VIA lecture, COSMOVIA, Paris, ketov@04.06.2010

INFLATION and QUANTUM GRAVITY

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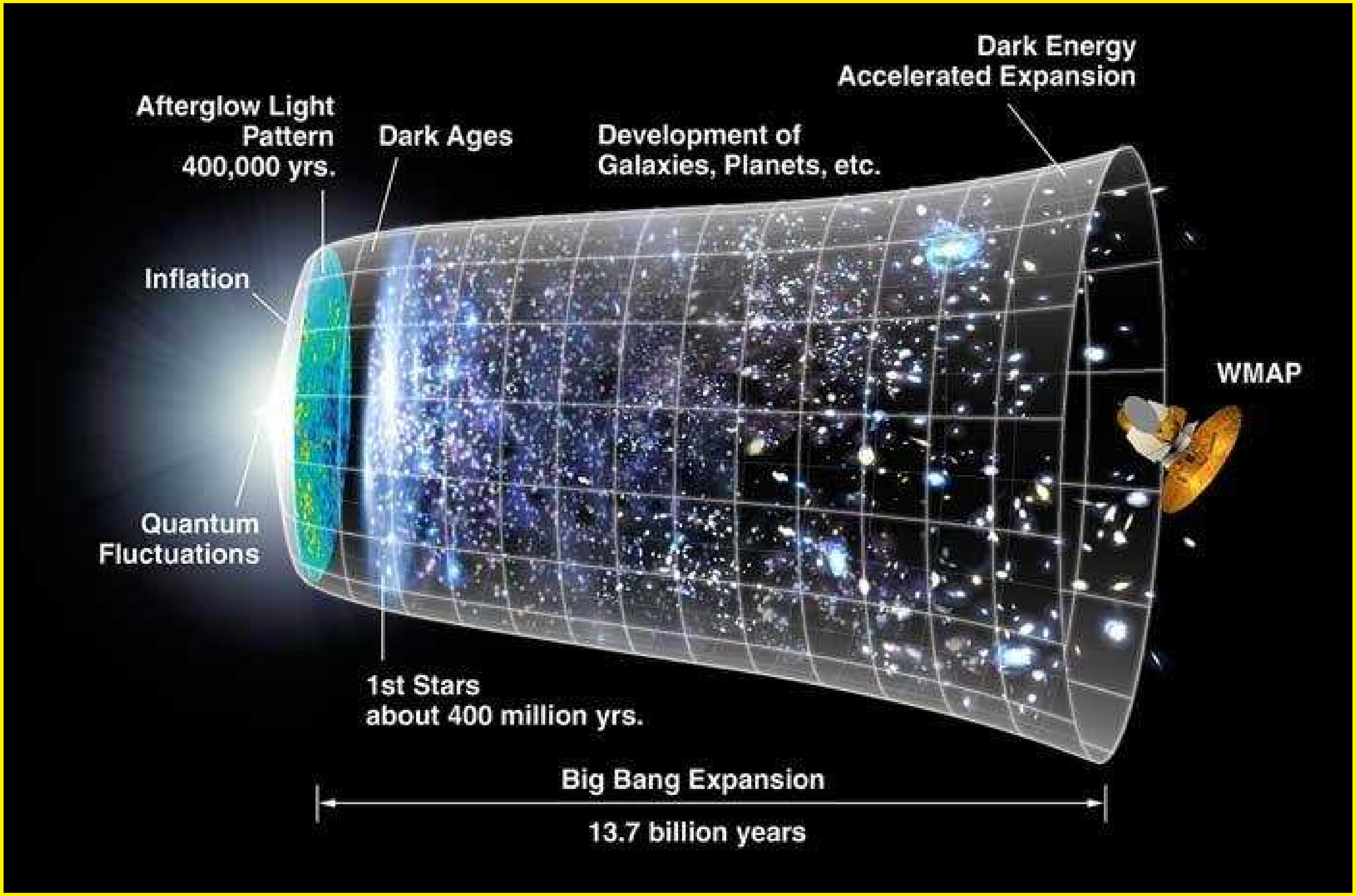
- Motivation: Big Bang, Inflation, Dark Energy and Dark Matter
- General challenges, prejudices and lessons
- Inflationary solutions and formation of structure
- Fundamental problems and new ideas
- Connections to String Theory
- Conclusion and Discussion

