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A GENERALIZED URBAN STORAGE FUNCTION MODEL CONSIDERING SPATIAL RAINFALL DISTRIBUTION

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The rainfall spatial variability has not been considered in the Storage Function (SF) models so far even though there exist various SF models including the Urban SF (USF) model, a relatively new SF model mainly for urban watersheds. Therefore, in this study, we aim to propose a generalized USF (GUSF) model for the storm runoff analysis by considering the spatial rainfall distribution in the basin. This was achieved by the introduction of a new parameter named as rainfall distribution factor (γ) in the USF model. The GUSF and USF models were examined and the results revealed that the GUSF model with γ exhibited higher hydrograph reproducibility associated with the lowest error evaluation criteria which emphasize the effect of parameter γ. Further, the Akaike information criterion (AIC) was used to establish the best model among two based on the number of optimized model parameters. The GUSF model received the lowest AIC score in calibration and validation which indicate that the inclusion of a single parameter, rainfall distribution factor, can substantially improve the performance of a model by representing the spatial rainfall distribution of basin in a better way.

Key Words: *USF model, generalized urban storage function model, rainfall distribution factor, hydrograph reproducibility, AIC criteria*

1. INTRODUCTION

Flooding is a crucial issue in both rural and urban areas, but the severity level of floods is greater in urban areas because most of the population is concentrated near floodplains[1)]. Therefore, flood modeling in urban watersheds is essential. For this purpose, the lumped Storage Function (SF) models, have been widely used in many parts of the world. The SF model was originally invented by Kimura[2)] with lag time which is still widely used in Japan. Subsequently, several improved SF models have been proposed in terms of how to express its nonlinearity, model structure, and the storage hysteresis loop[3), 4)]. However, all these models require effective rainfall as their input for the direct runoff prediction. Hence, it involves the problems of baseflow and effective rainfall component separation that may further add uncertainties to the model simulations. They also incorporated the runoff coefficient in SF models to account for the loss components in the basin. To overcome the problems associated with separation processes, Baba et al. introduced an SF model with loss mechanism which uses the observed rainfall and runoff[5]. Later, Takasaki et al. developed a new Urban SF (USF) model for the urban watersheds with the combined sewer system by considering urban runoff process[6].

Generally, in the conventional SF models, the rainfall is spatially averaged over the basin and assumed a spatially uniform rainfall across the basin. This spatially averaged rainfall was considered as the observed basin rainfall in the conventional models. However, in actual condition, the rainfall is spatially distributed over the watershed and this spatial variability will be quite high even in small urban watersheds[8]. The use of basin average rainfall will further result in the underestimation or overestimation of storm runoff based on the meteorological factors as well as the location of rainfall occurrence[9]. For example, if a localized rainfall with high intensity is occurring near the watershed outlet, the outlet will receive an immediate high magnitude response without any significant losses compared with a delayed and diminished outlet response resulted from the upstream rainfall. This effect will be profound in small urban watersheds due to the relatively short time of concentration and the high percent of impervious surfaces. The existing spatial variability in rainfall will contribute uncertainties to the basin average rainfall and finally to the model predictions, but to what extent is unknown.

The performance of different existing SF models have already been evaluated for an urban watershed and it was found that the USF model performs better as compared with conventional SF models[1), 7)]. The USF model well-simulated the discharge in small to medium-sized urban watersheds for single and multi-peak flood events with different meteorological factor and peak discharge[1), 7)]. The model also exhibited higher performance for different objective functions used during the model calibration[6]. However, the USF model assumed a spatially uniform rainfall over the basin likewise the conventional SF models. So far, the rainfall spatial variability has not been considered in the SF models and thereby an attempt has been made for the first time to address this issue by introducing a new parameter called rainfall distribution factor, hereafter termed as γ, in the USF model. This parameter will either increase or reduce the basin average rainfall to adjust it with the true basin rainfall which will further reduce the uncertainties involved to some extent. Further modifications are also possible using radar and dense rain gauge network data to gain a better understanding of the rainfall spatial variability.

