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NONLINEAR FILTERS AND STORAGE FUNCTION MODELS

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INTRODUCTION

This report compares the performance of nine combinations of approximate nonlinear filters and nonlinear storage function models for the real-time forecasting of runoff from rainfall on the basis of their forecasting accuracy and computation time. The models are those of Kimura, Prasad and Hoshi. Among the three models, Hoshi's model has been found to be superior (see, Hoshi and Yamaoka (1)). The three approximate filters are the extended Kalman filter (EKF), a second-order filter (SOF) and a single stage iteration filter (SSIF), which are derived using truncated Taylor series expansion to represent system nonlinearites. SSIF is found to be complex but superior if nonlinearites are significant. In the hope of obtaining better forecasting accuracy, several authors have combined a more complex approximate nonlinear filter with a less superior storage function model or *vice versa*. However, in spite of having these combinations of nonlinear filters and storage function models, there is no comparison which illustrates their relative advantages and disadvantages.

NONLINEAR RAINFALL-RUNOFF MODELS

With the three storage function models and the continuity equation

$$\frac{dS(t)}{dt} = CR(t) - Q(t) \tag{1}$$

where S is storage, C is a coefficient, CR represents the effective rainfall, and Q is runoff, the rainfall-runoff process can be represented by the following three differential equations

$$\text{MODEL I:} \quad K_1N_1Q^{N_1-1}(t)\frac{dQ(t)}{dt} + Q(t) = CR(t) \tag{2}$$

$$\text{MODEL II:} \quad K_2\frac{d^2Q(t)}{dt^2} + K_1N_1Q^{N_1-1}(t)\frac{dQ(t)}{dt} + Q(t) = CR(t) \tag{3}$$

$$\text{MODEL III:} \quad K_2\frac{d^2[Q^{N_2}(t)]}{dt^2} + K_1N_1Q^{N_1-1}(t)\frac{dQ(t)}{dt} + Q(t) = CR(t) \tag{4}$$

where K_1, K_2, N_1 and N_2 are coefficients. Model I is based on *Kimura's storage function model*, Model II is well known as the *Prasad model of rainfall-runoff*, and Model III is based on *Hoshi's storage function model* (Hoshi and Yamaoka (1)).

CASE STUDY

The nine algorithms, resulting from combining the three approximate nonlinear filters with the three nonlinear rainfall-runoff models, are applied to the flood on June 27 - July 3, 1979 in Akimatsu Sub-basin (area = 113 sq. km.) of the Onga River Basin, Kyushu, Japan. Hourly rainfall data are taken from Ookuma, Uchino, and Kawashima Gaging Stations and hourly runoff data from Akimatsu Gaging Station. Rainfall over the sub-basin is averaged using Thiesen network. The performance of the nine algorithms are compared on the basis of the root mean squared error $RMSE$.

RESULTS AND DISCUSSION

In all computations, $P(0|0)$ and R are kept constant for each rainfall-runoff model. To avoid outright filter divergence, small values of $P(0|0)$ are used. As shown in Table 1, six sets of values of Q_{11} and Q_{22}, the first and second diagonal elements of the matrix Q, are considered to assess the performance capabilities of the three filters when applied to each model. (All other elements of Q are zeroes.)

Table 1 summarizes the results of the performance assessment of the nine algorithms in

Table 1 Results of the performance assessment in terms of *RMSE* values. Note that the symbol * means diverge.

Model	Set	Q_{11}	Q_{22}	EKF	SOF	SSIF
I	1	0.0001	0.0	0.863	0.864	0.801
	2	0.001	0.0	0.781	0.781	0.769
	3	0.01	0.0	0.733	0.733	0.782
	4	0.1	0.0	0.702	0.702	0.711
	5	1.0	0.0	0.684	0.684	0.684
	6	10.0	0.0	0.679	0.679	0.679
II	1	0.0001	0.0001	0.693	0.692	0.695
	2	0.001	0.001	0.571	0.571	0.580
	3	0.01	0.01	0.558	0.558	0.566
	4	0.1	0.1	0.553	0.554	0.551
	5	1.0	1.0	0.545	0.545	0.541
	6	10.0	10.0	0.542	0.542	0.541
III	1	0.0001	0.0001	0.563	*	1.595
	2	0.001	0.001	0.524	0.527	0.521
	3	0.01	0.01	0.498	0.491	0.503
	4	0.1	0.1	0.487	*	0.481
	5	1.0	1.0	0.478	*	0.478
	6	10.0	10.0	0.476	*	0.478

terms of *RMSE* values. Observe in Set 6 of each model that there is a marked difference in *RMSE* values between models than between filters. In fact, SOF and SSIF yielded no improvement at all over EKF in almost all of the sets of Q when combined with the three models. This apparent lack of improvement in using SOF and SSIF suggests that there is insufficient amount of nonlinearity (Jazwinski (2)). As shown in Table 1, an optimum performance of EKF could be realized by varying the values of Q_{11} and Q_{22}. This means that the accuracy of runoff forecasts depends on the choice of constant values to be assigned to the noise matrix Q and not on the choice of the filter. This result is similarly reported by Puente and Bras (3). It is shown that the behaviour of the runoff forecasts are the same among the three nonlinear filters in Set 6 of Model I. In terms of computation time, SOF takes roughly 50% longer than EKF and SSIF about five times longer for three iterations.

As for model accuracy, Model III provides the best one-step ahead forecasts. In terms of *RMSE* values, there is an increase in accuracy of 20% between Models I and II and 11% between Models II and III. However, these percentages should not be taken as final, as the three models may perform differently in other flood hydrographs. Nevertheless, this result verifies the superiority of Model III. In terms of computation time, there is an increase of 56% between Models I and II and 92% between Models I and III. Moreover, the differential terms in storage models of Prasad and Hoshi can smooth away the undesirable effect of rainfall variations. These differential terms have provided Models II and III better estimations of the rising limb of the hydrograph.

CONCLUSIONS

Models II and III are found to reduce bias in the rising limb of the flood hydrograph. It is shown that the accuracy of runoff forecasts depends on the adequacy of the model and not on the complexity of the nonlinear filter. That is, the more accurate the model is, the more accurate the runoff forecasts are. Also, the accuracy of runoff forecasts depends on the choice of constant values to be assigned to the noise matrix Q and not on the choice of the filter. Based on the results involving single flood hydrograph example, it is shown that the combination of EKF and Model III is to be preferred over other combinations on the basis of accuracy of the model, simplicity of the filter and small computational requirement.

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