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A Comparison of Several Combinations of Nonlinear Filters and Storage Function Models for Real-Time Runoff Forecasting

by

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Abstract

The performance of nine combinations of approximate nonlinear filters and nonlinear storage function models for the real-time forecasting of runoff from rainfall are compared on the basis of their forecasting accuracy and computation time. The models are those of Kimura, Prasad and Hoshi, and the filters are the extended Kalman filter, a second-order filter and a single-stage iteration filter. Different sets of system and observation noise covariance matrices are considered to evaluate the three nonlinear filters' performance. Results show that the accuracy of runoff forecasts depends on the superiority of the model, on the choice of constant values to be assigned to the system noise matrix and not on the complexity of the filter. Also, the effect of catchment time lag which is the overall measure of the delay in catchment response to rainfall on the performance of each model is investigated. It is shown that, if the catchment time lag is accurately determined, Kimura's storage function model is as good as Prasad model or Hoshi's model, as it achieves almost the same level of accuracy and requires the smallest computation time. However, Prasad model and Hoshi's model are much more stable and accurate than Kimura's model in the presence of bias in rainfall response.

1. Introduction

The optimal operation of a flood control reservoir requires accurate real-time river flow forecasts. This is one of the reasons why many rainfall-runoff models have been developed in an attempt to obtain accuracy in runoff forecasting. Comparisons of these models are given by many authors[1)-4)]. Some of these models were developed based on expressions for storage (S) in the basin in terms of discharge (Q). Three of the equations for storage are

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given below:

$$S = K_1 Q^{N_1} \qquad \text{Kimura (1961)}^{5)} \quad (1)$$

$$S = K_1 Q^{N_1} + K_2 \frac{dQ}{dt} \qquad \text{Prasad (1967)}^{6)} \quad (2)$$

$$S = K_1 Q^{N_1} + K_2 \frac{dQ^{N_2}}{dt} \qquad \text{Hoshi and Yamaoka (1982)}^{2)} \quad (3)$$

where K_1, K_2, N_1 and N_2 are parameters. Combining the above expressions for storage with the continuity equation ($dS/dt = R - Q$) yields three nonlinear storage function rainfall-runoff models which are described in the next section: Model I is derived from Eq. (1), Model II from Eq. (2), and Model III from Eq. (3). They are widely used and studied in Japan because they consider the nonlinearity in many small basins in Japan. The nonlinearity is characterized by the very rapid response of flood runoff to heavy rainfall due to steep gradient and short water course of the river. Observed hydrographs are usually excessively peaked in shape and extremely short in duration. The merits of the models have also been discussed in many literatures[5)-8)]. Among the three nonlinear storage function models, Model III has been found to be superior[7)]. On the other hand, real-time forecasting of river flows is done using filtering techniques in combinaton with a rainfall-runoff model.

This study compares the performance of several combinations of three approximate nonlinear filters and three storage function models for the real-time forecasting of runoff from rainfall. The three approximate filters which are used to deal with nonlinearities in the resulting rainfall-runoff models are the extended Kalman filter (EKF), a second-order filter (SOF) and a single stage iteration filter (SSIF). Based on the same theoretical basis, the three approximate nonlinear filters were derived using truncated Taylor series expansion to represent system nonlinearities (see 9)-13) for the details of the derivations). Among the three filters, SSIF is the most complex, followed by SOF. Wishner and his colleagues[10)] compared their performance and concluded that, if nonlinearities are significant, SSIF is superior, followed by SOF, and that SSIF requires the longest computation time, followed by SOF. Puente and Bras[14)] have investigated the practical use of nonlinear filters on a single conceptual stochastic watershed model and found EKF to be as good as the other more complicated filters depending on the noise specifications. However, several authors have combined a less superior storage function model with a more complex approximate nonlinear filter or *vice versa* in the hope of obtaining better forecasting accuracy. Specifically, Duong and his colleagues[8)] have combined Model II and EKF, Hoshi and Yamaoka[15)] Model II and SOF, Hiramatsu and his colleagues[16)] Model I and SSIF and Hoshi[17)] Model III and EKF. In spite of having these combinations of nonlinear filters and storage function models, there is no comparison which illustrates their relative advantages and disadvantages.

Clearly, what the engineering hydrologist would like to know is whether or not the added complexity and computational requirements of nonlinear filters or storage function models, or both, as compared to the simplest combination of EKF and Model I, are justified by the hoped-for improvements in the forecasting accuracy. This paper answers this point.

Moreover, a major influence on the reliability of forecasts is the extent to which the three nonlinear storage function rainfall-runoff models represent the response of a catchment to rainfall[20)]. To consider this, the effect of catchment time lag which is the overall measure of the delay in catchment response to rainfall on the accuracy of runoff forecasts is discussed.

2. Nonlinear Storage Function Rainfall-Runoff Models and State Space Formulation

The development of the single-valued storage-discharge relationship in Eq. (1) assumes that unsteady flow effects are negligible. Prasad[6] modified this relation by adding the term $K_2 dQ/dt$ to allow for unsteady flow effects and to describe a looped storage-discharge relationship which is observed in natural channels. Hoshi and Yamaoka[7] incorporated a new parameter N_2 to Eq. (2), as shown in Eq. (3), by deriving Eq. (3) from the kinematic wave model. With the three storage equations and the continuity equation for a conceptual reservoir

$$\frac{dS(t)}{dt} = CR(t-T_L)-Q(t) \tag{4}$$

the rainfall-runoff process can be represented by the following three differential equations

$$\text{MODEL I}: K_1N_1Q^{N_1-1}(t)\frac{dQ(t)}{dt}+Q(t) = CR(t-T_L) \tag{5}$$

$$\text{MODEL II}: K_2\frac{d^2Q(t)}{dt^2}+K_1N_1Q^{N_1-1}(t)\frac{dQ(t)}{dt}+Q(t) = CR(t-T_L) \tag{6}$$

$$\text{MODEL III}: K_2\frac{d^2[Q^{N_2}(t)]}{dt^2}+K_1N_1Q^{N_1-1}(t)\frac{dQ(t)}{dt}+Q(t) = CR(t-T_L) \tag{7}$$

where C is a parameter, R is the average rainfall over the catchment, and T_L is the catchment time lag. Differentiating Eqs. (1), (2), and (3) and eliminating dS/dt from Eq. (4) yield Eqs. (5), (6) and (7), representing Model I, Model II and Model III. For these models, when the forecast lead time is shorter than T_L, the runoff forecasting can be implemented only with the observed rainfall[5]. When the lead time is longer than T_L, rainfall forecast is needed[20][21].