Based on the above discussions, this study aims to propose a generalized USF (GUSF) model with all possible loss components for the storm-runoff analysis by considering the spatial rainfall distribution in the basin by the introduction of a new parameter, γ. The performance evaluation of the GUSF model along with the USF model was conducted to examine the effectiveness of parameter γ in terms of hydrograph reproducibility and information criteria point of view. The Kanda river basin, a typical small to medium-sized urban watershed in Tokyo, was selected as the target basin.

2. MATERIALS AND METHODS

(1) Generalized USF (GUSF) model

The storage equation of GUSF model is the empirical representation of Hoshi's SF model[4] in which the observed river discharge Q is replaced by $Q + q_R$, which is the discharge including the drainage from the sewer system for the urban area, where, q_R is drainage from the basin through the combined sewer system (mm/min) and is given as:

$$s = k_1(Q + q_R)^{p1} + k_2 \frac{d}{dt}(Q + q_R)^{p2} \qquad (1)$$

where s: storage (mm), Q: observed river discharge (mm/min), t: time (min), k_1, k_2, p_1, p_2: model parameters. The GUSF model can also be applied in non-urban watersheds by omitting the q_R component and the storage equation will be the same as that proposed by Hoshi. Combining the above expression of storage with the following continuity equation yields the nonlinear expression of GUSF model.

$$\frac{ds}{dt} = \gamma R + I - E - O - Q - q_R - q_l \qquad (2)$$

where γ: rainfall distribution factor, R: basin average rainfall (mm/min), I: inflow from other basins (mm/min), E: evapotranspiration (mm/min), O: water intake from the basin (mm/min), q_l: loss to the groundwater (mm/min). The basin average rainfall should consider as a fraction based on the spatial variability in rainfall and γ will represent this fraction. Even though γ looks similar to the runoff coefficient in its expression, the purpose of its incorporation is completely different from that of runoff coefficient. The main intention of inclusion of parameter γ was to consider the spatial distribution of basin rainfall. Further, the outflow q_l was defined by considering the SMPT model[10] and is given by[6]:

$$q_l = \begin{Bmatrix} k_3(s - z) & (s \geq z) \\ 0 & (s < z) \end{Bmatrix} \qquad (3)$$

where k_3 and z are the parameters. The storm drainage q_R is controlled by the carrying capacity of the sewer. Hence, the maximum volume of q_R cannot exceed maximum carrying capacity $q_{R\,max}$. The q_R is calculated as[6],

$$q_R = \begin{cases} \alpha(Q + q_R - Q_0) & \alpha(Q + q_R - Q_0) < q_{R\,max} \\ q_{R\,max} & \alpha(Q + q_R - Q_0) \geq q_{R\,max} \end{cases} \quad (4)$$

where α is the slope of the linear relationship between total discharge $Q + q_R$ and the drainage q_R; and Q_0 is the initial river discharge just before the rain starts [6]. Substituting Eqs. (1) and (3) into Eq. (2) will lead to a second-order Ordinary Differential Equation (ODE). This second-order ODE is transformed into the first-order ODE and can be numerically solved. The river discharge Q will obtain from the solution.

The GUSF is an eight parameter model with parameters $k_1, k_2, k_3, p_1, p_2, z, \alpha, \gamma$. Additionally, to analyze the effect of parameter γ in the model, the USF model was considered without parameter γ (γ=1). The Shuffled Complex Evolution-University of Arizona (SCE-UA) global optimization method proposed by Duan[11] was used to identify the calibrated parameters of the two models with 100 generations and root mean square error (RMSE) as the objective function to be minimized. The search range of parameters for SCE-UA is set as, k_1(0-500), k_2(0-5000), k_3(0-1), p_1(0-1), p_2(0-1), z (0-300), α (0-1), and γ (0-10). Due to high spatial variability in rainfall, sometimes, the basin average rainfall will be very low even though high magnitude rainfall occurs near the basin outlet. Therefore, the basin average rainfall should consider as doubled, tripled, etc. to represent a high magnitude rainfall near the watershed outlet. Consequently, the maximum possible value of γ was set as ten to incorporate the effect of a ten times higher magnitude rainfall resulting from the spatial distribution of rainfall in the basin. Moreover, the model calibration was conducted using two data scenarios: (i) individual event-based scenario where individual flood events were used and (ii) all event-based scenario where all the available events were used for the model calibration. Further, all event-based parameters were used to validate the model.