History of our Universe



One Physics (Same Energy Scale) – One Theory

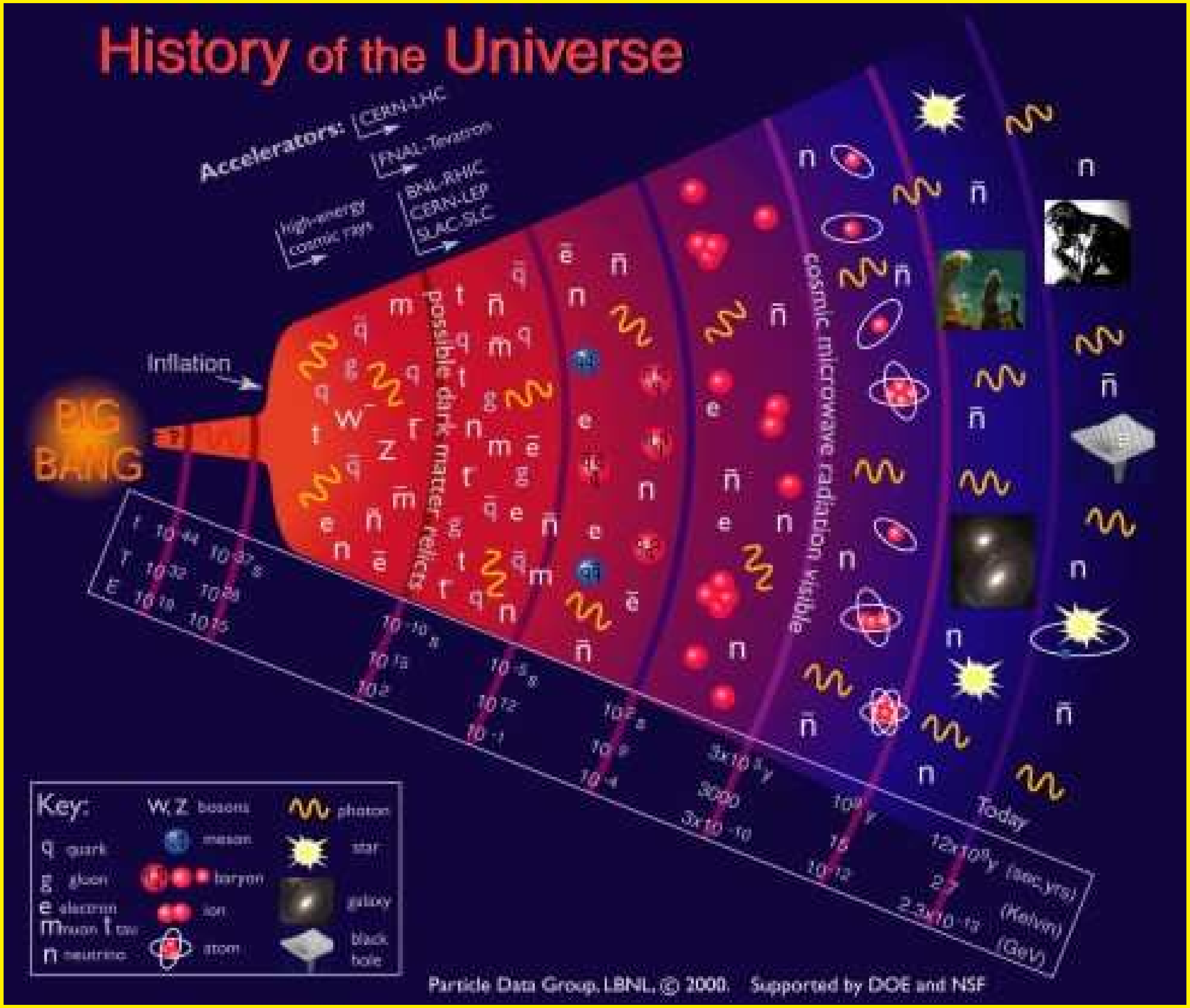
There are essentially two basic and conceptually different approaches to Physics: a phenomenological approach and a fundamental approach.

In the phenomenological (or bottom-up) approach, one interprets physical data from experiment within a theoretical model. In the fundamental (or up-to-down) approach one uses a fundamental theory to describe and predict physical data.

Ideally, we would like to describe ALL fundamental physics by a single theory. It is supposed to include Quantum Field Theory (or the Standard Model of elementary particles), and well as Quantum Gravity (or the Standard Model of Cosmology). There is no such theory yet, but there is a good candidate for it (String Theory!).

Message: there is nothing more practical than a good theory! (single explanation to many facts). Cosmological puzzles should also be resolved in a single package with High-energy Physics puzzles beyond the SM!

Universe and High-Energy Physics Scales



Inflation in early Universe

- Cosomological inflation (= phase of 'rapid' accelerated expansion) predicts homogeneity of our Universe at large scales, its large size and entropy, and an almost scale-invariant spectrum of cosmological perturbations (in agreement with the WMAP measurements of the CMB radiation spectrum).

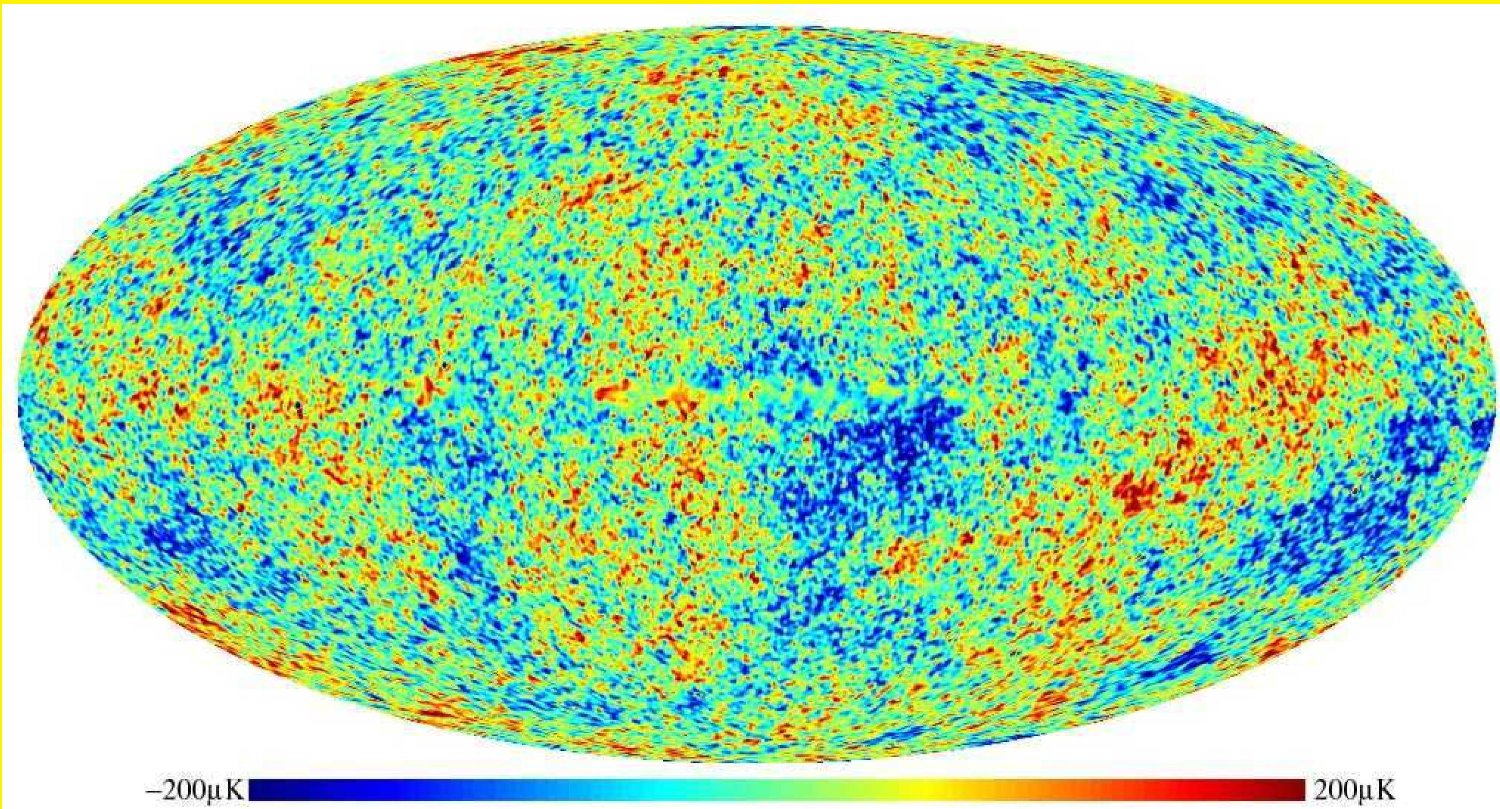
- Known mechanisms of inflation use a special slow-roll scalar field (called inflaton), with a proper scalar potential.