The nonlinear rainfall-runoff model dynamics need to be transformed in state space formulation of the approximate nonlinear filtering algorithms. The dynamic equation in state space form is written as

$$\dot{x}(t) = f[x(t)]+u(t) \tag{8}$$

By adding the stochastic noise component $u(t)$ to the dynamic equation, the models account for measurement errors (in observations and rainfall inputs) and errors in model structures and model parameters. The state variables or the elements of the $n\times 1$ state vector $x(t)$ for Model I are denoted as

$$x_1 = Q(t)\ ,\quad x_2 = K_1\ ,\quad x_3 = N_1 \quad \text{and} \quad x_4 = C \tag{9}$$

and the elements of the nonlinear dynamic function $f[x(t)]$ are given by

$$f_1 = [x_4R(t-T_L)-x_1]/x_2x_3x_1^{x_3-1} \quad \text{and} \quad f_2 = f_3 = f_4 = 0 \tag{10}$$

for Model II,

$$\begin{aligned} &x_1 = Q(t)\ ,\quad x_2 = dQ(t)/dt\ ,\quad x_3 = K_1\ ,\quad x_4 = 1/K_2\ ,\\ &x_5 = N_1 \quad \text{and} \quad x_6 = C \end{aligned} \tag{11}$$

and

$$\begin{aligned} &f_1 = x_2\ ,\quad f_2 = -x_2x_3x_4x_5x_1^{x_5-1}+x_4[x_6R(t-T_L)-x_1] \quad \text{and}\\ &f_3 = f_4 = f_5 = f_6 = 0 \end{aligned} \tag{12}$$

and for Model III,

$$x_1 = Q^{N_2}(t)\ ,\quad x_2 = d[Q^{N_2}(t)]/dt\ ,\quad x_3 = K_1\ ,\quad x_4 = 1/K_2\ ,\quad x_5 = N_1\ ,$$

$$x_6 = 1/N_2 \quad \text{and} \quad x_7 = C \tag{13}$$

and

$$f_1 = x_2 \ , \quad f_2 = -x_2x_3x_4x_5x_6x_1^{x_5x_6-1} + x_4[x_7R(t-T_L) - x_1^{x_6}] \quad \text{and}$$
$$f_3 = f_4 = f_5 = f_6 = f_7 = 0 \tag{14}$$

Eq. (8) may be discretized by expanding $x(t+\Delta t)$ up to second order in Δt.

$$\begin{aligned} x(t+\Delta t) &= x(t) + \dot{x}(t)\Delta t + \frac{1}{2}\ddot{x}(t)(\Delta t)^2 + \cdots \\ &= x(t) + f(x)\Delta t + \frac{1}{2}\frac{\partial f(x)}{\partial x}f(x)(\Delta t)^2 + \cdots \end{aligned} \tag{15}$$

where Δt denotes a small time step.

Defining $x_k \triangleq x(t)$ and $x_{k+1} \triangleq x(t+\Delta t)$, Eq. (15) may be written as

$$x_{k+1} = x_k + f(x_k)\Delta t + \frac{1}{2}A(x_k)f(x_k)(\Delta t)^2 + u_k \tag{16}$$

where $A(x_k)$ is defined in the next section and u_k also represents discretization errors besides those mentioned above and is assumed to be Gaussian, zero mean, and white:

$$E\{u_k\} = 0 \ , \quad E\{u_k u_j^T\} = U_k \delta_{kj} \tag{17}$$

The observation equation y_k is given as

$$y_k = h(x_k) + w_k \tag{18}$$

where k denotes the discrete time instant at which the observations are made, $h(x_k)$ denotes the nonlinear observation function, and w_k denotes a sequence of zero mean Gaussian white noise errors in the observations:

$$E\{w_k\} = 0 \ , \quad E\{w_k w_j^T\} = W_k \delta_{kj} \tag{19}$$

Note that the time step between observations is Δt. For Model I, $h(x)$ is defined by

$$h(x) = x_1 \quad (linear) \tag{20}$$

for Model II,

$$h(x) = x_1 \quad (linear) \tag{21}$$

and for Model III,

$$h(x) = x_1^{x_6} \quad (nonlinear) \tag{22}$$

3. Approximate Nonlinear Filters

In the derivations of the approximate nonlinear filters, Taylor series expansions of the two nonlinear functions $f(\cdot)$ and $h(\cdot)$, which appear in Eqs. (8) and (18), are employed. The Taylor series expansions are carried out to first-order terms for EKF and SSIF and to second-order terms for SOF.

Extended Kalman Filter

EKF for the nonlinear system (Eqs. (8) and (18)) consists of prediction via

$$\hat{x}_{k+1/k} = \hat{x}_{k/k} + f(\hat{x}_{k/k})\Delta t + \frac{1}{2}A(\hat{x}_{k/k})f(\hat{x}_{k/k})(\Delta t)^2 \tag{23}$$

$$P_{k+1/k} = \Phi_k P_{k/k} \Phi_k^T + U_k$$
$$\Phi_k = I + A(\hat{x}_{k/k})\Delta t + A^2(\hat{x}_{k/k})\frac{(\Delta t)^2}{2} \tag{24}$$

and at an observation

$$\hat{x}_{k+1/k+1} = \hat{x}_{k+1/k} + K_{k+1}[y_{k+1} - h(\hat{x}_{k+1/k})] \tag{25}$$

$$P_{k+1/k+1} = [I - K_{k+1}H_{k+1}]P_{k+1/k} \tag{26}$$

The Kalman gain is

$$K_{k+1} = P_{k+1/k}H^T_{k+1}[H_{k+1}P_{k+1/k}H^T_{k+1} + W_{k+1}]^{-1} \tag{27}$$

Here, $A(\hat{x}_{k/k})$ is the $n \times n$ Jacobian matrix of $f(\hat{x}_{k/k})$ with elements

$$[A(\hat{x}_{k/k})]_{\alpha\beta} \triangleq \left.\frac{\partial f_\alpha}{\partial x_\beta}\right|_{x=\hat{x}_{k/k}} \quad \alpha, \beta = 1, 2, \cdots, n \tag{28}$$

and $H(\hat{x}_{k+1/k})$ is the $m \times n$ Jacobian matrix of $h(\hat{x}_{k+1/k})$ with elements

$$[H(\hat{x}_{k+1/k})]_{\alpha\beta} \triangleq \left.\frac{\partial h_\alpha}{\partial x_\beta}\right|_{x=\hat{x}_{k+1/k}} \tag{29}$$