(2) Performance evaluation

The river discharge computed by the two SF models was compared during calibration and validation to assess the reproducibility in terms of the additional parameter γ using four error functions[12] of RMSE, Nash-Sutcliffe efficiency (NSE), Percentage Error in Peak (PEP), and Percentage Error in Volume (PEV). Further, the Akaike Information Criterion (AIC) [13] was also used to identify the best model by comparing them in both calibration and validation. The best model is then the model with the lowest AIC score and is given as:

$$AIC = 2K - 2\ln(\mathcal{L}(\hat{\theta}|y)) \quad (5)$$

where K: number of parameters to be estimated and $\ln(\mathcal{L}(\hat{\theta}|y))$: log-likelihood at its maximum point of the model estimated. Later, this concept was refined to correct for small data samples as[14]:

$$AIC_C = AIC + \frac{2K(K+1)}{n-K-1} \quad (6)$$

where n: sample size.

(3) Study area and data used

The target basin area at Koyo Bridge is about 7.7 km^2 as shown in **Fig.1**. The rainfall and water level data collected from the Bureau of Construction, Tokyo Metropolitan Government (TMG) at one minute interval during 2003-2006 were used for the present study because the USF model was successfully established for the five selected flood events during this period compared with conventional SF models[1]. Therefore, the same five flood events, whose 60-minute maximum rainfall (R_{60}) is greater than 30 mm, were used for the calibration of the selected models. In the same manner, three events that are not included in the model calibration were selected for model validation as given in **Table 1**. The basin average rainfall (R) was determined using the Thiessen polygon method from the eight rain gauges scattered over the basin. The inflow component I was fixed at 0.0012 mm/min based on the business annual report of the TMG. The outflow components O and E were set at zero. The maximum drainage, $q_{R\,max}$ was estimated at 0.033 mm/min using the Manning's equation[6].

Table 1. Characteristics of target events.

Event No.	Event date	R_{60} (mm/h)	Total R (mm)	Climatic factors
Calibration events (C)				
C1	13/10/2003	53.9	57.5	Intensive localized storm
C2	24~25/6/2003	42.8	46.2	Frontal event
C3	8~9/10/2004	42.0	261.1	Typhoon
C4	11/09/2006	32.7	37.9	Frontal event
C5	15/07/2006	31.5	31.5	Frontal event
Validation events (V)				
V1	25~26/8/2005	29.6	122.5	Typhoon
V2	15~16/6/2006	29.1	94.5	Frontal event
V3	29~30/9/2004	27.9	68.5	Typhoon

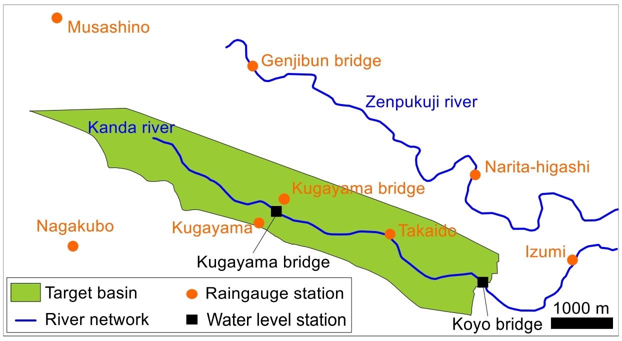

Fig.1 Index map of study area.

3. RESULTS AND DISCUSSIONS

(1) Hydrograph reproducibility

The SCE-UA method was applied for the parameter estimation of two models under the two selected data scenarios in the target basin. The individual event-based and the all event-based parameters of the models are shown in **Table 2**. It can be seen from the table that the parameter values are varying from event to event in both the models. The rainfall distribution factor, γ in the GUSF model is showing values greater than one during events 1, 2, and 3, whereas the values are less than one in the remaining events. Further, to check the significance of the optimized value of γ, the spatial variability of total rainfall was plotted by interpolating the rainfall received at eight gauging stations as shown in **Fig.2** using the Kriging interpolation technique in the Surfer mapping software. Only two events were plotted out of five due to the page constraints. **Fig.2 (a)** shows that high rainfall is occurring near the watershed outlet during calibrated event 3 which will produce an immediate and intensive response at the outlet. At this point, the basin average rainfall should consider at a higher magnitude which resulted in a γ value of 3.63 as shown in **Table 2**. On the contrary, the spatial rainfall distribution of event 4 illustrated in **Fig.2 (b)** revealed that the high rainfall is occurring at a specific location farther from the outlet point which will generate a delayed and diminished response. This further reduced the value of γ to 0.37 (**Table 2**). Therefore, it can be envisaged that the value of γ can be either less or greater than one, unlike in the USF model which is always one.

