II-89 Reconstructivity of system equations from a chaotic numerical time series

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1. INTRODUCTION

It is studied that many hydro-meteorological time series possess chaotic behavior(e.g., Kurths, 1987; Rodriguez-Iturbe et al.,1989; Mundt et al., 1991). And on the other hand, the studies on chaotic analysis (e.g. Lorenz, 1963; Rössler, 1976) indicate that certain mathematical models without any explicit random components are able to produce chaotic behaviors. These models can be described by the system of purely deterministic but non-linear equations. In order to reconstruct system equations from a numerical time series, Gouesbet(1991) studied the systems derived from the Rössler equations and Lorenz equations. For practical problems, usually only one time series is available, and the prediction of its future development is always a main aim of time series analysis. The reconstructivity and predictability for a chaotic numerical time series is examined by applying the extended Kalman filter(EKF)(Ueta et al.,1984) in this paper.

2. THE TACTICS OF THE STUDY

It is no doubt that chaotic behavior is generated from a certain continuous dynamic system. The outputs of its components show to be chaotic time series. Considering the situation that only one time series is available in practical research, the steps to find out chaotic system equations or equivalent system equations can be shown in Figure 1.
If original chaotic system equations are known beforehand, then a numerical calculation method can be employed from step (1) to step (2). The EKF is a suggested way from step (3) to step (4). For practical problems, the work should be started from step (2). Before EKF is used to evaluate parameters, there are two problems to be solved. One is to determine the number of basic variables(equations). The other one is to assume a proper structure of system equations. For the first problem a fractal dimension analysis method can be used, while the study for the second problem has been just started.

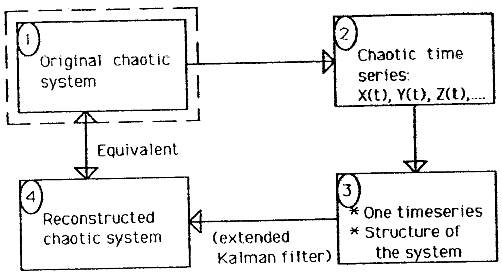

Figure 1. The tactics of the study

3. PREDICTION OF A CHAOTIC SYSTEM

3.1 Example of a chaotic system

In order to test the proposed approach, let us take Rössler equations as an example. The Rössler equations are:

$$\begin{cases} \dot{x} = -y-z \\ \dot{y} = x+ay \\ \dot{z} = b+z(x-c) \end{cases} \tag{1}$$

where $\dot{x}$, $\dot{y}$ and $\dot{z}$ are the derivatives of x, y and z with respect to time t. The parameters a , b and c are constants. The behavior of the system is very sensitive to the parameter values. Among three parameters, the a mainly affects chaotic behavior of the system, and the parameters b and c mainly affect the amplitudes of the x, y and z. The system obtained when $a=0.398$, $b=2.0$, and $c=4.0$, is often used as an example of a chaotic phenomenon (e.g., Gouesbet, 1991). If the time interval $\Delta t=0.05$, the initial values of x, y, and z are all 1.0, and the equations are solved by the Runge-Kutta Gill method, the time series x(t), y(t), z(t), and phase space portrait of the system exhibit the appearance shown in Figure 2. This is a typical example of a chaotic (or strange) attractor acting on the system.

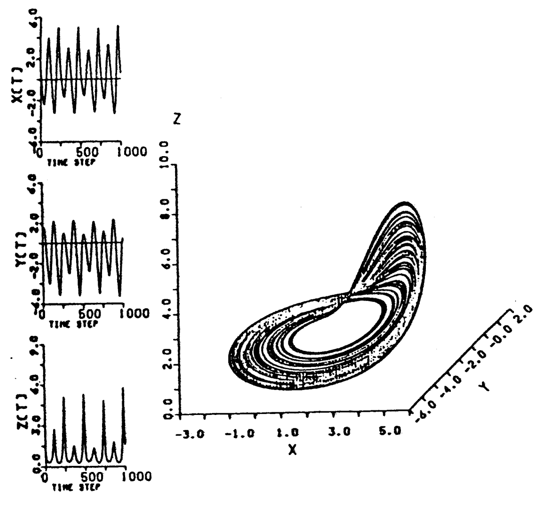

Figure 2. The time series(1000 steps) and phase space portrait(10000 steps) of the Rössler equations using the parameters: $a=0.398$, $b=2.0$, $c=4.0$.