Second-Order Filter

The purpose of SOF is to reduce the effect of the dynamic nonlinearity $f(x)$ and observation nonlinearity $h(x)$ by including second-order terms of $f(x)$ and $h(x)$ in the Taylor series expansions. Here, the second-order terms are defined by μ_k for $f(x_k)$ and π_{k+1} for $h(x_{k+1})$, which are the *bias correction terms* introduced by Athans and his colleagues[9]. The SOF algorithm for the nonlinear system (Eqs. (8) and (18)) consists of prediction via

$$\hat{x}_{k+1/k} = \hat{x}_{k/k} + f(\hat{x}_{k/k})\Delta t + \frac{1}{2}A(\hat{x}_{k/k})f(\hat{x}_{k/k})(\Delta t)^2 + \mu_k \tag{30}$$

$$\mu_k = \frac{1}{2}\sum_{i=1}^{n} \gamma_i \operatorname{trace}[B_i(\hat{x}_{k/k})P_{k/k}] \tag{31}$$

$$P_{k+1/k} = \Phi_k P_{k/k} \Phi_k^T + U_k \tag{32}$$

and at an observation

$$\hat{x}_{k+1/k+1} = \hat{x}_{k+1/k} + K_{k+1}[y_{k+1} - h(\hat{x}_{k+1/k}) - \pi_{k+1}] \tag{33}$$

$$\pi_{k+1} = \frac{1}{2}\sum_{j=1}^{m} \gamma_j \operatorname{trace}[D_j(\hat{x}_{k+1/k})P_{k+1/k}] \tag{34}$$

$$P_{k+1/k+1} = [I - K_{k+1}H_{k+1}]P_{k+1/k} \tag{35}$$

The Kalman gain is

$$K_{k+1} = P_{k+1/k}H^T_{k+1}[H_{k+1}P_{k+1/k}H^T_{k+1} + W_{k+1} + L_{k+1}]^{-1} \tag{36}$$

$$(L_{k+1})_{ij} = \frac{1}{2}\operatorname{trace}[D_i(\hat{x}_{k+1/k})P_{k+1/k}D_j(\hat{x}_{k+1/k})P_{k+1/k}] \tag{37}$$

where $(L_{k+1})_{ij}$ is the ij-th element of the $m \times m$ matrix L_{k+1}.

Here, $\gamma_1, \gamma_2, \cdots, \gamma_{n,m}$ denote the natural basis vectors in $R_{n,m}$, i. e.,

$$\gamma_1 \triangleq \begin{bmatrix} 1 \\ 0 \\ \cdots \\ 0 \end{bmatrix}, \gamma_2 \triangleq \begin{bmatrix} 0 \\ 1 \\ \cdots \\ 0 \end{bmatrix}, \cdots, \gamma_{n,m} \triangleq \begin{bmatrix} 0 \\ 0 \\ \cdots \\ 1 \end{bmatrix} \tag{38}$$

$B_i(\hat{x}_{k/k})$, for $i = 1, 2, \cdots, n$, is the Hessian $n \times n$ symmetric matrix whose elements involve the various second partial derivatives of $f_i(\hat{x}_{k/k})$, $i = 1, 2, \cdots, n$, i. e.,

$$[B_i(\hat{x}_{k/k})]_{\alpha\beta} \triangleq \left.\frac{\partial^2 f_i}{\partial x_\alpha \partial x_\beta}\right|_{x=\hat{x}_{k/k}} \quad \alpha, \beta = 1, 2, \cdots, n \tag{39}$$

and $D_j(\hat{x}_{k+1/k})$, for $j = 1, 2, \cdots, m$, is the Hessian $m \times m$ symmetric matrix whose elements are

$$[D_j(\hat{x}_{k+1/k})]_{\alpha\beta} \triangleq \frac{\partial^2 h_j}{\partial x_\alpha \partial x_\beta}\bigg|_{x=\hat{x}_{k+1/k}} \tag{40}$$

Single-Stage Iteration Filter

This local iteration algorithm attempts to reduce the effect of both the dynamic nonlinearity $f(x)$ and the observation nonlinearity $h(x)$. In EKF update equations, $h(x)$ is linearized around the predicted estimate $\hat{x}_{k+1/k}$. Since the updated state estimate $\hat{x}_{k+1/k+1}$ is close to the true state x_{k+1}, the observation linearization errors can be reduced by relinearizing around $\hat{x}_{k+1/k+1}$. The dynamic nonlinearity is reduced by smoothing one step back and relinearizing $f(x)$ around the smoothed estimate. These iterative relinearizations are made until little further improvements are realized from additional iterations. Given $\hat{x}_{k/k}$ and $P_{k/k}$, for $i = 1, 2, \cdots, l$, SSIF consists of the iteration

$$\eta_{i+1} = \hat{x}_{k+1/k} + K_{k+1}(\eta_i, \xi_i)[y_{k+1} - h(\eta_i) - H_{k+1}(\eta_i)(\hat{x}_{k+1/k} - \eta_i)] \tag{41}$$

$$\xi_{i+1} = \hat{x}_{k/k} + S_k(\xi_i)[(\eta_{i+1} - \hat{x}_{k+1/k})] \tag{42}$$

$$\eta_1 = \hat{x}_{k+1/k}\ , \quad \xi_1 = \hat{x}_{k/k}$$

where

$$\hat{x}_{k+1/k} = \xi_i + f(\hat{x}_{k/k})\Delta t + \frac{1}{2}A(\hat{x}_{k/k})f(\hat{x}_{k/k})(\Delta t)^2 + \Phi_k(\xi_i)[\hat{x}_{k/k} - \xi_i] \tag{43}$$

$$K_{k+1}(\eta_i, \xi_i) = P_{k+1/k}H^T_{k+1}(\eta_i)[H_{k+1}(\eta_i)P_{k+1/k}H^T_{k+1}(\eta_i) + W_{k+1}]^{-1} \tag{44}$$

$$P_{k+1/k} = \Phi_k(\xi_i)P_{k/k}\Phi^T_k(\xi_i) + U_{k+1} \tag{45}$$

$$\Phi_k(\xi_i) = I + A(\xi_i)\Delta t + A^2(\xi_i)\frac{(\Delta t)^2}{2} \tag{46}$$

$$S_k(\xi_i) = P_{k/k}\Phi^T_k(\xi_i)P^{-1}_{k+1/k} \tag{47}$$

Then

$$\hat{x}_{k+1/k+1} = \eta_l \tag{48}$$

$$P_{k+1/k+1} = [I - K_{k+1}(\eta_l, \xi_l)H_{k+1}(\eta_l)]P_{k+1/k}(\xi_l)[I - K_{k+1}(\eta_l, \xi_l)H_{k+1}(\eta_l)]^T + K_{k+1}(\xi_l, \eta_l)W_{k+1}K^T_{k+1}(\xi_l, \eta_l) \tag{49}$$

The following variables are used in the above three filters: $\hat{x}_{k/k}$ is estimate of $n\times 1$ state vector at k given $Y_k(=\{y_1, y_2, \cdots, y_k\})$, $P_{k/k}$ is $n\times n$ covariance matrix of the error in $\hat{x}_{k/k}$, Φ_{k+1} is $n\times n$ state transition matrix, U_{k+1} is $n\times n$ system noise covariance matrix, $\hat{x}_{k+1/k}$ is $n\times 1$ state estimate at $k+1$ given Y_k, $P_{k+1/k}$ is $n\times n$ covariance matrix of the error in $\hat{x}_{k+1/k}$, W_{k+1} is $m\times m$ observation noise covariance matrix, and K_{k+1} is $n\times m$ Kalman gain at $k+1$.