Further, these optimally estimated parameters of both the models were used to reproduce the hydrographs during calibration and validation as shown in **Fig.3**. Only three calibration and two validations events have been depicted in **Fig. 3** due to page limitation. It can be envisaged from **Fig.3 (a-c)** that the 8-parameter GUSF model almost overlaps with the observed river discharge and accurately reproducing the peak in the calibration events except for event 3. On the contrary, the USF model shows a great deviation in the reproduced hydrograph at the recession limb and exhibits considerable deflection from the observed peak discharge. It is clear from **Fig.3 (d-e)** that the GUSF model was able to reproduce the shape of the hydrograph, especially the rising and recession limbs during validation. However, the model slightly overestimated the highest peak discharge in all the events. In contrast, hydrograph reproduced by the USF model highly deviated from the observed hydrograph and lower predicted the highest peak although the remaining peaks were overestimated. The calibration and validation results exhibited that the GUSF model can more precisely reproduce the shape of the observed hydrograph as well as the peak discharge compared with the USF model. On the other hand, the USF model is not preserving the shape of the hydrograph as well as the peak discharge. The significant deviation demonstrated by the USF model at the recession limb can be attributed to the omission of spatial distribution factor γ. This indicates the necessity of parameter γ in the USF model to describe the rainfall spatial variability in the basin.

Table 2. Estimated model parameters for each model.

Para.	Model	Event 1	Event 2	Event 3	Event 4	Event 5	All event
k_1	USF	94.9	279.1	46.3	305.1	307.8	58.5
	GUSF	40.6	103.6	166.7	19.9	18.3	31.1
k_2	USF	241.5	464.3	539.7	2670.2	2954.0	973.0
	GUSF	774.7	4992.9	4292.7	4987.0	4995.5	482.8
k_3	USF	0.91	0.67	0.01	0.10	0.02	0.02
	GUSF	0.01	0.02	0.02	0.58	0	0.51
p_1	USF	0.12	0.03	0.39	0.02	0.02	0.34
	GUSF	0.49	0.34	0.34	0.70	0.50	0.31
p_2	USF	0.97	0.32	0.45	0.03	0.02	0.71
	GUSF	0.34	0.07	0.13	0.01	0.01	0.36
z	USF	202.5	275.6	4.4	299.3	299.8	28.3
	GUSF	8.24	18.7	35.0	231.7	5.9	157.0
α	USF	0.87	0.88	0.43	0.93	0.90	0.57
	GUSF	0.48	0.4	0.24	0.46	0.41	0.65
γ	USF	1	1	1	1	1	1
	GUSF	1.03	2.59	3.63	0.37	0.37	0.61

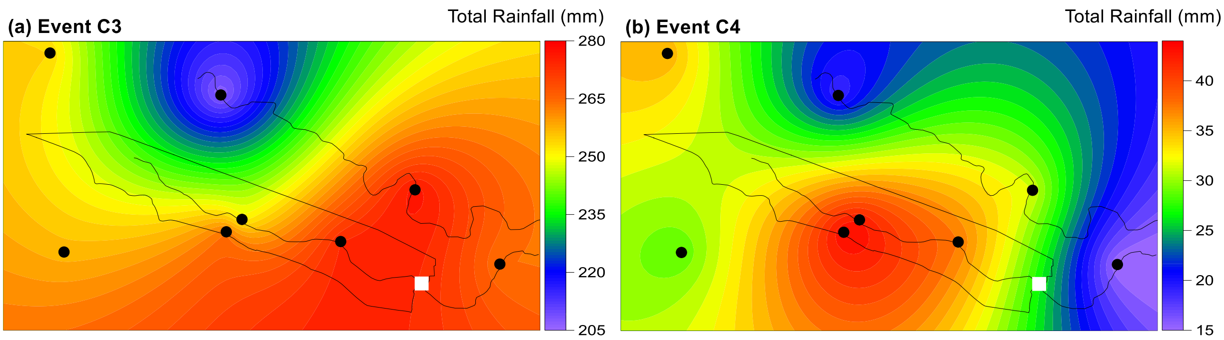

Fig.2 Spatial distribution of total rainfall during (a) event 3 and (b) event 4 (black circle and white square represent rain gauge and water level stations respectively).