- The scale of inflation is close to that of Grand Unification (ie. well beyond the electro-weak scale of 100 GeV).

- Inflation is driven by a vacuum energy, and it has to have an end. After inflation, the vacuum energy gets transformed into the energy of SM particles (reheating), and it gives rise to nucleosynthesis.

- The nature of the inflaton field (and its scalar potential) is still a mystery.

CMB radiation (Mona Lisa of Cosmology)



Universe Structure Formation (Theory)

- Microscopic quantum fluctuations of the inflaton field (about its vacuum state) in early Universe lead to macroscopic formation of structure. Quantum fluctuations during inflation are caught up in the rapid expansion and then stretched to huge sizes. Once the fluctuation is taken to such a large scale, it becomes frozen-in.

- As regards vacuum fluctuations of the quantized inflaton scalar field, $\phi = \phi_{cl} + \delta\hat{\phi}$ with $\langle \delta\hat{\phi} \rangle = 0$, in the FLRW background, one finds from QFT that $\boxed{\left|\delta\hat{\phi}\right| \equiv \sqrt{\langle (\delta\hat{\phi})^2 \rangle} = H/2\pi}$ at the horizon exit.

- show the movie: a supercomputer simulation of the structure formation.

Dark Energy & Dark Matter in present Universe

- Studies of large-scale distribution and evolution of galaxies and (distant) type-Ia supernovae (Perlmutter at al, Schmidt et al, 1998), have led to a discovery of DARK MATTER and DARK ENERGY in the present Universe, with

$$\rho_{total} = \rho_{\text{visible}} + \rho_{\text{dark matter}} + \rho_{\text{dark energy}}$$

in the proportion $4\% + 22\% + 74\% = 100\%$, respectively.

- The DE is needed to balance the energy budget of the present Universe and explain the accelerated rate of its expansion. The DE works against gravity to boost the expansion of the Universe. A small positive cosmological constant $\Lambda > 0$ may account for the present Universe accelerated expansion, due to the (experimentally dictated) DE-equation of state with $w = P/\rho = -0.97 \pm 0.07$

- The DM plays the key role in the structure formation in our Universe and holds the clusters and galaxies together. It does not interact electromagnetically but it does interact gravitationally. The DM should be Cold and non-baryonic. Possible candidates for the massive CDM particle include axion, gravitino and neutralino (= WIMP in MSSM).

Experimental evidence

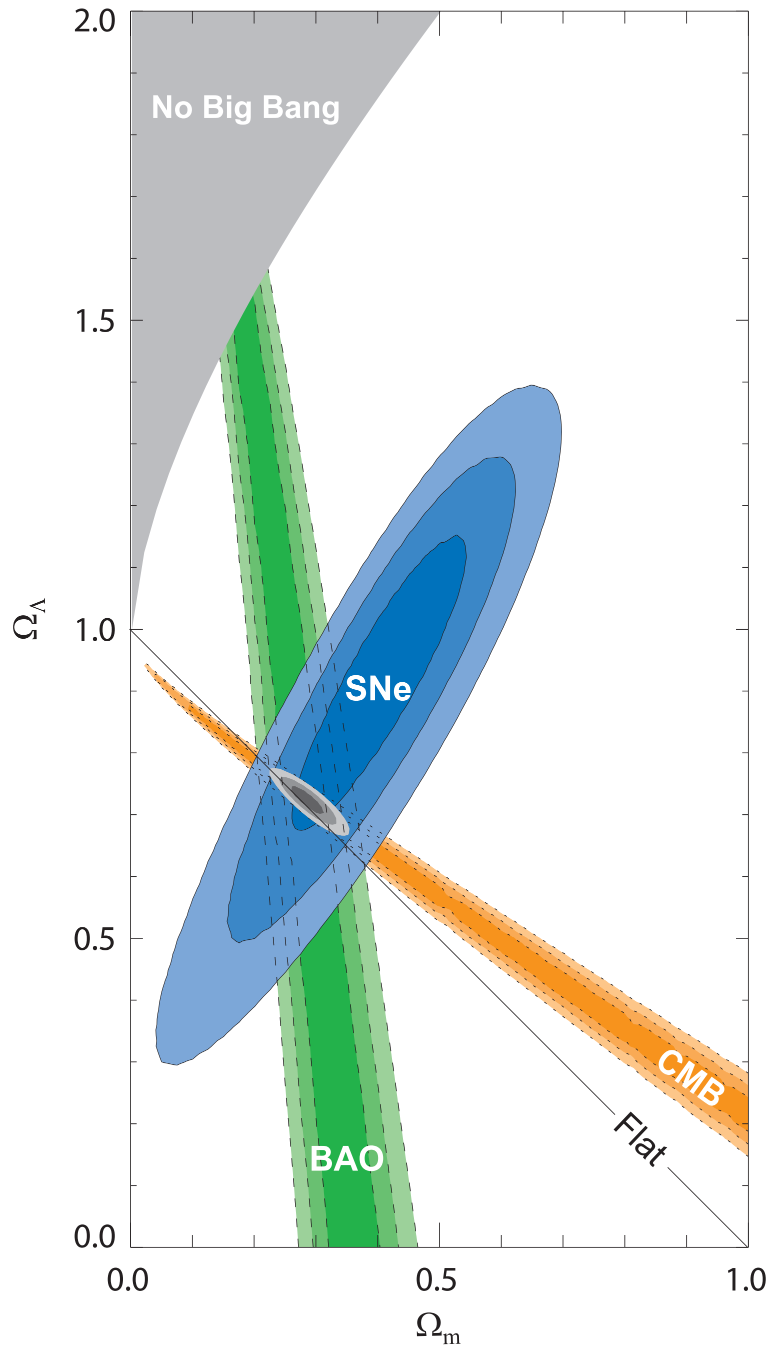
The experimental evidence for DM and DE comes from at least 3 independent sources:

- type Ia supernovae observations (S. Perlmutter, UC Berkeley, and B. Schmidt, ANU Canberra, 1998),

- precision measurements of CMB temperature fluctuations (BOOMERANG, MAXIMA, WMAP, 2000 and 2003)

- baryonic acoustic oscillations (Sloan Digital Skies Survey, 2005):

see the combination of the confidence level contours of 68.3%, 95.4% and 99.7% in the $\Omega_\Lambda - \Omega_m$ plane from the Cosmic Microwave Background, Baryonic Acoustic Oscillations and the Union SNe Ia set (assuming $w = -1$) , according to Kowalski et al. (Supernova Cosmology Project), Astrophys. J. 686 (2008) 749:



No Big Bang
SNe
BAO
CMB
Flat
2.0
1.5
1.0
0.5
0.0
0.0
0.5
1.0
Ω_Λ
Ω_m

DE and DM in Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi \, G_N \left(T_{\mu\nu}^{\text{visible}} + T_{\mu\nu}^{\text{DM}} \right)$$

or

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi \, G_N \left(T_{\mu\nu}^{\text{visible}} + T_{\mu\nu}^{\text{DM}} + T_{\mu\nu}^{\text{DE}} \right)$$

- The present evolution of the Universe is phenomenologically well (and accurately) described by the standard Λ-CDM scenario within Einstein theory,

but (similarly to inflation)

- exact nature of both DE and DM is unknown,
- underying microscopic derivation is absent.

Beyond Einstein Gravity

The late time acceleration of our Universe can be interpreted in two qualitatively different ways, either as

(i) the manifestation of some (new) DE component (dynamical or not), as above

or as

(ii) the indication of the breakdown of Einstein gravity at cosmological distances.

The known (and popular) extensions of Einstein gravity are

- adding higher-order curvature invariants to Einstein-Hilbert action,
- adding extra fields with non-minimal coupling to gravity,
- adding large extra dimensions for gravity.

Dynamical DE = quintessence, ie. replacing a cosmological constant Λ by a scalar field (or several scalars). It leads to a time- and space-dependent vacuum energy density in cosmology, which is desirable for describing a more accurate and complete history of our Universe (perhaps, with the vanishing cosmological constant).

Major Challenges

DE and DM are not the only challenges facing modern cosmology. In addition, one has (the 'old' ones) to

- resolve the classical cosmological singularity (Big Bang) in the Einstein-Friedmann Universe, which is accompanied by infinities in ρ and R,
- construct a consistent non-perturbative theory of Quantum Gravity,
- identify the inflaton particle in High-Energy Physics,
- resolve a conflict between the 'observed' value of Λ amd its 'natural' QFT value, in a unified theory of Elementary Particles and Gravity.

We would like to keep (i) exact diffeomorphism invariance (relativity principle), (ii) universality of gravity (or equivalence principle with high precision), and (iii) effective field theory description of gravity $(T \ll T_{\rm H})$.

There is *a priori* no reason to restrict the gravitational Lagrangian to the EH-term linear in curvature, unless it does not contradict an experiment. The first attempt of that kind was made as early as 1921 (Weyl). However, there exist too many possibilities beyond EH (some selection criteria are needed).

Common prejudices

There is no doubt that any theory of Quantum Gravity is going to include the higher-order curvature terms in the UV. Those terms are expected to be relevant near curvature singularities. It may be possible that some higher-derivative gravity, subject to suitable constraints, could be the effective action to quantized theory of gravity (Sakharov, 1967), cf. String Theory.

- Objection #1: “all the higher-derivative field theories, including the higher-derivative gravity theories, have ghosts”, because of Ostrogradski theorem (1850). However, the theorem does not directly apply to the degenerate (read: gauge) field theories (Woodard, 2007), and, in fact, a higher-derivative gravity does not always have ghosts (many explicit examples are known).
- Objection #2: “all the higher-order curvature terms are suppressed by the inverse powers of $M_{\rm Pl}$ and thus are irrelevant in IR”. However, the effective Planck scale may be brought down to TeV scale with large extra dimensions (A-HDD), there may be warp (RS) factors (eg., generated by fluxes in string theory) in a higher-dimensional metric, and there may be singular perturbations in unstable cases (see the next slide:)

Lesson from Navier-Stokes hydrodynamics

The Navier-Stokes differential equations describe classical dynamics of a non-ideal fluid with viscosity,

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla})\vec{v} = -\frac{1}{\rho}\vec{\nabla} P + \nu \Delta \vec{v}$$

The higher derivatives enter this equation with the kinematic viscosity parameter ν. Singular perturbations in the Navier-Stokes equation are well known, and give rise to a radically different behaviour of the solutions at late times, even for arbitrary small values ν of viscosity.

The simplest mathematical example is given by the equation

$$-\varepsilon \ddot{y} + \dot{y} + y = 0$$

When $\varepsilon = 0$, one gets exponentially decaying solutions $y \propto e^{-t}$, whereas for small $\varepsilon > 0$, the general solution grows exponentially with time as $e^{t/\varepsilon}$. One could now imagine that 'small' quantum effects (eg., driven by the higher-order curvature terms in the modified Einstein equations) in the early Universe may become 'large' and visible in the present Universe (eg., the Dark Energy)!