4. A Case Study

This report compares nine algorithms, resulting from combining three approximate nonlinear filters with three nonlinear rainfall-runoff models, by applying them to the flood on June 27 to July 3, 1979 in Akimatsu Sub-Basin (area = 113 km^2) of the Onga River Basin, Kyushu, Japan (see Fig. 1). Rainfall data (unit: mm/hr) are taken from Okuma, Uchino, and Kawashima Stations and runoff data (unit: mm/hr) from Akimatsu Gaging Station. Rainfall over the sub-basin is averaged using Thiesen network. In this study, the one-step ahead runoff forecasting exercise is considered enough to compare the performance of the different combinations of filters and models. One-step length is one hour.

The catchment time lag T_L can be obtained from the observed rainfall and runoff data according to the procedure described in the Lecture Notes on River Engineering Course prepared by Japan's Ministry of Construction and Japan International Cooperation Agency

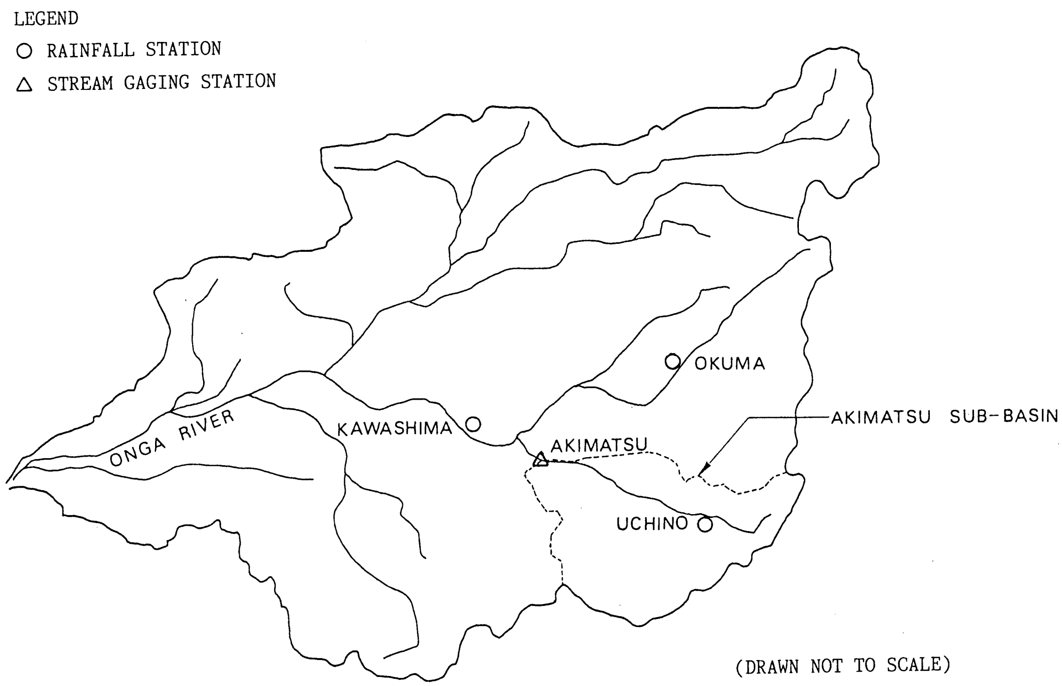


Fig. 1 Akimatsu Sub-Basin of the Onga River Basin.

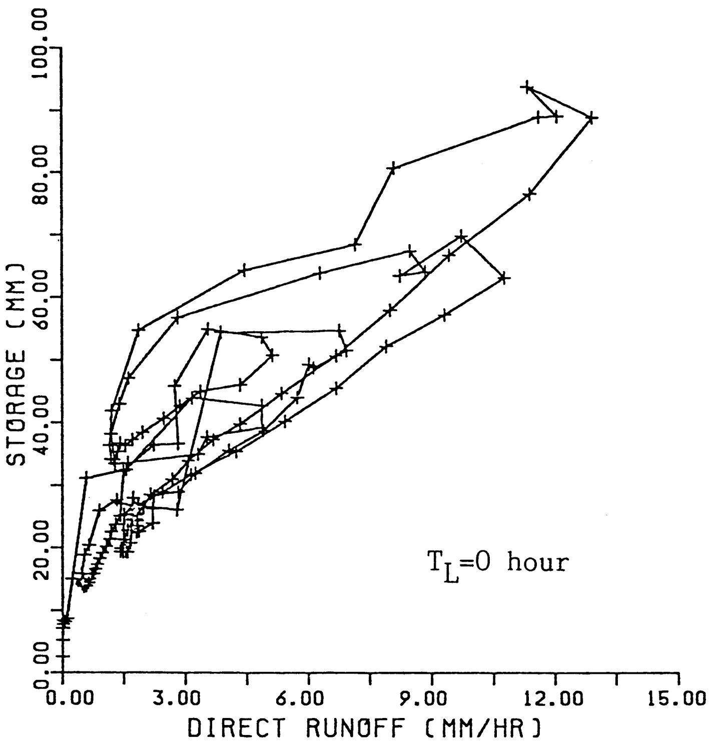
(1979). Fig. 2 shows the plots of the storage-direct runoff (discharge) relationships for $T_L = 0$ and $T_L = 1$ hour ($T_L = 2$ and 3 hours were also tested), which are derived using the procedure. The flood hydrograph considered in this study has five peaks which result in the multiple loops shown in this figure. This procedure specifies that the time lag which gives the smallest loop is the time lag for the basin. For the Akimatsu Sub-Basin, the time lag in catchment response to rainfall is thus determined to be equal to one hour. In this case, the observed rainfall value is used to forecast the one-hour ahead runoff value since the lead time is equal to T_L. The case of $T_L = 0$ assumes that the lead time is longer than T_L, and the rainfall value needs to be predicted to forecast the one-hour ahead runoff value. However, since this report aims not to investigate the rainfall forecasting problems, but to evaluate the performance of the different combinations of filters and models, given the same rainfall inputs, the one-hour ahead observed rainfall value is used instead. (The readers are referred to Puente and Bras[14] who have offered good discussions on problems related to rainfall prediction and filter performance.)