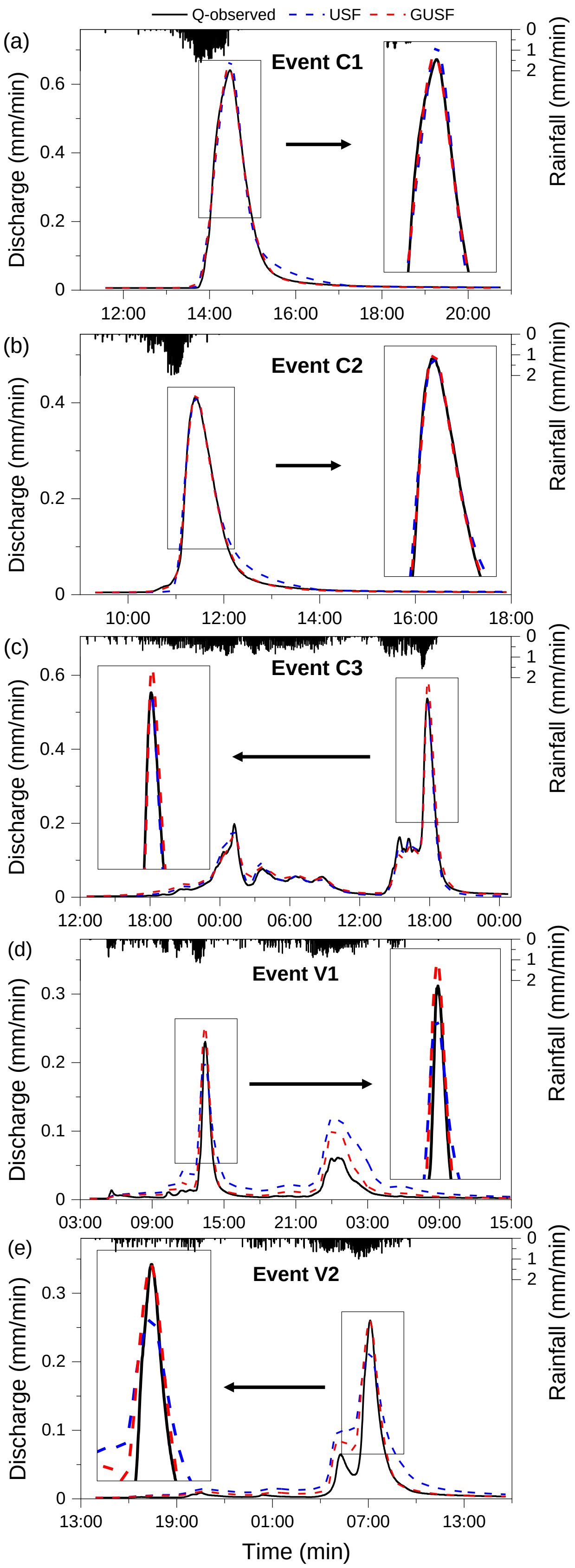

Fig.3 Reproduced hydrographs by both models during calibration and validation.

Further, the hydrograph reproducibility by the two models during calibration and validation was analyzed using error functions of RMSE, NSE, PEP,

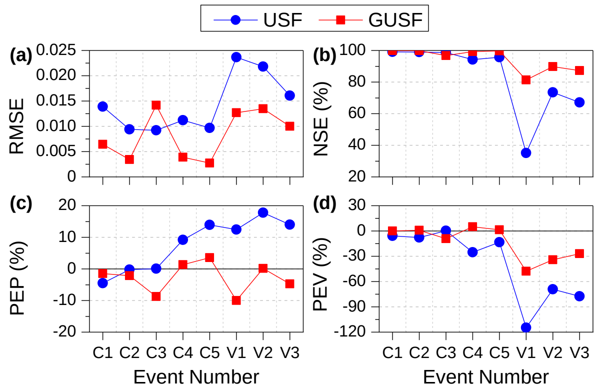

Fig.4 Comparison of RMSE (mm/min), NSE, PEP, and PEV by the USF and GUSF models.