Some lessons from superstrings

- Theory of superstrings is the leading candidate for a unified Theory of all fundamental interactions, it offers a consistent perturbative Quantum Gravity, and is capable to generate the SM of elementary particles and the cosmological SM. A fully non-perturbative superstring theory (or M-theory) is unknown. The perturbative superstrings are defined on-shell (in the form of quantum amplitudes), and they imply infinitely many higher-order curvature corrections to Einstein equations, to all orders in Regge slope parameter α' and string coupling g_s (in 10 space-time dimensions). Off-shell quantum corrections are largely unknown and ambiguous.
- String theory may resolve the Big-Bang singularity (see eg., pre-Big-Bang scenario of Gasperini-Vaneziano, string gas cosmology of Brandenberger-Vafa). String theory put limits on the maximal (Hagedorn) temperature $T \leq T_H$ with $T_H = 1/\pi(8\alpha')^{1/2}$ (for type-II strings) in the (free CFT, or Matsubara) string partition function (Atick, Witten, 1988), due to the infinite tower of massive states in the string spectrum.
- String theory also put limits on the maximal values of electric and magnetic fields in Born-Infeld electrodynamics, $\left|\vec{E}\right| \leq 1/b$ and $\left|\vec{H}\right| \leq 1/b$, with $b = 2\pi\alpha'$.
- String theory gives rise to UV/IR mixing via dualities and non-commutativity.

It is natural to expect similar features in the string-generated gravity.

Effective cosmology in 4D spacetime

Whatever the fundamental theory is, it should give rise to the effective field theory in curved 4D spacetime, that should be viable extension of the SM of elementary particles and the SM (Λ-CDM) of cosmology, as well as describe inflation.

So, let's first review the phenomenological description of inflation, and some 4D inflationary models that show promise of their origin from Quantum Gravity.

It is going to be the foundation for further upgrades of the inflationary theory to 4D supergravity and Superstring Landscape.

FLRW metric and cosmological acceleration

- The main Cosmological Principle of a spatially homogeneous and isotropic $(1+3)$-dimensional universe (at large scales) gives rise to the FLRW metric

$$ds^2_{\text{FLRW}} = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right]$$

where the function $a(t)$ is known as the scale factor in 'cosmic' (co-moving) coordinates (t, r, θ, ϕ), and k is the FLRW topology index, $k = (-1, 0, +1)$. The FLRW metric (1) admits a 6-dimensional isometry group G that is either $SO(1,3)$, $E(3)$ or $SO(4)$, acting on the orbits $G/SO(3)$, with the spatial 3-dimensional sections H^3, E^3 or S^3, respectively. Important notice: Weyl tensor $C^{\text{FLRW}}_{ijkl} = 0$.

- Present Universe acceleration and early Universe inflation are defined by

$$\ddot{a}(t) > 0 \ , \ \text{or equivalently} \ , \ \frac{d}{dt}\left(\frac{H^{-1}}{a}\right) < 0$$

where $H = \dot{a}/a$ is Hubble function. The amount of inflation (# e-foldings) is given by

$$N_e = \ln \frac{a(t_{\text{end}})}{a(t_{\text{start}})} = \int_{t_{\text{start}}}^{t_{\text{end}}} H\, dt \approx \frac{1}{M^2_{\text{Pl}}} \int_{\phi_{\text{end}}}^{\phi} \frac{V}{V'}\, d\phi$$

Simple example: Starobinsky model

In 4 dimensions, there are only 3 independent quadratic curvature invariants: $R^{\mu\nu\lambda\rho}R_{\mu\nu\lambda\rho}$, $R^{\mu\nu}R_{\mu\nu}$ and R^2. In addition,

$$\int d^4x\,\sqrt{-g}\left(R^{\mu\nu\lambda\rho}R_{\mu\nu\lambda\rho} - 4R^{\mu\nu}R_{\mu\nu} + R^2\right)$$

is topological for any metric, while

$$\int d^4x\,\sqrt{-g}\left(3R^{\mu\nu}R_{\mu\nu} - R^2\right)$$

is also topological for any FLRW metric. Hence, as regards the FLRW metrics, the most general quadratic curvature action is given by $(8\pi G_N = 1)$

$$S = -\frac{1}{2}\int d^4x\,\sqrt{-g}\left(R - 2\Lambda - \alpha R^2\right)$$

Starobinsky model ($\Lambda = 0$) has an exact inflationary solution $a(t) \propto e^{H_0 t}$ with $H_0^2 = (24\alpha)^{-1}$, which is dS-like. It is stable (attractor!) when $\alpha > 0$.

It is easy to get the observed (WMAP5) value of the CMB spectral index, $n_s = 0.960 \pm 0.013$, in the Starobinsky model. Its extension is known as $f(R)$-gravity.

$f(R)$ gravity

An $f(R)$ gravity is specified by the action $S_f = -\frac{1}{2\kappa^2}\int d^4x\, f(R)$ where R is the Ricci scalar curvature of a metric $g_{\mu\nu}(x)$, and κ is the gravitational coupling constant, $\kappa^2 = 8\pi G_N$. A matter action S_m, minimally coupled to the metric, is supposed to be added to S_f. We use the 'mostly minus' spacetime signature.

The gravitational equations of motion derived from the action $S_f + S_m$ read

$$f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} + g_{\mu\nu}\Box f'(R) - \nabla_\mu\nabla_\nu f'(R) = \kappa^2 T_{\mu\nu}$$

where the primes denote differentiation. Those equations of motion are the 4th-order differential equations with respect to the metric (ie. with the higher derivatives). Taking the trace of the equation above yields

$$\Box f'(R) + \frac{1}{3}f'(R)R - \frac{2}{3}f(R) = \kappa^2 T$$

Hence, in contrast to General Relativity having $f'(R) = const.$, in $f(R)$ gravity the field (or conformal mode of the metric) $\phi = f'(R)$ is dynamical and represents the independent propagating (scalar) degree of freedom. In terms of the fields $(g_{\mu\nu}, \phi)$ the equations of motion are of the 2nd order in the derivatives.