For the one-hour ahead runoff forecasting problem, the performance of the nine algorithms are compared on the basis of the root mean squared error

$$RMSE = \left[\frac{1}{N}\sum_{i=1}^{N}\{y_f(i) - y_0(i)\}^2\right]^{1/2} \tag{50}$$

where N is the number of data points, y_f is the forecasted runoff and y_0 is the observed runoff. Here, the smaller the *RMSE* value (unit: mm/hr), the better the algorithm's performance.

In all computations, $P(0|0)$ and W are kept constant for each rainfall-runoff model. To avoid outright filter divergence, small values of $P(0|0)$ are used. As shown in Table 1, six sets of values of U_{11} and U_{22}, the first and second diagonal elements of the marix U, are



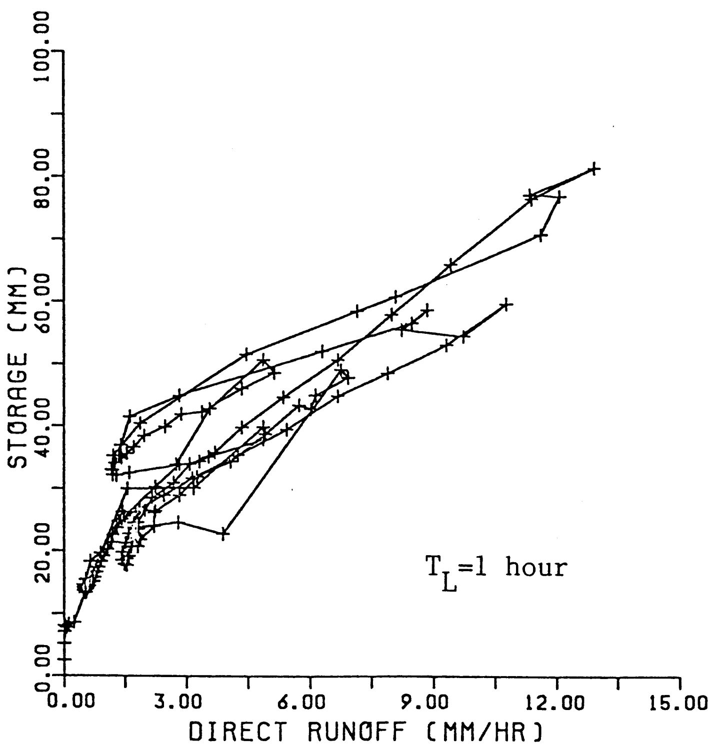


Fig. 2 Storage-direct runoff relationships.

Table 1 Results of the performance assessment in terms of RMSE values. Note that the symbol * means diverge.

Model	Set	U_{11}	U_{22}	$T_L=1$ EKF	$T_L=1$ SOF	$T_L=1$ SSIF	$T_L=0$ EKF	$T_L=0$ SOF	$T_L=0$ SSIF
I	1	0.0001	0.0	0.638	0.638	0.640	0.863	0.864	0.801
	2	0.001	0.0	0.614	0.614	0.620	0.781	0.781	0.769
	3	0.01	0.0	0.565	0.564	0.593	0.733	0.733	0.782
	4	0.1	0.0	0.516	0.516	0.526	0.702	0.702	0.711
	5	1.0	0.0	0.489	0.489	0.490	0.684	0.684	0.684
	6	10.0	0.0	0.478	0.478	0.478	0.679	0.679	0.679
II	1	0.0001	0.0001	0.763	0.762	0.799	0.693	0.692	0.695
	2	0.001	0.001	0.618	0.618	0.630	0.571	0.571	0.580
	3	0.01	0.01	0.552	0.552	0.612	0.558	0.558	0.566
	4	0.1	0.1	0.532	0.532	0.720	0.553	0.554	0.551
	5	1.0	1.0	0.523	0.522	0.882	0.545	0.545	0.541
	6	10.0	10.0	0.521	0.522	0.555	0.542	0.542	0.541
III	1	0.0001	0.0001	0.541	0.552	0.624	0.563	*	1.595
	2	0.001	0.001	0.485	0.486	0.527	0.524	0.527	0.521
	3	0.01	0.01	0.455	0.462	0.467	0.498	0.491	0.503
	4	0.1	0.1	0.438	0.568	0.452	0.487	*	0.481
	5	1.0	1.0	0.436	*	0.447	0.478	*	0.478
	6	10.0	10.0	0.437	*	0.447	0.476	*	0.478

considered to assess the performance capabilities of the three filters when applied to each model. (All other elements of U are zeroes.) These two elements represent errors in the rainfall inputs and in the model equations due to the incomplete knowledge of the nature of the system (check Eqs. (10), (12) and (14) for the elements of the function $f[x(t)]$ of the model equation). Different values of W are examined in combination with different values of U, and $W = 0.001$ gives the best performance for each combination of filter and storage function model and is thus used in combination with those values of U in Table 1. The reason for not showing the results on testing different values of W is that U is considered much more significant than W since dynamic nonlinearity is much more dominant than observation nonlinearity.

In the absence of prior information on the values of the different parameters of the models, K_1 and K_2 are initially estimated from the following empirical formulas given by Hoshi[19], with $N_1 = 0.6$ and $N_2 = 0.4648$.

$$K_1 = 4.57A^{0.24} \tag{51}$$

$$K_2 = 5.26A^{0.48}\bar{r}^{-0.2648} \tag{52}$$

where A is the drainage area (km^2) and $\bar{r}$ is the average rainfall intensity (mm/hr).