3.2 Reconstructed systems by Gouesbet

The paper(Xu et al, 1993) has shown that the EKF is effective on evaluating parameters of a chaotic system when exact system equations are used. Because system equations are usually unknown in practice, it is necessary to reconstruct them. Generally only one time series is available for analyzing a system, denoting it by x(t). Putting x(t) on special status, the three-dimensional reconstructed system equations can be:

$$\begin{cases} \dot{x} = Y \\ \dot{Y} = Z \\ \dot{Z} = F(x,Y,Z) \end{cases} \tag{2}$$

The identical equation (3) corresponding equation (1) is derived from equations (2) as follows. The mean of the time series x(t) can be found from it.

$$\dddot{x}-F(x,\dot{x},\ddot{x})=0 \qquad (3)$$

Taking the form of a reconstructed system as equations (2), we can directly derive F(x,Y,Z) from equation (1) as shown in equation (4). Equations (2) and (4) are an exactly reconstructed system of the original Rössler equations (1):

$$F(x,Y,Z)=ab-cx+x^2-axY+xZ+(ac-1)Y+(a-c)Z-Y(x+b-aY+Z)/(a+c-x) \qquad (4)$$

Another suggestion is that F(x,Y,Z) is taken as a ratio of polynomial expression that may be attempted in any case.

$$F(x,Y,Z)=\Big(\sum_{j+k+m=0}^{N_0} N_{jkm}x^jY^kZ^m\Big)/\Big(1+\sum_{j+k+m=1}^{N_0} D_{jkm}x^jY^kZ^m\Big) \qquad (5)$$

where N_0 is the order of polynomials. Gouesbet evaluated 39 parameters which considered up to third order polynomials. However the problems of accuracy and sensitivity to time series of the results appeared. By referring to the theoretical values of the parameters and the exactly reconstructed system (4), which can be found only when the original equations (1) are known beforehand, equation (6) is used instead.

$$F(x,Y,Z)=A+Bx+Cx^2+ExZ+GZ+Y(U+Vx+Wx^2+SY+TZ)/(1+Px) \qquad (6)$$

in which A,...,T are constant parameters.

3.3 Prediction of the reconstructed system

Using equations (2) and (6) and the time series x(t) obtained from the original Rössler equations, we can apply the EKF to predict the time series x(t), Y(t) and Z(t), and to identify the parameters A,...,T. In this case, the number of the system variables is 14 in the EKF. They correspond to the vector $[x,Y,Z;A,B,C,E,G,P,U,V,W,S,T]^T$ in equation (6). As results, the one step ahead prediction of the time series x(t), Y(t), and Z(t) are shown in Figure 3 when 1000 steps calculation is made.

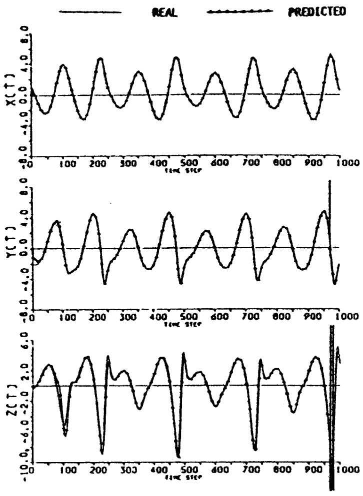

Figure 3. Predicted time series of the reconstructed system by use of extended Kalman filter

The results indicate that the prediction of the observed time series x(t) is quite good. The average of absolute errors is 0.0023. It is 0.175 percent of the average of absolute values of time series x(t). The partial predictions of the time series Y(t) and Z(t) are also reasonable. However it was also found that there is a possibility that an unstable behavior appears in the calculation. This is related to that the instability happens when the value of the denominator 1+px becomes near zero.

4. CONCLUSIONS

From the results of the study we can make conclusions as follows.

(1)The reconstructed system equations that Gouesbet suggested are related to the original ones. If original system equations are unknown beforehand, it is difficult to decide the structure of equations and to simplify terms of them.

(2)When the EKF is applied to identify parameters and predict time series based on the reconstructed system equations given by Gouesbet, sometimes an unstable behavior happened. The blame falls on the structure of the reconstructed system. The possibility that unstable situations appear can not be ignored when the EKF is used. In other words, The reconstructed system equations is not suitable for the Rössler equations in this case.

(3)Owing to these two reasons mentioned above, a new general form of reconstructed system equations should be studied, for example, taking the derivatives of x, y and z as the Taylor series expansions and so on.

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