$f(R)$ gravity and effective Einstein equations

The equations of motion in the $f(R)$-FLRW cosmology (generalizing Friedmann and Raychaudhuri equations, respectively) are given by

$$H^2 = \frac{\kappa^2}{3f'}\rho_R \quad \text{and} \quad \frac{\ddot{a}}{a} = -\frac{\kappa^2}{2f'}(\rho_R + 3P_R)$$

with the energy density ρ_R and pressure P_R are due to the curvature modification,

$$\rho_R = \frac{Rf' - f}{2} - 3H\dot{R}f'', \quad P_R = 2H\dot{R}f'' + \ddot{R}f'' + \frac{1}{2}\left(f - f'R\right) + f'''\dot{R}^2$$

The ρ_R and P_R identically vanish in Einstein gravity, where $f(R) = R$.

It is not diffiicult to choose the function $f(R)$ in order to get $\ddot{a} > 0$, with a desired equation of state for DE. A reconstruction of the function $f(R)$ from any desired scale factor (history) $a(t)$ is also possible, with $R = -6\left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right]$

$f(R)$ gravity $\subset$ scalar-tensor gravity

- A connection between $f(R)$ gravity and scalar-tensor gravity is given by a non-perturbative Legendre-Weyl transform. The action S_f is classically equivalent to

$$S_A = \frac{-1}{2\kappa^2} \int d^4x \sqrt{-g}\, \{AR - Z(A)\}$$

where the real scalar $A(x)$ is related to the scalar curvature R by the Legendre transformation

$$R = Z'(A) \qquad \text{and} \qquad f(R) = RA(R) - Z(A(R))$$

A Weyl transformation of the metric $g_{\mu\nu}(x) \to \exp\left[\frac{-2\kappa\phi(x)}{\sqrt{6}}\right] g_{\mu\nu}(x)$ with an arbitrary field parameter $\phi(x)$ yields

$$\sqrt{-g}\, R \to \sqrt{-g}\, \exp\left[\frac{-2\kappa\phi(x)}{\sqrt{6}}\right] \left\{ R + \sqrt{\frac{6}{-g}} \partial_\mu \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \phi \right) \kappa - \kappa^2 g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right\}$$

Hence, when choosing $A(\kappa\phi) = \exp\left[\frac{2\kappa\phi(x)}{\sqrt{6}}\right]$ and ignoring the total derivative, we can rewrite the above action to the equivalent form

$$S_\phi = \int d^4x \sqrt{-g}\left\{\frac{-R}{2\kappa^2} + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2\kappa^2}\exp\left[\frac{-4\kappa\phi(x)}{\sqrt{6}}\right] Z(A(\kappa\phi))\right\}$$

in terms of the physical (and canonically normalized) scalar field $\phi(x)$.

- Applicability of the Legendre-Weyl transform implies the invertibility of the function $f'(R)$ at the reference point R_0 ie. both $f'(R_0)$ and $f''(R_0) \neq 0$. The $R_0 = const.$ is a solution to the pure $f(R)$-gravity provided that $f'(R_0)R_0 = 2f(R_0)$. When considering small perturbations, $R = R_0 + Z$, and linearizing the equations of motion with respect to Z, one gets

$$\left(\Box + m^2\right) Z = 0 \qquad \text{with} \qquad m^2 = \frac{1}{3}\left[R_0 - f'(R_0)/f''(R_0)\right]$$

Hence, the R_0-background is stable when $m^2 > 0$ (Starobinsky,1988).

- After the Weyl transform, the gravity-coupled matter fields in S_m become conformally coupled to ϕ. It results in the extra force that needs to be locally suppressed (eg. within our Solar system).

Chaotic inflation in Starobinsky model

When $f(R) = R - R^2/M^2$, the inflaton vacuum energy emerges even with $\Lambda = 0$. In terms of the new variable and the parameter

$$-y = \sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}} \qquad \text{and} \qquad V_0 = \frac{1}{8}M_{\rm Pl}^2 M^2$$

the inflaton scalar potential reads (an end to inflation is guaranteed!)

$$V(y) = V_0 \left(e^y - 1\right)^2$$

The slow-roll inflation parameters are defined by

$$\varepsilon(\phi) = \frac{1}{2}M_{\rm Pl}^2\left(\frac{V'}{V}\right)^2 \qquad \text{and} \qquad \eta(\phi) = M_{\rm Pl}^2\frac{V''}{V}$$

where the primes denote the derivatives with respect to the inflaton field ϕ. The necessary condition for the slow-roll approximation is the smallness of the inflation parameters

$$\varepsilon(\phi) \ll 1 \qquad \text{and} \qquad |\eta(\phi)| \ll 1$$

Slow-roll inflation in $(R - R^2/M^2)$ gravity

The first slow-roll condition implies $\ddot{a}(t) > 0$, whereas the second guarantees that inflation lasts long enough, via domination of the friction term in the inflaton equation of motion

$$3H\dot{\phi} = -V'$$

Here H stands for the Hubble function $H(t) = \dot{a}/a$. It is to be supplemeted by the Friedmann equation

$$H^2 = \frac{V}{3M_{\rm Pl}^2}$$

It follows that

$$\dot{\phi} = -M_{\rm Pl}\frac{V'}{\sqrt{3V}} < 0$$

whose solution during the slow-roll inflation $(t_0 < t_{\rm start} \leq t \leq t_{\rm end})$ is

$$\phi(t) = -\sqrt{\frac{3}{2}}M_{\rm Pl}\ln\left[\frac{4\sqrt{V_0}}{3\sqrt{3}M_{\rm Pl}}(t - t_0)\right]$$

The resulting differential equation on the scale factor $a(t)$ has a solution

$$a(t) = e^{H_0 t}\left[\frac{t - t_0}{\text{const.}}\right]^{-3/4}$$

where we have introduced the notation $H_0 = M/\sqrt{24}$. The presence of a singularity at $t = t_0$ is harmless because this inflationary solution is only valid during the slow-roll inflation when $t \geq t_{\text{start}} > t_0$, so that it does not apply to the Big Bang. A resolution of the Big Bang singularity is supposed to require the higher-order curvature terms in the gravitational effective action.