5. Results and Discussion

Table 1 summarizes the results of the performance assessment of the nine algorithms in terms of *RMSE* values. For both cases of $T_L = 1$ and $T_L = 0$, SOF and SSIF yield no improvement at all over EKF in almost all of the sets of U when combined with the three models. This apparent lack of improvement in using SOF and SSIF suggests that there is insufficient amount of nonlinearity[11]. It also implies that the nonlinearity in the basin under study is not dominant. As illustrated in Table 1, an optimum performance of EKF could be realized by varying the values of U_{11} and U_{22}. This means that the accuracy of runoff

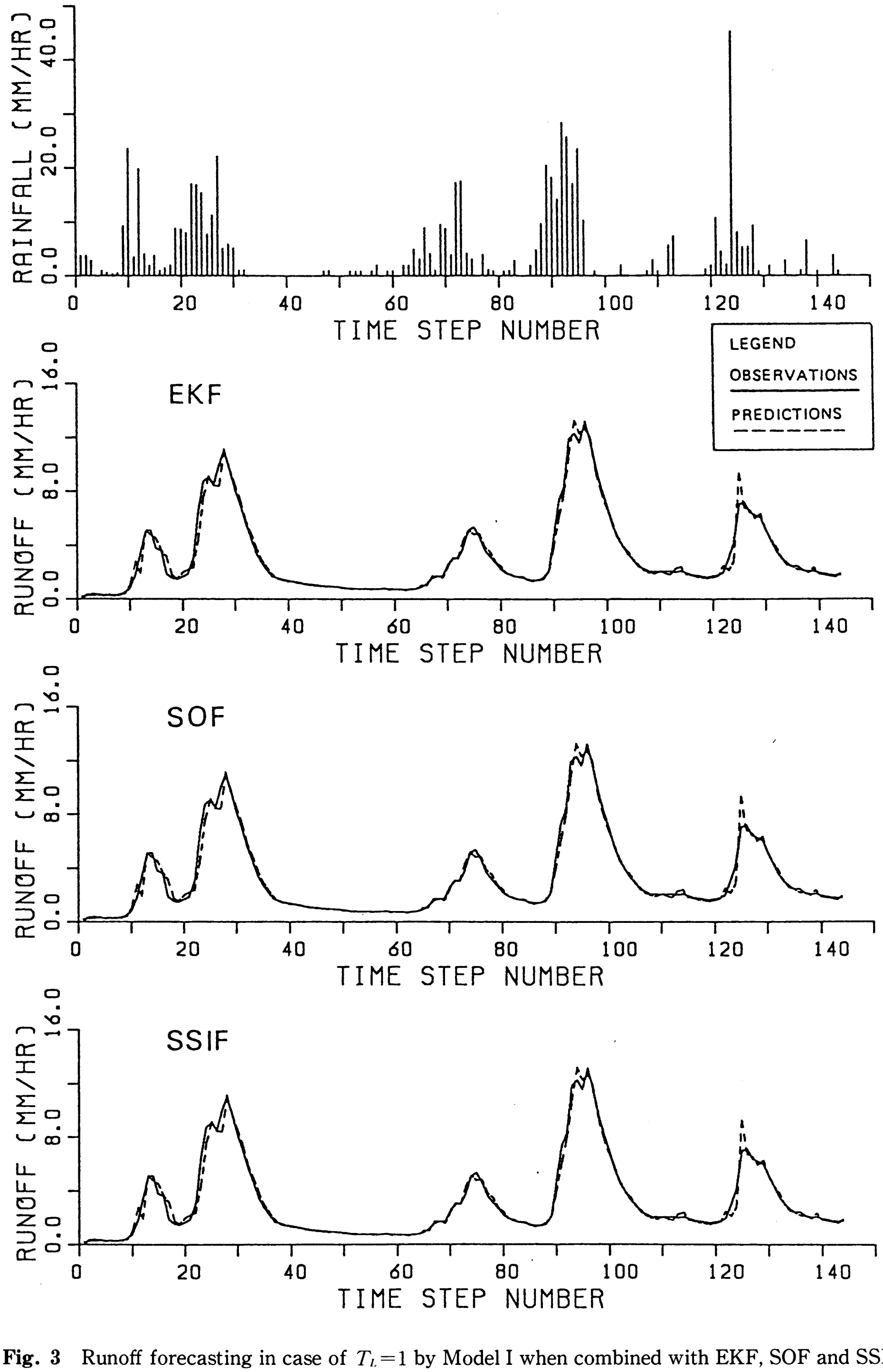


Fig. 3 Runoff forecasting in case of $T_L=1$ by Model I when combined with EKF, SOF and SSIF, U of Set 6, time step length one hour.

forecasts depends on the choice of constant values to be assigned to the noise matrix U and not on the choice of the filter. This result is similarly reported by Puente and Bras[14]. Fig. 3 demonstrates the similarity of the behaviour of the runoff forecasts among the three nonlinear filters for U in Set 6 of Model I under $T_L = 1$. Table 2 shows the normalized computation time with the combination of EKF and Model I taking about 125 milliseconds on a FACOM M780/20 for 144 data points using computer programs written in PL/I language. As shown in this table, SOF takes roughly 17% longer than EKF and SSIF about five times longer for three iterations.

Fig. 4 shows the time series plots of the parameters of Model I as estimated by EKF, SOF, and SSIF for $T_L=1$. This figure demonstrates that, when the *RMSE* values in Table 1 are roughly the same among the three nonlinear filters, the behaviour of the parameter

Table 2 Normalized computation time.