and PEV as shown in **Fig.4**. From **Fig.4 (a)** and **(b)**, we can see that the GUSF model generates low RMSE close to zero and high NSE close to 100% during calibration as well as validation, except for calibration event 3. The RMSE and NSE values were close for both the models during event 3 even though the USF model exhibited slightly better performance. Calibration event 3 is a multi-peak event with the largest number of observations and the use of 100 generations may not be sufficient for its optimal parameter search using the SCE-UA method in the GUSF model. The PEP and PEV become positive for underestimation and **Fig.4 (c)** depicts that the PEP estimated by GUSF model is very low and not greater than 10% during both calibration and validation. Conversely, the USF model largely varies in its PEP values and lower predicted the peak flow in validation as well as in calibration events 4 and 5. Likewise the PEP, the GUSF model shows the best ranges of PEV values in **Fig.4 (d)** which is close to zero during calibration and reaches a maximum of 60% during validation. Simultaneously, the USF model generates higher values of PEV, especially during validation. The higher values of NSE coupled with the lower values of RMSE, PEP, and PEV for GUSF model in calibration and validation indicated that the hydrograph reproducibility by GUSF is the highest compared with the USF model.

(2) AIC aspect

Further, the AIC aspect was also used to determine the best model for calibration and validation. **Fig.5** shows the AIC_C values for the two models and it can be seen that the GUSF model received the lowest AIC_C except for calibration event 3. This higher AIC_C value of GUSF model for event 3 can be attributed to its higher RMSE during the event as shown in **Fig.4 (a)**. The lower AIC values of GUSF model in most of the events can be explained as the effect of incorporated rainfall distribution factor, γ. Therefore, the GUSF is much more effective than the USF model with an additional optimized parameter. The

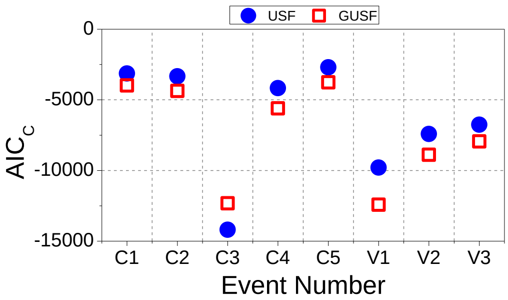

Fig.5 The corrected AIC (AIC_C) values during calibration and validation.

inclusion of parameter γ in the GUSF model aids to have a steep recession in the considered urban watershed and consequently found to be more suitable over the USF model for the use in urban watersheds. The rainfall distribution is temporally as well as spatially varying and the value of γ will depend on meteorological factors, basin geology and geomorphology, etc. Not only parameter γ is subject to change, but the remaining parameters of GUSF model will also vary in each event based on the meteorological factor and hence the real-time application of the model using the calibrated parameters is a challenging task. However, one solution to tackle this issue is the real-time prediction of the model parameters using data assimilation techniques which will improve the model effectiveness in an operational context even though the GUSF model performed well in validation.

4. CONCLUSIONS

A generalized USF (GUSF) model was proposed to account for the basin rainfall spatial variability by introducing a new parameter γ in the existing USF model. The GUSF model was applied in the Kanda basin, Tokyo along with USF model to evaluate the effectiveness of γ in the GUSF model. The results revealed that GUSF model has the least RMSE (high NSE) compared with the USF model during calibration and validation which further shows that the SCE-UA method has successfully identified the optimal parameters. The lower values of PEP and PEV received by GUSF model further indicate that the incorporation of spatial distribution factor can drastically improve the performance of the model. In addition, the summary of AIC results shows that the GUSF received the lowest AIC values in calibration and validation compared with the USF model which make it the parsimonious model. As a conclusion, the GUSF model can be considered as the best not only for the hydrograph reproducibility but also the most parsimonious based on the AIC perspective during calibration and validation in an urban watershed when compared with the USF model.

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