$\varepsilon(\phi)$ first approaches 1 at $\phi_{\text{end}} = \sqrt{\frac{3}{2}} M_{\text{Pl}} \ln\left(2\sqrt{3} - 3\right) \approx -0.94\ M_{\text{Pl}}$, since $|\eta(\phi)|$ approaches 1 later, at $\phi_{\text{end}} = -\sqrt{\frac{3}{2}} M_{\text{Pl}} \ln \frac{5}{3} \approx -0.62\ M_{\text{Pl}}$. Then we find

$$N_e = \frac{3}{4}\left(e^{-y} + y\right) - \frac{3}{4}\left(\exp\left[\sqrt{\frac{2}{3}} \cdot 0.94\right] - \sqrt{\frac{2}{3}} \cdot 0.94\right) \approx \frac{3}{4}\left(e^{-y} + y\right) - 1.04$$

and

$$\varepsilon = \frac{4e^{2y}}{3\left(1 - e^y\right)^2} \qquad \text{and} \qquad \eta = \frac{-4e^y(1 - 2e^y)}{3\left(1 - e^y\right)^2}$$

Inflation observables in $(R - R^2/M^2)$ gravity

An analytic approximation can be obtained by using the expansion with respect to the inverse number of e-foldings. It yields

$$\varepsilon = \frac{3}{4N_e^2} + \mathcal{O}\left(\frac{\ln^2 N_e}{N_e^3}\right)$$

and

$$\eta = -\frac{1}{N_e} + \frac{3\ln N_e}{4N_e^2} + \frac{5}{4N_e^2} + \mathcal{O}\left(\frac{\ln^2 N_e}{N_e^3}\right)$$

The primordial spectrum in a power-law approximation takes the form of k^{n-1} in terms of the comoving wave number k and the spectral index n. In particular, the slope n_s of the scalar power spectrum, associated with the density perturbations, is given by (Liddle, Lyth)

$$n_s = 1 + 2\eta - 6\varepsilon \ ,$$

the slope of the tensor primordial spectrum, associated with the gravitational waves, is given by $n_t = -2\varepsilon$, whereas the scalar-to-tensor ratio is given by $r = 16\varepsilon$.

Spectral indices in $(R - R^2/M^2)$ gravity

As an extension of the leading (Mukhanov-Chibisov) terms, we find

$$n_s = 1 - \frac{2}{N_e} + \frac{3\ln N_e}{2N_e^2} - \frac{2}{N_e^2} + \mathcal{O}\left(\frac{\ln^2 N_e}{N_e^3}\right)$$

In addition, the amplitude of the initial perturbations, $\Delta_R^2 = M_{\rm Pl}^4 V/(24\pi^2\varepsilon)$, is yet another physical observable, whose experimental value is

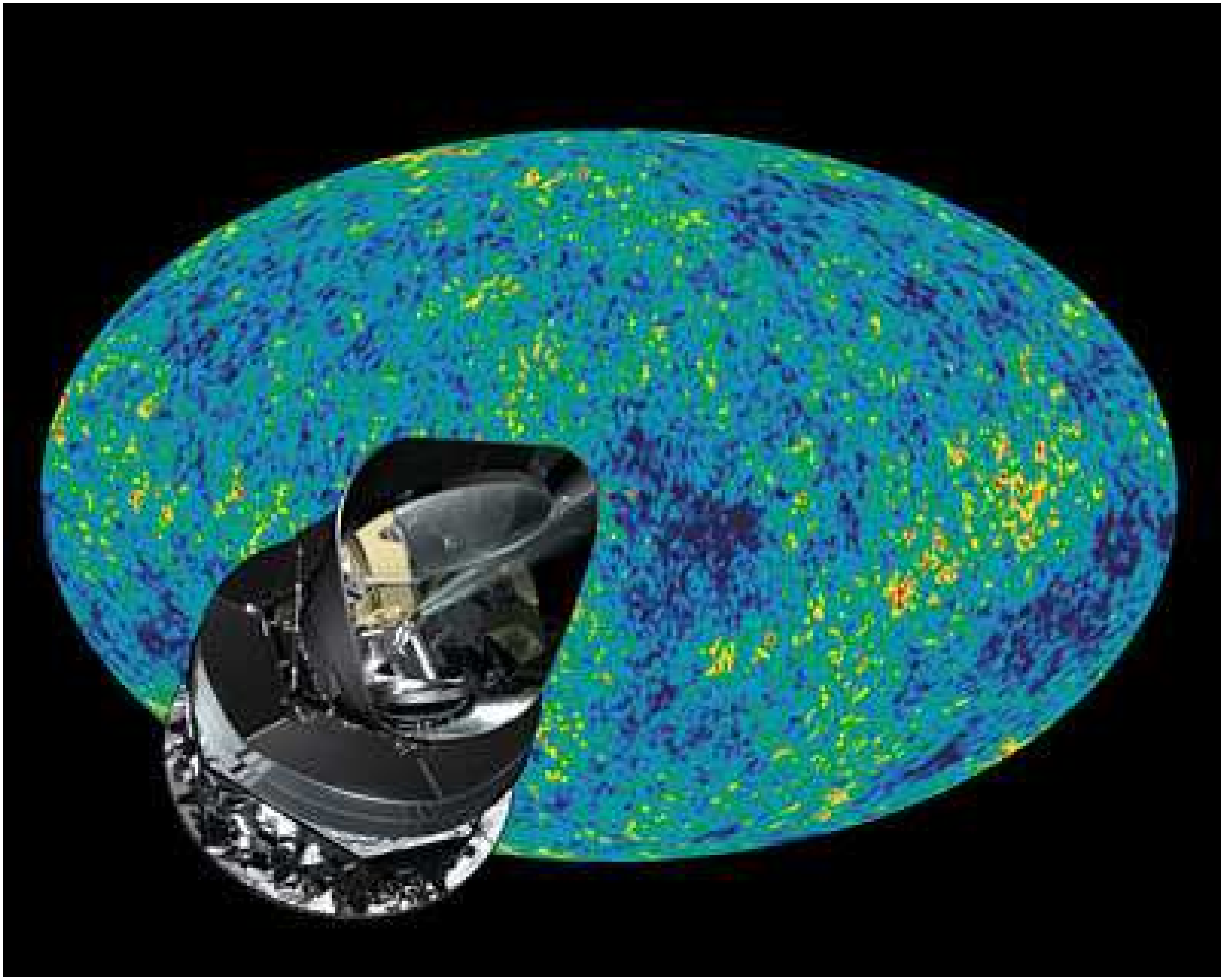
$$\left(\frac{V}{\varepsilon}\right)^{1/4} = 0.027\, M_{\rm Pl} = 6.6\times 10^{16}\ \text{GeV}$$