Model	EKF	SOF	SSIF
I	1.00	1.17	4.93
II	1.56	1.90	7.56
III	1.92	3.02	9.44

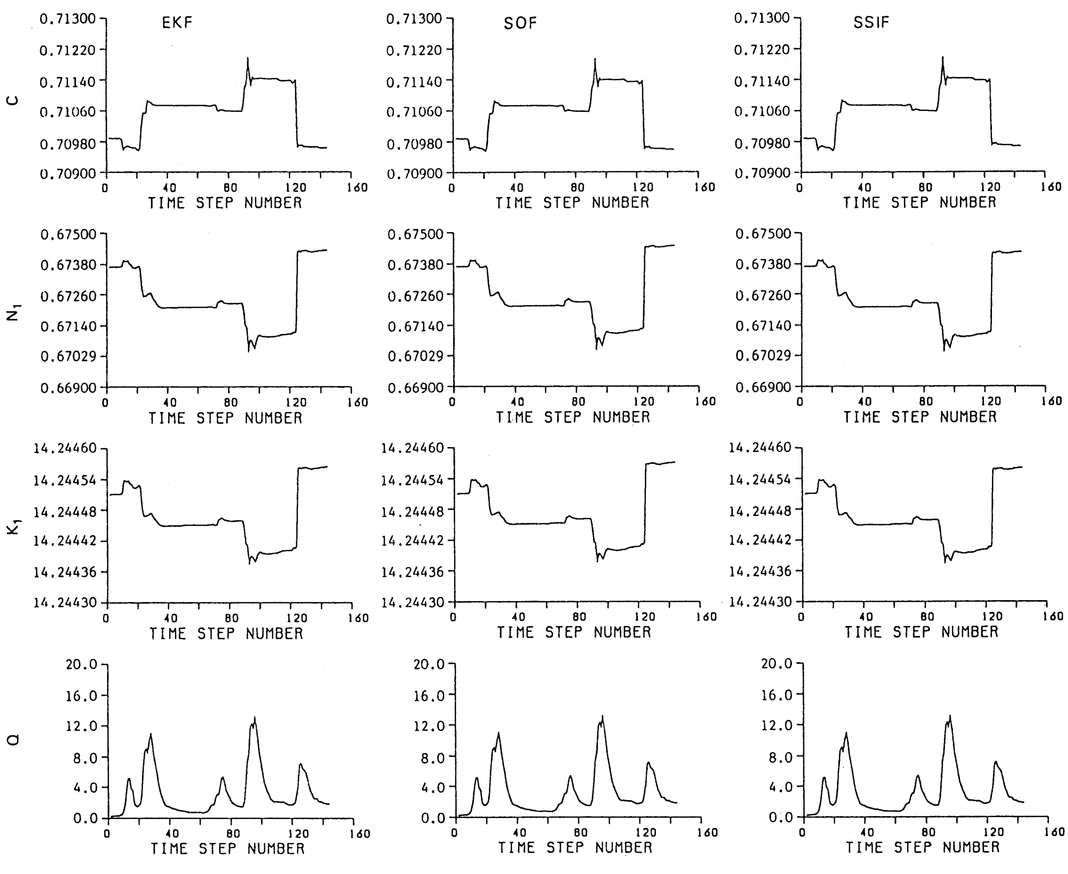


Fig. 4 Parameters of Model I as estimated by EKF, SOF and SSIF, U of Set 6, time step length one hour.

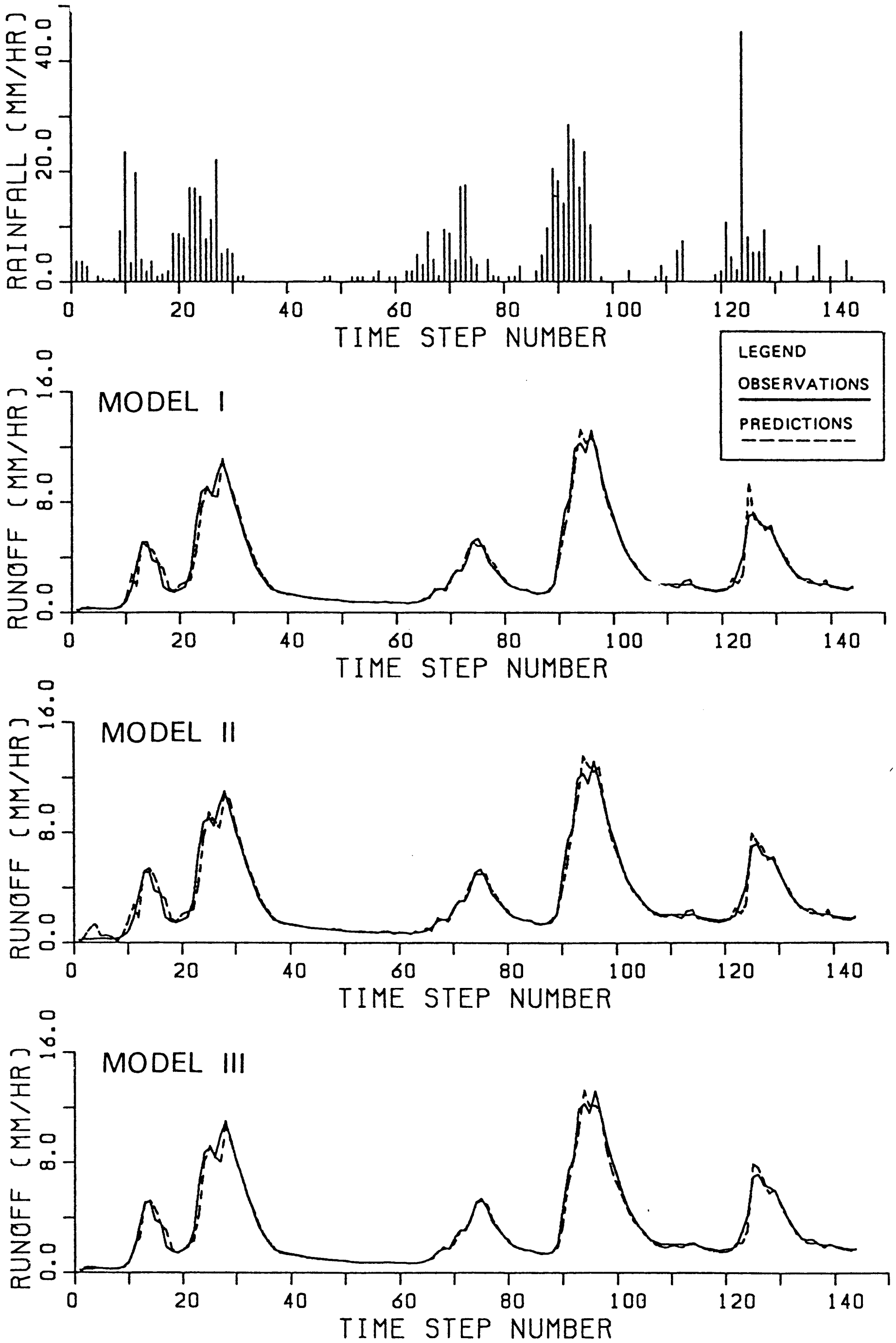


Fig. 5 Runoff forecasting in case of $T_L = 1$ by Models I, II and III when combined with EKF, U of Set 6, time step length one hour.

