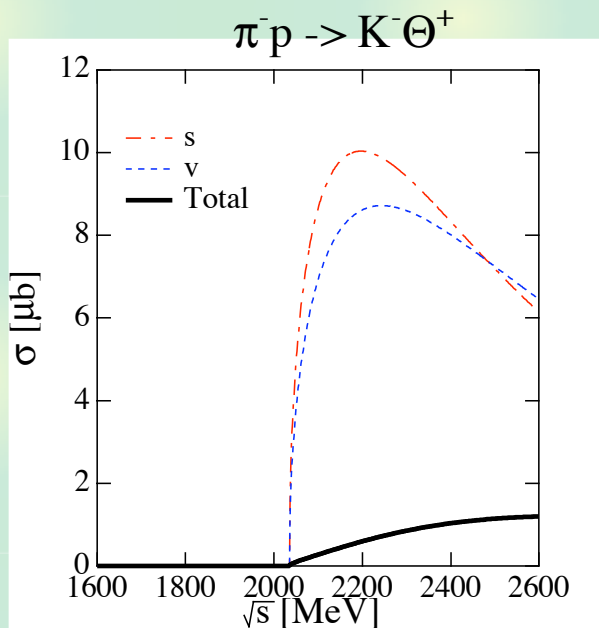
Two structures should be added coherently.



—> interference effect among  $s$  and  $v$ .

# $\Theta$ production

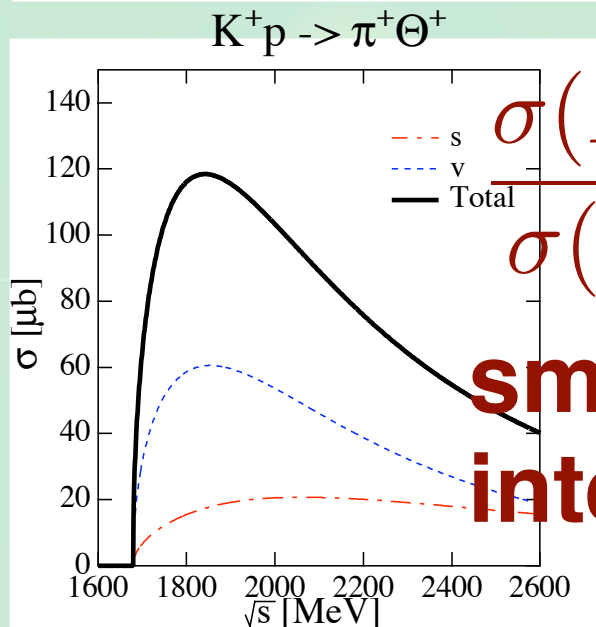
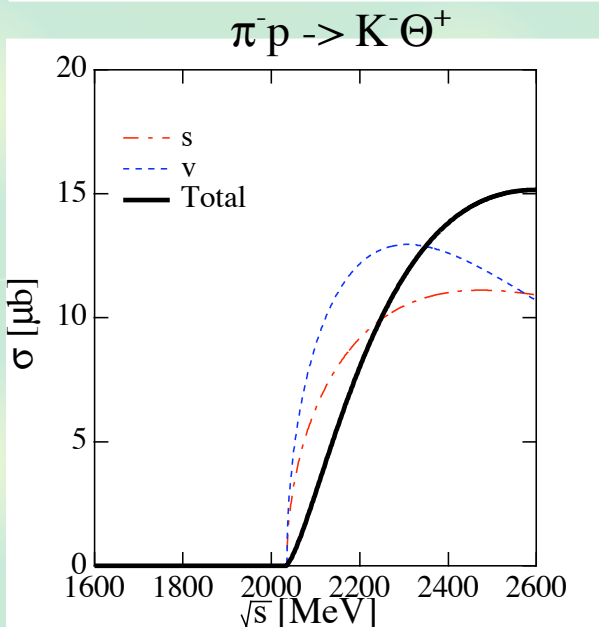
$1/2^+$



$$\frac{\sigma(K^+)}{\sigma(\pi^-)} \sim 50$$

large interference

$3/2^-$



$$\frac{\sigma(K^+)}{\sigma(\pi^-)} \sim 3$$

small interference

## Summary 1 : mixing scheme

We examine  $8-\overline{10}$  mixing scheme for the exotic and non-exotic baryon resonances.

- Masses of  $\Theta(1540)$  and  $\Xi(1860)$  are well fitted in the  $8-\overline{10}$  mixing scheme with  $J^P = 1/2^+$  or  $3/2^-$  baryons.
- A very narrow width of  $\Theta$  can be obtained for the  $J^P = 3/2^-$  case.
- For both  $J^P$ , the mixing angle is close to the ideal angle.

## Summary 2 : Two-meson coupling and $\Theta$ production

Based on the mixing scheme, we evaluate the two-meson coupling of  $\Theta$ , and calculate the reaction process for  $\Theta$  production



There is an **interference effect** between two amplitudes, which is prominent for  $1/2^+$  case and rather moderate for  $3/2^-$  case

$J^P$	$g^s$	$g^v$	$\sigma_{K^+}/\sigma_{\pi^-}$	$\text{Re}\Sigma_{\Theta}$
$1/2^+$	1.59	-0.27	50	-78 MeV
$3/2^-$	0.104	0.209	3	-23 MeV