It determines the normalization of the R^2-term in the action as

$$\frac{M}{M_{\rm Pl}} = 4\cdot\sqrt{\frac{2}{3}}\cdot(2.7)^2\cdot\frac{e^y}{(1-e^y)^2}\cdot 10^{-4} = (3.5\pm 1.2)\cdot 10^{-5}$$

We find that the WMAP5 experimental bounds on the scalar spectral index are satisfied provided that the e-foldings number N_e lies between 35.9 and 71.8, with the middle value of $\bar{N}_e = 53.8$. There is also the noticable suppression of tensor fluctuations as $|r| < 8.2\cdot 10^{-3}$ and $|n_t| < 10^{-3}$.

Planck mission: 0.5% accuracy in CMB expected



Local tests of $f(R)$ gravity

- An IR modification of Einstein gravity is to be consistent with local physics constraints, eg., those coming from our Solar System. There are many choices of $f(R)$ that obey all existing tests, eg., $f(R) = R + \lambda R_0 \left[\left(1 + \frac{R^2}{R_0^2} \right)^{-n} - 1 \right]$ with properly chosen parameters $R_0 \sim H_0^2$, $\lambda > 0$ and $n > 0$ (Starobinsky et al,2009)

- A scalar ϕ conformally coupled to matter, leads to extra 5th force ($\propto \nabla\phi$) violating the equivalence principle and resulting in the different accelerations of Earth and Moon towards Sun (observational (Cassini) bound is $\frac{\Delta a}{a} < 10^{-13}$). The effective local screening of ϕ may be possible due to the Chameleon effect (Khoury, Veltman, 2004), ie. a dependence of the scalar mass upon local environment. The characteristic (Chameleon) distance is about 10 pc for the Sun and about 100 Kpc for a galaxy. Thus, the scalar mass can be sufficiently large on Earth but, on the cosmological scales, can be of order H_0, thus generating the present Universe acceleration. The scalar couplings to matter are of the order one, as is expected in String Theory! Measuring Newton's constant in space vs. that on Earth would be a good test!

Next step: Supergravity

- Supersymmetry is the symmetry between bosons and fermions, it is well motivated in particle physics beyond the SM, and it is needed for consistency of strings. Supergravity is the theory of local supersymmetry. Supergravity is the only known consistent route to couple spin-3/2 particles (gravitinos).
- Most of studies of superstring- and brane- cosmology are based on their effective description in the 4-dimensional $N = 1$ supergravity.
- An $N = 1$ locally supersymmetric generalization of $f(R)$ gravity is possible (Gates Jr., SVK, 2009). It is non-trivial because, despite of the apparent presence of the higher derivatives, there should be no ghosts, and the auxiliary freedom (Gates Jr., 1996) is to be preserved. The modified supergravity action is classically equivalent to the standard $N = 1$ Poincaré supergravity coupled to a dynamical chiral superfield whose Kähler potential and superpotential are dictated by a single holomorphic function. The inflaton arises as the superconformal mode of a supervielbein, and the quintessence is holomorphic (or complex).

A possible connection to the Loop Quantum Gravity was investigated by Gates Jr., N. Yunes and SVK, in Phys. Rev. D80 (2009) 065003, arXiv:0906.4978 [hep-th].

Conclusion

- Many mechanisms of inflation were proposed (chaotic, hybrid, topological, bouncing, K-inflation, string- and brane-inflation, etc.). There are many candidates for inflaton field, as well as inflaton scalar potentials. The main problem is not just finding yet another inflationary model (though, it may be worthy to write a paper about it!), but get it in the context of High-energy Physics (beyond the SM), Supergravity and Superstring Theory, from the first principles.

- We are now entering the decade of precisional cosmology (Planck mission, etc.). It is going to sharpen the observational values of CMB (the spectral index, etc.) and presumably rule out many inflationary models proposed so far.

- $f(R)$ gravity is quite natural as the effective high-energy theory of inflation in early Universe, both from the phenomenological and fundamental viewpoints (and it may also be used for describing the present Universe accelaration). A derivation of a specific model of $f(R)$ gravity from Superstrings remains an open problem.

- A manifestly $N = 1$ supersymmetric extension of $f(R)$ gravity exist, it is chiral and is parametrized by a holomorphic function. An $F(R)$ supergravity is classically equivalent to the standard theory of a chiral scalar superfield (with a non-trivial Kähler potential and a chiral superpotential) coupled to the $N = 1$ Poincaré supergravity in four spacetime dimensions

- Any $F(R)$ supergravity results in a spacetime torsion. It also has two candidates for a CDM-particle: a massive gravitino or a massive axion. We conjectured the identification of the dynamical chiral superfield in $F(R)$ supergravity with the dilaton-axion chiral superfield in Superstring Theory

- The inflaton scalar potential in $F(R)$ supergravity is derivable via Legendre–Kähler-Weyl transform in superspace. The Kähler potential, the superpotential and the scalar potential are all governed by a single holomorphic function

- The no-scale $F(R)$ supergravity with the vanishing cosmological constant and spontanenous supersymmetry breaking also exist, and it does not need fine-tuning.

Comments and Discussion