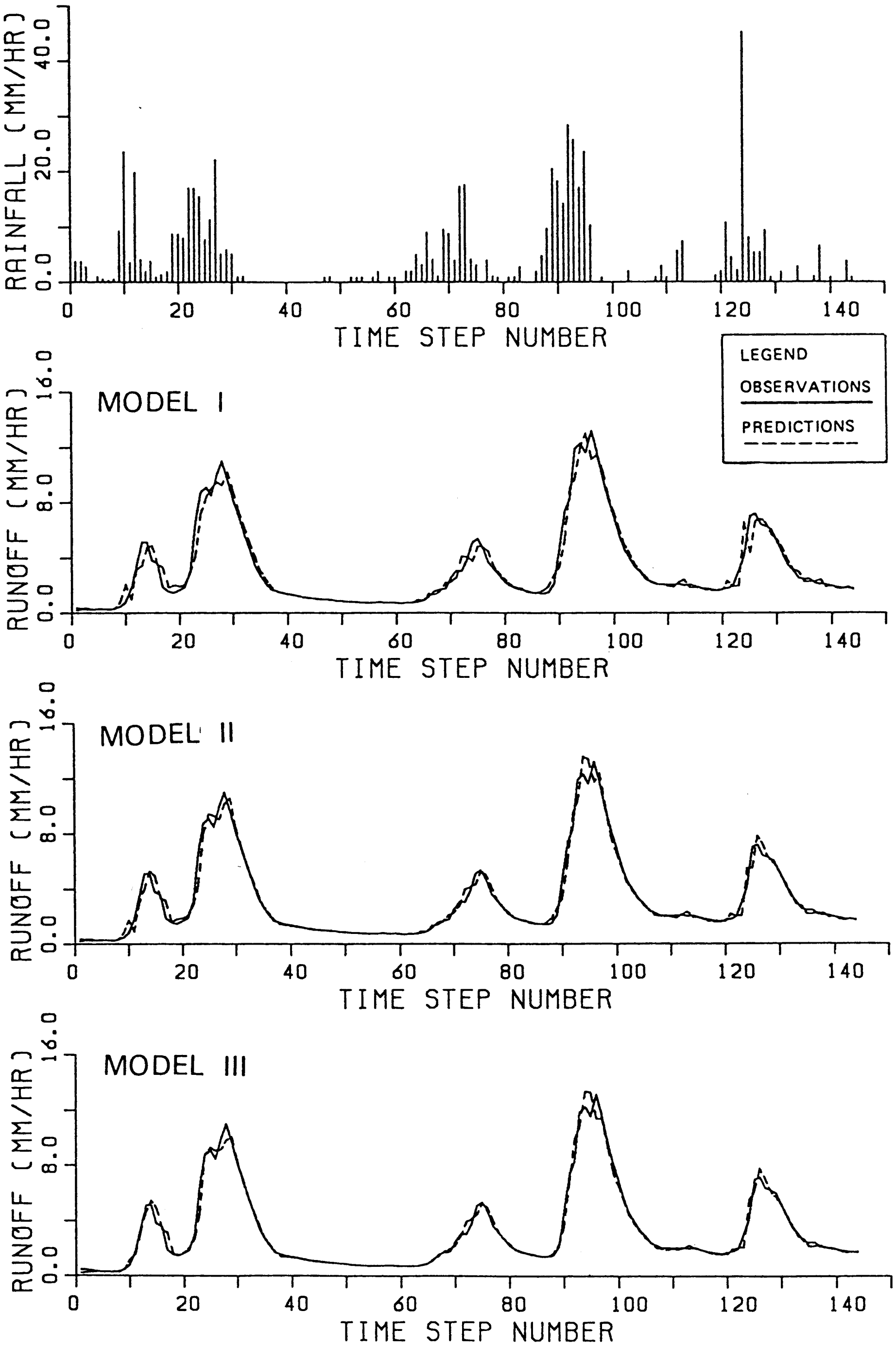


Fig. 6 Runoff forecasting in case of $T_L = 0$ by Models I, II and III when combined with EKF, U of Set 6, time step length one hour.

estimates are identical.

Plots of the observed runoff outputs and one-hour ahead forecasts by Models I, II, and III when combined with EKF are presented in Fig. 5 for the case of $T_L = 1$ and in Fig. 6 for the case of $T_L = 0$. Although Model III provides the best one-hour ahead forecasts as indicated by the *RMSE* values for the case of $T_L = 1$ in Table 1, visual inspection of the behaviour of the observed and forecasted runoff values in Fig. 5 would indicate that the differences are not that impressive. Notice also in Table 1 that Model III is only 9% more accurate than Model I. Being simple and requiring the least computational requirement, Model I is as good as the other models provided that T_L is accurately determined. This is so because, as shown in Fig. 2, $T_L = 1$ results in an approximately single-valued storage-discharge relationship which is the basis of Model I. It follows that Model I provides accurate runoff forecasts if a single-valued storage-discharge relationship can be achieved. However, if T_L is neglected, the efficiency of Model I decreases while those of Models II and III are essentially the same. This is demonstrated by comparing the behaviour of the forecasted runoff values in Fig. 5 with those in Fig. 6. This result is also shown in Table 1 : Model I has a significant increase (42%) in *RMSE* value between $T_L = 1$ and $T_L = 0$, while there are only slight increases for Models II (4%) and III (9%). This suggests that Models II and III are much more stable and accurate than Model I in the case of neglecting any time delays in catchment response to rainfall ($T_L = 0$). It also implies that the omission of T_L is compensated by the differential terms in storage equations (2) and (3), since these terms are added specifically to describe the looped storage-discharge relationships in Fig. 2 ($T_L = 0$). Prasad[6] has shown that the parameter K_2 of these differential terms depends on the characteristics of rainfall. Moreover, Models II and III are expected to work better than Model I if there are errors in rainfall predictions (such as overpredictions and underpredictions of rainfall amounts or nonpredictions of rainfall events) in the case of the forecast lead time being longer than T_L.

On the other hand, there is a good timing of peak runoff forecast in the case of $T_L = 1$ than in the case of $T_L = 0$, which is obvious. In connection with this, the timing errors in peak runoff forecasts are neither caused nor corrected by both filters and models, but dependent on the choice of T_L. Moreover, it can be inferred from Figs. 5 and 6 that the differential terms in storage equations (2) and (3) can limit the undesirable effect of the abrupt rainfall variation at time step $k = 124$. Notice also in Fig. 6 that these differential terms have provided Models II and III better estimates of the rising limb of the hydrograph. Although the results show that Model III is slightly better than Model II, the difference is not very convincing. Referring again to Table 2, there is an increase of 56% in computation time between Models I and II and 92% between Models I and III.

6. Conclusions

Using a single flood hydrograph example, nine combinations of nonlinear filters and nonlinear storage function rainfall-runoff models are compared on the basis of their runoff forecasting accuracy and computation time. It is shown that the accuracy of runoff forecasts depends on the superiority of the model and not on the complexity of the nonlinear filter. The more superior the model used is, the more accurate the forecasts are. The accuracy also depends on the choice of constant values to be assigned to the noise matrix U and not on the choice of the filter. This means that the rainfall-runoff process in the basin under study is not as highly nonlinear as to warrant the inclusion of the second-order term or the use of local iteration. It is further shown that the inclusion of the catchment time lag T_L can improve the performance of the different combinations. Also, if the catchment time lag is accurately determined, the combination of EKF and Model I is as good as the other combinations as it achieves almost the same level of accuracy and requires the smallest computation time. Models II and III are found to reduce bias in the rising limb of the flood hydrograph and are much more stable and accurate than Model I in the presence of bias in rainfall response.

It should be borne in mind that the analysis is confined to only one flood hydrograph. If complete verification of the performance of the nine algorithms is needed, they should be tested in many floods from basins with different climatologic and physiographic characteristics. However, the results of this study should contribute to the discussion on the accuracy of nonlinear filters and nonlinear storage function models in rainfall-runoff analysis in particular and on the accuracy of model computations in hydrology in general.

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